

# BRKGA ADAPTED TO MULTIOBJECTIVE UNIT COMMITMENT

## *Solving Pareto frontier for UC multiobjective Problem using BRKGA SPEA2 NPGA and NSGA II techniques*

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Abstract: *The environmental concerns are having a significant impact on the operation of power systems. The traditional Unit Commitment problem, which to minimize the fuel cost is inadequate when environmental emissions are also considered in the operation of power plants. This paper presents a Biased Random Key Genetic Algorithm (BRKGA) approach combined with non-dominated sorting procedure to find solutions for the unit commitment multiobjective optimization problem. In the first stage, the BRKGA solutions are encoded by using random keys, which are represented as vectors of real numbers in the interval [0, 1]. In the subsequent stage, a non-dominated sorting procedure similar to NSGA II is employed to approximate the set of Pareto solution through an evolutionary optimization process. The GA proposed is a variant of the random key genetic algorithm, since bias is introduced in the parent selection procedure, as well as, in the crossover strategy. Test results with the existent benchmark systems of 10 units and 24 hours scheduling horizon are presented. The comparison of the obtained results with those of other Unit Commitment (UC) multiobjective optimization methods reveal the effectiveness of the proposed method.*

## 1 INTRODUCTION

The power system generation scheduling is composed of two tasks: On the one hand, the traditional unit commitment (UC) that involves scheduling the turn-on and turn-off of the thermal generating units; on the other hand, the economic dispatch (ED), which assigns, the amount of power that should be produced by each on-line unit in order to minimize the total operating costs for a specific time generation horizon. The traditional configuration of this problem was modified when environmental concerns arised due to the goals imposed by Kyoto protocol. The carbon emissions produced by fossil-fueled thermal power plants should also be minimized. Hence, it is necessary to consider the emission as another objective. Therefore, we are in the presence of problem with two, usually conflicting, objectives.

Several methods have been reported in the literature concerning to the environmental/economic dispatch problem. However, to obtain an optimal solution, it is important to consider not only the output generation level of each generating unit but also the

turn on/off schedule, due to start-up costs/emissions that have significant influence in the problem solution.

In (Graneli et al., 1992) the problem is formulated as single objective with an emission limit constraint. The disadvantage of such an approach is that it does not allow for obtaining solutions with a tradeoff between costs and emissions. In addition, this type of approach leads to solutions maximizing the profit but disregarding possible solutions with CO<sub>2</sub> reduction.

The  $\epsilon$ -constraint method for multiobjective optimization was presented in (Hsiao et al., 1994). This method is based on preferences of the objectives. The most important objectives are considered while the other objectives are treated as constraints bounded by some allowable levels. The main disadvantage of this approach is to find weakly non-dominated solutions.

Some of the previous studies of unit commitment problems including emission constraints have been solved using lagrange relaxation methods (Wang et al., 1995; Yamin et al., 2007). In (Wang et al., 1995) an augmented lagrange relaxation is used to solve a unit commitment considering the typical system constraints such as power balance, min-

imum up/down time and ramp rate constraints and adding transmission and environmental constraints. In (Yamin et al., 2007) lagrange relaxation is combined with evolutionary programming.

Current research is directed to handle both objectives simultaneously as competing objectives instead of simplifying the multiobjective problem to a single objective problem. Three multiobjective evolutionary algorithms (MOEAs) have been applied to the Economic Dispatch (ED) problem with meaningful success (Abido, 2003c; Abido, 2003b; Abido, 2003a). Since they use a population of solutions in their search, multiple Pareto-optimal solutions can, in principle, be found in one single run. Different MOEAs like Niche Pareto Genetic Algorithm (NPGA) (Horn et al., 1994), Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler and Thiele, 1998) and Non-dominated Sorting Genetic Algorithm (NSGA) (Srinivas and Deb, 1994) have been applied to this problem. These models can be efficiently used to eliminate most of the difficulties of classical methods. However, the quality and diversity of the nondominated solutions presented in (Abido, 2003c; Abido, 2003b; Abido, 2003a) have not been measured and assessed quantitatively. In (Abido, 2006) a comparative study among MOEA techniques was developed to evaluate their potential to solve the multiobjective ED problem. The potential of MOEA to handle this problem is investigated and their effectiveness to solve the ED multiobjective problem was shown. It is important to refer that new versions of MOEAs were presented such as NSGA-II (Deb et al., 2002), and SPEA2 (Zitzler et al., 2001). The NSGA-II algorithm was also applied to the ED multiobjective problem in (Basu, 2008).

Gonçalves and Resende (2010) introduce the tutorial on the implementation and use of Biased Random Key Genetic Algorithm (BRKGA) for solving combinatorial optimization methods. More recently, in (Roque et al., 2011) the BRKGA approach is used to find solutions for single objective Unit Commitment problem.

