Synchronisation in vehicle routing: classification schema, modelling framework and literature review

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Abstract

The practical relevance and challenging nature of the Vehicle Routing Problem (VRP) have motivated the Operations Research community to consider different practical requirements and problem variants throughout the years. However, businesses still face increasingly specific and complex transportation requirements that need to be tackled, one of them being synchronisation. No literature contextualises synchronisation among other types of problem aspects of the VRP, increasing ambiguity in the nomenclature used by the community. The contributions of this paper originate from a literature review and are threefold. First, new conceptual and classification schemas are proposed to analyse literature and re-organise different interdependencies that arise in routing decisions. Secondly, a modelling framework is presented based on the proposed schemas. Finally, an extensive literature review identifies future research gaps and opportunities in the field of VRPs with synchronisation.

Keywords: routing, synchronisation, literature review, classification schema, mathematical programming

Highlights:

- Synchronisation in vehicle routing is defined through a classification schema.
- A general modelling framework is provided for routing problems with synchronisation.
- A literature review on the topic of synchronisation in vehicle routing is performed.

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1 Introduction

Businesses face increasingly fierce competition where lead times and flexibility are crucial factors for customer satisfaction. In this context, transportation processes need to account for increasingly specific and complex requirements, which may be achieved by maintaining transportation processes *as-is* and investing in additional resources (e.g., inventory, vehicles) to buffer the impacts. However, this approach can be economically unsustainable for some business sectors, thus triggering the need for alternative approaches that "squeeze" the most out of the available resources. Furthermore, new practical settings for routing problems have been emerging throughout the years where the specialisation of resources, vehicles or staff becomes a key and unavoidable factor in planning processes (e.g., Paraskevopoulos, Laporte, Repoussis, & Tarantilis, 2017).

Synchronisation of vehicles constitutes a clear and practical opportunity for these operational realities. It enables the coordination of vehicles for specific routing tasks that are dependent on other vehicles. In complex supply chains, where multiple vehicles, crews, materials and other resources are involved, synchronisation can be a catalyst for more efficiently combining different operations that require different resources by decreasing unproductive times. This can eventually result in a decrease in the overall resources required.

The topic of synchronisation in vehicle routing was first approached systematically by Drexl (2012), proposing a classification schema of synchronisation aspects where interdependencies arise in the routing decisions concerning two or more different vehicles. The survey characterised the VRP with Multiple Synchronisation Constraints (VRPMS) as a VRP where routes were dependent on one another, as opposed to standard VRPs. Consequently, a change in one route may affect other routes, which, in a worst-case scenario, may render all other routes infeasible. The classification schema of Drexl (2012) was built upon a specific case study of the VRP with Trailers and Transhipments (VRPTT). Through induction, five main types of synchronisation were proposed: task, operation, movement, load and resource synchronisation.

Despite the systematisation provided by Drexl (2012), the concept of synchronisation is still used in a broad manner (e.g., Bolduc, Renaud, & Montreuil, 2006; Gschwind, 2015; Russell, Chiang, & Zepeda, 2008), beyond the scope of interdependence between different routes. There is a lack of consensus on the different types of synchronisation one can find in a VRP.

The ambiguity behind the concept of synchronisation in VRPs constitutes a major obstacle to consolidating knowledge in this field. In the opinion of the authors, the concept of synchronisation in vehicle routing is not as broad as the literature on the topic would lead it to believe. Considering the concept of route interdependency that is intrinsic to this concept, this research considers that there are, at most, two distinct types of synchronisation: operation synchronisation and movement synchronisation. Operation synchronisation refers to the temporal synchronisation of tasks within given time offsets to perform an operation correctly. Movement synchronisation refers to the need to have a vehicle move from one task to another when another vehicle does the same. Despite limiting synchronisation to only these two types, this does not mean that synchronisation is isolated and not impacted by other problem requirements that a given problem may have to consider. As previously stated by Drexl (2012), synchronisation typically requires additional developments, both in terms of modelling as well as in terms of solution algorithms (e.g., Grimault, Bostel, & Lehuédé, 2017). These additional requirements may be significantly impacted by the interaction of synchronisation with other common problem requirements, which makes it relevant to provide a study of the most common routing problem aspects that are integrated with synchronisation.

To that effect, the purpose of this paper consists in answering the following research questions:

1. How can one define synchronisation in the Vehicle Routing Problem and distinguish it from

other problem aspects?

2. What are the upcoming trends and research opportunities in the field of VRPs that contain synchronisation aspects?

All of the scientific output of this paper is grounded on a literature review. The survey performed by Drexl (2012) provided a stepping stone toward this goal, which the authors intend to contribute further.

The first research question of this paper intends to re-focus the concept of synchronisation on what derives from the definition from Drexl (2012): routes are interdependent, i.e., vehicles are not independent of each other – consequently, changes in a route may affect the feasibility of other routes. To that effect, we build on the concepts of Drexl (2012). We provide a simple mathematical formulation for the VRP with Synchronisation where the different types of synchronisation are modelled and mathematically defined. Afterwards, to define boundaries between synchronisation and non-synchronisation aspects, a classification schema for common routing aspects is proposed, which builds on existing perspectives in the literature, such as the classification schema of Irnich, Toth, and Vigo (2014) for common VRP aspects such as "local" or "global" constraints, as well as the categorisation of synchronisation aspects provided by Drexl (2012). This clear separation of synchronisation aspects from other problem aspects is the main distinctive factor of our classification schema, which, to the best of our knowledge, is not present in any other classification schema in the literature. Leveraging this paper's classification schema, the mathematical formulation is also extended to a more general VRP with Synchronisation where interactions between synchronisation and other common problem aspects can be modelled and explored.

The second research question intends to update the state of the art of this research stream. Using the previously proposed classification schema for classifying different problem aspects in the literature, relevant publications considering routing problems with synchronisation are organised, taking into account their similarities. Furthermore, we provide an overview of adopted solution approaches for these problems, among other relevant aspects and research gaps. From the performed literature review, opportunities and trends are identified, which may be explored by the research community in the future.

The remainder of this paper is organised as follows. Section 2 consists of a problem statement section for the Vehicle Routing Problem with Synchronisation by defining the concept of synchronisation and presenting a mathematical formulation for this problem. Section 3 presents a classification schema for classifying different problem aspects in VRPs, focusing on the classification of synchronisation aspects. Section 4 describes the conducted literature review and presents its main results on the topic of VRPs with synchronisation based on the previously presented schema. Finally, Section 5 provides a general discussion of the results obtained in this paper and suggests possible research avenues for future work.

2 The Vehicle Routing Problem with Synchronisation

This section states the scope of the concept of synchronisation in vehicle routing by presenting a mathematical definition of the problem. It also serves as a glossary for the nomenclature that is later used throughout the paper.

2.1 Essential concepts

Due to the additional complexity that VRPs with Synchronisation entail compared to a standard VRP in terms of problem formulation, we start by defining essential concepts which will be used throughout this paper, presented in Table 1.

Table 1: Defining and exemplifying essential concepts

Concept	**Definition**	**Examples**
Locations	Real-world sites that exist for performing tasks.	Depot; Customer A; Pickup Location B; Delivery Location C
Tasks	Duties, usually mandatory, that must be done at a given location.	Finishing route at the depot; Delivering load to customer A; Picking up load at location B; Delivering load at location C
Operations	Associations of two tasks.	Pickup and delivery request (p, d); Task pair (i, j) subject to synchronisation
Vehicles	Real-world entities whose purpose is to perform routes.	Trucks; Staff members; Machines
Routes	Ordered sequences of tasks that need to be performed by a vehicle.	–
Resources	Entities of limited availability, available for consumption by one or more vehicles in order for them to fulfil tasks.	Demand of a customer; Vehicle capacity; Limited route length

In the context of this problem, the distinction between locations and tasks is of the utmost importance. In VRPs with synchronisation, synchronisation aspects may require locations to have more than one task associated with it, meaning that these terms cannot be used interchangeably.

Distinguishing tasks from operations is also relevant. Operations are an abstraction of the task concept, used to link different tasks that are dependent in some form. By our definition, any task being performed by more than one vehicle is not a task but instead an operation composed of tasks. Also, any association of more than two tasks is not, by our definition, an operation; however, these associations can be split into two or more operations with two tasks each.

