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Small-signal stability and decentralized control design for electric energy systems with a large penetration of distributed generators

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ABSTRACT

This paper investigates small-signal stability and decentralized control design for distribution electric energy systems with a large penetration of distributed generators. Two real world distribution systems are studied in this paper. The first system is the IEEE 30-node distribution system and the second one is the distribution system on Flores Island, one of the western group islands of the Azores Archipelago.

The Block Gerschgorin Theorem and Liapunov function-based stability criteria are applied to formally state sufficient conditions for small-signal stability. The results illustrate that when the governor control of distributed generators is designed without considering interactions between generators, small-signal instability could occur in the system and even the sufficient conditions for stability would not be satisfied.

In the next step, the paper assesses control design to stabilize potentially unstable distribution systems. The main focus is on designing enhanced decentralized control based on the introduced stability criteria. The findings illustrate that implementing the enhanced decentralized control could ensure stability and support a large penetration of distributed generators.

1. Introduction

New pressures for cleaner, more efficient and simply smarter use of energy will lead to large deployment of smaller-scale power plants closer to the end users. These units are broadly referred to as distributed generation (DG) and they offer many advantages. For instance, combined heat and power (CHP) DG units offer increased efficiency through waste heat recovery, and high reliability and security (Zerriffi, Dowlatabadi, & Apt, 2004). Low-head hydro, wind and solar power sources provide clean electricity. In addition, all DG systems hold the potential to reduce transmission and distribution losses (Nazari & Ilić, 2010).

In general, a large penetration of distributed generators could pose new technical problems for legacy distribution systems. The main focus of this paper is on potential small-signal instability problems due to DGs. To support large integration of DG units, it is essential to (1) introduce a structure-based dynamic model to assess the small-signal stability of distribution systems with abundant DGs, (2) identify potential technical problems which might arise by large integration of DGs; and (3) introduce technically innovative ways and theoretically sufficient conditions to ensuring stability of future distribution systems. This paper investigates these issues and illustrates them on two real world distribution systems.

In Section 2 a plausible structure of future distribution systems with DG units providing a large portion of electricity is described first. Then, in Section 3 it is shown that a large penetration of DG units sending power into an electric distribution system may cause small-signal instability problems not perceived in today distribution systems. This new problem has been recently observed by several authors including Cardell, Ilić, and Tabors (1998), Cardell and Ilić (2004) and Guttromson (2002). However, the explanation of the main cause of these problems and possible solutions still need further investigation.

In Section 3 the paper introduces a structure-based dynamic model for future distribution systems. The proposed model provides a better insight of the decentralized nature of these systems, where each DG represents a sub-system of the whole system. Furthermore, the Block Gerschgorin Theorem and Liapunov function-based stability criteria are used to precisely describe sufficient conditions for stability. The findings illustrate that interestingly two methods result in the same conditions.

In Section 4 an advanced decentralized control is introduced to enhance frequency stability of future distribution systems. The proposed control is designed based on the sufficient conditions for stability. The paper closes in Section 5 with a brief discussion of the overall findings and the future outlook of advanced decentralized control systems.

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2. Future electric distribution systems

In the past electric power systems have had vertical structures in which electricity is produced by large central power plants and delivered to the end users by means of transmission and distribution networks. It is believed by many that producing electricity locally by DG units has many advantages for electric power systems. For instance, CHP technologies have higher efficiency (up to 85%) compared with centralized power plants (around 30%). In addition, many DGs are falling under the category of renewable energy resources and they are green. Most of DG technologies could also provide ancillary services for the grid (Pepermans, Driesen, Haeseldonckx, Belmans, & Dhaeseleer, 2005).

Recent trends have been toward transforming traditional electric power systems into systems with many small generators placed on the distribution side of the grid. Figs. 1 and 2 illustrate schematics of distribution systems with DGs placed close to the end users. The first system (shown in Fig. 1) is created by varying the IEEE 30-node distribution system in complementation with new DGs. Shown in black is a point of connection between the transmission and distribution system modeled as an ideal power source. For demonstration represented in gray are two Combustion-Turbines (C-T) connected to nodes 13 and 14. In the future the same system might be expected to have a small hydro plant and/or a small wind plant. The data for the IEEE 30-node distribution system is available in Kersting (1991).

The second system (shown in Fig. 2) is the electric power distribution system on Flores Island, one of the western group islands of the Azores Archipelago (a colony of Portugal). The electric network on Flores consists of a 15-kV radial distribution network. Total demand on the island is around 2 MW and three DGs are supplying the demand. More than 50% of the electric

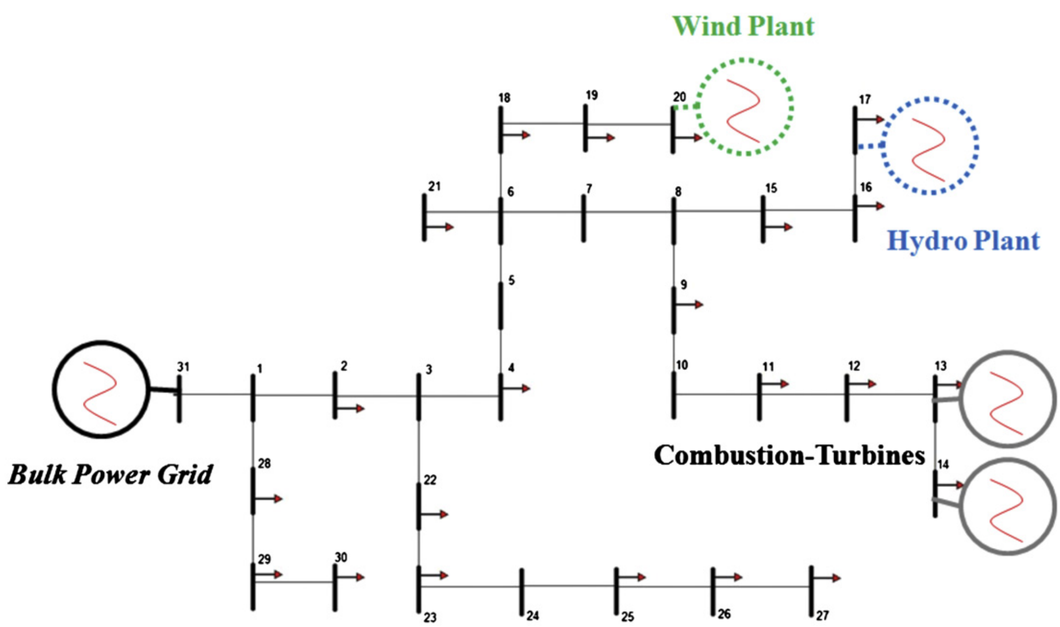

Fig. 1. Schematics of the modified IEEE 30-node distribution system.

