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USING CIRCLE COVERING TO TACKLE NESTING REPRESENTATIONS LIMITATIONS

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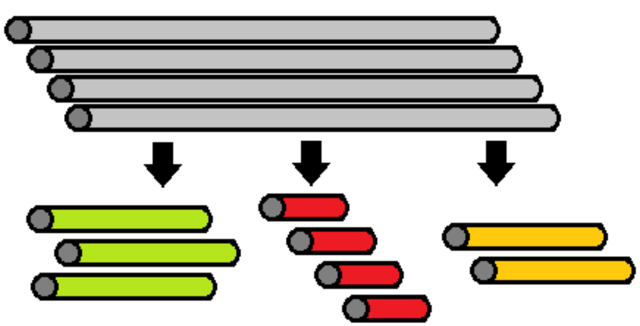
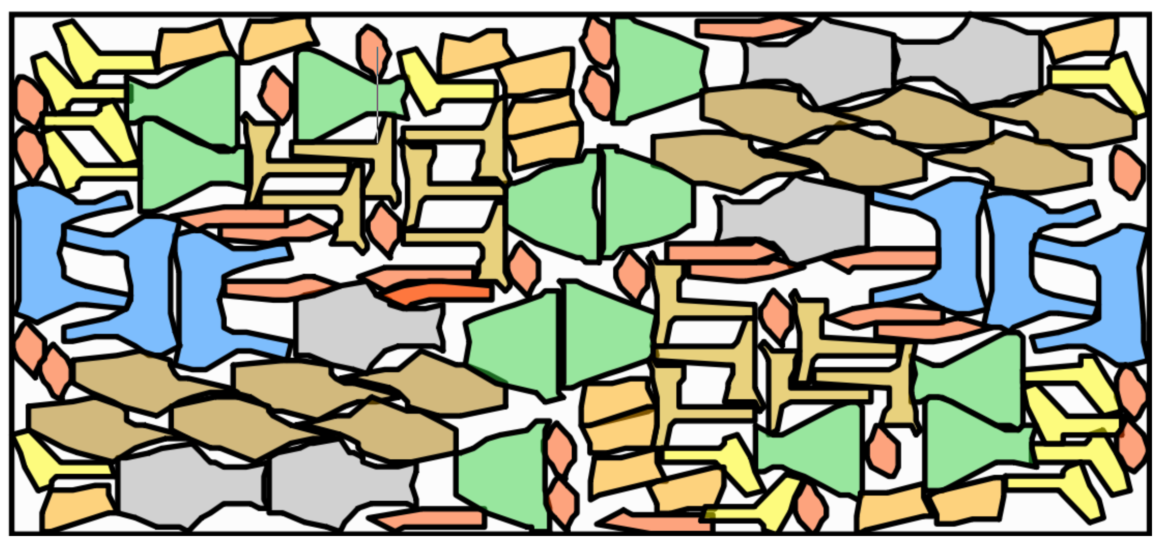
Motivation

Cutting and Packing Problem

Efficient cutting of raw material in small pieces is a complex and important task

Strong impact in industrial production costs (energy, raw material savings, environmental benefits)





Cutting and Packing Problem

- Aims to find a good fit to minimize wasted space
- Hard combinatorial and geometric problem
- Pieces are cut from, or placed inside a set of larger bins, in a non-overlapping configuration
- If pieces have irregular outlines it is defined as a Nesting problem

Nesting Problem

- Also known as Irregular Shapes Placement Problem
- Characteristics:
  - 2D problem
  - One big item, may have defects
  - High number of pieces to place
  - Great diversity on the size of the pieces
  - No overlap between pieces
  - Complex shapes (multi-connected regions, curves, …)
  - Continuous and/or discrete admissible orientations