In this paper the BRKGA algorithm combined with nondominated sorted procedure and MOEA techniques is applied to the two standard 10-unit 24-hour test systems presented in (Winter et al., 2003) and (Sawaragi et al., 1985). The proposed approach BRKGA is combined with a ranking selection method, that is used to focus on different levels of the nondominated solutions, and a sharing fitness procedure as in NSGA.

The paper is organized as follows: Following the description of the problem formulation, which is given in section 2, an explanation on the BRKGA and

its implementation to the UC bi-objective problem, is given in section 3. Section 4 provides test results and finally in Section 5 some conclusions are drawn.

## 2 UC MULTIOBJECTIVE PROBLEM FORMULATION

In the multiobjective UC problem one needs to determine an optimal schedule, which minimizes the production cost and emission of atmospheric pollutants over the scheduled time horizon subject to system and operational constraints. Due to its combinatorial nature, multi-period characteristics, and nonlinearities, the UC problem is a hard optimization problem, which involves both integer and continuous variables and a large set of constraints.

Let us now introduce the parameters and variables notation.

### Decision Variables:

$Y_{t,j}$ : Thermal generation of unit  $j$  at time period  $t$ , in [MW];

$u_{t,j}$ : Status of unit  $j$  at time period  $t$  (1 if the unit is on; 0 otherwise);

### Auxiliary Variables:

$T_j^{on/off}(t)$ : Time periods for which unit  $j$  has been continuously on-line/off-line until time period  $t$ , in [hours];

### Parameters:

$T$ : Number of time periods (hours) of the scheduling time horizon;

$t$ : Time period index;

$N$ : Number of generation units;

$j$ : Generation unit index;

$R_t$ : System spinning reserve requirements at time period  $t$ , in [MW];

$D_t$ : Load demand at time period  $t$ , in [MW];

$Y_{min,j}$ : Minimum generation limit of unit  $j$ , in [MW];

$Y_{max,j}$ : Maximum generation limit of unit  $j$ , in [MW];

$N_b$ : Number of the base units;

$T_{min,j}^{on/off}$ : Minimum uptime/downtime of unit  $j$ , in [hours];

$T_{c,j}$ : Cold start time of unit  $j$ , in [hours];

$S_{H/C,j}$ : Hot/Cold start-up cost of unit  $j$ , in [S];

$SD_j$ : Shut down cost of unit  $j$ , in [S];

$S_{e,j}$ : Start-up atmospheric pollutant emission of unit  $j$ , in [ton-CO<sub>2</sub>] if CO<sub>2</sub> or [mg/Nm<sup>3</sup>] if nitrogen oxides;

$\Delta_j^{dn/up}$ : Maximum allowed output level decrease/increase in consecutive periods for unit  $j$ , in [MW];

### 2.1 Objective Functions

As already said, in the multi-objective problem formulation, two important objectives in electrical thermal power system are considered. These are economy and environmental impacts.

On the one hand, the first objective is to minimize the system operation costs composed by generation and start-up costs. The generation costs, i.e. the fuel costs, are conventionally given by a quadratic cost function as in equation (1),

$$F_j(Y_{t,j}) = a_j \cdot (Y_{t,j})^2 + b_j \cdot Y_{t,j} + c_j, \quad (1)$$

where  $a_j, b_j, c_j$  are the cost coefficients of unit  $j$ .

Therefore, the cost incurred with an optimal scheduling is given by the minimization of the total costs for the whole planning period, as in equation (2).

$$\text{Minimize } \sum_{t=1}^T \left( \sum_{j=1}^N \{F_j(Y_{t,j}) \cdot u_{t,j} \right. \quad (2)$$

$$\left. + SU_{t,j} \cdot (1 - u_{t-1,j}) \cdot u_{t,j} \right. \quad (3)$$

$$\left. + SD_j \cdot (1 - u_{t,j}) \cdot u_{t-1,j} \right).$$

where  $S_{t,j}$  and  $SD_{t,j}$  are the start-up and shut-down costs of unit  $j$  at time period  $t$ , respectively.

On the other hand, the second objective is to minimize the total quantity of atmospheric pollutants emission. The emissions are generally expressed as a quadratic function:

$$E_j(Y_{t,j}) = \alpha_j \cdot (Y_{t,j})^2 + \beta_j \cdot Y_{t,j} + \gamma_j, \quad (4)$$

where  $\alpha_j, \beta_j, \gamma_j$  are the emission coefficients of unit  $j$ .

So, the total emission of atmospheric pollutants is expressed as follows:

$$\text{Minimize } \sum_{t=1}^T \left( \sum_{j=1}^N \{E_j(Y_{t,j}) \cdot u_{t,j} \right. \quad (5)$$

$$\left. + Se_{t,j} \cdot (1 - u_{t-1,j}) \cdot u_{t,j} \right).$$

where  $Se_j$  is the start-up atmospheric pollutant emissions of unit  $j$  at time period  $t$ .