2.2 Synchronisation in vehicle routing

In the context of vehicle routing, synchronisation concerns problem aspects that may impact the feasibility of routes other than the one that is being changed. They trigger an *interdependency between tasks of different routes*. In the event that a given route is changed by any means, these problem aspects may impact not only the feasibility of the route being changed but also of other routes that are "linked" with it.

Considering this definition, synchronisation aspects can be summarised in two main categories, based on the nomenclature proposed by Drexl (2012):

- *Operation synchronisation* – refers to the interdependencies between tasks of different routes that need to be performed within some temporal offset;
- *Movement synchronisation* – refers to the interdependencies between sequences of tasks among different routes.

2.2.1 Operation synchronisation

The purpose of operation synchronisation is to ensure that certain tasks, being temporally synchronised in different routes, are performed within given time limits.

It can be formally defined as follows:

Consider an operation (a, b). If its composing tasks a and b are being performed by two different routes k and k', respectively, then tasks a and b must be performed within a given temporal offset.

Operation synchronisation is useful for operations that require more than one vehicle to visit a certain customer to accomplish it successfully.

A problem that acknowledges operations synchronisation is the Home Health Care Routing and Scheduling Problem (HHCRSP) (e.g., Bredström & Rönnqvist, 2008; Mankowska, Meisel, & Bierwirth, 2013). Other typical applications concern aircraft scheduling or vessel scheduling at ports (e.g., Bakkehaug, Rakke, Fagerholt, & Laporte, 2016; Bélanger, Desaulniers, Soumis, & Desrosiers, 2006).

2.2.2 Movement synchronisation

Movement synchronisation concerns the inter-route dependency between two vehicles whose task sequences need to be synchronised. Movement synchronisation is a frequent requirement with routing problems acknowledging vehicles of different types, namely vehicles that cannot move autonomously and vehicles that need to transport these non-autonomous vehicles (Drexl, 2012). It can be formally defined as follows:

Consider an operation (i, j), composed of tasks i and j, henceforth called passive operation. Let us also consider a set of n operations $\{(a_1, b_1), ..., (a_n, b_n)\}$, henceforth called active operations. If a route performs task j immediately after task i, then there must exist at least one route that performs task b immediately after task a, with (a, b) belonging to the set of active operations.

Typically, movement synchronisation occurs between passive and active operations. A passive operation is one that requires that an active operation be performed in order for it to be performed as well. In turn, an active operation is one that may be accompanied or not by a passive operation. However, movement synchronisation can also occur only between passive operations, meaning that both operations require the other one for them to be both performed. Movement synchronisation between passive operations can be viewed as both movement synchronisation between active operations and passive operations applied reciprocally.

The concepts of active and passive operations can be easily translated into different problem variants from the literature, although usually, they apply these concepts to the vehicles themselves rather than the tasks being performed, as explained below. For the sake of this schema, vehicles are not classified as called active or passive. In fact, a vehicle could exhibit both active and/or passive behaviour at different points of its route. Instead, the active/passive attribute should be associated with operations, depending on the movement synchronisation constraints being enforced.

The most typical problem variants with movement synchronisation are the Truck and Trailer Routing Problem (TTRP) (e.g., Chao, 2002; Parragh & Cordeau, 2017) and the Active-Passive VRP (e.g., Meisel & Kopfer, 2012; Tilk, Bianchessi, Drexl, Irnich, & Meisel, 2018).

The classification schema of Drexl (2012) defined movement synchronisation as the simultaneous traversal of arcs, which maps into our proposed schema through both movement and operation synchronisation. In most real-life applications, movement synchronisation typically implies that operation synchronisation is also performed, as it is necessary that the arcs be traversed simultaneously. However, in a general context, this may not always be the case.

2.3 Mathematical formulation

The VRP with Synchronisation is now characterised by means of a mixed integer linear program.

2.3.1 Assumptions

The mathematical formulation that is henceforth presented consists of a simplified version containing the essential constraints of a routing problem. To that effect, several assumptions were considered, which are described ahead.

We consider a generic vehicle routing problem base formulation, whose purpose is to perform a set of n tasks, geographically dispersed through m locations, by means of a set of routes being performed by a set of K vehicles. The routes start and end in a depot through tasks 0 and $n+1$, respectively.

Especially when modelling synchronisation aspects in the same location, the problem's transportation network may not necessarily correspond to a real-life transportation network. However, for the sake of simplicity, this formulation considers that each location has one single task. We will continue with the nomenclature adopted in the previous section. Therefore we will use the term "tasks" to refer to the vertices of the transportation network and use the term "locations" to refer to real-world locations.

In this formulation, it is also assumed that each vehicle performs only one route, and therefore these concepts may be used interchangeably. Furthermore, there are no optional tasks, meaning that all tasks must be performed, and any vehicle is able to perform a given task. No additional problem requirements are considered, such as capacity or time window restrictions.

An extended mathematical formulation for the VRP with Synchronisation is presented in Appendix A of this paper, where most relevant assumptions are removed, and synchronisation is modelled in conjunction with other common problem requirements. The Appendix also instantiates the proposed framework to common routing problems with synchronisation.

2.3.2 Sets, parameters, decision variables and objective function

The sets and parameters of the problem are enumerated in Table 2.

The transportation network of the problem, $\mathcal{G} = (\mathcal{N}_0, \mathcal{A})$ consists of a directed and incomplete graph of tasks, whose set of arcs can be defined as in Equation (1).

$$\mathcal{A} = \{(i,j) \in \mathcal{N}_0 \times \mathcal{N}_0 : i \neq j, i \neq n+1, j \neq 0\} \tag{1}$$

The decision variables of the problem are the ones that follow.

$$x_{ij}^k \quad \begin{cases} 1 & \text{if arc } (i,j) \in \mathcal{A} \text{ is traversed by vehicle } k \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij}^{kk'} \quad \begin{cases} 1 & \text{if operation } (i,j) \in \mathcal{R}\text{, with tasks } i \text{ and } j\text{, is performed by routes } k \text{ and } k'\text{, respectively} \\ 0 & \text{otherwise} \end{cases}$$

w_i^k Arrival time of vehicle $k \in \mathcal{K}$ at task $i \in \mathcal{N}_0$

The objective function to be considered is the one present in Equation (2), which consists of the minimisation of the total travel costs.

$$\min F = \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} c_{ij}^k x_{ij}^k \tag{2}$$

Table 2: List of sets and parameters

Sets		
$\mathcal{L}$	Set of locations	
$\mathcal{K}$	Set of vehicles	$\mathcal{K} = \{k_1, k_2, ..., k_K\}$
$\mathcal{N}$	Set of tasks	$\mathcal{N} = \{1, 2, ..., n\}$
$\mathcal{N}_0$	Set of tasks with depot tasks	$\mathcal{N}_0 = \mathcal{N} \cup \{0, n+1\}$
$\mathcal{A}$	Set of arcs (i, j)	$\mathcal{A} \subset \mathcal{N}_0 \times \mathcal{N}_0$
$\mathcal{R}$	Set of all operations (i, j)	$\mathcal{R} \subset \mathcal{N}_0 \times \mathcal{N}_0$
$\mathcal{R}^\lambda$	Set of operations (i, j) subject to lower-bounding synchronisation constraints	$\mathcal{R}^\lambda \subseteq \mathcal{R}$
$\mathcal{R}^\mu$	Set of operations (i', j') subject to upper-bounding synchronisation constraints	$\mathcal{R}^\mu \subseteq \mathcal{R}$
$\mathcal{R}^\rho$	Set of passive operations (i, j) subject to movement synchronisation	$\mathcal{R}^\rho \subset \mathcal{R}, \mathcal{R}^\rho \subset \mathcal{A}$
$\mathcal{R}^\alpha_{ij}$	Set of active operations (i', j') whose movement can be synchronised with passive operation $(i, j) \in \mathcal{R}^\rho$	$\mathcal{R}^\alpha_{ij} \subset \mathcal{R}, \mathcal{R}^\alpha_{ij} \subset \mathcal{A}$
Parameters		
c^k_{ij}	Cost for traversing arc $(i, j) \in \mathcal{A}$ by vehicle $k \in \mathcal{K}$	
t_{ij}	Travel time from task i to task j	
s_i	Service time of task $i \in \mathcal{N}_0 \setminus \{n+1\}$	
λ_{ij}	Min. time offset between the arrival times of synchronised operation $(i, j) \in \mathcal{R}^\lambda$	
μ_{ij}	Max. time offset between the arrival times of synchronised operation $(i, j) \in \mathcal{R}^\mu$	
T	Planning horizon	