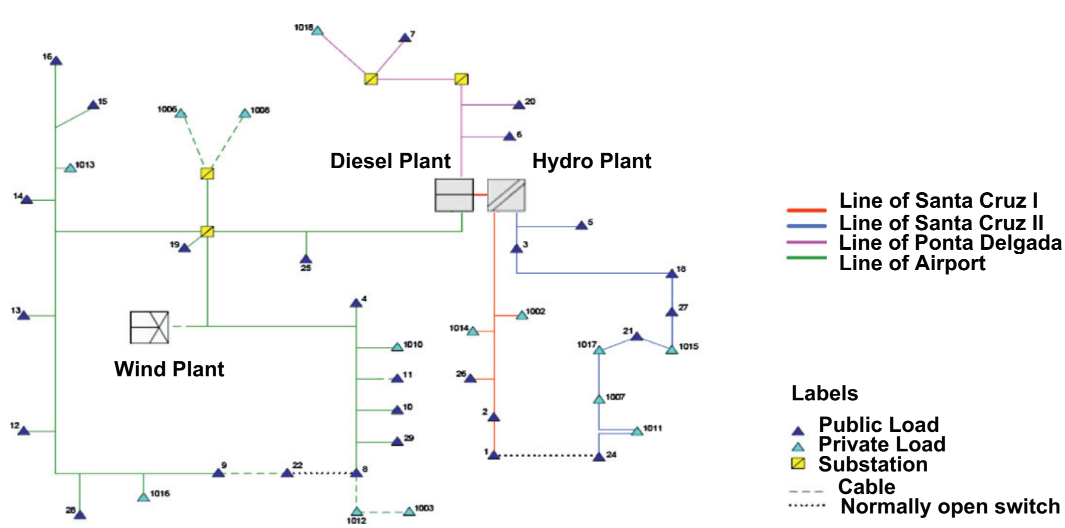

Fig. 2. Schematics of the distribution system on Flores Island (EDA Report, 2009).

energy is provided by four diesel units whose total capacity is 2.5 MW. Around 35% of the demand is supplied by four hydro plants with overall capacity of 1.65 MW. Moreover, two synchronous wind plants with the total capacity of 0.65 MW are providing the rest of the demand (15%) (EDA Report, 2009). The general policy of EDA (Electricidade Dos Açores) is to make Flores Island sustainable. Therefore, in the future diesel generators will be replaced by renewable sources of energy such as wind plants. The data for the distribution system on Flores is available in Nazari (2012, chap. 3).

In order to ensure stability of new distribution systems, it is necessary to establish systematic modeling and develop analyzing tools. Such tools do not exist at present. This paper is an effort toward fulfilling this need. To start with, it is important to understand that at present utilities do not have any systematic methodology for sitting and operating DGs into the existing distribution systems. If the DGs are very small, the design is such that they are expected to supply highly localized parts of end users. Therefore, it is likely that they reduce overall power losses relative to the delivery losses seen in today distribution systems (Nazari & Ilić, 2010). Moreover, no significant dynamic interactions between clusters of local DGs and consumers on one side, and the rest of the network, on the other, take place. As a result, these highly distributed small-scale DGs do not require any major change in operating practices. In particular, it is essential to ensure that the response of such system to small disturbances is unlikely to lead to any significant system-wide instabilities of frequency and voltage.

On the other hand, when DGs of larger capacity are placed within an existing distribution system, the tradeoff between delivery loss reduction and system stability with respect to disturbance is harder to manage (Nazari & Ilić, 2009). For delivery loss reduction purposes the tendency is to reduce distribution lines flows by sending electricity back to the grid. This, however, could create robustness problems, since the system-wide flows created by the DGs are no longer negligible.

In an earlier work the authors have shown that placement of medium size DGs for delivery loss reduction could lead to an operating conditions that is not robust (Nazari & Ilić, 2009). This could result due to strong interaction of DG units.

It is this inter-dependence of placing DGs for efficient steady state stabilization and the robustness by each systems that points into needs for introducing more formal approaches to sitting medium-size DGs and stability analysis for identifying robustness problems. Notably the role of systematic control design for provably stable future distribution systems becomes key in moving forward.

The remained of this paper concerns modeling sufficient conditions for ensuring robustness of given systems with medium-size DG units and ultimately the control design for stabilizing potentially unstable systems. A mathematical model of the interconnected system is needed to assess instabilities caused by interactions of electrically close DGs. The effect of interactions must be quantified and control must be designed to compensate instabilities caused by each process.

In addition, it is important to better understand the control design specifications, which must be placed on DGs themselves to ensure that the system as a whole remains robust. Having methods to evaluate complexity and cost tradeoff between different control designs for achieving this will become more critical in the future. Many questions lie ahead. For example, how small and how localized DGs should be before one worries about robustness issues at all? In fact, introduction of micro-grid and plug-and-play solutions to integrating highly distributed energy resources of 1 kW order such as micro-CHPs and PV panels are currently not perceived to be likely to cause unstable system response. The theoretical question is whether this is indeed true with a very high penetration of such small devices. In particular, if clusters of very small resources in large numbers are displaced further away from consumers, would each system begin to experience an emerging instability? Moreover, if larger DGs are deployed, would this cause instabilities and why?

