Most Relevant Measurements for State Estimation According to Information Theoretic Criteria

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Abstract* — *This work presents a methodology for selecting the most relevant measurements for real-time power system monitoring. A genetic algorithm is employed to find the meter plan, composed of relevant, real-time measurements and pseudo-measurements that present the best compromise between investment costs and state estimation performance. This is achieved by minimizing both the number of real-time measurements in the power network and the degradation of the estimated states. Performance measures based on the Information Theory are investigated. Simulation results illustrate the performance of the proposed method.

Keywords—Measurement Design, State Estimation, Information Theory, Stochastic Optimization.

I. INTRODUCTION

Present day Energy Management Systems (EMS) and Distribution Management Systems (DMS) include functions for monitoring, control, contingency analysis, and optimization through advanced computational applications. Among these, the State Estimation function (SE) intends to filter small, ordinary measurement errors, and eliminate possible bad data introduced by t data acquisition system malfunctioning [1], [2].

SE deals with active\reactive power flows, injections and magnitudes of bus voltages and branch currents. These measurements are commonly provided by remote terminal units (RTUs) at power substations. More recently, synchronized phasor measurements have started to become available at selected substations by means of phasor measurement units (PMUs).

To accomplish the desired tasks, a sufficiently accurate network model and redundant measurements should be provided. Redundancy can be interpreted as the excess of data in mandatory number for the estimation of state variables and depends on the location, type, quality and quantity of measurements in the power network. The availability of measurements plays an important role on the SE performance, mainly on bad data processing [1], [2].

However, the supply of measurements is usually constrained by economic and operational factors such as communication infra-structure, metering cost, and real-time data processing capability. These constraints become more significant in DMS, in which the network topology, size and availability of fewer real-time meters makes both SE process and the measurement design engineering challenges.

Additionally, with the advent of smart grids and distributed generation, the increasing need of monitoring, control and optimization of distribution networks requires the implementation of efficient DMS solutions. Therefore, the measurement design problem plays a critical role in the development and implementation of DMS. A commonly adopted strategy is to employ pseudo-measurements obtained from forecasted load profiles [3]. However, a SE process strongly relying on the use of pseudo-measurements can become unreliable, providing poor quality estimated quantities. Therefore, whenever the inclusion of pseudo-measurements becomes a necessity, one should like to be able to select a combination of pseudo- and real-time measurements that can provide a high quality estimation of the state variables. One way of doing that is by selecting a set of real-time measurements, so that a given expected measure of the SE quality becomes the closest to the one that would be obtained by a fully real-time metered network.

The meter placement problem has been addressed by many authors. In [4]-[8], the system observability and minimization of investment costs were the major concern, whereas [8]-[11] focus on the robustness of a metering system. Methods based on intelligent system techniques have also been proposed. A steady-state genetic algorithm (GA) [9] has been employed in order to design metering plans with bad data processing capability. In [10], a GA is employed to design the metering system, to meet observability and data criticality requirements considering the loss of measurement units, while in [12] a constructive heuristic was proposed for the same problem, but also considering different topology scenarios. Although these methods address system observability, economic and operational constraints, the SE quality is not directly considered in the metering design.

This paper proposes a method for designing metering systems for SE. The methodology aims to select the best

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combination of real-time and pseudo measurements that minimizes the loss of performance of the SE process. A performance measure for state estimation, based on information theory is also investigated. Tests with the IEEE-14 bus test system and a real distribution system are presented and their results discussed. In Section 2 the technical background required for the proposed methodology will be presented. Section 3 presents a methodology for provision of relevant measurements for power system state estimation. In Section 4 simulations results using the IEEE-14 bus test system and a three-phase distribution system with unbalanced loads are presented. Section 5 presents the conclusions of this research work.

II. TECHNICAL BACKGROUND

The classical SE process is divided into 4 steps: network configuration, at which the bus-branch model is obtained from the breaker and switch statuses; observability analysis, determines whether the system is observable or not, considering the current available measurements; SE filtering step – core of the process – state and measurements estimates are obtained, usually through the weighted least squares (WLS) estimator; and the bad data processing step, at which data inconsistencies are identified and corrected provided that there is enough measurement redundancy. The main aspects involving these steps can be found in [1] and [2].