2.3.3 Constraints

The model will be subject to the constraints that follow.

$$\sum_{k \in \mathcal{K}} \sum_{i:(i,j) \in \mathcal{A}} x^k_{ij} = 1 \qquad \forall j \in \mathcal{N} \tag{3}$$

$$\sum_{i:(i,j) \in \mathcal{A}} x^k_{ij} - \sum_{i:(j,i) \in \mathcal{A}} x^k_{ji} = 0 \qquad \forall j \in \mathcal{N}, k \in \mathcal{K} \tag{4}$$

$$\sum_{j:(0,j) \in \mathcal{A}} x^k_{0j} = \sum_{i:(i,n+1) \in \mathcal{A}} x^k_{i,n+1} \qquad \forall k \in \mathcal{K} \tag{5}$$

$$\sum_{j:(0,j) \in \mathcal{A}} x^k_{0j} \leq 1 \qquad \forall k \in \mathcal{K} \tag{6}$$

$$w^k_j \leq T \sum_{i:(i,j) \in \mathcal{A}} x^k_{ij} \qquad \forall j \in \mathcal{N}_0, k \in \mathcal{K} \tag{7}$$

$$w^k_i + s_i + t_{ij} \leq w^k_j + T\left(1 - x^k_{ij}\right) \qquad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K} \tag{8}$$

$$\sum_{l:(l,i) \in \mathcal{A}} x^k_{li} + \sum_{l:(l,j) \in \mathcal{A}} x^{k'}_{lj} - 1 \leq y^{kk'}_{ij} \qquad \forall (i,j) \in \mathcal{R}, k, k' \in \mathcal{K} \tag{9}$$

$$y^{kk'}_{ij} \leq \sum_{l:(l,i) \in \mathcal{A}} x^k_{li} \qquad \forall (i,j) \in \mathcal{R}, k, k' \in \mathcal{K} \tag{10}$$

$$y^{kk'}_{ij} \leq \sum_{l:(l,j) \in \mathcal{A}} x^{k'}_{lj} \qquad \forall (i,j) \in \mathcal{R}, k, k' \in \mathcal{K} \tag{11}$$

$$x^k_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K} \tag{12}$$

$$y^{kk'}_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{R}, k, k' \in \mathcal{K} \tag{13}$$

$$0 \leq w^k_i \leq T \qquad \forall i \in \mathcal{N}_0, k \in \mathcal{K} \tag{14}$$

Constraints (3) ensure that all tasks must be performed exactly once.

Constraints (4) establish the inflow and outflow conservation: vehicles entering a task node must also exit it. Constraints (5) ensure that every vehicle starts and ends its route at the depot. Constraints (6) establish that a vehicle can only leave the depot once.

Constraints (7)–(8) correctly establish the values that variables w_i^k must take. Constraints (7) are linking constraints between variables x_{ij}^k and w_i^k; they impose that the arrival time of vehicle k at task i cannot be different from zero if the vehicle does not perform said task $\left(x_{ij}^k = 0 \implies w_i^k = 0\right)$. Constraints (8) establish vehicle arrival times to task nodes. They also serve as sub-tour elimination constraints.

Constraints (9)–(11) are linking constraints that set the values of decision variables $y_{ij}^{kk'}$ based on the values of variables x_{ij}^k. Constraints (9) set variable $y_{ij}^{kk'}$ to 1 if tasks i and j of operation $(i,j) \in \mathcal{R}$ are being performed. Constraints (10) and (11) set the opposite cases. When task i is not being performed, constraints (10) forcefully set the value of $y_{ij}^{kk'}$ to zero. When task j is not being performed, constraints (11) forcefully set the value of $y_{ij}^{kk'}$ to zero.

Finally, constraints (12)–(14) establish the domain and nature of the decision variables. Variables x_{ij}^k and $y_{ij}^{kk'}$ are defined as binary variables through constraints (12) and (13), respectively. Constraints (14) define variables w_i^k as continuous, whose values cannot exceed the established planning horizon.

The synchronisation constraints for this model are presented ahead.

2.3.4 Synchronisation constraints

Operation synchronisation For modelling purposes, it is relevant to distinguish two types of operation synchronisation constraints: lower-bounding (LB) and upper-bounding (UB) constraints.

As their names imply, LB constraints set a lower bound on the start time of the second task. This lower bound is calculated from the time that the first task starts being executed. For these constraints, the time offset between the start times of task pairs (i,j) is represented by λ_{ij}. On the other hand, UB constraints set an upper bound on the start time of the second task. For these constraints, the time offset between the start times of task pairs (i,j) is represented by μ_{ij}.

The LB and UB constraints are defined as follows.

$$w_i^k + \lambda_{ij} \leq w_j^{k'} + T\left(1 - y_{ij}^{kk'}\right) \qquad \forall (i,j) \in \mathcal{R}^{\lambda}, k, k' \in \mathcal{K} : k \neq k' \tag{15}$$

$$w_i^k + \mu_{ij} + T\left(1 - y_{ij}^{kk'}\right) \geq w_j^{k'} \qquad \forall (i,j) \in \mathcal{R}^{\mu}, k, k' \in \mathcal{K} : k \neq k' \tag{16}$$

Constraints (15) are LB constraints. They ensure that if a given synchronised operation $(i,j) \in \mathcal{R}^{\lambda}$ is performed, the start time of task j can only be performed λ_{ij} time units after the start time of task i. Constraints (16) are UB constraints. They state that if a given synchronised operation $(i,j) \in \mathcal{R}^{\mu}$ is performed and task i precedes j (or is simultaneous), then task j must start being performed up to μ_{ij} time units after i begins to be performed.

By applying these synchronisation constraints, as well as different combinations of them, we are able to model a variety of synchronisation requirements, identical to the ones present in Dohn, Rasmussen, and Larsen (2011).

Figure 1 illustrates the possible combinations of synchronisation constraints in a Gantt chart-like form. A given operation (i,j) requires that tasks i and j be performed at their corresponding locations. We assume that they will be visited by two distinct vehicles k and k', respectively. In this figure, it is possible to visualise the inadmissible service time allocations (marked in dashed form) that each of the synchronisation types entails. For example, in Figure 1a, the diagram shows that task i can only be delayed up to a certain time without also having to delay task j; analogously, task j can only be moved up to a certain time without also having to move up task i.

For the sake of simplicity, we will use the following notation $i \preceq j$ to indicate that task i is visited before task j (or simultaneously).

Minimum difference (Figure 1a) is the result of applying a lower-bounding synchronisation constraint between tasks i and j. It is a synchronisation type that forces $i \preceq j$ and can be found in several real-world applications, such as in large items transportation, where delivery and installation vehicles must be synchronised such that the delivery of the merchandise must occur before the installing team arriving (e.g., Hojabri, Gendreau, Potvin, & Rousseau, 2018). This happens because the time offset $\lambda_{ij} \geq 0$ is implemented with respect to the arrival time of vehicle k at task i, w_i^k. A dependence relationship is established between i and j, where the visit to task i can be moved up as much as desired, and the visit to task j can be delayed as much as necessary.

Maximum difference (Figure 1b) is the result of applying an upper-bounding constraint between tasks i and j. Unlike the minimum difference type, this synchronisation type does not force a given order of visits. One of the most representative examples of this synchronisation type concerns the pickup and delivery of perishable items involving multiple vehicles, where the time lapse between having the pickup vehicle pick up the merchandise and having the delivery vehicle deliver it must be constrained (e.g., Anaya-Arenas, Prodhon, Renaud, & Ruiz, 2019). This is due to the fact that in this synchronisation type we are only concerned with maintaining a maximum offset between task arrival times. In the case illustrated in Figure 1b, one can observe that tasks i and j can only be moved up or delayed up to a given threshold $\mu_{ij} \geq 0$, respectively. However, tasks i and j can be delayed or moved up without restrictions, respectively, meaning that an order of visits is not forced. In conclusion, this synchronisation type will only be binding if $i \preceq j$.