In order to answer these questions a structure-based dynamic model is introduced which lends itself to the needed analysis. The paper starts by observing that poor system stability could be caused by either (A) a very sensitive response of a DG itself to the local disturbances; or (B) interactions of electrically interconnected unstable DGs; or (C) interactions of electrically interconnected stable DGs. To assess the nature of possible instabilities, the paper starts by recognizing fundamental changes in distribution systems. In particular, instabilities may occur in the newer distribution systems which may have several DGs interacting. On the other hand, in today distribution networks power is typically supplied by a single large substation source for which there is no frequency instability concern at all.

As one proceeds to assess necessary communication and control complexity needed to ensure robustness of future distribution systems, potential sources of instabilities should be related to physical causes (A)–(C). This is generally hard to do, but since the complexity of cyber over-laid on top of the physical grid greatly depends on this understanding, such methods are essential to introduce.

In general, it is possible to assess the robustness of the response of a DG itself with respect to disturbances. Intuitively, smaller DGs are more sensitive to small disturbances than larger generators. To quantify this sensitivity for a given specific type of a power plant, it is possible to use the well-understood droop characteristics of power plants (Ilić & Zaborszky, 2000).

Next the droop characteristics of any electric power plant are derived starting from the dynamical model of the power plant. Since this paper is primarily concerned with frequency instability, a so-called real-reactive power decoupling assumption is made (Pai, Gupta, & Padiyar, 2004). Any stand-alone power plant comprising a prime mover, electric generator and their primary control can be written as a dynamic system of the form

$$\frac{d\tilde{X}_{LC}^{(i)}}{dt} = f_{LC}^{(i)}(\tilde{X}_{LC}^{(i)}, \tilde{Z}_{LC}^{(i)}, \tilde{d}_{LC}^{(i)}(t)) \tag{1}$$

where $\tilde{X}_{LC}^{(i)}$ is the set of internal state variables, such as frequency ($\tilde{\omega}_G^{(i)}$), fuel flow ($\tilde{W}_F^{(i)}$) and fuel control ($\tilde{V}_{CE}^{(i)}$), in a closed-loop power plant i with its primary control; $\tilde{Z}_{LC}^{(i)}$ is a vector of coupling variables between power plant i and the electrical system to which the plant is connected. In general, the coupling variables are active power out of generators ($\tilde{P}_G^{(i)}$). Moreover, $\tilde{d}_{LC}^{(i)}(t)$ is a vector of local disturbances seen by the power plant. A linearized model of (1) takes on the form

$$\frac{dX_{LC}^{(i)}}{dt} = A_{LC}^{(i)}X_{LC}^{(i)} + C_M^{(i)}Z_{LC}^{(i)} + D_P^{(i)}d_{LC}^{(i)}(t) \tag{2}$$

where

$$X_{LC}^{(i)} = \tilde{X}_{LC}^{(i)} - \overline{X}_{LC}^{(i)}, \quad Z_{LC}^{(i)} = \tilde{Z}_{LC}^{(i)} - \overline{Z}_{LC}^{(i)}, \quad d_{LC}^{(i)} = \tilde{d}_{LC}^{(i)} - \overline{d}_{LC}^{(i)}$$

Parameters with bar denote the equilibrium point of the system. As an illustration, consider a linearized model of the diesel generator shown in Fig. 2

$$\frac{d}{dt}\begin{bmatrix} \omega_{G_d} \\ m_{B_d} \\ P_{C_d} \end{bmatrix} = \begin{bmatrix} \frac{-D_d}{M_d} & \frac{C_c}{M_d} & 0 \\ \frac{-C_dK_d}{T_dR_c} & \frac{-1}{T_d} & \frac{-C_dK_d}{T_d} \\ -K_I & 0 & 0 \end{bmatrix}\begin{bmatrix} \omega_{G_d} \\ m_{B_d} \\ P_{C_d} \end{bmatrix} + \begin{bmatrix} \frac{-1}{M_d} \\ 0 \\ 0 \end{bmatrix} P_{G_d} + D_{P_d}\, d(t) \tag{3}$$

In this model, ω_{G_d} is the frequency, m_{B_d} is the fuel rate and P_{C_d} is the fuel control of the diesel generator. In addition, M_d and D_d

are the inertia and damping coefficients respectively. C_d and K_d are the transfer function coefficients for the fuel system, T_d is the time constant of the fuel system, and K_I is the gain of the governor (Sharma, Islam, & Pryor, 2000). The data for the dynamic model of the diesel generator is available in Table B7. Furthermore, the linearized dynamic models of the hydro and wind plants on Flores and those of the C-Ts on the IEEE 30-node system are presented in Appendix A.

It follows from this example that $Z_{LC}^{(i)} = P_G^{(i)}$. In other words, independent of the type of the power plant, its linearized dynamics can be expressed in terms of its own $X_{LC}^{(i)}$ while the coupling variable is real power ($P_G^{(i)}$) generated by the plant. For hydro plants internal state variables $X_{LC}^{(i)}$ are different than for the thermal power plants, and are a function of power generation type. However, the coupling variable between a thermal power plant and a hydro power plant is the same (Ilić & Zaborszky, 2000).

It is important to observe that model (2) is of the same form for any type of a power plant. The numerical parameters will determine both robustness of a power plant with respect to large disturbances and with respect to small disturbances.

A linearized model of (2) can be used to assess small-signal stability of a stand-alone power plant in response to small disturbances. In particular, in order to define the basic sensitivity of the plant with respect to its coupling variable $P_G^{(i)}$ and the rest of the system, the sensitivity of frequency ($\omega_G^{(i)}$) with respect to active power deviation ($P_G^{(i)}$) known as the droop characteristics is considered (Ilić & Zaborszky, 2000)

$$\Sigma^{(i)} = -\frac{\partial \omega_G^{(i)}}{\partial P_G^{(i)}} \tag{4}$$

Calculating the droop characteristics of the plants shown in Figs. 1 and 2 with and without closed loop frequency control system illustrates that DGs without closed loop control have larger droop characteristics (shown in Table 1). This implies that in order to minimize the sensitivity of frequency to deviations in real power, at least larger DGs need to be equipped with closed loop G-C systems.