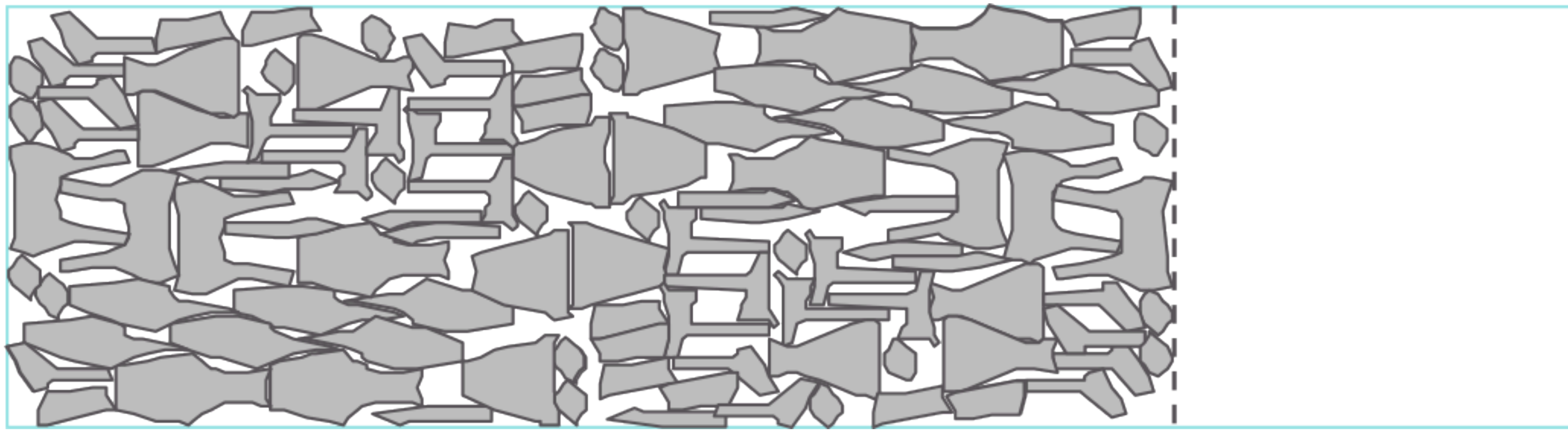
Industrial Applications

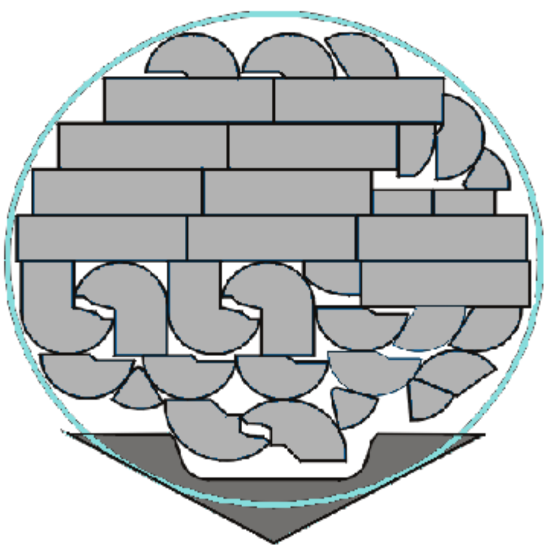

Garment

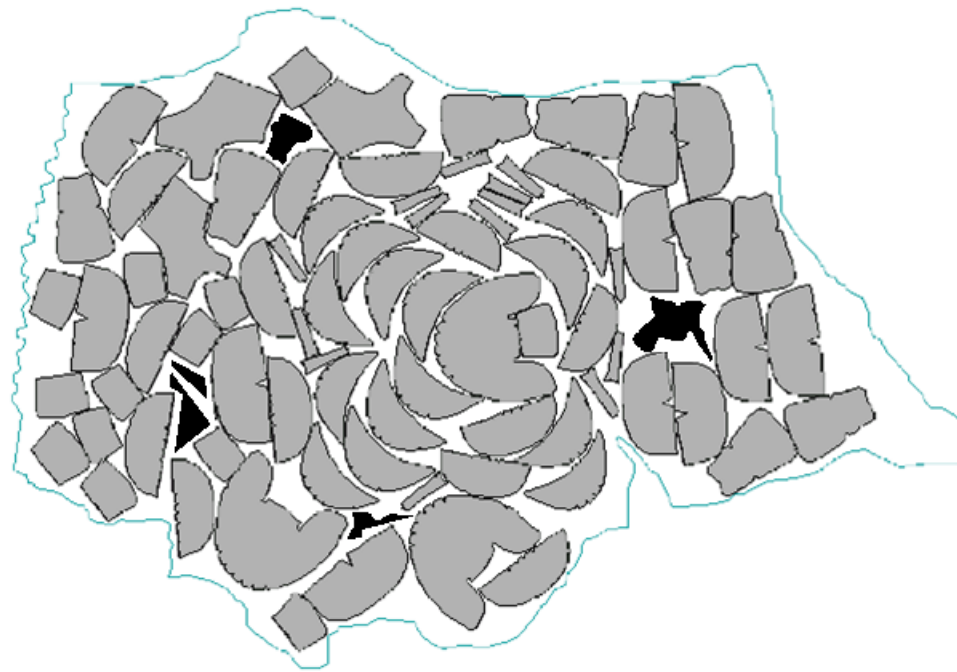

Footware


Metal sheet cutting


Furniture







Main Challenges

Nesting Challenges

- Obtain adequate representations for the nesting problem
- Achieve faster and more efficient solutions
- Efficiently represent the relative positions between pieces

Geometrical Challenges

- Efficiently represent non-rectilinear outlines
- Deal with free rotations

- Lack of solutions limit geometric tools
- Other problems with similar challenges (ex. collision detection in games/physics engine based simulations)

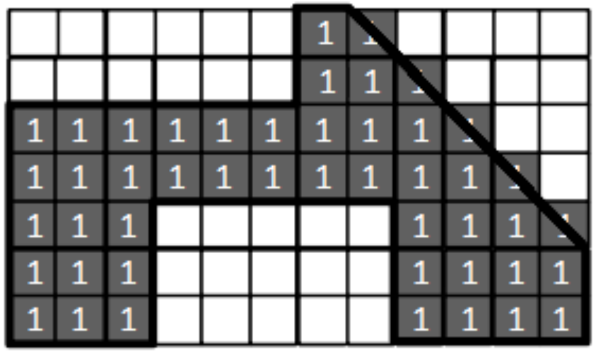
Geometrical Representations

Discrete Representations

Grid

Representation through discretization of geometrical outline (Bin/ Pieces)

- How is it used?
  - Pieces coded into a matrix, 0 = empty, 1 = non-empty
  - Overlap verification done though analysis of each discretized element
  - Pieces are placed on a discretized bin
    - if (**element >= 1**) then **Overlap!!**



- Advantages / disadvantages:
  - Easy to check feasibility of layout
  - Only 90° rotations
  - Can represent any outline
  - Approximation errors (curves, non-orthogonal segments)