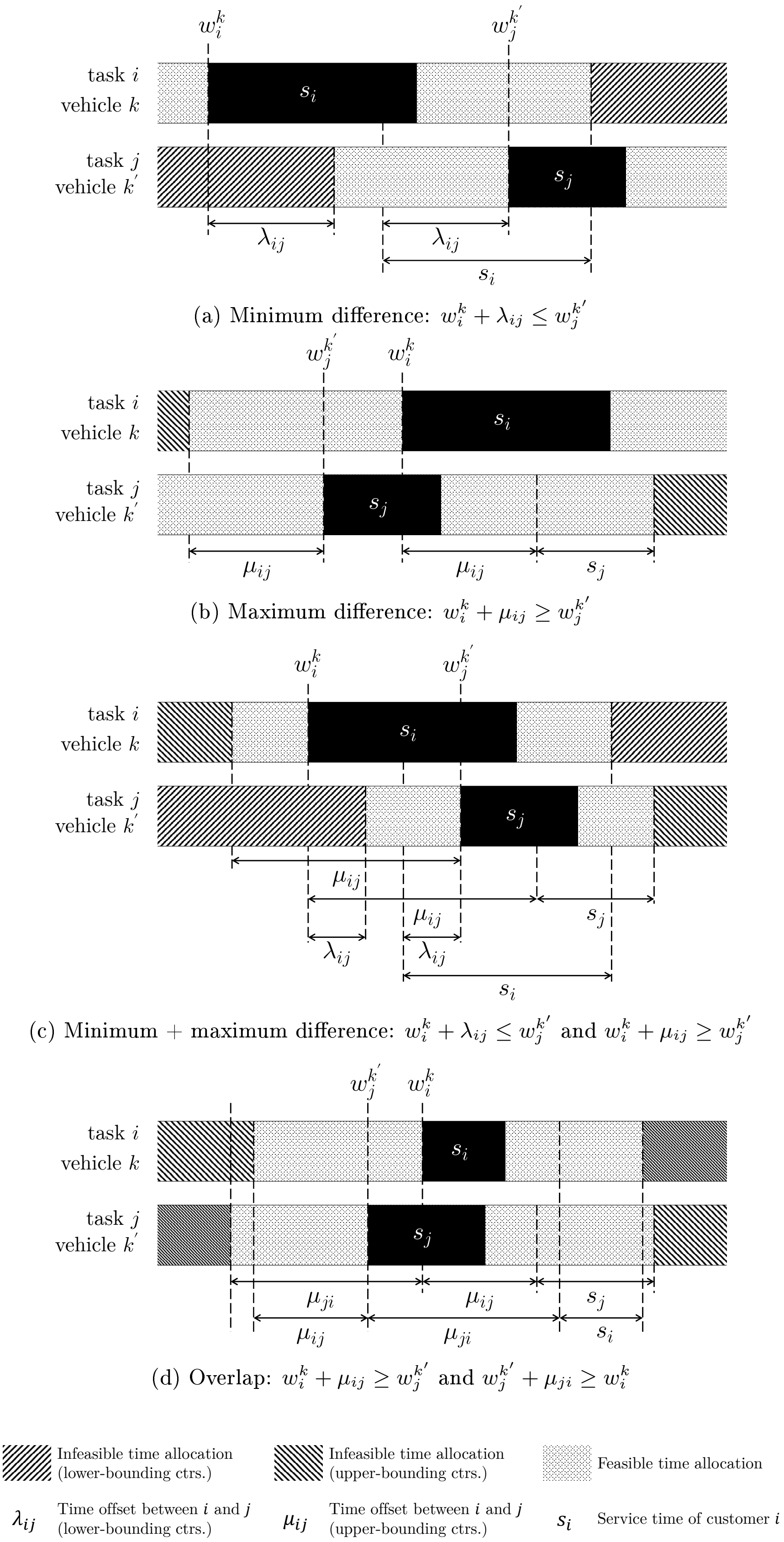
The synchronisation types that follow consist of different possible combinations of the constraints presented above. Figure 1c illustrates a case of minimum + maximum difference, which combines lower-bounding and upper-bounding constraints. Unlike the previous cases, arrival times are now constrained within a closed interval due to combining both synchronisation types. We continue to force a given order of visits, and we require that services not be performed too far apart. A special case of this synchonisation type is exact synchronisation, where $\lambda_{ij} = \mu_{ij}$. For this special case, the time offset between the arrivals of i and j must be exact. One may find this synchronisation type in several applications in the literature, such as home health care routing (e.g., Mankowska et al., 2013), where two or more staff members may be required to be present simultaneously at a patient's location, in which case $\lambda_{ij} = \mu_{ij} = 0$.

Finally, Figure 1d illustrates a case of overlap, which results from the combination of two upper-bounding constraints: one between i and j and another one between j and i. Overlap is relatively similar to the minimum + maximum difference type; however, there is one major difference. Although the arrival times at tasks are also constrained within a closed interval, in overlap, we do not force a given order of visits.

We present a summary of these synchronisation types in Table 3 with regard to the main aspects of each operation synchronisation type.

It should be noted that, although we have used the arrival time as the temporal variable to be synchronised, this could also be applied *mutatis mutandis* to other time variables (e.g., a task's completion time) without significantly changing the theoretical aspects of each synchronisation type.

For operations (i, j) subject to both lower-bounding and upper-bounding constraints, these pairs are included both in sets $\mathcal{R}^\lambda$ and $\mathcal{R}^\mu$. In the case of the overlap type, both operations (i, j) and (j, i) are added to set $\mathcal{R}^\mu$.



(a) Minimum difference: $w_i^k + \lambda_{ij} \leq w_j^{k'}$

(b) Maximum difference: $w_i^k + \mu_{ij} \geq w_j^{k'}$

(c) Minimum + maximum difference: $w_i^k + \lambda_{ij} \leq w_j^{k'}$ and $w_i^k + \mu_{ij} \geq w_j^{k'}$

(d) Overlap: $w_i^k + \mu_{ij} \geq w_j^{k'}$ and $w_j^{k'} + \mu_{ji} \geq w_i^k$

Figure 1: Types of synchronisation

Table 3: Characteristics of different types of operation synchronisation

Synchronisation type	Constraints			Order of visits	Time lapse between visits	
	LB (i,j)	UB (i,j)	UB (j,i)		Minimum	Maximum
Minimum difference	•	–	–	$i \preceq j$	$\lambda_{ij} \geq 0$	–
Maximum difference	–	•	–	–	–	$\mu_{ij} \geq 0$ (if $i \preceq j$)
Min. + max. difference	•	•	–	$i \preceq j$	$\lambda_{ij} \geq 0$	$\mu_{ij} \geq 0$
Overlap	–	•	•	–	–	$\mu_{ij} \geq 0$ (if $i \preceq j$) $\mu_{ji} \geq 0$ (if $j \preceq i$)
Exact synchronisation	•	•	–	$i \preceq j$	$\lambda_{ij} \geq 0, \lambda_{ij} = \mu_{ij}$	$\mu_{ij} \geq 0, \mu_{ij} = \lambda_{ij}$

Movement synchronisation Movement synchronisation is achieved by considering set $\mathcal{R}^{\rho}$, which contains the passive operations to be synchronised, as well as sets $\mathcal{R}^{\alpha}_{ij}$, which contain the active operations that each passive operation (i,j) can be synchronised with. The movement synchronisation constraints are the ones that follow.

$$x^k_{ij} \leq \sum_{(i',j') \in \mathcal{R}^{\alpha}_{ij}} x^{k'}_{i'j'} \qquad \forall (i,j) \in \mathcal{R}^{\rho}, k, k' \in \mathcal{K} : k \neq k' \tag{17}$$

Constraints (17) state that a passive operation (i,j) can only be performed if there is also a corresponding active operation (i',j') being performed.

3 Classification Schema

3.1 Schema outline

As previously stated, this research considers that only two major types of synchronisation respect the route interdependency concept that is pivotal to synchronisation. This thesis was supported by evaluating how synchronisation can be distinguished and framed among other commonly known problem aspects.

To provide the much-needed clarification for the concept of synchronisation, a classification schema for different routing problem aspects was devised, building on nomenclature and previously existing conceptual schemas of the literature. The motivations for this approach are two-fold. First, it enables the contextualisation of synchronisation among other problem categories. Second, since our definition of synchronisation is more restrictive than other ones present in the literature, we are able to classify other problem aspects that we do not consider to be synchronisation into their corresponding categories, thus eliminating ambiguities that may arise.