In fact, the main focus of this paper is on stability of interconnected systems with many DGs. At a system level, dynamics of these systems could become very sensitive to very small fluctuations around operating points. In particular, this paper shows that strong interaction between DG units or poor tuning of governor control systems could lead to overall small-signal frequency instability. It is critical to observe that such analysis is not done by today distribution companies. This is not done because in the old systems with one power plant no dynamic instabilities are expected.

In preparation for the control design of interest, it is perceived that the commonly used droop characteristics definition includes the effects of primary control of power plant, governor control in particular. At present typical primary control is fully decentralized since governor affects valve position on a steam power plant, for example, in response to deviations of local frequency from the desired frequency set point (ω_G^{ref}). In the next section stability conditions are selected assuming decentralized control.

Table 1
Droop characteristics of the plants in Figs. 1 and 2.

Type of power plants	Combustion turbine (pu)	Diesel generator (pu)	Hydro generator (pu)
With closed loop frequency control	0.075	1e−4	0.45
Without closed loop frequency control	0.5	0.05	1.78

3. Stability conditions with decentralized control

In this section, the paper assesses stability of future distribution systems with abundant DGs equipped with decentralized control. In particular, sufficient conditions for stability of such evolving distribution systems are derived using Block Gerschgorin Theorem as well as Liapunov function-based stability criteria.

Earlier work has indicated that potential small-signal frequency instabilities may occur in distribution networks with a large penetration of MW-scale DG units (Cardell & Ilić, 2004; Cardell et al., 1998; Guttromson, 2002). However, no fundamental causes of these instabilities are discussed.

This paper stresses that it is possible to identify fundamental causes of potential frequency instabilities in terms of the strength of electrical interactions between DGs and the damping magnitude (real part of the eigenvalue) contributed by the state variables of DG units. To this end, the paper introduces a structure-based dynamic model whose state variables are local state variables $X_{LC}^{(i)}$ of all individual DGs and their coupling variables $Z_{LC}^{(i)}$. This model builds on the model first proposed in Liu and Ilić (1994) and then used for stability analysis of distribution systems with DGs in Cardell et al. (1998) and Cardell and Ilić (2004). The model has a system matrix whose decomposable structure lends itself to deriving sufficient conditions for stability by assessing stability of its sub-systems and the strength of their interactions. Next this model is derived.

Recalling from earlier in this paper, the coupling variable of a DG is its real power output, namely $Z_{LC}^{(i)} = P_G^{(i)}$. Moreover, since the coupling variables of all power plants are subject to the real power flow network constraints, it is possible to express their dynamics in terms of local state variables of the DGs, in particular their frequencies ($\omega_G^{(i)}$) (Liu & Ilić, 1994)

$$\frac{dP_G^{(i)}}{dt} = \sum_{j=1}^{n} Kp_{ij}\omega_G^{(i)} \tag{5}$$

here n denotes the number of power plants in the system and Kp is the coupling matrix, defined as follows:

$$Kp = J_{GG} - J_{GL} J_{LL}^{-1} J_{LG}$$

The elements in each sub-matrix J_{ij} are defined as $\partial P_i / \partial \delta_j$ where G represents generator nodes and L represents load nodes. By adding (5) to (2) a full system model representing dynamics of the distribution system with n DGs is obtained as (Cardell et al., 1998)

$$\frac{d}{dt}\begin{bmatrix} X_{LC}^{(1)} \\ \vdots \\ P_G^{(1)} \\ \vdots \end{bmatrix} = \begin{bmatrix} A_{LC}^{(1)} & 0 & C_M^{(1)} & 0 \\ 0 & \ddots & 0 & \ddots \\ Kp_{11} S_{\omega_G^{(1)}} & \cdots & 0 & \cdots \\ \vdots & \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} X_{LC}^{(1)} \\ \vdots \\ P_G^{(1)} \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ Dp^{(1)} \\ \vdots \end{bmatrix} \frac{dP_L}{dt} \tag{6}$$

where the matrix $S_{\omega_G^{(i)}}$ includes 0's and 1 and relates $\omega_G^{(i)} = S_{\omega_G^{(i)}} X_{LC}^{(i)}$. In addition, dP_L/dt represents changes in the system loads, modeled as disturbances to the grid.

The desired system model is obtained by changing the order of state variables of (6) and by ordering the state variables as the internal state variables of DGs and their coupling variable (real power out of the corresponding DG)

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ X_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \end{bmatrix} \frac{dP_L}{dt} \tag{7}$$

where

$$X_i = \begin{bmatrix} X_{LC}^{(i)} \\ P_G^{(i)} \end{bmatrix}, \quad A_{ii} = \begin{bmatrix} A_{LC}^{(i)} & C_M^{(i)} \\ Kp_{ii}S_{\omega_G^{(i)}} & 0 \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & 0 \\ Kp_{ij}S_{\omega_G^{(j)}} & 0 \end{bmatrix}$$

$$\gamma_i = \begin{bmatrix} 0 \\ Dp^{(i)} \end{bmatrix}$$

3.1. Block Gerschgorin stability conditions

As noted earlier, the particular interest is to identify sufficient conditions for stability of new distribution systems with DGs. These conditions are obtained by applying Block Gerschgorin Theorem to the new system model presented in (7) (Feingold & Varga, 1962)

$$(\|(A_{ii}-sI_i)^{-1}\|_\infty)^{-1} \le \sum_{\substack{j=1 \\ j \ne i}}^{n} \|A_{ij}\|_\infty \quad \forall i \in [1,n] \tag{8}$$

where $\|\cdot\|_\infty$ represents the infinity norm of the indicated matrix. The left hand side of (8) represents the set of complex-valued numbers that all the eigenvalues of the full system matrix lie in the union of these sets (Feingold et al., 1962). Also, the right hand side of (8) is calculated by adding the infinity norms of the off-diagonal matrices.