## 2.2 Constraints

The constraints can be divided into two sets: the demand constraints and the technical constraints. Regarding the first set of constraints it can be further divided into load requirements and spinning reserve requirements, which can be written as follows:

### 1) Power Balance Constraints

The total power generated must cover the total load demand, for each time period.

$$\sum_{j=1}^N Y_{t,j} \cdot u_{t,j} \geq D_t, t \in \{1, 2, \dots, T\}. \quad (6)$$

### 2) Spinning Reserve Constraints

The spinning reserve is the total amount of real power generation available from on-line units net of their current production level.

$$\sum_{j=1}^N Y_{max,j} \cdot u_{t,j} \geq R_t + D_t, t \in \{1, 2, \dots, T\}. \quad (7)$$

The second set of constraints includes unit output range, minimum number of time periods that the unit must be in each status (on-line and off-line), and the maximum output variation allowed for each unit.

### 3) Unit Output Range Constraints

For each time period  $t$  and unit  $j$ , the real power output of each generator is restricted by maximum and minimum production limits.

$$Y_{min,j} \cdot u_{t,j} \leq Y_{t,j} \leq Y_{max,j} \cdot u_{t,j}. \quad (8)$$

### 4) Ramp rate Constraints

Due to the thermal stress limitations and mechanical characteristics, the output variation levels of each on-line unit in two consecutive periods are restricted by ramp rate limits.

$$-\Delta_j^{dn} \leq Y_{t,j} - Y_{t-1,j} \leq \Delta_j^{up}. \quad (9)$$

### 5) Minimum Uptime/Downtime Constraints

If the unit has already turned on or off, there will be a minimum uptime/downtime time before it is shut-down or started-up, respectively.

$$T_j^{on}(t) \geq T_{min,j}^{on} \text{ and } T_j^{off}(t) \geq T_{min,j}^{off}. \quad (10)$$

## 3 MULTIOBJECTIVE UC OPTIMIZATION

### 3.1 Decoding procedure

The decoding procedure is commonly used in all four multiobjective optimization algorithms. For each chromosome, the corresponding solution is performed in two main stages, as it can be seen in Figure 2 in (Roque et al., 2011). Firstly, the output generation level matrix for each unit and period is computed from random key value. In this solution, the units production is proportional to their priority, which is given by the random key value. By doing so, each element of the output generation matrix,  $Y_{t,j}$  is given as the product of the percentage vectors by the periods demand  $D_t$ . Here each component of the percentage vectors are given by corresponding random key entrie divided by the sum of the all random key values as illustrated in algorithm 1 (Roque et al., 2011). Then, these solutions are checked for constraints satisfaction using a repair algorithm presented in (Roque et al., 2011).

### 3.2 Repair algorithm

The idea of this technique is to convert any infeasible individuals to a feasible solution by repairing the sequential possible violations constraints in the UC

problem. The repair algorithm is composed by several steps. Firstly, the output levels are adjusted in order to satisfy the output range constraints. Next, we have the adjustment of output levels to satisfy ramp rate limits. It follows the repairing of the minimum up-time/downtime constraints violation. Afterwards, the output levels are adjusted in order to satisfy spinning reserve requirements. Finally, the output levels are adjusted for demand requirements satisfaction at each time period. For details about the repairing mechanisms, the reader is referred to (Roque et al., 2011).

### 3.3 NSGA

A fast and elitist non-dominated sorted genetic algorithm (NSGA II) (Deb et al., 2002) is used to approximate the set of Pareto solution. In this approach, the ranking selection method is used to focus on non-dominated solutions while the crowding distance is computed to ensure diversity along the nondominated front. The population of size  $N_p$  is used for selection, crossover, and mutation to create a new offspring population of equal size. The rank procedure is employed by different levels of domination until all individuals in the intermediate combined population, of size  $2N_p$ , are ranked. Firstly, the nondominated solutions are assigned with same rank value and thereafter the crowding distance is computed. The nondominated solutions must be emphasized more than any other solution. In order to find individuals of the next front, the solutions of the first front are temporarily ignored, and the above procedure is repeated to find subsequent fronts. The individuals of the new population are selected from the intermediate population, they are chosen from subsequent nondominated fronts in the order of their ranking. To choose exactly the population members, the solutions of the last front are sorted considering the crowding distance by descending order. The NSGA-II approach proposed by (Deb et al., 2002) was implemented as follows:

- Generate random population, decoding the individuals and evaluate the solutions;
- Sort the population using non-domination-sort. For each individual, rank and crowding distance are assigned;
- For each generation the follows steps are given: Select the parents, which are fit for reproduction by using the binary tournament selection based on the rank and crowding distance; genetic operators copy, simulated binary crossover and mutation are applied under selected parents; the offspring population is combined with parents (the size of intermediate population is the double); selection is

performed to set the individuals of the next generation; after sorting the intermediate population, only the best individuals are selected based on its rank and crowding distance; a new generation is then obtained maintaining the population size fixed; the algorithm stop criterium is the maximum number of generations previously established.

### 3.4 NPGA

A Niche pareto genetic algorithm was presented in (Horn et al., 1994). This technique involves the addition of two specialized genetic operators: Pareto domination tournaments and fitness sharing. These operators allow for selection based on partial ordering of the population, as well as, to preserve diversity in the population.

