J. Plasma Physics (2010), *vol.* 76, *part* 6, *pp.* 833–843.
doi:10.1017/S0022377810000425

On the quantum space–time structure of light

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(Received 29 June 2010, revised 13 July 2010 and accepted 13 July 2010, first published online 17 August 2010)

Abstract. We extend the quantum theory of Time Refraction for a generic spatial and temporal modulation of the optical properties of a medium, such as a dielectric or a gravitational field. The derivation of the local Bogoliubov transformations relating the global electromagnetic modes (valid over the entire span of space and time) with the local modes (valid for the vicinity of each spatial and temporal position) is presented and used in the evaluation of vacuum photon creation by the optical modulations of the medium. We use this approach to relate and review the results of different quantum effects such as the dynamical Casimir effect, space and Time Refraction, the Unruh effect and radiation from superluminal non-accelerated optical boundaries.

1. Introduction

In recent years there has been an increasing interest in the study of the interplay between the geometry or curvature of space and time and the quantum aspects of the fields that exist on top of it, and in particular, the electromagnetic vacuum. Models, such as the Fulling–Unruh–Davies radiation [1, 2] and the mechanism of black-hole evaporation proposed by Hawking [3, 4], predict vacuum photon creation as the result of particular space–time geometries. In parallel, similar effects in non-stationary optical systems, which include the dynamical Casimir effect [5–7] and Time Refraction [8, 9], also predict the generation of photons out of the vacuum. These two groups of phenomena are connected because a varying dielectric medium is optically equivalent to a varying gravitational field [10]; however, the relation between them at a more fundamental level and within a common and self-consistent formulation is not fully established.

On one hand, the Unruh–Davies radiation explores the equivalence between gravitation and acceleration, and demonstrates the existence of a thermal radiation spectrum produced by an accelerated boundary, in the same way as the gravitational field at the horizon of a black hole produces the Hawking radiation [3, 4].

On the other, the dynamical Casimir effect [5–7] results from an extension of the double plate geometry of the famous Casimir effect [11], in which the boundary conditions of the field vary in time. Time Refraction results from the symmetry between space and time, extending the usual concept of refraction into the time domain. Recently, Guerreiro et al. [12] showed that an non-accelerated but superluminal optical boundary emits radiation that is similar to but distinct from the Unruh radiation.

Photon creation appears to be associated with some forms of symmetry breaking or loss of invariance of the medium where the electromagnetic field exists; however, not all types of symmetry breaking lead to photon production. For example, in ordinary refraction, light interacts with the boundary between two distinct optical media and suffers a deflection and reflection of the direction of propagation. This is a consequence of loss of translational invariance of the medium and is related with the non-conservation of photon momentum. The static Casimir effect [11] also results from the breaking of translational invariance imposed by two metallic plates and the spatial modulation of the zero-point fluctuations. However, in both these processes no photons are created from the vacuum.

Time Refraction is the temporal analog of ordinary or space refraction and consists in an instantaneous temporal change of the refractive index of the medium that produces a temporal jump of the photon. Although the photon momentum is conserved in the process, the photon frequency is shifted and extra photons are created. The model of Time Refraction has also been extended to smooth or more complex modulations of refractive index yielding similar results [13].

In the case the of photon creation in complex space–time curvatures, such as the Unruh effect [1], or when we consider general optical modulations in dielectric media, the losses of spatial and temporal invariance are entangled and the analysis becomes more difficult.

In this paper we present a work which unifies these different effects and models, presenting a newer insight on the impact on the quantum properties of the electromagnetic vacuum of the spatial and temporal structure of the optical medium that embeds the electromagnetic vacuum – being either a material medium or the curvature of space and time itself. To some extent, we present a generalization of the Quantum Theory of Time Refraction (proposed and developed in the past years by Mendonça, Guerreiro and Martins) to include general modulations of an optical medium and to encompass effects, such as the Unruh and the Hawking effects. In Sec. 2, using the covariant formalism, we investigate how a general spatial and temporal modulation of the properties of the optical medium impacts on local fluctuations and the quantum state of light. In the following sections, we apply the previous model to compute different effects, including space and Time Refraction and the Unruh effect. Finally, in Sec. 7, we present our conclusions.