In particular, off diagonal matrices denote the coupling matrix between the ith DG and other DGs in the system. By inspection, it can be simply obtained that the infinity norm of off-diagonal matrices equals to the norm of their coupling variables. That is

$$\|A_{ij}\|_\infty = |Kp_{ij}|$$

where $|Kp_{ij}|$ denotes the electrical interaction between the ith and the jth DGs. Hence, (8) can be re-written as follows:

$$\min\{|s-\lambda_{i,1}| \cdots |s-\lambda_{i,m}|\} \le \sum_{j \ne i} |Kp_{ij}| \quad \forall i \in [1,n] \tag{9}$$

here m denotes the number of state variables for the ith sub-system (A_{ii}). Condition (9) states that eigenvalues of the full system lie within the circles centered at eigenvalues of a sub-system and the radius equals to the sum of the electrical interaction between the sub-system and other sub-systems. Fig. 3 demonstrates the schematic of the circles in which eigenvalues ($\lambda_i = r_i + jv_i$) of the full system lie. The blue crosses represent eigenvalues of sub-systems (A_{ii}).

One can simply conclude from Fig. 3 that stability of the full system is satisfied when all the circles lie in the left hand side of the complex plane. That is, (1) eigenvalues of all sub-systems lie in the left hand side of the complex plane (sub-systems are asymptotically stable); (2) the real part of the slowest eigenvalue of a sub-system (the closest eigenvalue to the imaginary axis) is greater than the sum of the electrical interaction between the sub-system and other sub-systems.

The physical interpretation of this theorem is that when local DGs are asymptotically stable and the strength of electrical interaction between DGs is less than the damping magnitude (real part) of the slowest eigenmode of DGs, then the whole system always remains asymptotically stable. This also implies that the main cause of frequency instability in distribution systems with abundant DGs is (1) low damping magnitude of the eigenmode of local DGs; (2) strong coupling between DGs. In general, low damping results from poor tuning of the governor control of DGs. Furthermore, strong coupling between DGs is caused by strong electrical interaction between them. Coupling between DGs is measured by the norm of the off-diagonal terms of the coupling matrix ($\sum_{j \ne i} |Kp_{ij}|$). If this value is greater than the damping magnitude of the ith DG, then the DG is strongly coupled to other generators.

In the next step, the Gerschgorin stability criteria are applied to formally state sufficient conditions for stability of the systems shown in Figs. 1 and 2. The first system consists of two C-Ts controlled by traditional G-C systems. The G-Cs are tuned in a conventional fashion. They are responding to perturbations in local frequency and are designed to provide DGs with a 5–7% droop characteristics. The paper shows that tuning G-Cs in a traditional way, without considering interactions between the plants, may lead to small-signal instability problems.

Next, the small-signal stability of the 30-node distribution system is investigated. The results illustrate that the full system has two eigenvalues in the right hand side of the complex plane. This implies that the system can become very sensitive to even very small disturbances. Fig. 4 shows how frequency deviation of C-Ts becomes unstable when a small perturbation equal to 0.1 pu occurs at node 15. Eigenvalues of the full system and the sub-systems are presented in Table B1. In addition, the coupling matrix of the distribution system is shown in Table B3.

The results of stability analysis illustrate that the 30-node distribution system with a traditional control design is not satisfying the second condition for stability with the deficit of 1.15 pu/s. This explains why the system has two eigenvalues in the right hand side of the complex plane.

In a similar fashion, the small-signal stability of the distribution system on Flores is investigated. On Flores, the diesel and hydro plants are equipped with traditional G-C systems with similar control logic to the G-C of the C-Ts. However, the wind plant has a basic pitch control system.

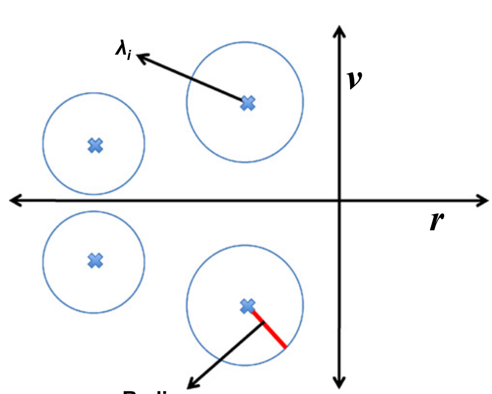

Fig. 3. Illustration of Gerschgorin Circles in which eigenvalues of the full system matrix lie.

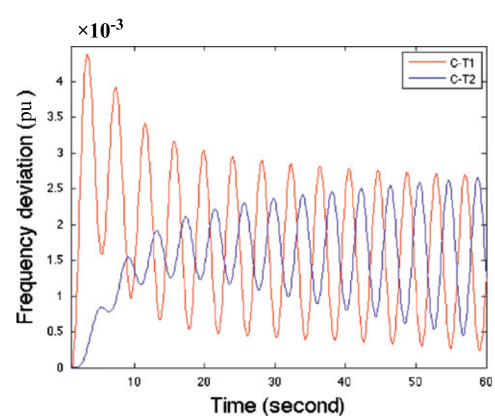

Fig. 4. Frequency response of the IEEE 30-node system after a small perturbation at node 15.

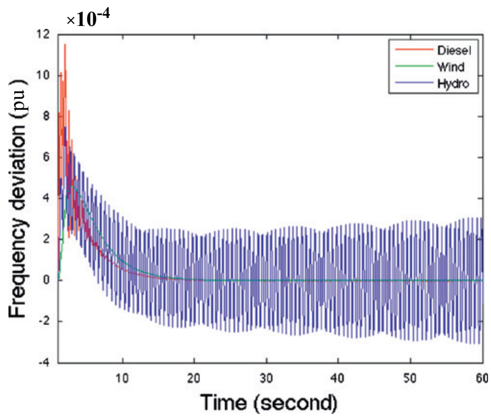

Fig. 5. Frequency response of the plants on Flores after a small perturbation on the island.