The classification schema we present can take on every problem aspect of a routing problem and map it into one of four categories. This categorisation is linked with the direct scope of the interdependency that a problem aspect induces. The scope of an interdependency can be four-fold: within the same route, between routes, among the whole routing problem or between the routing problem and another optimisation problem. The resulting classification schema reconciles several taxonomies in the literature, such as the ones in Irnich et al. (2014), Drexl (2012) and Bektaş, Laporte, and Vigo (2015).

Figure 2 presents an overview of the classification schema and visually represents it.

The categories of the schema are:

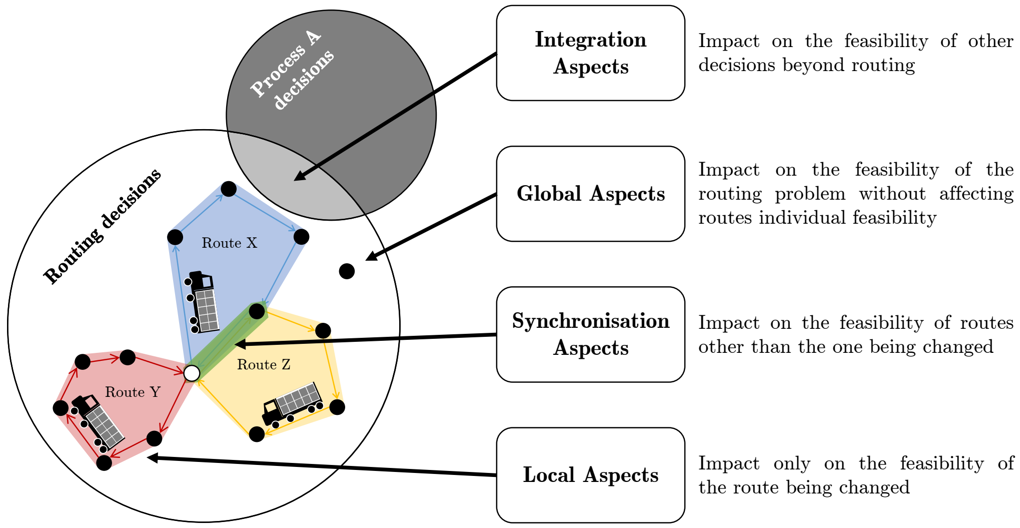


Figure 2: Visual representation of the proposed problem aspect classification schema

- **Local Aspects** – concerns the problem aspects that, in the event a route is changed, only impact the feasibility of the route being changed;
- **Synchronisation Aspects** – concerns the problem aspects that, besides impacting the route being changed, also impact other related routes;
- **Global Aspects** – concerns the problem aspects that, although they do not impact the individual feasibility of any route, affect the feasibility of the overall routing problem;
- **Integration Aspects** – concerns the problem aspects that can impact the feasibility of decisions beyond the scope of routing.

Each of these four categories contains several well-known problem aspects from the VRP literature. The extended modelling framework presented in Appendix A addresses and is able to model, besides synchronisation, the most common local and global problem aspects, guided by this categorisation. Table 4 summarises these problem aspects modelled in this framework, as well as their corresponding constraints for reference.

Table 4: Common routing aspects and their correspondence in the modelling framework

Problem aspect		**Description**	**Constraints**
Global aspects	Task constraints	Tasks are performed at most once	(19)
		Tasks are mandatory/optional	(20)
	Operation constraints	Operations are performed at most once	(22)
		Operations are mandatory/optional	(23)
		Operation must be performed by different vehicles	(24)
		Operation must be performed by the same vehicle	(25)
	Resource constraints	Customer demand must be satisfied (split deliveries)	(26)
Synchronisation aspects	Operation synchronisation	Lower-bounding constraints	(15)
		Upper-bounding constraints	(16)
	Movement synchronisation	Active-passive behaviour between operations	(17)
Local aspects	Capacity constraints	Vehicle capacity limits	(27)–(28)
		Vehicle capacity limits (multi-trip VRP)	(29)–(30)
	Operation constraints	Restrict route sequencing	(31)–(33)
	Time window constraints	Bound vehicle arrival time to customer	(34)
	Route length	Maximum route length	(35)–(36)
		Maximum route length (multi-trip VRP)	(37)–(40)

3.2 Comparison with existing schemas in the literature

Local aspects correspond to problem aspects that Irnich et al. (2014) would classify as "local" or "intra-route constraints". These constraints are confined to a single route and establish whether or not it can be considered feasible. Taking into account the definition from Irnich et al. (2014), they can be checked as soon as the route sequence is determined, regardless of the design of other routes. Common local aspects found in the literature include capacity constraints, precedence constraints for operations occurring in the same route, time windows constraints or route length constraints.

Synchronisation aspects correspond to problem aspects that rely on the concept of interdependent routes, as defined by Drexl (2012). These constraints are confined to a set of interdependent routes and establish whether or not this group of routes can be considered feasible. As soon as all interdependent routes are determined, they can be checked for their feasibility without considering the design of other routes. Synchronisation aspects found in the literature consist of operation synchronisation and movement synchronisation, which have been previously described in this paper.

Global aspects correspond to problem aspects that Irnich et al. (2014) would classify as "global" or "inter-route constraints". These constraints are not confined to any specific route. Instead, they depend on the overall design of the routes. Taking into account the definition provided by Irnich et al. (2014), the feasibility of these constraints can be checked once all routes have been determined. Therefore, they do not depend on the feasibility of any particular route or group of routes. These constraints are typically found in global requirements that the routing problem must comply with, such as ensuring that tasks or operations are performed under their correct conditions or ensuring that the consumption of a shared resource is limited.

Irnich et al. (2014) consider synchronisation a part of global problem aspects. The proposed schema considers that their underlying differences justify that synchronisation should be considered in its own category since global aspects consist of problem requirements that are not indexed to any particular route – unlike synchronisation aspects.

This narrower categorisation of synchronisation also means that certain synchronisation aspects proposed by Drexl (2012) – task, resource and load synchronisation – are, instead, considered as global aspects. In fact, when used in an isolated manner, these problem aspects do not trigger an interdependency between specific routes. Instead, they establish global requirements for given entities or resources of the problem – such as ensuring that tasks are performed exactly once by some vehicle ("task synchronisation"), guaranteeing that customer demand, to be delivered among different vehicles, is satisfied ("load synchronisation"), or by ensuring that the consumption of a shared resource by all vehicles is scarce and limited ("resource synchronisation"). In sum, it can be argued that global aspects trigger the need for vehicles to coordinate but not to synchronise.

Integration aspects refer to problems where the interdependency is not established between the typical VRP entities but between VRP entities and other external decisions, such as production or inventory decisions. These constraints link the routing problem with other optimisation problems. The problem aspects that fit in this category are found in the general class of Integrated VRPs, considering the definition provided by Bektaş et al. (2015).

4 Overview of the State of the Art

An overview of the state of the art on the VRP with synchronisation is presented by resorting to the outputs provided by the literature review. Each collected reference is evaluated according to their problem application, the objective function components being optimised, the categories of problem aspects found, the solution approaches, as well as an indication of the sources of uncertainty and dynamism (if any).

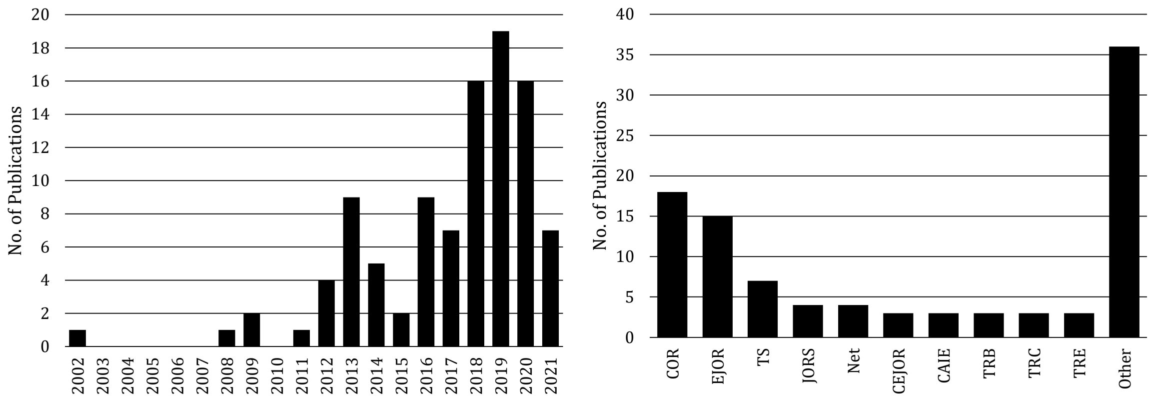
4.1 Methodology

The conduction of the literature review followed an approach similar to the one in Seuring, Müller, Westhaus, and Morana (2005), comprising four main stages: material collection, descriptive analysis, category selection and material evaluation. Despite these well-defined stages, the overall process was highly iterative, given the research questions of this paper and the integrative nature of the review. The scope of the literature review concerned VRPs with at least one synchronisation aspect.