2. Light in optical media

We start from Maxwell's equations in an inhomogeneous, unbounded and non-stationary optical medium written in covariant formalism:

$$\partial_\mu D^{\mu\nu} = j^\nu, \tag{2.1}$$

where $D^{\mu\nu}$ is the displacement four-tensor, $\partial_\mu \equiv \partial/\partial x^\mu$ is the four-gradient, $\mathbf{x} \equiv x^\mu = (ct, \vec{r})$ is the four-position and j^ν is the four-current. We admit that the constitutive relations of the medium can be approximately given as

$$D^{\mu\nu} = \epsilon^{\mu\nu}_{\alpha\beta} F^{\alpha\beta}, \tag{2.2}$$

corresponding to the first term of the Taylor expansion of the displacement four-tensor as a functional of the electromagnetic four-tensor $F^{\alpha\beta}$ for weak electromagnetic fields and assuming that the field has neither permanent polarization nor permanent magnetization. As the optical medium is both inhomogeneous and non-stationary, the generalized dielectric constant $\epsilon^{\mu\nu}_{\alpha\beta}$ (which can be expressed in terms of the dielectric constant and magnetic permeability of the medium) must be understood as a function of the four-position $\mathbf{x}$.

A similar approach can be used to write the four-current in terms of $F^{\alpha\beta}$ as

$$j^{\nu} = \sigma^{\nu}_{\alpha\beta} F^{\alpha\beta}, \tag{2.3}$$

where $\sigma^{\nu}_{\alpha\beta}$ can be interpreted as a generalized conductivity of the medium. Combining (2.1), (2.2) and (2.3), yields

$$\left[\epsilon^{\mu\nu}_{\alpha\beta}(\mathbf{x})\frac{\partial}{\partial x^{\mu}} - \tilde{\sigma}^{\nu}_{\alpha\beta}(\mathbf{x})\right] F^{\alpha\beta}(\mathbf{x}) = 0, \tag{2.4}$$

where we have expressed the explicit dependence on the four-position and where $\tilde{\sigma}^{\nu}_{\alpha\beta}(\mathbf{x}) = \sigma^{\nu}_{\alpha\beta}(\mathbf{x}) - \partial_{\mu}\epsilon^{\mu\nu}_{\alpha\beta}(\mathbf{x})$ is the effective conductivity. The solutions of (2.4) can be written as linear combinations of the elements of a set of mutually orthogonal functions $f^{\alpha\beta}_{i}(\mathbf{x})$, which correspond to the eigen-mode expansion of the electromagnetic field, say

$$F^{\alpha\beta}(\mathbf{x}) = \sum_{i} \alpha_i f^{\alpha\beta}_{i}(\mathbf{x}), \tag{2.5}$$

with $\alpha_i = \alpha^{*}_{-i}$ and $f^{\alpha\beta}_{i}(\mathbf{x}) = f^{*\alpha\beta}_{-i}(\mathbf{x})$ to assure that $F^{\alpha\beta}(\mathbf{x})$ is a real quantity.

Following the usual second quantization procedure, we can easily establish the electromagnetic field operator

$$\hat{F}^{\alpha\beta}(\mathbf{x}) = \sum_{i} \hat{a}_i f^{\alpha\beta}_{i}(\mathbf{x}) = \frac{1}{2}\sum_{i} \hat{a}_i f^{\alpha\beta}_{i}(\mathbf{x}) + h.c., \tag{2.6}$$

where $\hat{a}_i$ and $\hat{a}^{\dagger}_{i}$ are, respectively, the annihilation and creation operators for each mode $f^{\alpha\beta}_{i}(\mathbf{x})$ satisfying $\hat{a}_i = \hat{a}^{\dagger}_{-i}$.

Now we introduce the idea of local field modes $f^{\alpha\beta}_{i}(\mathbf{y};\ \mathbf{x})$ of the electromagnetic field at position $\mathbf{y}$ as the solutions of the differential equation

$$\left[\epsilon^{\mu\nu}_{\alpha\beta}(\mathbf{y})\frac{\partial}{\partial x^{\mu}} - \tilde{\sigma}^{\nu}_{\alpha\beta}(\mathbf{y})\right] F^{\alpha\beta}(\mathbf{x}) = 0, \tag{2.7}$$

corresponding to the electromagnetic modes obtained in a homogeneous and stationary optical medium with generalized dielectric constant identical to the value at position $\mathbf{y}$ of the original medium. These local and global modes correspond, respectively, to the quasimodes and true modes introduced by Dalton et al. [14] to describe the transmission of light through a beam splitter at a quantum level, but now extended into a fully relativistic context.