Tournament selection is used to adjust selection pressure by changing the tournament size. Two candidates are chosen at random from the current population. A comparison set of  $tdom$  individuals is also chosen randomly. Each of the candidates are compared to each individual in the comparison set. If a candidate is dominated by the comparison set, and other is not, it loses the competition. If there are tournament ties, i.e. neither or both are dominated by the comparison set, the decision is based on the fitness sharing of individuals, using niche counts as calculated for the objective space in (Horn et al., 1994). Each candidate niche count is computed in the objective space, using its evaluated objective values. The candidate with lowest niche count wins the tournament. Tournaments are held until the next generation is filled. Then crossover and mutation operators are applied to the new population. As already said, in the case of a tie, the population density around each candidate is computed within a specified distance, known as the niche radius  $\sigma_{share}$ . The niche count for candidate  $i$  is given by:

$$m_i = \begin{cases} \sum_{j \in Pop} \left(1 - \frac{d_{i,j}}{\sigma_{share}}\right) & \text{if } d_{i,j} < \sigma_{share} \\ 0 & \text{if } d_{i,j} \geq \sigma_{share} \end{cases}, \quad (11)$$

The winner of the tied tournament is the competitor with the lowest niche count. As in (Horn et al., 1994), the fitness sharing is updated continuously, once the niche counts are calculated using individuals in the partially filled population of the next generation, rather than that of the current generation.

### 3.5 SPEA

The Strength Pareto Evolutionary Algorithms (SPEAs) was introduced in (Zitzler and Thiele, 1998) and an improved version, known as SPEA II is given in (Zitzler et al., 2001). In this algorithm, nondominated solutions are stored in an external set. The individuals are assigned according to the Pareto dominance concept. When the nondominated solutions exceeds a previously fixed size for the external set, the number of individuals in the external set is reduced by means of a truncation technique, as in (Zitzler et al., 2001). If the number of nondominated individuals is less than the predefined external set size, the external set is filled up by dominated individuals. The fitness assignment occurs in two different stages. The individuals are assigned by the strengths of its dominators in both the external set and the population. Strength represents the number of individuals in the population and in the external set covered by individual considered. The fitness of each individual is given by the sum of the strengths of its dominators in the external set and in the population. If individuals have equal fitness value, the density estimation technique, as given in SPEA2 (Zitzler et al., 2001), is used. This technique results from an adaptation of the  $k$ -th nearest neighbor method. The basic idea of the truncation procedure is to remove the individual which has the minimum distance to another individual. If there are several individuals with minimum distance, the individuals with second smallest distances to another individual are removed and so on. The SPEA-II approach proposed by (Zitzler et al., 2001) implements the following steps:

- Generate the initial population, decoding the individuals and evaluate the solutions and create the empty external Pareto-optimal;
- Compute fitness values of individuals in the population and in the external set;
- Copy nondominated individuals of the population to the external set;
- For each generation: Update the external set keeping only the nondominated solutions. When the number of nondominated solutions is higher than the specified size for the external set, it is reduced by applying the truncation technique. If the number of nondominated individuals is less than the external set size, the external set is filled up by dominated individuals;
- The mating pool is filled using binary tournament selection with replacement on the updated external set;

- After the recombination, of the mating pool, the crossover and mutation operators are applied and a new population is created;
- The algorithm stops when the maximum number of generations is reached.

### 3.6 BRKGA adapted to multiobjective UC optimization

We also use the ranking selection method for ordering the nondominated solutions according to the Pareto domination concept while the crowding distance is used to break ties by choosing the best individuals to be included in new population. For details about the BRKGA approach, the reader is referred to (Gonçalves and Resende, 2010; Roque et al., 2011). The initial population with size  $N_p$  is created by generating the random keys. Given population of chromosomes (random keys) a decoding procedure is applied such that at each chromosome corresponds a feasible UC solution, that is an output generation level matrix and the corresponding unit status matrix both satisfying the UC constraints. The fitness function used to evaluate the solutions includes both the total operational costs and CO<sub>2</sub> emissions. We adopt a fitness procedure similar to that of NSGA-II, given in (Deb et al., 2002). Therefore, the population is sorted based on the nondomination. Each solution is assigned a fitness (rank) equal to its nondomination level. The biased selection and biased crossover operators and the introduction of mutants are used to create a offspring population, also of size  $N_p$ . On the one hand, the biased selection ensures that one of the parents used for mating comes from a subset containing the best solutions of the current population. On the other hand, the biased crossover chooses with higher probability an allele from the best parent. Mutants are generated as the initially population and are introduced directly on the next generation.

We start by combining the current population with the newly obtained one. The combined population size is the double ( $2N_p$ ) of the current population and it is sorted by the nondomination criterion (Fast Nondominated Sorting Approach).