A. Non-Linear State Estimation

The relation between state variables and acquired measurements can be modeled by:

$$z = h(x) + \varepsilon \tag{1}$$

where z ($m\times1$) is the vector of received measurements; $h(\cdot)$ is the observation function vector; x ($n\times1$) is the state vector and ε is the measurement error vector.

The equality constrained WLS estimator [2] is the solution of the optimization problem established by:

$$min\, J(x) = [z - h(x)]^T W [z - h(x)] \tag{2}$$

where W is the weighting matrix, the inverse of the measurement error covariance matrix $R = E(\varepsilon\varepsilon^T) = W^{-1}$.

The estimated state $\hat{x}$ that solves (2) can be obtained by the iterative process as follows:

$$G(x_k)\Delta x_k = H(x_k)^T W[z - h(x_k)] \tag{3}$$

where $H = \partial h / \partial x$, is the Jacobian matrix of the acquired measurements and $G = H^T W H$ ($n\times n$) is the Gain matrix.

It is important to remark that the system observability in (3) can be evaluated via the singularity analysis of the Gain matrix. A system is considered numerically observable, if its Gain matrix has full rank, at the flat start [1].

B. Effect of Pseudo-measurements in State Estimation

Pseudo-measurements are employed in SE to enhance/maintain system observability when some real-time measurements are temporarily or permanently unavailable. These virtual measurements are usually generated by forecasting routines, based on historical data, load forecasting models, and tracking estimators. It is a common practice to assign large variances to pseudo-measurements, resulting in very low weights in the SE process [2].

The replacement of a real-time measurement by a pseudo-measurement can be represented by a sequence of rank-k updates in the Gain matrix. Considering an observable network composed only by real-time measurements, with a respective gain matrix G_0, the removal of k real-time measurements results in the modified matrix G_1, given by:

$$G_1 = G_0 - H_p^T W_p H_p \tag{4}$$

where H_p ($k\times n$) and W_p ($k\times k$) are the Jacobian and weighting matrices associated with the set of removed measurements p, respectively.

Replacing the removed measurements by their respective pseudo-measurements results in the gain matrix G_2:

$$G_2 = G_1 + H_p^T \tilde{W}_p H_p \tag{5}$$

where $\tilde{W}_p$ ($k\times k$) is the weighting matrix of the added pseudo-measurements.

The combination of (4) and (5) gives the rank-k update for the Gain matrix:

$$G_2 = G_0 + H_p^T [\tilde{W}_p - W_p] H_p \tag{6}$$

From (6), it can be concluded that the substitution of a real-time measurement by a pseudo-measurement affects the Gain matrix in a fashion similar to a measurement removal, since the condition (7) is always satisfied for pseudo-measurements, that is the difference $[\tilde{W}_p - W_p]$ is negative definite. Additionally, (6) resemblances (4) when the weights associated to the added pseudo-measurements are very small.

$$\tilde{W}_p \le W_p \tag{7}$$

C. Relative Entropy

Information Theory (IT) is a branch of the applied mathematics that involves the quantification of information. Its foundations were developed by Shannon [13] for the design of efficient and reliable communication systems. IT has been further applied to a variety of fields from statistical inference to neurobiology and artificial intelligence. The application of IT learning concepts to SE was firstly suggested in [14], [15].

An important information quantifier is the *Kullback-Leibler* divergence (I_{KL}) or relative entropy, which measures the similarity between two probability densities. For continuous random variables, I_{KL} is defined by:

$$I_{KL} = D(P\,||\,Q) = \int_{-\infty}^{+\infty} ln\left(\frac{p(x)}{q(x)}\right) p(x)dx \tag{8}$$

where P and Q are random variables with probability densities $p(x)$ and $q(x)$ respectively.