Another equivalent way to interpret these local modes is to consider that they correspond to the electromagnetic field measured by a *gedanken* detector placed at point $\mathbf{y}$. Such a detector is similar to the one used by Unruh [1] to study the spontaneous excitation of accelerated detectors (i.e. a non-relativistic n-level system, linearly coupled to the external relativistic field). Note that, if the detector and the medium are in relative motion, then (2.4) must be expressed in the proper frame of the detector.

The field detected by the local detector corresponds to the overlap between the global and local modes

$$U_{ij}(\mathbf{y}) = \frac{1}{|f_j^{\alpha\beta}(\mathbf{y})||f_j^{\alpha\beta}(\mathbf{y};\ \mathbf{x}-\mathbf{y})|} \int d\mu\ f_i^{*\alpha\beta}(\mathbf{y})\ f_j^{\alpha\beta}(\mathbf{y};\ \mathbf{x}-\mathbf{y}), \tag{2.8}$$

where μ is a measure describing the sensitivity of the detector and $|g|^2 = \int d\mu\ g^*(\mathbf{y})$ $g(\mathbf{y})$.

Now we can relate the annihilation and creation operations of the global modes ($\hat{a}_i$ and $\hat{a}_i^\dagger$, respectively) and of the local modes ($\hat{b}_i$ and $\hat{b}_i^\dagger$, respectively):

$$\hat{b}_i(\mathbf{y}) = \sum_i U_{ij}(\mathbf{y})\hat{a}_i = \sum_{i>0} U_{ij}(\mathbf{y})\hat{a}_i + \sum_{i>0} U_{-ij}(\mathbf{y})\hat{a}_i^\dagger. \tag{2.9}$$

This equation defines the Bogoliubov transformation between local and global field operators.

A consequence of the previous equation is that the global and local vacuum states do not necessarily match, resulting in a local modulation of zero-point fluctuations as perceived by the detector. In the following sections we explore in detail some of the implications of these modulations by analyzing some examples.

3. Space refraction

Let us consider a sharp boundary between two stationary dielectric media with dielectric constants equal to ϵ_1 and ϵ_2, respectively, located at $x^1 = 0$ and described by the dielectric function $\epsilon(\mathbf{x}) = \epsilon_1 H(-x^1) + \epsilon_2 H(x^1)$, where $H(x^1)$ is the Heaviside function. In each medium, the local modes can be expanded in terms of plane wave modes with linear polarization four-vector $e_i^{\alpha\beta}$ as

$$f_j^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) = i\sqrt{\frac{\hbar\omega_j(\mathbf{y})}{2\epsilon(\mathbf{y})}} exp\left[-k_j^\mu(\mathbf{y})x_\mu\right] e_j^{\alpha\beta}(\mathbf{y}), \tag{3.1}$$

where the wave four-vector is given by $k^\mu = (\omega/c, \vec{k})$ and satisfies the dispersion relation $\omega_j(\mathbf{y}) = \sqrt{k_j^2/\epsilon(\mathbf{y})}$.

Instead, the global modes must match the boundary conditions for the electric and magnetic induction fields at the interface between the two media, corresponding to the well-known Fresnel equalities. Then, a global mode describing photons with wave four-vector k_i^μ incident in the interface between the two media and coming from medium one is

$$f^{\alpha\beta}(x) = \begin{cases} f_i^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) + r\ f_r^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) & for\ x^1 < 0 \\ t\ f_t^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) & for\ x^1 > 0 \end{cases}, \tag{3.2}$$

where t and r are the Fresnel amplitude coefficients. As expected, the incident wave is coupled to a transmitted (for $x^1 > 0$) and a reflected (for $x^1 > 0$) waves. The refraction of light at the interface preserves the photon frequency $\omega_i = \omega_r = \omega_t$ but changes the wave vector according to the Snell equality.