The nondomination criterion leads to several levels of nondominated fronts. For the first level, the nondominated individuals of the combined population are chosen. Second level, corresponds to a front containing individuals only dominated by the individuals of the first level front. All other levels are defined in a similar way, that is, in each level a front containing individuals dominated by all previous nondominated fronts is obtained. In order to obtain the new population we go through the generated fronts, in as-

ending order of level, and include all its individuals until we reach  $N_p$ . At the last nondominated front level to be included if only some of the individuals are to be chosen, the descending order of crowding distance is used as a selection criterium.

The multiobjective BRKGA flowchart is illustrated in Figure 1.

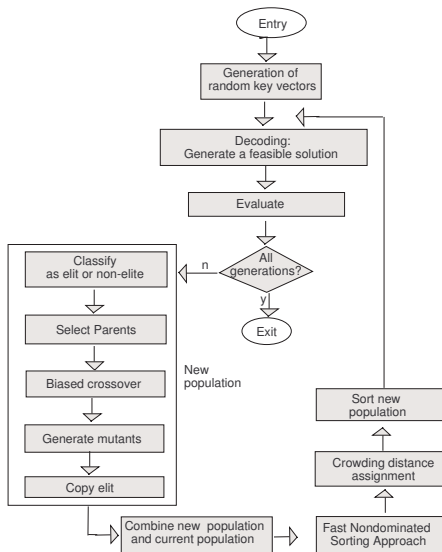


Figure 1: Flowchart of BRKGA multiobjective algorithm.

### 3.6.1 Genetic operators in BRKGA

**Biased Selection:** Pair of parents are selected from parent population. The parent population is divided into two sets: The elit set, comprising the best individuals, and the non-elit set, comprising the remaining individuals. One parent is selected from the elit set, while the other parent is chosen from the remaining, non-elite, individuals.

**Biased Crossover:** Given two parents and a specified probability of crossover, the crossover interchanges the genes or alleles to produce a new individual. As already mentioned, genes are chosen by using a biased uniform crossover, that is, for each gene a biased coin is tossed to decide on which parent the gene is taken from. This way, the offspring inherits the genes from the elite parent with higher probability (0.7 in our case).

**Mutants:** To ensure diversity and to avoid premature convergence, we introduce a percentage of new individuals, called mutants, in the population. These individuals are randomly generated as was the case for the initial population.

## 4 COMPUTATIONAL EXPERIMENTS AND RESULTS

### 4.1 GA parameters

#### 4.1.1 BRKGA Configuration

The BRKGA final parameter values were decided after some empirical experiments have been performed. The experimented values were chosen using the guidelines provided by (Gonçalves and Resende, 2010; Deb et al., 2002), as well as, the computational experiments in (Roque et al., 2011). The current population of solutions is evolved by the GA operators onto a new population as follows:

Elit set is formed by 20% of best solutions; 40% of the new population is obtained by introducing mutants; Finally, the remaining 60% of the population is obtained by biased reproduction, which is accomplished by having both a biased selection and a biased crossover. Moreover, we set the number of generations to 100 ( $10N$ ), the population size to  $40(4N)$  and the crossover probability to 0.7.

#### 4.1.2 SPEA, NSGA, and NPGA Configurations

The algorithms are implemented according to their description in the literature. The other operators (recombination, mutation, sampling) remain identical. To ensure the same conditions of application of the method BRKGA, identical population size of 40 and number of generations of 100 to BRKGA are used for each algorithm.

The NPGA, NSGA II, and SPEA2 parameters values are chosen using the guidelines proposed in (Deb et al., 2002). Some complementary computational experiments are performed, where other appropriate values of the GA parameters are arrived at based on the satisfactory performance of trials conducted for this application with different range of values. For NPGA, the niche radius  $\sigma_{share} = 0.1$  was chosen as in (Horn et al., 1994) and several computational experiments were made in order to choose the size of the comparison set  $t_{dom}$ . The parameter ranges between 5% and 30%. The results obtained have shown a favorable value of  $t_{dom}$  to be 10%.

For NPGA and NSGA II real coding an intermediate crossover similar to Matlab crossover operator has been employed. The childs are obtained as  $Child_1 = Parent_1 + rand.ratio.(Parent_2 - Parent_1)$  and  $Child_2 = Parent_2 - rand.ratio.(Parent_2 - Parent_1)$  where  $rand$  is random number in the interval  $[0, 1]$ , the ratio crossover was set 1.2 and the crossover probability to 0.8. The Gaussian mutation is used as in Matlab

Toolbox Optimization with  $scale = 0.1, shrink = 0.5$ . The mutation rates has been set to 0.2.