If P and Q are random variables parameterized by the vector θ used in (9), then combining (8) ,(9), and replacing $f(x \mid \theta_0 + \delta)$ by a second order Taylor expansion around θ_0, the following expression is obtained:

$$\begin{aligned} P &= p(x,\theta) = f(x \mid \theta_0 + \delta) \\ Q &= q(x,\theta) = f(x \mid \theta_0) \end{aligned} \tag{9}$$

$$D(P \,||\, Q) \approx \frac{1}{2}\delta^T \left[\nabla^T \nabla \, ln \, f(x \mid \theta_0)\right]\delta \tag{10}$$

where $\nabla^T \nabla \, ln \, f(x \mid \theta_0)$ is called Fisher Information Matrix.

The Fisher Information is a local information measure for parametric statistical models. Additionally, it is related with the amount of information lost when we use the approximate model $f(x \mid \theta_0 + \delta)$ instead of the true model $f(x \mid \theta_0)$. Fisher Information naturally arises in the derivation of WLS estimator and can be used for the quantification of information measures for SE.

III. PROPOSED METHODOLOGY

A. Problem Statement

A relevant measurement is defined in this work as the measurement whose uncertainty greatly impacts the SE quality and performance. As a consequence, differently from the classical meter placement problem, the focus is in the set of measurements that minimally degrades the quality of SE process, when replaced by pseudo-measurements. This can be generally formulated as the following optimization problem

$$\begin{aligned} & min \;\; \alpha \Delta P(z) + \beta C_z \\ & s.t \\ & g(z) = 0 \end{aligned} \tag{11}$$

where z is the vector representing the set of real-time measurements (location, type and quality) ; C_z is the investment cost associated with the set of measurements; $\Delta P(z)$ is the decrease of SE performance; and $g(z)$ are the functions related with constraints in the measurement set.

The constraints in (11) can be already known power system quantities such as zero power or current injections or measurements that are obligatory real-time. The weights α and β must be chosen in order to provide the best compromise between the objectives.

The problem established (11) is a combinatorial in nature. Therefore, metaheuristic based methods are suited for solving this class of problems [9]-[11], [17]. In this paper, a Genetic Algorithm (GA) will be employed in the solution of (11). A complete description of GA and its application to combinatorial optimization problems can be found in [19].

B. SE Performance Measure

In order to identify relevant measurements, metrics for SE performance and quality must be specified. In [3], [16] – [18] metrics are proposed for different purposes. In [16], the final value of WLS performance index defined in (2) is employed as quality measure, whereas the performance of the process is evaluated by the number of iterations in (3) until convergence. However, these measures require the running of the SE process, and are not clearly related with the characterization of the measurement set, as in the case of the number of iterations. The work presented in [17] establishes a relation between the condition number of the gain matrix and different combinations of measurements. Additionally, the condition number plays an important role in the convergence of the iterative process (3), being a better measure than the number of iterations. Singh [3] proposes a performance measure based on the covariance matrix of estimated states.

Meaningful measures of SE performance can be obtained from the IT. Assuming the measurement errors are independent, normally distributed, and uncorrelated [1], [2], the relative entropy can be approximated given:

$$I_{KL} \approx \frac{1}{2}\varepsilon^T E_x(G(x))\varepsilon \tag{12}$$

Since G is normal and positive semi-definite, (12) can be rewritten as:

$$I_{KL} \approx \frac{1}{2}\varepsilon^T Q\Lambda Q^T \varepsilon = \frac{1}{2}\sum_{i=1}^{n} \lambda_i \alpha_i^2 \tag{13}$$

where $Q\Lambda Q^T$ is the spectral decomposition of $E_x(G(x))$, and $\alpha = Q^T \varepsilon$.

From (10), it can be inferred that the relative entropy is dependent on the spectrum of the Fisher matrix. Additionally, (13) states that:

1 – the eigenvalues of G must be as large as possible;

2 – its spectrum must be as uniform as possible.

The first requirement is directly related to the *Cramér-Rao* bound: the higher the eigenvalues of the Gain matrix, the smaller the eigenvalues of its inverse and, consequently, smaller the variances of the estimated states. The second addresses the observability and ability to process bad data, as it enforces all estimated states to stay as close as possible of the true ones.