When we use (2.9) to match the local and the global modes we obtain

$$\hat{b}_r = r\hat{b}_i, \ \hat{b}_t = t\alpha\hat{b}_i, \tag{3.3}$$

with $\alpha = \sqrt{\epsilon_1/\epsilon_2}$. Using this relation, we compute the relation between the number operator $\hat{N} \equiv \hat{b}^\dagger\hat{b}$ for the local modes as

$$\hat{N}_i = \hat{N}_r + \hat{N}_t, \tag{3.4}$$

implying that for a generic quantum state of the radiation field, the expectation value of the incident number operator N_i is identical to the sum of the expectation values of the reflected and transmitted photons. Also, because the photons maintain their frequency upon reflection and refraction, the expectation value of the total energy operator given as $\hat{W} \equiv \hbar\omega\hat{N}$ is also conserved: $W_i = W_r + W_t$. In this case, the zero-point fluctuations around the space boundary between the two media do not lead to photon creation. On the other hand, the expectation of the photon momentum $\hat{\vec{P}} \equiv \hbar\vec{k}\hat{N}$ is not conserved in agreement with the results of the classical theory: $\vec{P}_i \neq \vec{P}_r + \vec{P}_t$.

The quantum model of space refraction can be used to study more complex configurations of spatial breaking, such as the quantum model of a beam splitter [14] or the Casimir effect in dielectrics [15]. In the latter case, though the spatial modulation of the vacuum fluctuations does not lead to the vacuum photon creation, it originates as a macroscopic force between two dielectric plates.

4. Time refraction

Let us consider a time discontinuity in an infinite dielectric medium such that at instant $x^0 = 0$ the dielectric constant changes from ϵ_1 to ϵ_2. This can be described by a time-dependent dielectric constant $\epsilon(\mathbf{x}) = \epsilon_1 H(-x^0) + \epsilon_2 H(x^0)$. Again, the local modes can be expanded in terms of plane wave modes given by (3.1), but now the global modes must match the boundary conditions for the dielectric displacement and magnetic fields at $x^0 = 0$. The global mode describing photons with wave four-vector k_i^μ incident for $x^0 < 0$ is

$$f^{\alpha\beta}(x) = \begin{cases} f_i^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) & \mathit{for}\ x^0 < 0 \\ t'\ f_t^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) + r'\ f_r^{\alpha\beta}(\mathbf{y};\ \mathbf{x}) & \mathit{for}\ x^0 > 0 \end{cases}, \tag{4.1}$$

where $t' = (1-\alpha)/2\alpha^2$ and $r' = (1+\alpha)/2\alpha^2$ are the temporal Fresnel amplitude coefficients [16]. This expression shows that each field mode existing for $x^0 < 0$ is coupled with two modes existing for $x^0 > 0$, corresponding to a transmitted and reflected wave, in a way similar to the usual space refraction. Time Refraction is accompanied by a shift of the photon frequency after $x^0 = 0$ because light satisfies a different dispersion relation due to the change of the dielectric constant. The relation between the initial and the final photon frequencies is $\omega_i\sqrt{\epsilon_1} = \omega_t\sqrt{\epsilon_2} = \omega_r\sqrt{\epsilon_2}$ and can be called the Snell's Law for Time Refraction. On the other hand, the wave vector satisfies $\vec{k}_i = \vec{k}_t = -\vec{k}_r$, indicating that the photon momentum is conserved.

When we use (3.1) to match the local and the global modes, we obtain

$$\hat{b}_i = A\hat{b}_t + B\hat{b}_r^\dagger, \tag{4.2}$$

with $A = (1+\alpha)/2\sqrt{\alpha}$ and $B = (1-\alpha)/2\sqrt{\alpha}$.

For a general quantum state of the radiation field, the incident number operator $\hat{N}_i$ satisfies

$$\hat{N}_i = A^2\hat{N}_r + B^2(\hat{N}_t + 1) + AB(\hat{b}_r\hat{b}_t + \hat{b}_r^\dagger\hat{b}_t^\dagger). \tag{4.3}$$

Hence, in principle, the expectation value of the number of photons is not preserved: $N_i \neq N_r + N_t$, and new photons are created.

The model of Time Refraction has also been extended to include more complex temporal modulations of the refractive index. For example, Mendonça et al. considered a temporal analog of the beam splitter [9], consisting of medium that is suddenly perturbed and after some time returns to its initial condition after suffering two opposite time refractions. In a temporal beam splitter with a duration τ, the photon creation process is also present and the rate of creation resonantly depends on the photon frequency and the time interval. The existence of such a temporal resonance can then be used to build up a temporal resonance cavity for light amplification.