For SPEA2, we use a population of size 40 and an external population of size 40, so that overall population size becomes 80. The uniform crossover and simulated binary crossover operators are applied with probability 0.7 and 0.9, respectively. For real-coded crossover, the probability distribution used in the simulated binary crossover operator has been set up distribution indice  $\eta_c$  of 5. Like in (Deb and Agrawal, 1995), we use the polynomial mutation described as follows: if  $x_i$  is the decision variable selected for mutation with a probability  $p_m$ , the result of the mutation is the new value  $x'_i$  obtained by a polynomial probability distribution  $P(\delta) = \frac{1}{2} \cdot (\eta_m + 1) (1 - |\delta|)$ .  $x_i^L$  and  $x_i^U$  are the lower and upper bound of  $x_i$ , respectively, and  $r_i$  is a random number in the interval  $[0, 1]$ . Hence, we have

$$x'_i = x_i + (x_i^U - x_i^L) \cdot \delta_i$$

with

$$\delta_i = \begin{cases} (2r_i)^{\frac{1}{\eta_m+1}} - 1 & \text{if } r_i < 0.5, \\ 1 - |2(1-r_i)|^{\frac{1}{\eta_m+1}} & \text{if } r_i \geq 0.5. \end{cases} \quad (12)$$

The distribution index  $\eta_m$  was set to 15 and the mutation probability to 0.1. Table 1 has the population size, the crossover and mutation probabilities, and the number of generations used in each approach.

Table 1: GA Parameters.

	BRKGA	NSGAII	NPGA	SPEA2
Population size	40	40	40	40
Crossover probability	0.7	0.8	0.8	0.9
Mutation probability		0.2	0.2	0.1
N. Generations	100	100	100	100

## 4.2 Case 1 results

Here, we present the results obtained for case study 1. The problem data is provided in Appendix A. The BRKGA has the most widely spread front, as it can be seen in Figure 2, and the average values of the coverage metric measure (Zitzler and Thiele, 1999), over 10 optimization runs, as shown in Table 2. We can observe that the nondominated solutions of BRKGA covers relatively higher percentages of the other solutions.

On the one hand, as can be seen in Table 2, on average the nondominated set achieved by BRKGA dominates about 67.3 % of the nondominated solutions found by NSGA II. However, the front obtained by NSGA II only dominates in less than 13.9 % of the nondominated solutions produced by BRKGA.

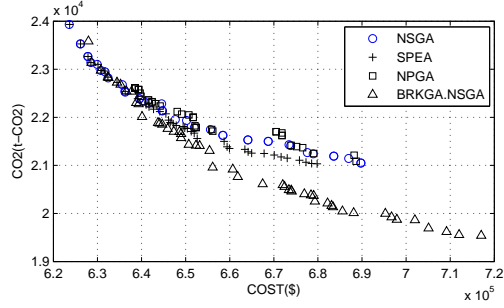


Figure 2: Pareto-optimal fronts obtained from different algorithms in a single run.

Table 2: Percentage of Nondominated Solutions of set B covered by those in set A.

set A / set B	BRKGA	NSGA II	NPGA	SPEA2
BRKGA		67.3	99.5	70.3
NSGA II	13.9		76	8.8
NPGA	0	10.3		0
SPEA2	13.9	61.3	98.8	

On the other hand, with regard to NPGA, a BRKGA front dominates on average 99.5% of the corresponding NPGA front, while the nondominated set produced by NPGA never dominates the front obtained by BRKGA. Finally, the nondominated set achieved by BRKGA dominates about 70.3% of the nondominated solutions found by SPEA2 while the front obtained by SPEA2 dominates only in less than 13.9%.

## 4.3 Case 2 results

In this section, we provide the results obtained for case study 2. For problem details see Appendix B and the reference therein. The BRKGA average values of the coverage metric measure over 10 optimization runs are showed in Table 3. We can observe that the nondominated solutions of SPEA2 and BRKGA covers relatively higher percentages of the other solutions.

Table 3: Percentage of Nondominated Solutions coverages.

set A / set B	BRKGA	NSGA II	NPGA	SPEA2
BRKGA		88.5	75	30.3
NSGA II	11		49	4
NPGA	22.3	40		10.8
SPEA2	84.8	98.5	92.5	

In Table 3, we can observe that, on average, the nondominated set achieved by BRKGA domi-

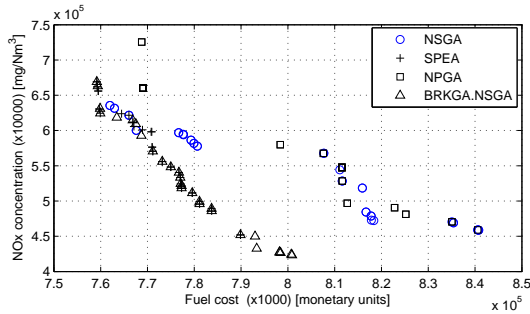


Figure 3: Pareto-optimal fronts obtained from from different algorithms in a single run.

brates 88.5% of the nondominated solutions found by NSGA II, while the front obtained by NSGA II dominates less than 11 % of the nondominated solutions produced by BRKGA. Moreover, the BRKGA front dominates on average 75% of the corresponding NPGA front while the nondominated set produced by NPGA dominates less than 22.3 % of the nondominated solutions produced by BRKGA. Finally, SPEA2 front dominates on average 84.8% of the corresponding BRKGA front while the nondominated set produced by BRKGA dominates 30.3 % of the nondominated SPEA2 solutions.