Material collection The collection of the references that served as a basis for the literature review was performed from two sources of information. The first source of information was the Scopus database in January 2022 with the following search query: `("vehicle routing" OR "arc routing" OR "pickup and delivery" OR "truck and trailer") AND ("synchronization" OR "temporal dependencies" OR "temporal interdependencies")`. The selection of these keywords was performed iteratively by testing several combinations of keywords commonly used in the VRP field and assessing the overall relevance of the obtained references. Additionally to the results obtained from Scopus, we added all the references that cited Drexl (2012), as this was the first seminal paper that approached synchronisation aspects in the VRP in a systematic manner. Additional criteria for the search included filtering the results by only including publications written in the English language. From these criteria, a total of 294 references were initially retrieved, which were later reduced to a shortlist of 99 references in subsequent stages of the review, based on the criteria described below.

Descriptive analysis In order to evaluate the relevance and interest of the topic, the metadata of the references were analysed. The focus of this analysis was mainly on the year of publication and the types of journals where the research was published. Figure 3 shows the distribution of the papers that were reviewed by year of publication and by journal. The increasing trend in the number of published papers throughout the years, observable in Figure 3a, supports the claim that synchronisation is a growing research stream. Although two publications clearly stand out through the analysis of Figure 3b, one can observe a great dispersion of the papers throughout several journals. The most commonly found journals are *Computers and Operations Research* and the *European Journal of Operational Research*. These journals support the hypothesis that the research community is aware that additional work on tackling the topic of synchronisation in VRPs is necessary, as these journals have a high focus on methodological contributions for decision-making.

Category selection The category selection phase was instrumental in designing the classification schema proposed in this paper. This stage aimed to devise a categorisation of each problem aspect one may find in a VRP. It was established that the most appropriate way to clarify synchronisation aspects and assess the main problem aspects that accompany them would be to classify each reference according to the classification schema that was previously presented in this paper. In parallel with this classification of problem aspects, the references were also aggregated into major categories of problems, taking into account their similarities. The ultimate categories that were produced were the following:



(a) by year of publication (b) by journal

Legend: COR – Computers and Operations Research; EJOR – European Journal of Operational Research; TS – Transportation Science; JORS – Journal of Operational Research Society; Net – Networks; CEJOR – Central European Journal of Operations Research; CAIE – Computers and Industrial Engineering; TRB – Transportation Research Part B: Methodological; TRC – Transportation Research Part C: Emerging Technologies; TRE – Transportation Research Part E: Logistics and Transportation Review.

Figure 3: Distribution of the reviewed papers by journal and year of publication

- problems whose focus is to obtain feasible schedules for a given set of tasks;
- problems where synchronisation is required for transfer or cross-docking operations, i.e., involving the exchange of load or other entities between vehicles;
- problems where synchronisation is motivated by the presence of unmanned vehicles; and
- problems where synchronisation is used to allow for the movement of trailers or passive vehicles.

The discussion and analysis of the references are guided by these problem categories.

Material evaluation Each of these references was manually filtered through an analysis of the abstracts and the type of publication. Criteria for exclusion of a publication could be one of the following: (i) the publication did not showcase a routing problem with synchronisation aspects, according to our classification schema; (ii) the publication was not a research paper containing original research (e.g., the publication was a survey/review); (iii) the publication lacked a formal definition of the problem being tackled, either through a mathematical model or a detailed problem statement section (or equivalent); (iv) the publication consisted of ongoing or unfinished research work (e.g., technical reports, conference proceedings). Despite these exclusion criteria, references relating to ongoing or unfinished work were considered in the review if they are dated from 2020 onwards. This exception was considered to incorporate recent works that could potentially not have yet sufficient time to be published.

The authors reached a shortlist of 99 references using these filtering criteria, which were then categorised according to the classification schema. Additional information was taken regarding the application of the problem being tackled, the objective function being optimised, the adopted solution approach, if proposed, and the existence of unknown or uncertain problem parameters.

4.2 Routing with synchronisation of schedules

A significant part of the publications found tackle problems where the focus of the problem is not so much on determining a routing plan for a given fleet of vehicles but rather on obtaining a feasible schedule for a given set of tasks that need to be performed, subject to operation synchronisation. Supporting this discussion, Table 5 presents each reference framed in this problem category, indicating the problem application, the objective function components being optimised, the categories of problem aspects found, the solution approaches, as well as an indication of the sources of uncertainty and dynamism (if any).

Predominant applications In this category, Home Health Care is arguably the most prevalent problem application that can be found (e.g., Bredström & Rönnqvist, 2008). Routing problems in this sector typically consider a fleet of staff elements that must serve a set of patients (customers) by performing certain tasks/services at their homes. Some of these services require more than one staff person to be present at the customer location, thus requiring synchronisation. The operation synchronisation requirements can be either simultaneity (e.g., staff members must perform their tasks simultaneously) or precedence (e.g., a task must be performed by a given staff member before a second task is started). The synchronisation of schedules is also especially relevant in routing problems that rely on the efficient use of scarce resources. In these applications, it is common to use synchronised scheduling between vehicles to ensure that these resources are not being utilised simultaneously. Bakkehaug et al. (2016) present a problem in the field of ship routing, whose purpose consists in maintaining capacity at ports feasible throughout time, therefore obtaining vessel schedules that ensure vessel departures and arrivals occur within predetermined time offsets. Fedtke and Boysen (2017) presents a synchronised scheduling problem in a modern rail-rail transhipment yard, requiring the synchronisation of gantry cranes and shuttle cars. Froger, Jabali, Mendoza, and Laporte (2022) apply schedule synchronisation to a routing problem involving electric vehicles, where the arrival times to charging stations must be synchronised. There are various other applications considering the synchronisation of resource schedules, such as in the construction sector (e.g., An, Byon, & Cho, 2018; Grimault et al., 2017), log-truck scheduling (e.g., Hachemi, Gendreau, & Rousseau, 2013) or in humanitarian and military logistics (e.g., Lam & Hentenryck, 2016).

Objective functions The results of Table 5 show that the objective functions of routing problems involving the synchronisation of schedules can vary greatly. Within the home health care application, one of the most common objective function components is the maximisation of vehicle-customer preferences (e.g., Afifi, Dang, & Moukrim, 2015; Ait Haddadene, Labadie, & Prodhon, 2019; Haddadene, Labadie, & Prodhon, 2016), as an attempt to improve service quality to patients providing them with the caregivers they most prefer. In a more general manner, routing problems with synchronisation of schedules also consider the minimisation of scheduling-related components, such as the total travel time (e.g., Frifita & Masmoudi, 2020; López-Aguilar, Boyer, Salazar-Aguilar, & Sbihi, 2018; Polnik, Riccardi, & Akartunalı, 2020), the total waiting time (e.g., Doulabi, Pesant, & Rousseau, 2020) or even the total tardiness (e.g., Mankowska et al., 2013). Other less common objectives can be found, such as the maximisation of the number of performed tasks (e.g., Dohn, Kolind, & Clausen, 2009). Nevertheless, the classical objective of minimising the total distance costs is still frequently found (e.g., Cappanera, Requejo, & Scutellà, 2020; Ghilas, Demir, & Van Woensel, 2016; Parragh & Doerner, 2018; Rasmussen, Justesen, Dohn, & Larsen, 2012), as well as the minimisation of vehicle fixed costs (e.g., Liu, Tao, & Xie, 2019; Nguyen, Crainic, & Toulouse, 2013).