The matrix determinant or trace, which is also the product of the eigenvalues of a matrix, can be used as an information measure, since it can evaluate the magnitude of the matrix eigenvalues which agrees with the results obtained in [3]. In this paper the matrix determinant will be employed since it provides information about the off-diagonal elements, differently from the matrix trace.

C. Solution Encoding

The encoding used to describe the proposed solutions should be easy to be manipulated by a GA and provide a complete description of the different possible scenarios.

Measurements are usually obtained through RTUs and PMUs installed at power substations and power devices. Single measurements, such as power and current injections and voltage magnitudes, can be available through smart meters and protection devices. In order to represent different combinations of measurements, the proposed encoding for the GA is a binary vector of size (n_{rtu}+m)x1, where n_{rtu} and m are the number of available RTUs and single measurements in the network, respectively. The value 0 indicates that the measurement (or RTU) referenced in the solution is a pseudo measurement(or the measurements associated with the RTU are pseudo-measurements), while real-time measurements(or RTU) are assigned with value 1, as shown in Fig. 1.

0	1	1	…	0	1	…	1	1
RTU *1*	RTU *2*	RTU *3*		RTU n_{rtu}	Meas *1*		Meas *m-1*	Meas *m*

Fig. 1 – Solution Encoding

D. Fitness Function

The fitness function is responsible for guiding the GA optimization process. Therefore, an adequate formulation – covering all objectives/constraints and establishing a compromise between them – must be chosen. In this work, the fitness function is given by

$$fobj = -\alpha\, log_{10}\left|det\left(G_2\right)\right| + \beta \frac{nm_{rt}}{nm} \qquad (14)$$

where nm is the number of measurements nm_{rt} is the number of real-time measurements and G_2 is the Gain matrix after replacing the real time measurements by pseudo-measurements, which is obtained using (6). The parameters α and β should be adjusted according with the problem objectives.

The determinant is only evaluated at flat start, since the Gain matrix eigenvalues are expected to naturally decrease during the iterative process expressed by (3). This procedure intends to avoid computational costs involved in obtaining the estimated states for each solution proposed by the GA. Also, since the metering design problem is structural rather than numerical, the condition (12) can be relaxed.

E. Algorithm Proposed

The procedure for the GA based optimization assumes that a set of candidate real-time measurements are available and the network is observable for this set. The search process will try to find the best combination of pseudo-measurements that minimize the fitness presented in (11). The main steps of the proposed algorithm for the provision of pseudo-measurements are described in Fig. 2.

The proposed algorithm differs from other meter placement algorithms found in the literature in two aspects. At first, a measurement set that provides full network observability must be provided. In a classical meter placement problem an (at least) observable plan is the objective of the search process. Secondly, the main concern of this algorithm is to define the group of measurements whose uncertainty does not strongly impact on the estimation process, while in the classical algorithms this aspect is, in general, not clearly addressed. For instance, it is already known that injection measurements are better than power flows if observability is the main concern. However, if the focus is the quality of the filtering process or bad data identification, power flows measures are preferable to power injections [1], [2].

Step 1- Form matrix G_0, assuming all candidate measurements as real-time measurements and using the flat start x_0;

Step 2- if G_0 is singular, stop – the network is unobservable; else, go to Step 3;

Step 3- Set the GA parameters;

Step 4- Set the initial pool of solutions according with the solution encoding described in Section III.C;

Step 5- Find the best solution using GA, evaluating the fitness of each solution according with (14);

Step 6- if an acceptable solution is found, stop; else

Fig. 2 – Proposed Algorithm

Since GA is a stochastic optimization method, there is no guarantee about how close the obtained solution is from the global optimal solution. Therefore, the best solution found must be analyzed and if not acceptable, a new run of the GA with different parameters must be done. The evaluation and acceptance of the proposed solution must be based on the designer experience and knowledge. In the next section, results obtained with a SE benchmark test system and a real distribution network exemplifies the proposed method.

IV. RESULTS

A standard SE routine is employed for the IEEE-14 bus test system and a three-phase SE model is used for the distribution network. The proposed methodology was implemented in MATLAB and the optimization has been carried out using the GA from the global optimization toolbox. The GA parameters used in simulation are presented in Table 1.