## 5 CONCLUSIONS

In this paper a new approach is used to find Pareto sets for multiobjective unit commitment problem. The proposed algorithm combines the biased selection and biased crossover of the BRKGA approach with non-dominated sorting procedure and crowded comparison operator used in NSGA II technique.

The algorithm maintains a finite-sized archive of nondominated solutions which gets iteratively updated in the presence of new solutions based on the concept of pareto dominance. The multiple Pareto optimal solutions can be found in one simulation run such as in other multiobjective techniques.

The proposed approach has been assessed through a comparative study, for two case study problems, with other multiobjective optimization techniques. The best results are obtained for SPEA2 and BRKGA approaches. The results shows that BRKGA can be an effective method for producing tradeoff curves. Tradeoff curves such as those presented here may give

decision makers the capability of making better decisions. Given that the approaches have similar decode procedures, the improvement in performance is most likely due to elitism. Elitism also guarantees that no good solutions are lost.

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## REFERENCES

- Abido, M. A. (2003a). Environmental/economic power dispatch using multiobjective evolutionary algorithms. *IEEE Trans. Power Syst.*, '18:1529–1537.
- Abido, M. A. (2003b). A niched pareto genetic algorithm for multiobjective environmental/economic dispatch. *Electr. Power Energy Syst.*, '25:97–105.
- Abido, M. A. (2003c). A novel multiobjective evolutionary algorithm for environmental/economic power dispatch. *Electr. Power Syst. Res.*, '65:71–81.
- Abido, M. A. (2006). Multiobjective evolutionary algorithms for electric power dispatch problem. *IEEE Transactions on Evolutionary Computation*, '10:315–329.
- Basu, M. (2008). Dynamic economic emission dispatch using non-dominated sorting genetic algorithm-ii. *Electric Power Energy System*, 30:140–149.
- Deb, K. and Agrawal, R. B. (1995). Simulated binary crossover for continuous search space. *Complex Systems*, 9:115–148.
- Deb, K., Pratab, A., Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE Trans. Evol. Comput.*, 6:182–197.
- Gonçalves, J. F. and Resende, M. G. C. (2010). Biased random-key genetic algorithms for combinatorial optimization. *Journal of Heuristics*, 17:487–525.
- Granelli, G. P., Montagna, M., Pasini, G. L., and Maranino, P. (1992). Emission constrained dynamic dispatch. *Electr. Power Syst. Res.*, '24:56–62.
- Horn, J., Nafpliotis, N., and Goldberg, D. E. (1994). A niched pareto genetic algorithm for multiobjective optimization. In *1st IEEE Conf. Evol. Comput., IEEE World Congr. Comput. Intell.*, volume 1, pages 67–72.
- Hsiao, Y. T., Chiang, H. D., Liu, C. C., and Chen, Y. L. (1994). A computer package for optimal multiobjective var planning in large scale power systems. *IEEE Trans. Power Syst.*, '9:668–676.
- Roque, L., Fontes, D. B. M. M., and Fontes, F. A. C. C. (2011). A biased random key genetic algorithm approach for unit commitment problem. *Lecture Notes in Computer Science*.



Sawaragi, Y., Nakayama, H., and Tanino, T. (1985). *Theory of multiobjective optimization*. Orlando: Academic Press.

Srinivas, N. and Deb, K. (1994). Multiobjective function optimization using nondominated sorting genetic algorithms. *Evol. Comput.*, 2:221–248.

Wang, S., Shahidehpour, M., Kirschen, D. S., Mokhtari, S., and Irissari, G. (1995). Short-term generation scheduling with transmission and environmental constraints using an augmented lagrangian relaxation. *IEEE Trans Power Systems*, 10:1294–300.

Winter, G., Greiner, D., Gonzalez, B., and Galvan, B. (2003). Economical and environmental electric power dispatch optimisation.

Yamashita, D., Niimura, T., Yokoyama, R., and Marmiroli, M. (2010). Pareto-optimal solutions for trade-off analysis of CO<sub>2</sub> vs. cost based on dp unit commitment.

Yamin, H. Y., El-Dwairi, Q., and Shaihidehpour, S. M. (2007). A new approach for genco profit based unit commitment in day-ahead competitive electricity markets considering reserve uncertainty. *Int J Elec Power Energy Systems*, 29:609–16.

Zitzler, E., Laumanns, M., and Thiele, L. (2001). Spea2: Improving the strength pareto evolutionary algorithm. *TIK-Rep.* 103.

Zitzler, E. and Thiele, L. (1998). An evolutionary algorithm for multiobjective optimization: The strength pareto approach. *TIK-Rep.*, 43.

Zitzler, E. and Thiele, L. (1999). Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach. *IEEE Trans. Evol. Comput.*, 3:257–271.

## APPENDIX A: DATA OF THE CASE STUDY 1

For more details see (Sawaragi et al., 1985; Yamashita et al., 2010).