Predominant problem aspects The types of operation synchronisation found in these problems can be either of precedence or simultaneity. However, a subset of references only considers simultaneous service as admissible synchronisation constraints (e.g., Afifi et al., 2015; Polnik et al., 2020). The increased restrictiveness in these problems in terms of operation synchronisation makes the route interdependency challenge more prevalent, which, in theory, turn these problems harder to solve. Nevertheless, some references attempt to attenuate these impacts by considering alternative, less restrictive requirements that also are intelligible in the scope of home health care. For example, Decerle, Grunder, Hassani, and Barakat (2018) tackles a problem that considers simultaneously synchronised visits. However, time window and synchronisation constraints are modelled as soft constraints. A penalty function is defined and incorporated into the problem's objective function with the purpose of minimising the time window violations, as well as the time offsets between synchronised visits. With a similar approach, Doulabi et al. (2020) consider a routing problem applied to both home health care and operating room scheduling where the upper bounds of time windows can be violated but also incur a penalty. The acknowledgement of penalties for certain constraints is not exclusive to problems with only simultaneous synchronisation. Mankowska et al. (2013) also consider penalty values in the tardiness that results from overshooting time window upper-bounds.

Precedence constraints in these problems are also common, although the literature typically does not require that these operations be necessarily performed by different vehicles (e.g., Masmoudi, Hosny, Demir, & Cheikhrouhou, 2018). This is due to the lack of global task constraints that specifically dictate that operations subject to precedence relationships be performed by different vehicles. This means that, if feasible, operations where one of its tasks must be performed before the other can be performed by the same vehicle, as long as no other problem constraint limits this behaviour and none of the task service times overlap, since, by definition, a vehicle can only perform one task at a time.

In terms of problem requirements extraneous to synchronisation, it is usual for time windows to be considered in these problems. This can be justified not only by the operational relevance of this problem requirement in general routing problems. Depending on the width of time windows at synchronised tasks, time window constraints can potentiate simplification and pre-processing procedures that can make these problems easier to solve. Although these problems could be viewed as a generalisation of the VRP with Time Windows, very few references consider capacity constraints. Parragh and Doerner (2018) consider a routing problem where capacity constraints can be considered to the detriment of staff qualifications. Doulabi et al. (2020), on the other hand, assume that providing a service to a given customer may consume a given quantity of a resource that is transported by the staff member, thus triggering the need for demand satisfaction and capacity constraints.

In terms of global constraints present in these problems, most references consider that not all vehicles (staff members) may be able to perform certain services. This type of global task constraints is motivated by the high degree of specialisation that certain home health care tasks require, which must be performed by qualified staff (e.g., Mankowska et al., 2013). At the same time, most references allow for preceding services to be performed by the same vehicle. Most references also consider that all tasks are mandatory and, therefore, must have a vehicle perform them. Very few references assume optional services (e.g., Parragh & Doerner, 2018; Rasmussen et al., 2012) at the expense of an additional cost to be considered in the objective function.

In sum, the literature on home health care routing presents several problem settings and variations interacting with operation synchronisation, which can ultimately make the route interdependence challenge more or less difficult to address. However, other problem applications reveal several similarities to home health care routing. Routing problems in the public utilities sector also exhibit many of the characteristics of health care routing. Goel and Meisel (2013) address a routing problem in the electricity sector, whose purpose is to schedule and synchronise maintenance operations at multiple points of an electricity network. In this particular problem, the operations being synchronised are in different locations since the maintenance operations require the presence of technicians at both ends of a power line. Hà et al. (2020) address a similar scheduling problem in the scope of an internet service provider. Ali, Côté, and Coelho (2021); Hanafi, Mansini, and Zanotti (2019); Hojabri et al. (2018) tackle a routing problem with synchronisation for the delivery and installation of large items to final customers, where the installation vehicles must arrive at the customer after the items have been delivered by another vehicle. Scheduling with synchronisation issues can also be found in problems involving the transportation of perishable items (e.g., Anaya-Arenas et al., 2019) or railway maintenance (e.g., Pour, Marjani Rasmussen, Drake, & Burke, 2019).

Solution approaches The solution approaches found in this category of problems are quite diverse. One of the first works that resorted to exact methods was Dohn et al. (2011), which proposed Dantzig-Wolfe decompositions to allow for column generation approaches. Since then, other column generation approaches were envisaged (e.g., Emadikhiav, Bergman, & Day, 2020; J. Li, Qin, Baldacci, & Zhu, 2020; Luo, Qin, Zhu, & Lim, 2016; Tilk, Drexl, & Irnich, 2019), most of them only referring to problems with operation synchronisation. Standalone branch-and-cut approaches are also commonly found in the literature (e.g., Bianchessi, Drexl, & Irnich, 2019; Doulabi et al., 2020; Hanafi et al., 2019; Luo et al., 2016).

For defining (or solving) these problems, most of the literature resorts to mixed-integer programming. This can easily be explained by the fact that mixed-integer programming is the state-of-the-art mathematical programming approach that is considered in the routing field. A limited number of references resorts to constraint programming models. Hà et al. (2020) use constraint programming to define a routing problem where certain customers must be visited by two types of vehicles within given time offsets. Hachemi et al. (2013) also resort to this methodology for solving and obtaining the daily schedules of the log-truck scheduling problem. Pour et al. (2019) resort to both mixed-integer and constraint programming to solve a crew scheduling problem. Constraint Programming may be a worthwhile possibility to be considered in future research, although it is seldom used in the routing field. This can be justified by the fact that routing problems with synchronisation of schedules exhibit similar constraints to the ones found in scheduling problems, which, in turn, constitutes one of the most successful use cases of constraint programming (Baptiste, Pape, & Nuijten, 2001; Col & Teppan, 2022).

Nonetheless, the most predominant solution approaches for routing problems with synchronisation are heuristic concepts. Large Neighbourhood Search (LNS) is the most popular of them all, probably due to the general popularity of this approach in routing problems, especially of its adaptive version presented by Pisinger and Ropke (2007). LNS approaches usually use well-known removal and insertion operators from the literature (e.g., Liu et al., 2019; Parragh & Doerner, 2018; Roozbeh, Hearne, & Pahlevani, 2020), such as Worst Removal, Shaw Removal, Best Insertion or k-Regret Insertion. However, the interdependence problem of synchronisation typically constitutes a non-negligible factor that may generate infeasible solutions, which in an LNS framework requires workarounds for either "repairing" solutions on each iteration or evaluating the feasibility of the candidate. For example, Ghilas, Demir, and Woensel (2016) propose an Adaptive LNS for a rich pickup and delivery problem with transfers, where after a removal and insertion iteration, a repair procedure is invoked to account for transfer requirements. A similar procedure is adopted for Bakkehaug et al. (2016) in a routing problem with voyage separation requirements.

In works involving operation synchronisation, population-based heuristics can also be found (Ait Haddadene et al., 2019; Masmoudi et al., 2018), although they are less common. Matheuristic approaches are also rarely found (e.g., Bredström & Rönnqvist, 2008), which indicates that further work may be envisaged on this front.

In terms of problem instances, the instances used by Bredström and Rönnqvist (2008) have been used frequently in a subset of references (e.g., Ait Haddadene et al., 2019; Decerle et al., 2018). In order to allow for the replicability of results and more objectively benchmark different solution approaches, it is expected that future research will trend towards the adoption of a set of common instances.

Given the diversity of solutions methods that the literature has adopted, it is expected that research of solution approaches for these problems continues on multiple methodological possibilities.

Routing under uncertainty and/or dynamism Another future challenge for the VRP with synchronisation is the real-life implementation of routing plans with vehicle synchronisation and its reactivity to unforeseen events or routing requests. While for traditional VRPs, dynamic routing usually needs only to reconstruct portions of the initial solution when faced with an unexpected event or request because routes are independent, this is not the case for the VRP with synchronisation. It is expected that, as more synchronisation aspects are introduced to a problem, the more changes a reconstructed routing solution will have compared to the initial plan.

Within the scope of the review, only one reference considered a dynamic routing problem with synchronisation aspects. Rousseau, Gendreau, and Pesant (2013) study a dynamic VRP where a constraint programming model is envisaged, which is then used for implementing an insertion heuristic to include new customers into the initial solution. The work acknowledges the increasing difficulty when trying to insert new customers that require synchronisation.