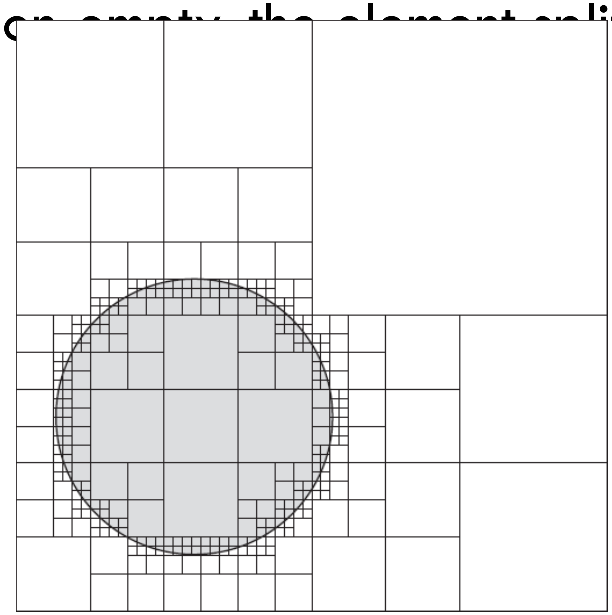
Adequate for integer sized elements & orthogonal orientations

Discrete Representations

Quad-Tree

Tree data structure where each node has 4 children, used to organize and access spatial information

- How is it used?
  - Irregular outlines are decomposed in non-uniform elements
  - When an element is both empty and non-empty, the element splits

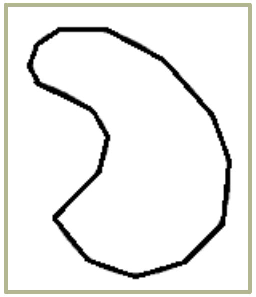


- Advantages / disadvantages:
  - Very fast searches (overlap)
  - Allows dynamic discretization
  - Less memory consumption
  - Similar to grids

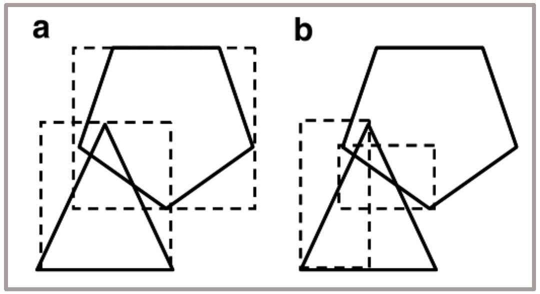
Polygonal Representations

Polygons

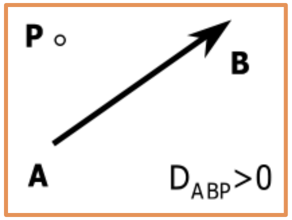
Closed circuit of straight segments



- How is it used?
  - Shape is represented through straight segments
  - Bounding boxes are used for initial overlap detection
  - D-functions used for direct polygonal comparison



- Advantages / disadvantages:
  - Overlap detection computationally expensive
  - Rotations possible but not efficient
  - Numerical precision problems
  - Curves approximated by straight lines, tangent to the curve
  - Approximation error is controlled

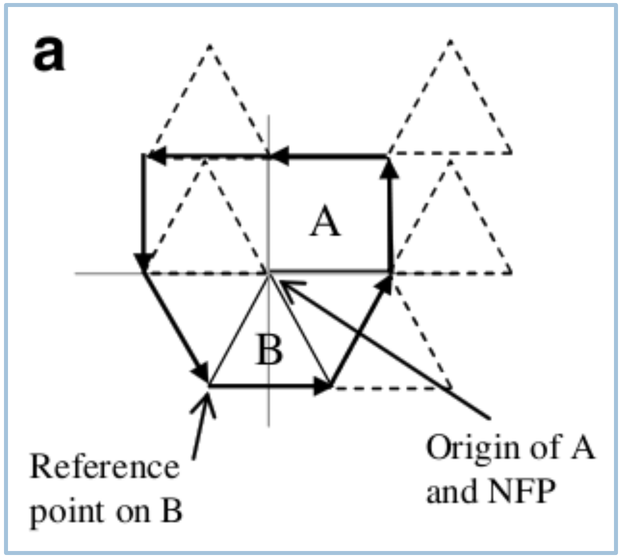


Polygonal Representations

No-Fit-Polygon

Points traced by a reference point from an orbital piece, with fixed orientation, while sliding along the external contour of a static piece

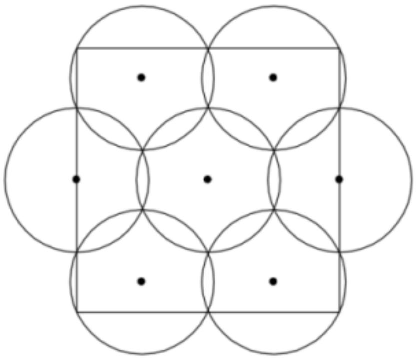
- How is it used?
  - Compares a vertex with a polygon
    - If (**vertex inside of polygon**) then **Overlap**!
- Advantages / disadvantages:
  - Allows faster overlap detection
  - Numerical precision problems
  - Rotations are computationally expensive
  - Discrete rotations
  - Pre-computation



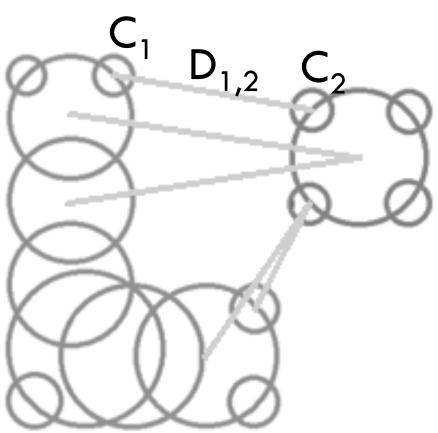
Circle Covering Representations

Circle Covering

Set of identical/non-identical circles that fully or partially cover an irregular outline



- How is it used?
  - Pieces are replaced by sets of circles
  - Mathematical models only for identical circles
  - Overlap detection → distance between circles
    - If ($\mathbf{R_1+R_2<D_{1,2}}$) then **Overlap**!