The results of small-signal study demonstrate when penetration of renewable energy resources is high, the island has unstable dynamic behavior. Fig. 5 illustrates frequency response of the plants after a small perturbation (0.1 pu) on the island. Eigenvalue analysis of the system shows that there are two eigenvalues in the right hand side of the imaginary axis. Table B5 shows the results of the eigenvalue analysis. Table B4, also, presents the coupling matrix of the island.

Note that because of frequency oscillations caused by fast fluctuations of intermittent resources, penetration of renewable energy resources (mostly wind) is limited on Flores Island. This problem is pronounced during peak hours (EDA Report, 2009; Hamsic et al., 2007).

The technical findings furthermore illustrate that the original system with conventional control systems is not satisfying the Gerschgorin stability criteria. In fact, the stand-alone hydro plant (the third sub-system) is not asymptotically stable. In addition, the system is not satisfying the second condition for stability with the deficit of more than 1.37 pu/s.

3.2. Liapunov stability conditions

An alternative approach to determine sufficient conditions for stability is using Liapunov stability method, fully elaborated in Siljak (2007). In this section the Block Gerschgorin Theorem-based and Liapunov-based stability criteria are compared and it is shown that these conditions are identical when the Liapunov equation is defined using the knowledge of system eigenvalues.

To this end, Eq. (7) is re-arranged by using nonsingular transformation ($X_i = T_i\hat{X}_i$). Furthermore, the energy function and the Liapunov equation of the system are introduced as follows (Siljak, 2007):

$$\frac{d\hat{X}_i}{dt} = \Lambda_{ii}\hat{X}_i + \sum_{j \neq i} \Delta_{ij}\hat{X}_j \quad \forall i \in [1,n] \tag{10}$$

$$\nu_i(\hat{X}_i) = (\hat{X}_i^T \hat{H}_i \hat{X}_i)^{1/2} \tag{11}$$

$$\Lambda_{ii}^T \hat{H}_i + \hat{H}_i \Lambda_{ii} = -\hat{G}_i \tag{12}$$

where

$$\Lambda_{ii} = T_i^{-1} A_{ii} T_i, \quad \Delta_{ij} = T_i^{-1} A_{ij} T_j$$

The solution to (12) is obtained as

$$\hat{H}_i = I_i \quad \text{and} \quad \hat{G}_i = -2\,\mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$$

where σ_i is the absolute real part of the ith eigenvalue of A_{ii}. The sufficient condition for the stability of the system shown in (10) is satisfied when W-matrix is Metzler. The W-matrix for the choice of Liapunov function takes on the form (Siljak, 2007)

$$w_{ij} = \begin{cases} -\sigma_M^i, & i = j \\ \lambda_M^{1/2}(\Delta_{ij}^T \Delta_{ij}), & i \neq j \end{cases} \tag{13}$$

here σ_M^i denotes the maximum real part of the eigenvalues of A_{ii} and λ_M represents the maximum eigenvalue of the indicated matrix. Since the second term of the W-matrix is the Euclidean norm of Δ_{ij}, it is possible to re-write Eq. (13) as follows:

$$w_{ij} = \begin{cases} -\sigma_M^i, & i = j \\ \|\Delta_{ij}\|_2, & i \neq j \end{cases} \tag{14}$$

As transformation matrices (T_i and T_j) are unity matrices, the Euclidean norm of Δ_{ij} is the same as the Euclidean norm of A_{ij}. By inspection, it is trivial to show that $\|A_{ij}\|_2 = |KP_{ij}|$. Therefore, the W-matrix takes on the form

$$w_{ij} = \begin{cases} -\sigma_M^i, & i = j \\ |KP_{ij}|, & i \neq j \end{cases} \tag{15}$$

The new W-matrix is Metzler if the following conditions hold:

$$\begin{cases} -\sigma_M^i < 0 \\ |\sigma_M^i| > \sum_{j \neq i} |KP_{ij}| \end{cases} \tag{16}$$

In summary, the results in this section claim that both Block Gerschgorin Theorem-based and Liapunov-based criteria are equivalent when the Liapunov equation is defined using the knowledge of system eigenvalues. Furthermore, both conditions have an intuitive physical interpretation of mathematical conditions which state that as long as local control reacts sufficiently fast to counteract the dynamics of interactions with the rest of the system, the decentralized control will be sufficient to stabilize the system frequency. This follows by analyzing Eqs. (2) and (15) simultaneously, and recalling that the coupling variable is $Z_{LC}^{(i)} = P_G^{(i)}$.

4. Enhanced decentralized control with Gerschgorin logic

In this section, an enhanced decentralized control system with Gerschgorin logic is introduced to improve the stability of future distribution systems. The new control is designed based on shifting all the Gerschgorian Circles of the full system to the left hand side of the complex plain. Eq. (16) illustrates the mathematical formulation of the proposed control system

$$\frac{dX_i}{dt} = A_{ii}X_i + B_iU_i + \sum_{j \neq i} A_{ij}X_j \quad \forall i \in [1,n] \tag{17}$$

where

$$U_i = -K_iX_i$$

and

$$|\sigma_M(A_{ii} - B_iK_i)| > \sum_{j \neq i} \|A_{ij}\|_\infty = \sum_{j \neq i} |Kp_{ij}|$$

For the C-Ts shown in Fig. 1 the control signal inputs to the fuel control state ($V_{CE}^{(i)}$) and therefore the control matrix takes on the form

$$B_{CT} = [0\ \ 1\ \ 0\ \ 0\ \ 0]^T$$

Likewise, for the plants on Flores the control matrices are obtained as

$$B_{Diesel} = [0\ 0\ 1\ 0]^T, \quad B_{Wind} = [1\ 0]^T, \quad B_{Hydro} = [0\ 0\ 0\ 1\ 0]^T$$

The decentralized control signal is superposed to the primary control of DGs and it responds to both disturbances of the internal state variables $X_{LC}^{(i)}$ as well as the coupling variable $Z_{LC}^{(i)} = P_G^{(i)}$. Fig. 6 demonstrates a block diagram of the existing closed-loop dynamics improved by the enhanced decentralized control loop.