Table 4: Generation constraints in case study 1.

Unit	$Y_{max,j}$ (MW)	$Y_{min,j}$ (MW)	$T_{on\ min,j}$ (h)	$T_{off\ min,j}$ (h)	Ramp rate (MW/h)
1	455	150	8	8	250
2	455	150	8	8	250
3	130	20	5	5	80
4	130	20	5	5	80
5	162	25	6	6	100
6	80	20	3	3	80
7	85	25	3	3	85
8	55	10	1	1	55
9	55	10	1	1	55
10	55	10	1	1	55

Table 5: Data fuel costs evaluation in case study 1.

Unit	$A_j$ (\$/MW <sup>2</sup> h)	$B_j$ (\$/MWh)	$C_j$ (\$/h)	startup cost (\$)
1	0.000528	17.809	1100	4950
2	0.000341	18.906	1067	5500
3	0.0022	18.26	770	605
4	0.002321	18.15	748	616
5	0.004378	21.67	495	990
6	0.007832	24.486	407	187
7	0.008069	30.514	528	286
8	0.004543	28.512	726	33
9	0.002442	29.997	731.5	33
10	0.001903	30.569	737	33

Table 6: Data fuel costs evaluation in case study 1.

Unit	$a_j$ ( $r-CO_2/MW^2h$ )	$b_j$ ( $r-CO_2/MWh$ )	$c_j$ ( $r-CO_2/h$ )	startup $CO_2$ ( $r-CO_2$ )
1	2.240E-05	0.7557	46.677	210.0
2	1.446E-05	0.8056	45.276	233.3
3	9.335E-05	0.7748	32.674	25.67
4	9.848E-05	0.7701	31.740	26.13
5	3.197E-05	0.1582	3.6157	7.231
6	5.720E-05	0.1788	2.9729	1.365
7	7.282E-05	0.2557	4.4248	2.396
8	3.807E-05	0.2389	6.0841	0.2765
9	2.046E-05	0.2513	6.1302	0.2765
10	1.594E-05	0.2561	6.1765	0.2765

Table 7: Load demand (MW) in case study 1.

Hour	Load demand (MW)	Hour	Load demand (MW)
1	700	13	1400
2	750	14	1300
3	850	15	1200
4	950	16	1050
5	1000	17	1000
6	1100	18	1100
7	1372	19	1200
8	1314	20	1400
9	1271	21	1300
10	1400	22	1100
11	1450	23	900
12	1500	24	800

## APPENDIX B: DATA OF THE CASE STUDY 2.

For more details see (Winter et al., 2003).

Table 8: Data fuel costs evaluation in case study 2.

Unit	$Y_{max,j}$ (MW)	$T_{on\ min,j}$ (h)	$T_{off\ min,j}$ (h)	$I_s$ (h)	$A_j$ ( $m.u./MW^2$ )	$B_j$ ( $m.u./MW$ )	$C_j$ ( $m.u.$ )
1	320	8	4	-5	0.0085	19.566	4437.2
2	320	5	2	-6	0.0050	20.927	1044.20
3	280	5	2	3	0.0253	18.995	1236.9
4	200	5	2	-3	0.0091	23.107	416.58
5	150	5	3	-7	0.0106	20.765	485.69
6	150	4	2	3	0.0116	22.251	300.86
7	120	4	2	5	0.0212	15.031	315.44
8	100	4	2	1	0.0254	15.031	262.87
9	80	3	1	-1	0.0356	10.375	222.16
10	60	3	1	-1	0.0454	9.9214	159.33

Table 9: Start-up costs, shut down costs and  $NO_x$  emissions coefficients in case study 2.

Unit	$a_j$ ( $m.u.$ )	$b_j$ ( $m.u.$ )	$c_j$ ( $m.u.$ )	$SD_j$ ( $m.u.$ )	$D_j$	$E_j$	$F_j$
1	267	34.75	0.09	75	-0.245	154.16	-1154.6
2	187	38.62	0.13	70	-0.002	16.414	-691.1
3	176	27.57	0.15	42	-0.069	36.931	-1626
4	227	26.64	0.11	62	0.1313	-20.77	1885.6
5	113	18.64	0.18	29	-0.005	16.287	-321.4
6	282	45.48	0.09	49	0.1686	-20.0	1361.8
7	94	10.65	0.18	32	0.016	1.7774	276.59
8	114	22.57	0.20	40	0.0193	1.7774	230.49
9	101	20.59	0.20	25	-1.793	246.71	-2636
10	85	20.59	0.20	15	-2.286	235.92	-1890

Table 10: Load demand (MW) in case study 2.

Hour	Load demand (MW)	Hour	Load demand (MW)
1	1459	13	1154
2	1372	14	1138
3	1299	15	1124
4	1280	16	1095
5	1271	17	1066
6	1314	18	1037
7	1372	19	993
8	1314	20	978
9	1271	21	963
10	1242	22	1022
11	1197	23	1081
12	1182	24	1459