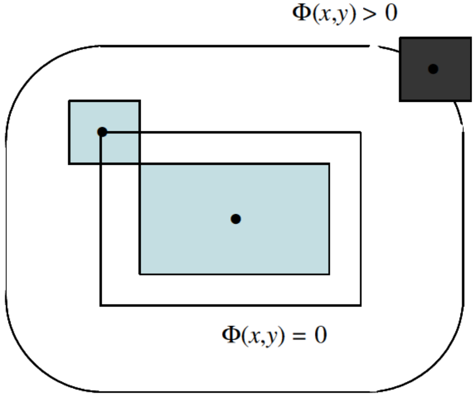


- Advantages / disadvantages:
  - Overlap detection is simple and fast
  - Needs circle positioning method
  - Continuous rotations are trivial
  - Numerical precision problems

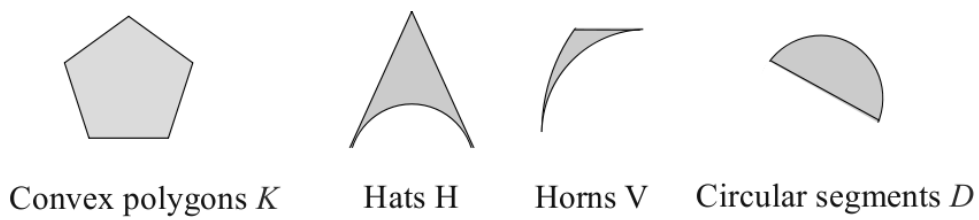
Phi-Functions

Phi-Functions

Mathematical expression that represents all mutual positions between two objects



- How is it used?
  - The function returns a value
    - If (**value is negative**) then **Overlap**!
- Advantages / disadvantages:
  - Complex objects are decomposed into basic shapes:



  - All basic elements have as their primitives:
    - Circles, triangle, rectangle, regular and other convex polygons, and their complements
  - Shapes represented by unions and intersections of functions

Circle Covering Approach

Circle Covering Approach

- Collision detection with circles:
  - Games and Physical Engine Based Simulations
    - Computation speed is priority
    - Approximation error secondary
    - Fixed sets of shapes → Circles are manually placed
  - Nesting
    - Higher precision (smaller approximation error)
    - Involves much contact between pieces
    - Variable sets of shapes → Manual placement not viable
    - Good automatic method is needed

- How to position the circles?
- How to deal with the tradeoff:
  - Number of circles
  - Approximation error

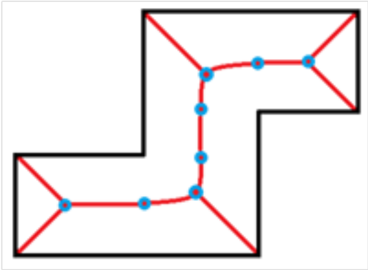
→ Automatic Circle Covering Method

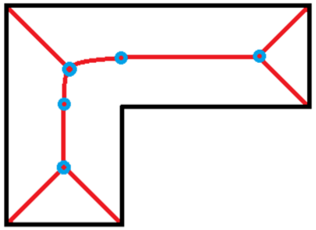
Medial Axis

Topological Skeleton

- Set of all points having more than one closest point on the objects boundary
- Any circle, with its center placed on the skeleton, will always be the biggest circle inside it

- How is it used?
  - Defines the equidistant axis to the outline of the shape
  - The biggest circles can be placed on the skeleton



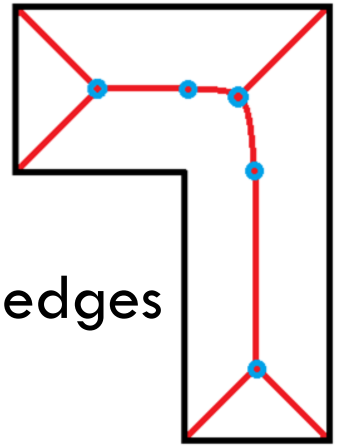


- Advantages / disadvantages:
  - Reduces complexity of circle placement problem
  - Numerical precision problems