Applying the new control strategy to the C-Ts of the IEEE 30-node system demonstrates that the system will restore its dynamic stability and can satisfy stability criteria. In fact, the stability margin of the new system (the distance of the Gerschgorin circles to the imaginary axis) is approximately 0.3 pu/s. Fig. 7 illustrates the frequency response of the system after implementing the enhanced decentralized control. As shown in Fig. 7, overall overshoot of the system is less than 2% and the whole system settles gradually. This implies that the enhanced control can ensure both stability and dynamic performance of the system.

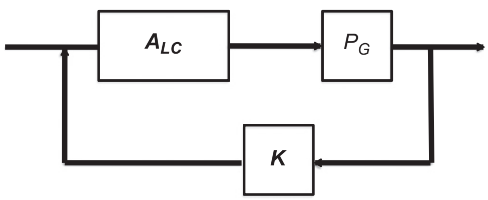

Fig. 6. The block diagram of the new control system.

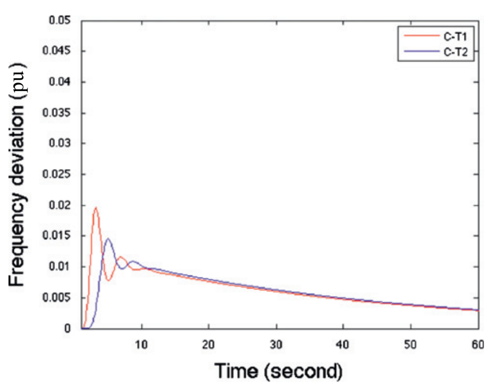

Fig. 7. Frequency response of the IEEE 30-node system after implementing the enhanced decentralized control.

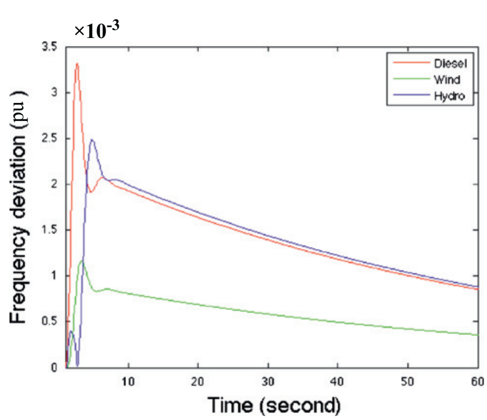

Fig. 8. Frequency response of Flores after implementing the enhanced decentralized control.

In the next step, dynamic stability of Flores is investigated considering the new control is implemented on the DG units. The technical findings illustrate that the island will be small-signal stable even with a high penetration of renewable energy resources. In this condition, the stability criteria are satisfied with the margin of around 0.2 pu/s. Fig. 8 demonstrates frequency deviation of the new system after a small perturbation (0.1 pu) occurs on the island. The results demonstrate that the overall overshoot of the new system is less than 0.35% and frequency deviations settle gradually. Eigenvalues of both scenarios are presented in Tables B2 and B5.

5. Conclusions and future outlook

This paper shows that poor tuning of G-C systems and strong electrical interaction between DGs are two main causes of potential small-signal instabilities in future distribution systems. If G-Cs of DGs are tuned in a conventional way, without considering interactions between the plants, DGs may start interacting against each other, especially when they are electrically close to each other. This can lead to small-signal instability problems.

To ensure stability of future distribution systems, an advanced decentralized control is introduced. The proposed control responds to the disturbances of internal states as well as coupling variables. The logic of the new control is to move the Gerschgorin Circles of the full system to the left hand side of the complex plane.

The proposed control is applied to two real world distribution systems with abundant DGs. The results demonstrate that the original systems are very sensitive to even very small perturbations in real power. On the other hand, the new systems with the enhanced decentralized control will be stable with appropriate stability margin.

The paper recommends that enhanced decentralized control systems are essential for stability of future distribution systems; otherwise, today policies, by which private operators can simply install and operate DG units without any constraint, is likely to lead to significant stability problems. In summary, the proposed control with Gerschgorin logic enables flexibility of DGs and brings many advantages such as

1. simple control systems;
2. no need for system-wide sensing and communications; and
3. relatively inexpensive control equipment.

However, decentralized control has inherent drawbacks such as it requires that all decentralized controllers operate as expected. Failure of some decentralized controllers to respond could lead to system-wide instabilities. The deployment of AMI (Automatic Meter Infrastructure) could resolve the problem by metering the actual actions of controllers. AMI enables two-way communication between DGs and SCADA (control center and data acquisition). The main advantage of this two-way communication is to stabilize the whole system at globally optimal equilibrium point. The future outlook of this paper is to deploy AMIs, operating in conjugate with DGs, in order to ensure both stability and efficiency of future distribution systems.

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Appendix A

In Appendix A, the state space model of the C-Ts on the IEEE 30-node system is presented in Eq. (18). In addition, the dynamic model of the hydro and wind plants on Flores are illustrated in Eqs. (19)–(21)

$$\frac{d}{dt}\begin{bmatrix} \omega_{G_{CT}} \\ V_{CE_{CT}} \\ W_{F_{CT}} \\ W_{Fd_{CT}} \end{bmatrix} = \begin{bmatrix} \frac{-D_{CT}}{M_{CT}} & 0 & \frac{c}{M_{CT}} & 0 \\ \frac{-K_D}{b} & \frac{-1}{b} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{-\delta}{\alpha} & \frac{-\beta}{\alpha} \end{bmatrix} \begin{bmatrix} \omega_{G_{CT}} \\ V_{CE_{CT}} \\ W_{F_{CT}} \\ W_{Fd_{CT}} \end{bmatrix} + \begin{bmatrix} \frac{-1}{M_{CT}} \\ 0 \\ 0 \end{bmatrix} P_{G_{CT}} + D_{P_{CT}}\, d(t) \tag{18}$$

where $V_{CE_{CT}}$ is the fuel control, $W_{F_{CT}}$ is the fuel flow, and $W_{Fd_{CT}}$ is the derivative of the fuel flow. In addition, α, β, δ and c are the transfer function coefficients for the fuel system, and K_D is the gain of the G-C (Cardell et al., 1998). The data for the coefficients of Eq. (18) is available in Cardell et al. (1998).