In another work [13], it was shown that the ideas and results of Time Refraction could also be extended to smooth and arbitrary temporal modulation of the refractive index of a medium. It also clarified the relation between Time Refraction and the dynamical Casimir effect in an oscillating cavity, showing that these two effects are equivalent.

Temporal refraction is a powerful conceptual and formal tool to analyze and compute the impact of temporal modulations of optical properties of the medium on quantum properties of the electromagnetic field. It establishes an important equivalence in the quantum scenario between space and time through the symmetry between spatial and temporal refractions. Just like spatial refraction, Time Refraction is also responsible for a macroscopic quantum effect: vacuum photon creation. However, this symmetry is not trivial because the non-existence of waves reflected backwards in time means that the impact of the time discontinuity of the refractive index can only affect properties of the electromagnetic field in the future. Hence this constrain is somewhat stronger, forcing photon creation that does not occur in space refraction. Nevertheless, this symmetry holds at a more fundamental level because, while Time Refraction conserves momentum, but not energy and space refraction conserves energy, but not momentum.

5. Space–Time Rrefraction

A more general class of problems considers moving optical boundaries; however, we need to distinguish two situations: optical boundaries slower and faster than light.

For subluminal optical boundaries, a simple approach to the problem can be made. First, we make a Lorentz boost to the frame co-moving with the optical boundary. In that frame we recover the case of space refraction even though the constitutive relations of the media differ from those in the original frame. Repeating the treatment for space refraction and making the inverse Lorentz transformation back to the original frame, one concludes that the relation between the incident, transmitted and reflected modes and field operators is identical to those obtained for the classical fields. The only meaningful difference introduced by the motion of the optical boundary is to shift the frequency and wave vector of different modes as a consequence of the simultaneous break of spatial and temporal symmetries. In this case, only the photon momentum perpendicular to the direction of motion of the optical boundary is preserved.

Optical boundaries can also have apparent velocities $\vec{u}$ larger than the speed of light without violating Einstein's principle of causality. They can be produced, for example, via the interaction of a short laser pulse with a plasma [17, 18] or by a laser or electron beam sweeping across a gas [19]. In all these cases, none of the actual particles in the medium is moving faster than c. Instead, the perturbation profile only appears to move due to delay in the arrival, at different points of the medium, of the laser or electron beam that causes the optical perturbation. For superluminal optical boundaries, it is impossible to find a Lorentz boost that can transform it into a space refraction [12], instead we can convert it into a Time Refraction using a boost velocity $v = c^2/u < c$. As expected, because a superluminal optical boundary is equivalent to Time Refraction, this process can extract photons out of an initial vacuum state:

$$N = N_r + N_t = \left[\frac{\omega_i - \omega_t - c^2(k_i - k_t)/u}{\omega_i + \omega_t - c^2(k_i + k_t)/u}\right]^2 . \tag{5.1}$$

This result has also been extended to the case of an optical boundary with an arbitrary spatial profile and moving with a constant but superluminal velocity [13], showing that resonant excitation of photon pairs from vacuum can also occur. This new radiation process resembles a kind of the Unruh radiation for superluminal boundaries. When the velocity of the optical perturbation matches the phase velocity of light, there occurs a resonant coupling between the optical perturbation and the electromagnetic vacuum. This resonance suggests this effect as a potential candidate for experimental research, as discussed in Sec. 7.

6. Unruh effect

The notion of space–Time Refraction introduces the idea that optical interfaces or optical heterogeneities can move relatively to the medium. Another class of problems considers moving detectors, *i.e.* situations where the local and global modes are considered in different reference frames. If the two frames are related by an ordinary Lorentz boost (like in the case of space and time refractions), then the matching between the two sets of modes is somewhat trivial. However, this is not necessarily the case for an arbitrary detector trajectory. For example, Unruh predicted [1] that an accelerated detector registers black-body radiation whereas an inertial detector does not observe any.