Medial Axis Construction

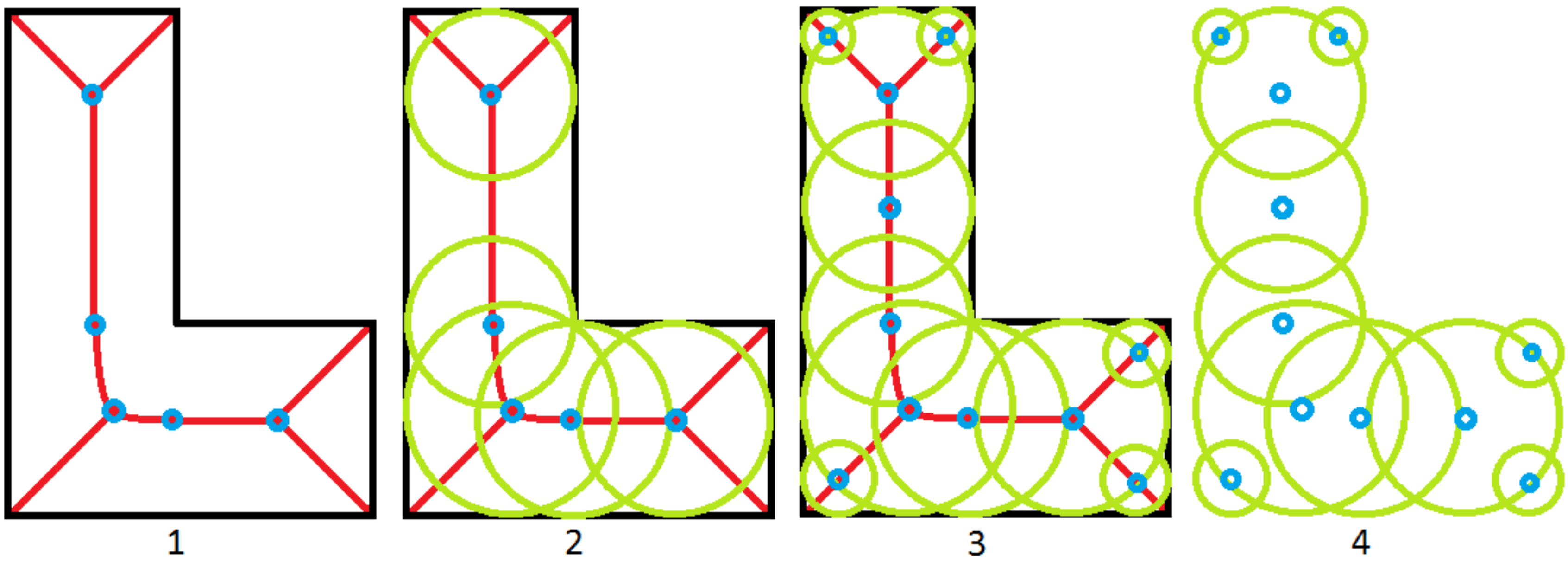
- How is it constructed?
  - Bissection from every pair of sequential segments are intersected, creating a new spawn point, that is the base for a new bissection
  - Iteratively repeat previous step, until no more bissections remain

- Bissection types:
  - Two straight edges → Straight bissection
  - Straight edge + vertex → Arc of parabola
  - Two vertexes → Straight bissection



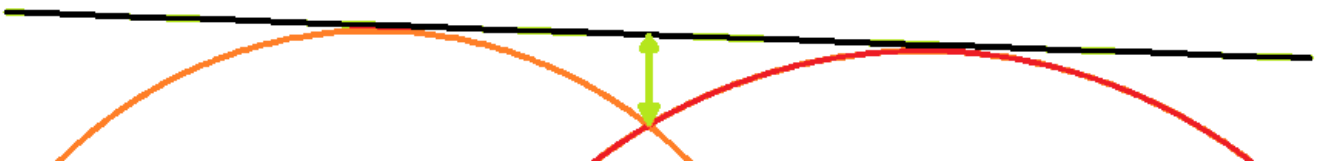
- Convex outline → Skeleton with straight edges
- Irregular outline → Skeletons with curves