The dynamics of the hydro plant on Flores are presented in Eq. (19)

$$\frac{d}{dt}\begin{bmatrix} \omega_{G_h} \\ q_h \\ \nu_h \\ a_h \end{bmatrix} = \begin{bmatrix} \frac{-D_h}{M_h} & \frac{K_q}{M_h} & 0 & \frac{-K_w}{M_h} \\ \frac{1}{T_f} & \frac{-1}{T_d} & 0 & \frac{1}{T_w} \\ 0 & 0 & \frac{-1}{T_e} & \frac{r'}{T_e} \\ \frac{-1}{T_s} & 0 & \frac{1}{T_s} & \frac{-(r_h+r')}{T_s} \end{bmatrix} \begin{bmatrix} \omega_{G_h} \\ q_h \\ \nu_h \\ a_h \end{bmatrix} + \begin{bmatrix} \frac{-1}{M_h} \\ 0 \\ 0 \\ 0 \end{bmatrix} P_{G_h} + D_{P_h}\, d(t) \tag{19}$$

here q_h is the penstock flow, ν_h is the governor droop, and a_h is the gate position. Moreover, T_f, T_d, T_w, and T_e are the time constants of the hydro plant. T_s is the time constant of the servomotor, and r_h and r' are the permanent and transient speed droop, respectively (Cardell et al., 1998).

A wind plant is a synchronous machine connected to the grid through a power electronic interface. The mechanical part of the plant consists of a rotating mass and a wind turbine with a pitch control system. Eqs. (20) and (21) illustrate the dynamics of the rotating mass and the wind turbine, respectively (Pierik et al., 2004)

$$\frac{d\omega_{G_w}}{dt} = \frac{1}{M_w}P_{m_w} - \frac{D_w}{M_w}\omega_{G_w} - \frac{1}{M_w}P_{G_w} \tag{20}$$

where

$$P_{m_w} = -K_m \omega_{G_w} \tag{21}$$

here P_{m_w} is the mechanical power, D_w is the damping coefficient, and K_m is the proportional gain of the pitch control system (Pierik et al., 2004). The data for the state space models shown in Eqs. (19)–(21) are available in Nazari (2012, chap. 3).

Table B1
Eigenvalues of the IEEE 30-node system.

Full system	The 1st C-T (at node 13th)	The 2nd C-T (at node 14th)
−19.9943	−19.9943	−19.9943
−19.9943	−0.0045 + 1.510i	−0.0057 + 1.5090i
0.0081 + 1.5217i	−0.0045 − 1.510i	−0.0057 − 1.5090i
0.0081 − 1.5217i	−1.0714	−1.0775
−0.0186 +1.4984i	−0.0993	−0.0907
−0.0186 − 1.4984i		
−1.1387		
−0.9989		
−0.1969		
−0.0037		

Appendix B

In Appendix B, the eigenvalues and coupling matrices of the systems investigated in Sections 3 and 4 are presented (Tables B2–B4). The data for the dynamic model of the diesel plant (shown in Eq. (3)) is also illustrated in Tables B6 and B7.

Table B2
Eigenvalues of the IEEE 30-node system after implementing the enhanced decentralized control.

Full system	The 1st C-T	The 2nd C-T
−19.9920	−19.9943	−19.9943
−19.9966	−1.9930	−1.9070
−3.4869	−1.7140	−1.7750
−2.8475 + 1.5735i	−1.450 + 1.510i	−1.5700 + 1.5090i
−2.8475 − 1.5735i	−1.450 − 1.510i	−1.5700 − 1.5090i
−1.6033 + 2.2518i		
−1.6033 − 2.2518i		
−0.5083 + 1.6159i		
−0.5083 − 1.6159i		
−0.0239		

Table B3
Coupling matrix of the IEEE 30-node system.

Type of power plants	C-T$_1$	C-T$_2$
C-T$_1$	1.2545	−1.1534
C-T$_2$	−1.1519	1.1519

Table B4
Coupling matrix of the distribution system of Flores.

Type of power plants	Diesel/hydro	Wind
Diesel/hydro	13.9056	−1.4073
Wind	−1.3461	1.3461

Table B5
Eigenvalues of the distribution system of Flores.

Full system	Diesel generator	Wind plant	Hydro generator
1.0e+02 *	1.0e+02 *	−21.7622	1.0e+02 *
(−0.0069 + 1.0174i	(−0.007 + 1.0174i	−0.7097	(−1.1094
−0.0069 − 1.0174i	−0.007 − 1.0174i		0.0001 + 0.1036i
−1.1094	−0.0031		0.0001 − 0.1036i
−0.2176	−0.0000)		−0.0060
0.0001 + 0.1035i			−0.0045)
0.0001 − 0.1035i			
−0.0029			
−0.0071			
−0.0060			
−0.0045			
−0.0000)			

Table B6
Eigenvalues of Flores after implementing the enhanced decentralized control.

Full system	Diesel generator	Wind plant	Hydro generator
1.0e+02 *	1.0e+02 *	−21.7622	1.0e+02 *
(−0.017+1.017i	(−0.017+1.017i	−1.7097	(−1.1094
−0.017−1.017i	−0.017 − 1.017i		−0.015+0.104i
−1.1094	−0.0140		−0.015−0.104i
−0.2176	−0.0150)		−0.0145
−0.015+0.1041i			−0.0160)
−0.015 − 0.1041i			
−0.0405			
−0.009+0.0174i			
−0.009−0.0174i			
−0.0002			
−0.0171)			

Table B7
Electromechanical parameters of the diesel plant on Flores.

M_d (s)	D_d (pu)	T_d (s)	K_d (pu)
1.133	0.005	0.6	40
R_c (pu)	C_d (pu)	K_I (1/s)	C_c (pu)
0.03	1	10	1

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