In the proper frame, a detector with acceleration a is better described by the Rindler coordinates ρ and τ defined according to $ct = \rho sinh\tau$ and $x^1 = \rho cosh\tau$. When matching the global modes calculated in an inertial frame and the local modes calculated in the Rindler frame [1], we obtain

$$\hat{a}_i = A\hat{b}_l - B\hat{b}_r^{\dagger}, \tag{6.1}$$

where $A = \sqrt{1 - exp(-\beta\hbar\omega)}$, $B = exp(-\beta\hbar\omega/2)\sqrt{1 - exp(-\beta\hbar\omega)}$ and $\beta = 2\pi c/\hbar a$. The indices l and r refer, respectively, to the Rindler left and right modes that take the place of the reflected and transmitted waves of space and time refractions. Unruh's *gedanken* detectors only register positive frequency modes, hence the left Rindler modes, which nave negative frequencies, must be traced out. Nevertheless, expressions (3.1) and (6.1) establish a fundamental equivalence between Time Refraction and the Unruh effect.

An alternative interpretation of the Unruh effect is that curved space–time works as an effective dielectric constant that changes the dispersion relation of light from point to point and forces the electromagnetic field to accommodate itself. In fact, the impact of a (quasi-static) gravitational field on the electromagnetic wave propagation is equivalent to the change of the dielectric constant of vacuum ϵ_0 to $\epsilon_0/\sqrt{g_{00}}$, and a similar change of the magnetic permeability of vacuum, μ_0 to $\mu_0/\sqrt{g_{00}}$ [20]. In terms of Maxwell's equations, (2.1) is replaced by [21]

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}D^{\mu\nu}) = j^\nu, \tag{6.2}$$

with $g = det\ g^{\mu\nu}$, which reduces back to (2.4) if we make $\tilde{\sigma}^\nu_{\alpha\beta}(\mathbf{x}) = \sigma^\nu_{\alpha\beta}(\mathbf{x}) - 1/\sqrt{-g}\ \partial_\mu\sqrt{-g}\epsilon^{\mu\nu}_{\alpha\beta}(\mathbf{x})$.

Another example of such type of effect is the well-known Hawking radiation which consists of thermal radiation with a black body spectrum predicted to be emitted by massive bodies, such as black holes [3, 4].

Neither the Unruh effect nor the Hawking mechanism of black-hole evaporation can ever be observed. For example, it would be necessary to create acceleration gradients of 10^{20} m s^{-2} to obtain Unruh's radiation corresponding to a temperature of 1 K. On the other hand, the equivalence between these effects and Time Refraction, as well as the potential of the later in producing large photon counts, suggests that Time Refraction is a far more interesting candidate for experimental research.

7. Toward experimental tests of vacuum radiation

In the previous sections we discussed how the generalization of the idea of refraction in a space–time scenario can be used to better understand, at a fundamental level, different quantum effects, usually associated with photon vacuum creation. Though many of these effects constitute pivotal breakthroughs in the understanding of the interplay between the space–time structure of a medium and the quantum nature of the fields that exist in it, the fact remains that direct experimental observation is very difficult. Such hurdle reflects the extreme physical conditions necessary to yield any measurable effect. Still, several experimental schemes were proposed using accelerated optical boundaries to excite anomalous vacuum fluctuations. For example, Yablonovich [22] suggested the use of ionization fronts in dielectrics to produce the Unruh-type radiation, whereas Chen and Tajima [23] proposed using plasma wakefields to accelerate electrons which would quiver under the influence of the non-trivial vacuum fluctuations, and Darbinyan et al. [24] put forward the idea of using crystal channeling phenomena. We may ask whether a fundamental insight, such as Time Refraction, can help toward implementing experimental tests of vacuum photon generation.

Time Refraction is not a purely quantum effect, rather it has a classical counterpart: Photon Acceleration or Phase Modulation, as is usually known by the plasma community and by the laser and optics community, respectively. Photon Acceleration explores the shift in photon frequency and momentum produced by a moving optical modulation in a laser pulse. As a classical effect occurring for