Circle Covering Construction

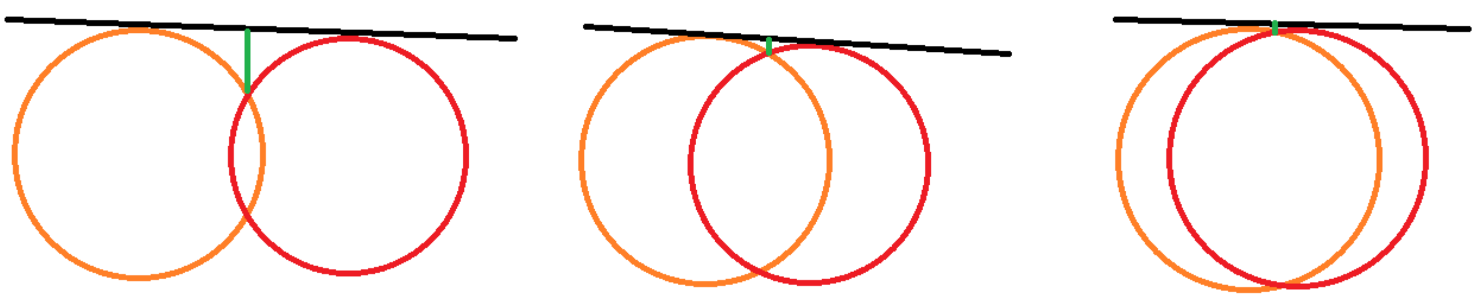


Circle Covering Approach

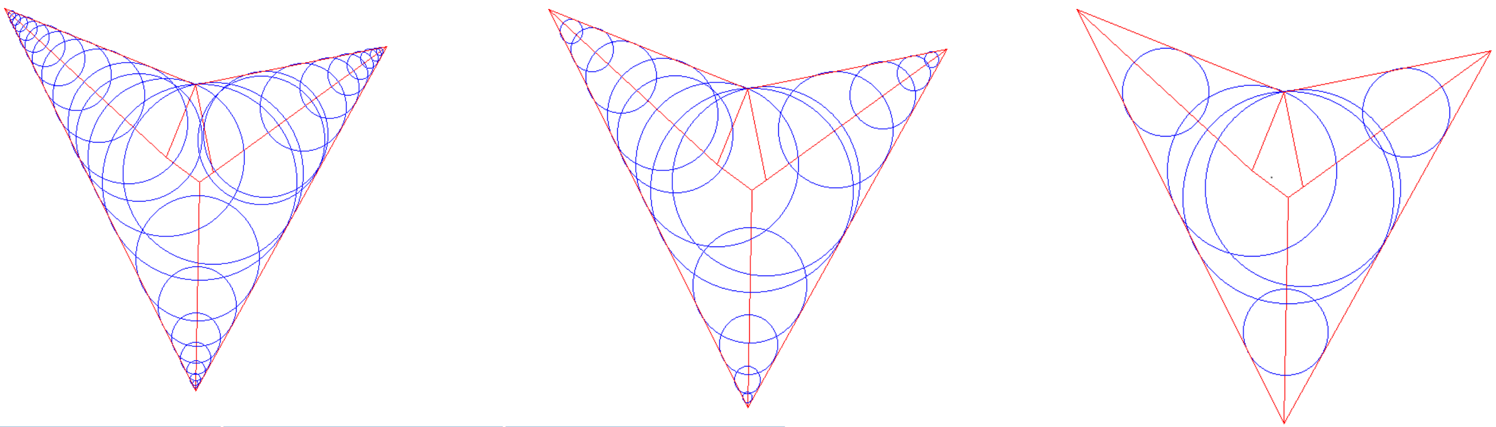
- How is the approximation controlled?
  - Error is controled by a threshold, which regulates the approximation to the shape outline



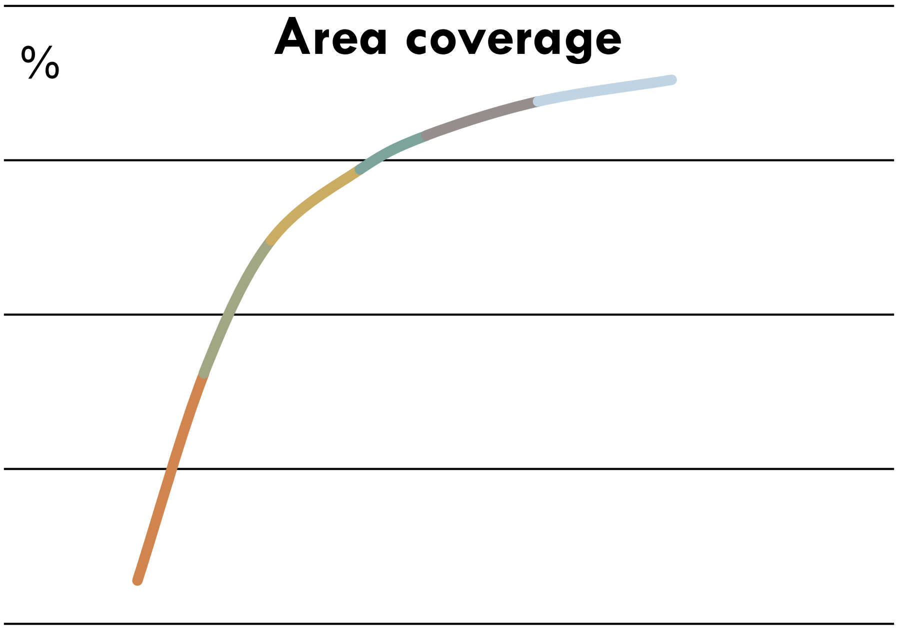
  - Lower threshold = better approximation = less error = more circles