TABLE 1 – OPTIMIZATION PARAMETERS

Parameter	Value
Population Size	3*(n_{rtu}+m)
Selection Type	Tournament
Crossover Type	Two point Crossover
Crossover Rate	0.95
Mutation Type	Uniform
Number of Generations	200

A. Case 1 – IEEE 14- Bus Test System

The IEEE 14 bus test system for SE studies is presented in Fig.3. For this system, it is assumed that all RTUs can provide power flows, injections and voltage magnitude measurements. Bus 7 is considered as having zero power injections, which are modeled as equality constraints. It is assumed that the variance of the pseudo-measurements is 100 times greater than the variance of the corresponding real-time measurements. The results obtained after 100 runs of the proposed algorithm are given in Table 2. The optimal meter plan is presented in Fig. 3.

TABLE 2 – OPTIMIZATION RESULTS: CASE 01

	#1
Parameters (14)	α=1; β=10^2
Number of real-time measurements	6
$\|G_2\|^{-1}$	3.25e-25
log10(k(G_2))	7.75

The results show that the proposed methodology succeeded in reducing the number of real-time measurements needed for SE. The determinant of the Gain matrix remained very small. Using some indicators presented in [18] it is also possible to assess the obtained metering system with respect to the presence of critical measurements and critical sets. In this case, the measurement system, if considered only formed by real-time measurements, presents no critical measurements or sets. This is shown in Table 4, in which $P(Unobs|C1)$ is related to the presence of critical measurements and $P(Unobs|C2)$ is related to the presence of critical sets. The higher values such indicators assume, more critical measurements and sets are present in the metering system.

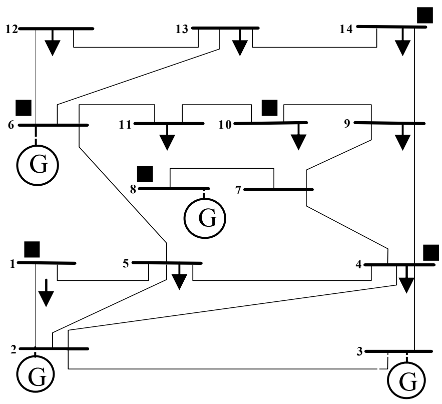

Fig. 3 – IEEE 14 –Bus Test System. The black squares indicate the real-time measurements proposed by the methodology.

B. *Case 2 – 33 Bus Distribution Test System*

In this case, a real 33 bus Distribution Feeder, with 32 branches and unbalanced loads is employed. The radial structure of Distribution Systems makes the observability constraints tighter than in meshed networks, which generally requires a large number of metering units in classical approaches [11]. It is assumed that RTUs located at load buses only possess power injection measurements, while the ones at generator buses have injection and voltage magnitude measurements. Additionally, the RTU installed at the distribution substation is supposed to contain power flows, injections and voltage magnitude measurements, and it is considered delivering real-time measurements all the time. Power injections at buses with no load and/or generation are treated as zero injection measurements. The pseudo-measurements variances are assumed to be 100 times greater than the ones associated to real-time measurements.

A three-phase, equality constrained, SE formulation was used. This model, combined with the system size, makes the problem ill-conditioned, requiring a more refined adjustment of the parameters in (14). The results obtained with the proposed methodology after ten runs, for two different settings, are given in Tables 3 and 4. The setting #1 prioritizes the reduction of the number of real-time measurements (high β), while #2 emphasizes SE quality (high α). Fig. 4 and 5 presents the meter plan obtained for setting #1 and #2 respectively.

In both simulations, a large reduction in the condition number happened (the initial condition number was $log_{10}(k(G_0))$ = 28.13). This large condition number is explained by the presence of only injections and voltage magnitude measurements. The determinant remains very close to zero for both settings. It can be observed that the increase in performance is achieved at the cost of more measurements in the network. For the setting #1, the metering system is unobservable considering only the real time measurements. However, for setting #2, the methodology placed measurements at the beginning and end of the feeders, enhancing the system observability. The system is observable, with critical measurements and sets, which is shown by the observability indices [18] in Table 4. These results clearly show the impact of measurement redundancy in state estimation quality and observability.