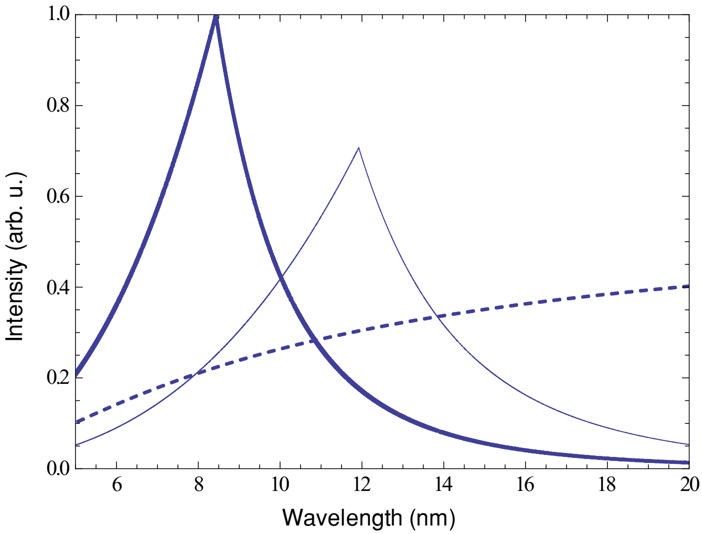

Figure 1. (Colour online) Emitted spectrum produced by the superluminal space–Time Refraction effect for an ionization front in an initial neutral gas with $c/u - 1 = 10^{-11}$ (bold), $c/u - 1 = 2 \times 10^{-11}$ (thin) and the thermal background noise (dotted). After the ionization front has passed, the medium is characterized by a final electron density of 10^{20} cm^3 ($\omega_p = 10^{12}$ Hz) and an electron temperature of 10^2 eV.

intense laser beams (large photon number), Photon Acceleration mainly focusses on the spectral effects and neglects effects associated with the quantum nature of light.

The development of intense laser systems has allowed for many experimental possibilities of Photon Acceleration, usually using plasmas as the background optical medium, among which we distinguish the frequency shift induced by relativistic ionization fronts [25, 26], flash ionization [22], nonlinear perturbations, and wake fields [27].

One of the methods of Photon Acceleration is produced by a superluminal optical boundary [17–19] exactly what is necessary to produce superluminal space–Time Refraction and excite photon out of the vacuum state. The two experiments would be exactly the same, except that in the quantum version there would be no probe laser beam: The electromagnetic vacuum would be the probe beam. Moreover, in dense media, such as plasmas, the optical perturbation does not need to be faster than the speed of light in the vacuum. Instead, because the refractive index can be quite large, it suffices the optical boundary to have an apparent speed faster than the seed of light in the medium, which can be quite smaller than c. Therefore, other experimental methods of Photon Acceleration can also be adapted, including the interaction of a short laser pulse with an active media [18] or an electron beam sweeping across a gas [19], as well as variations of the original proposals for detecting the Unruh effect [22, 27].

In Fig. 1, we consider the effects of a superluminal ionization front in an initially neutral gas producing a plasma with a final electron density of 10^{20} cm^3 ($\omega_p = 10^{12}$ Hz) and with a temperature of 10^2 eV and compare the vacuum emission with the thermal background radiated by the plasma. We notice that the peaks of the vacuum emission have a contrast of about 4 relative to the thermal spectrum.

These peaks result from the matching between the apparent velocity of the optical perturbation and the phase velocity of light, as discussed in Sec. 5. As the plasmas are dispersive (with a monotonic dispersion relation), there is always a wavelength satisfying the resonance condition and, as we change the apparent velocity of the perturbation, the emission spectrum is shifted. These features promote plasmas as the optimal optical media for testing these effects.

8. Conclusions

In this paper we have presented a mind frame that unifies and generalizes, both conceptually and formally, a set of effects associated with the electromagnetic vacuum and generated by the spatial and temporal structure of the medium – either an optical material medium or the curvature of space and time. This model summarizes part of the work developed for more than 10 years on the topic of the Quantum Theory of Time Refraction and relates it to even earlier results such as the Unruh radiation and the Hawking effect.

The concept of Time Refraction was initially developed when attempting to transpose the results of the models of Photon Acceleration [28, 29] and self-phase and crossed-phase modulation into a quantum scenario [30]. This concept is so fundamental that it can cross the gap between classical and quantum theories, as well as among different research fields, such as electromagnetism, optics, relativity and gravitation. This concept is simple in definition, universal in application (it can also be used in physical fields other than the electromagnetic) and leads to rich and deep physical phenomena, such as vacuum photon creation and the Casimir forces. These effects constitute a class of macroscopic quantum phenomena associated with zero-point fluctuations of the quantum vacuum.

The potential of the approach presented in this paper and of the concept of Time Refraction is not yet fully explored. Future work will investigate different ways of increasing vacuum photon creation and relate it with the generation of entangled light.

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