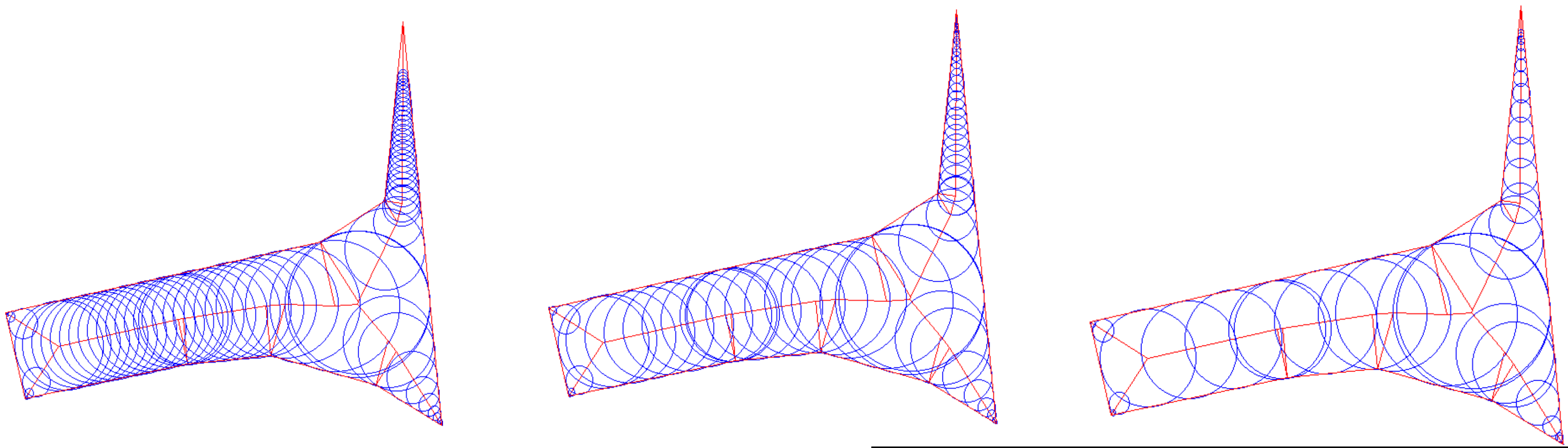
Current Results



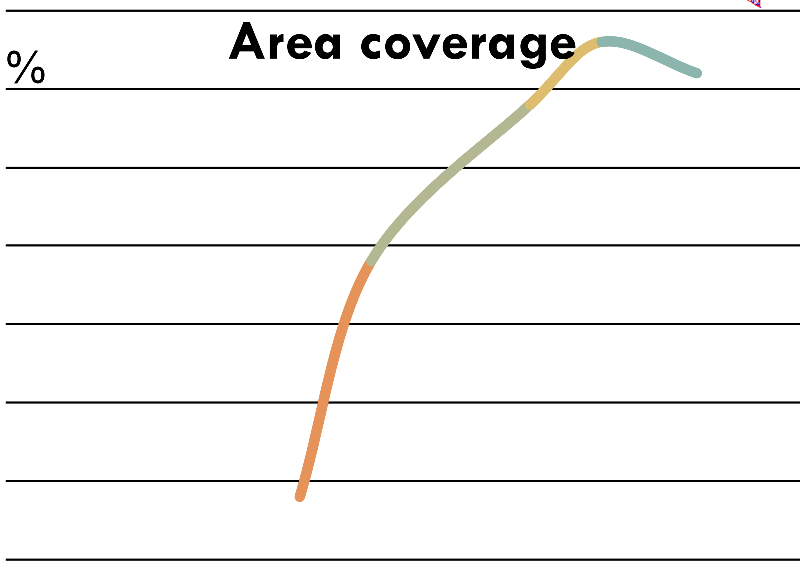
Threshold	Circles	Area (%)
0,75	6	81,4
0,50	9	88,1
0,25	12	92,4
0,15	16	94,7
0,10	19	95,8
0,05	24	96,9
0,01	30	97,6