TABLE 3 – OPTIMIZATION RESULTS: CASE 02

	#1	#2
Parameters (14)	α=10^{-3}; β=10^2	α=10^{-1}; β=10^2
Number of real-time measurements	15	25
$\|G_2\|^{-1}$	1.48e-25	7.4e-196
log10(k(G_2))	21.23	18.54

TABLE 4 – OBSERVABILITY INDICES [16]

	Case 1	Case 2	
	#1	#1	#2
P(Unobs|C1)	0	1	0.219
P(Unobs|C2)	0	1	0.391

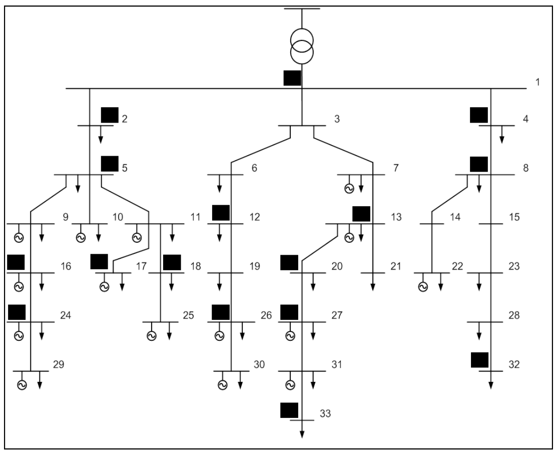

Fig. 4 – 33-Bus Distribution System and Optimal Plan #1

V. CONCLUSIONS

This paper introduced a new approach, based on Information Theoretic concepts, to design the topologic distribution of metering systems for real-time power systems monitoring, with the aim to serve a State Estimation process. A specific objective is to determine the best set of pseudo measurements that minimize the degradation of real-time SE performance. The adoption of a quality criterion to assess SE performance, based on Information Theory, is a major breakthrough: it provides a theoretic foundation to evaluate SE quality, instead of resorting to heuristic engineering inspired criteria. Furthermore, the model displays similarities and builds bridges to other suggested measures and this facilitates its understanding and interpretation.

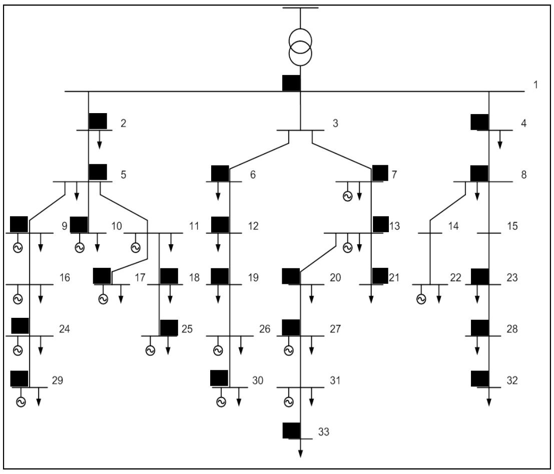

Fig. 5 – 33-Bus Distribution System and Optimal Plan #2

The success of the method was achieved via a careful mathematical development coupled with an algorithmic technique based on Genetic Algorithms with a suitable chromosome encoding. The problem has a combinatorial nature, which justifies the choice of a meta-heuristic as solver. The fitness function proved to be flexible, straightforward and adjustable to the designer optimization objectives. The problem solution is set as a compromise between cost and SE quality and therefore, although not discussed in this paper, a Pareto Optimal border may be explored by manipulating the trade-off between the two objectives – i.e., the new approach allows a theoretical foundation for a concept of trade-off between the variation in cost of a measuring system and its impact on SE performance.

Test results were obtained with the IEEE 14-bus system and with a real Distribution Feeder. These results illustrate the general character of the new model. In particular, it proved useful for real distribution networks, since the observability constraints usually require a large number of real-time measurements for an optimal solution, which is hard to achieve in distribution networks.

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