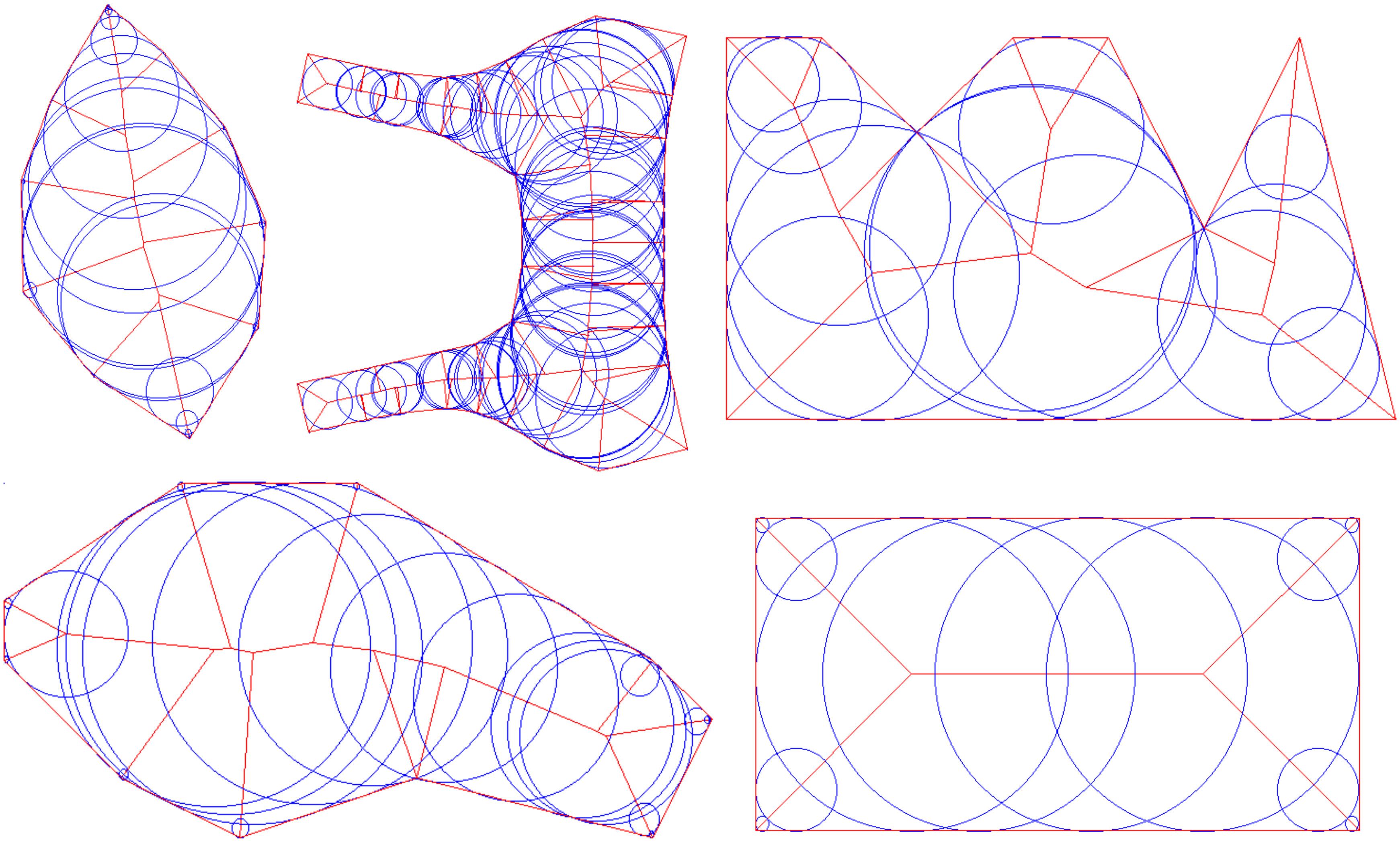
Current Results



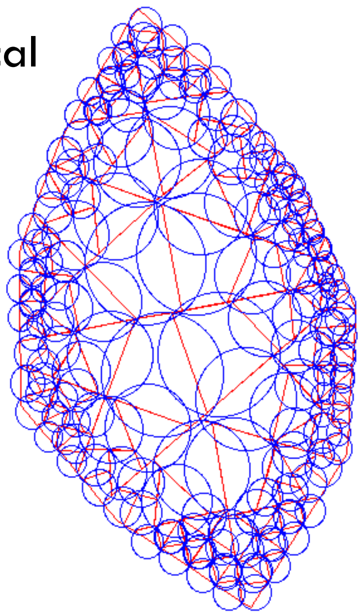
Threshold	Circles	Area (%)
10	37	95,4
5	46	96,9
2	66	97,9
1	75	98,3
0,5	87	98,1

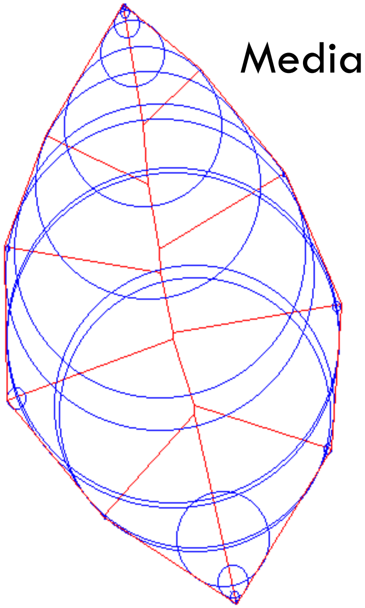


Other Shapes



Improvements



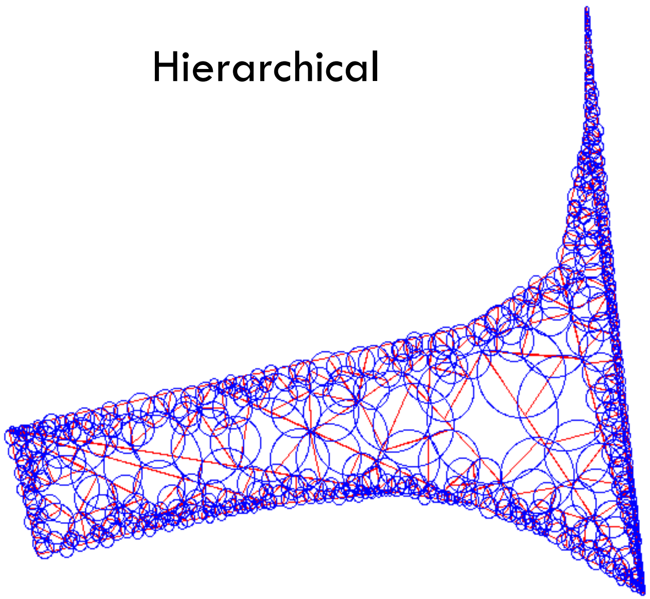


Threshold %	Circles	Area (%)
20%	52	85,4
10%	133	92,2
5%	299	96,1
2%	594	98,1

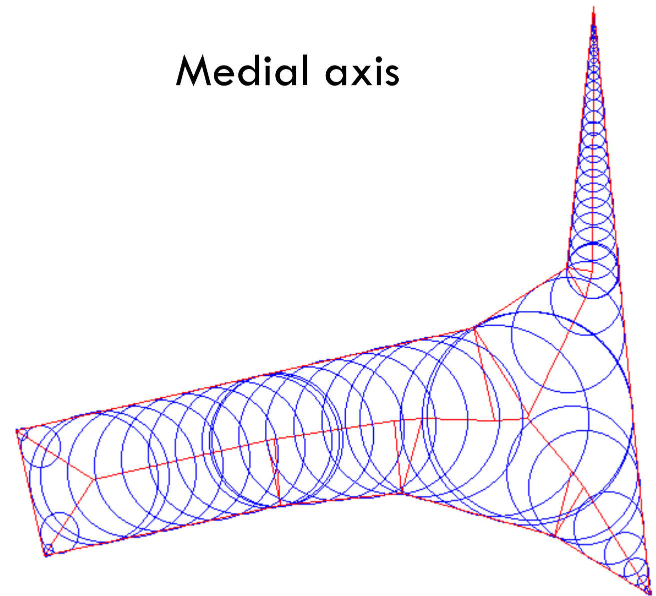
Threshold	Circles	Area (%)
25	15	97,1
2	24	98,2

Improvements

Hierarchical



Medial axis



Threshold %	Circles	Area (%)
20%	124	81,1
10%	351	92,3
5%	554	95,5
2%	1029	98,2

Threshold	Circles	Area (%)
10	37	95,4
2	66	97,9

Final Remarks

- Substantial improvement over previous hierarchical method
- Circle covering with medial axis useful for polygon representation
- Numerical precision problems cause many difficulties

Future Work

- Increase reliability to numerical errors
- Expand to deal with holes
- Support geometric outlines with curves
- Compare with other C.D. approaches