Routing with sources of uncertainty is also a largely unexplored topic. Work on problems considering parameters as random variables, such as service times or travel times between locations, is very limited. Furian, O'Sullivan, Walker, and Vössner (2018) consider the home health care routing case with operation synchronisation and propose an optimisation procedure embedded in a discrete-event simulation framework that handles stochasticity. Also, in the field of health care, Doulabi et al. (2020) consider a VRP with synchronised visits and stochastic travel and service times. Shi, Zhou, Ye, and Zhao (2020) present a routing problem with synchronised operation synchronisation considering greenhouse gas emissions, which resorts to robust optimisation for two uncertainty sets of travel and service times. In the context of city logistics, Anderluh, Larsen, Hemmelmayr, and Nolz (2019) use a Monte Carlo simulation with an optimisation approach for a 2-echelon VRP involving the synchronisation of vans and bicycles at satellite locations considering uncertain travel times.

Stochastic VRPs with synchronisation have already been acknowledged as a relevant research gap to be addressed (Parragh & Doerner, 2018). However, it is expected that advances on this front naturally surge as future research advances the state of the art on solution approaches for problems in a deterministic setting.

Table 5: References pertaining to routing problems with synchronisation of schedules

Reference	Application	Objective function	Problem aspects	Solution method	Uncertainty and/or dynamism
Afifi et al. (2015)	Home Health Care	min TT, max VCP, O	L-TW, S-O, G-T, G-O	SA	–
Ait Haddadene et al. (2019)	Home Health Care	min DC, max VCP	L-TW, L-RL, S-O, G-T, G-O	GA	–
Ali et al. (2021)	Large items delivery and installation	min DC	S-O, G-T, L-O	MS, ALNS	–
An et al. (2018)	Construction sector	min FC, OT, TT	S-O, G-T, G-O	MS	–
Anaya-Arenas et al. (2019)	Perishable items transportation	min RD	L-O, L-TW, L-RL, S-O, G-T, G-O	ILS	–
Bakkehaug et al. (2016)	Ship routing	O	L-O, L-C, L-TW, S-O, G-T, G-O	ALNS	–
Bianchessi et al. (2019)	Not specified	min DC, FC, RD	L-C, L-TW, S-O, G-T, G-O, L-O	BC	–
Bredström and Rönnqvist (2008)	Home Health Care	min TT	L-O, L-TW, S-O, G-T, G-O	MH	–
Cappanera et al. (2020)	Home Health Care	min DC	L-O, L-TW, L-RL, S-O, S-M, G-T, G-O	MH	–
Decerle et al. (2018)	Home Health Care	min DC	L-O, L-TW, S-O, G-T, G-O	MA	–
Dohn et al. (2009)	Not specified	max PT	L-TW, S-O, G-T, G-O	DW	–
Dohn et al. (2011)	Not specified	min DC	L-C, L-TW, S-O, G-T, G-O	BC, BP	–
Doulabi et al. (2020)	Home Health Care	min DC, FC, D, WT	L-O, L-C, L-TW, L-RL, S-O, L-O	MS, BC	U-TT, U-ST
Emadikhiav et al. (2020)	Calibration of measuring instruments	min TT	L-O, S-O, G-T, G-O	BP	–
Fedtke and Boysen (2017)	Railway transportation	min M	L-O, S-O, G-T, G-O	DA, H, MS	–
Frifita and Masmoudi (2020)	Home Health Care	min TT	L-O, L-TW, S-O, G-T, G-O	VNS	–
Froger, Mendoza, Jabali, and Laporte (2017)	Charging of electric vehicles	min RD	L-C, L-O, S-O, G-T, G-O	MH	–
Ghilas, Demir, and Woensel (2016)	Freight transportation	min DC	L-O, L-C, L-TW, S-O, G-T, G-O	ALNS	–
Ghilas, Demir, and Van Woensel (2016)	Freight transportation	min DC, O	L-O, L-C, L-TW, S-O, G-T, G-O	ALNS	–
Goel and Meisel (2013)	Electricity power line maintenance	min DC, O	L-O, S-O, G-T, G-O	MH, LNS	–
Grimault et al. (2017)	Construction sector	min DC, FC, RD	L-O, S-O, G-T, G-O, G-R	ALNS	–
Hà et al. (2020)	Public Utilities; Installation and maintenance services	min DC	L-C, L-TW, S-O, G-T, G-O	MS, MH, ALNS	–
Hachemi et al. (2013)	Log truck scheduling	min DC	L-O, S-O, G-T, G-O	MS, MH	–
Haddadene et al. (2016)	Home Health Care	max VCP	L-TW, S-O, G-T, G-O	GRASP, ILS	–
Hanafi et al. (2019)	Large items delivery and installation	O	L-RL, S-O, G-T, G-O	BC	–
Hojabri et al. (2018)	Large items delivery and installation	min DC	L-O, L-TW, S-O, G-T, G-O	ALNS	–
Lam and Hentenryck (2016)	Military	min DC	L-C, L-O, L-TW, S-O, G-T	BP	–
J. Li et al. (2020)	Household services	min DC	L-C, L-TW, L-RL, L-O, S-O	BC, BP, DW	–
Liu et al. (2019)	Not specified	min DC, FC	L-TW, S-O, G-T, G-O	ALNS	–
López-Aguilar et al. (2018)	Not specified	min TT	L-TW, S-O, G-T, G-O	MS	–
Luo et al. (2016)	Not specified	min DC, FC	S-O, G-T, G-O	BP, BC	–
Mankowska et al. (2013)	Home Health Care	min DC, D, O	L-O, L-TW, S-O, G-T, G-O	VNS	–

Continued on next page

Table 5: References pertaining to routing problems with synchronisation of schedules

Reference	Application	Objective function	Problem aspects	Solution method	Uncertainty and/or dynamism
Masmoudi et al. (2018)	Home Health Care	O	L-O, L-TW, S-O, G-T, G-O	ABC	–
Nguyen et al. (2013)	Not specified	min DC, FC	L-O, L-TW, S-O, G-T, G-O	TS	–
Parragh and Doerner (2018)	Home Health Care	min DC, O	L-C, L-TW, S-O, G-T, G-O	ALNS	–
Polnik et al. (2020)	Home Health Care	min TT	L-TW, S-O, G-T, G-O	MH	–
Pour et al. (2019)	Railway maintenance	min TT	L-TW, S-O, G-T, G-O	DA, MS	–
Rasmussen et al. (2012)	Home Health Care	min DC	L-O, L-TW, S-O, G-T, G-O	DW	–
Roozbeh et al. (2020)	Not specified	O	L-TW, S-O, G-T, G-O	ALNS	–
Rousseau et al. (2013)	Not specified	min DC	L-C, L-TW, S-O, G-T, G-O	H	D-T
Shi et al. (2020)	Not specified	min DC, FC, O	L-TW, S-O, G-T, G-O	SA, TS	U-TT, U-ST
Tilk et al. (2019)	Not specified	min DC, FC, O	S-O, G-T, G-O, L-O	BP, BC	–

Legend: Objective function – DC: Distance costs, FC: Vehicle fixed costs; RD: Route duration; M: Makespan; D: Delays; VCP: Vehicle-customer preferences; TT: Travel time; OT: Operation time; PT: No. of performed tasks; WT: Wait time; O: Other. **Problem aspects** – *Local aspects (L-):* C: Capacity constraints; TW: Time Window constraints; RL: Route Length constraints; *Synchronisation aspects (S-):* O: Operation constraints; M: Movement constraints; *Global aspects (G-):* T: Task constraints; O: Operation constraints; R: Resource constraints. **Solution method** – ABC: Artificial Bee Colony; ACO: Ant Colony Optimisation; ALNS: Adaptive Large Neighbourhood Search; BC: Branch-and-Cut; BP: Branch-and-Price; DA: Decomposition Approach; DP: Dynamic Programming; DW: Dantzig-Wolfe Decomposition; GA: Genetic Algorithm; GRASP: Greedy Randomised Adaptive Search Procedure; H: Heuristic approach; ILS: Iterated Local Search; LNS: Large Neighbourhood Search; MA: Memetic Algorithm; MCS: Monte-Carlo Simulation; MH: Matheuristic; MS: Mathematical Solver (MIP/CP); PSO: Particle Swarm Optimisation; SA: Simulated Annealing; TS: Tabu Search; VNS: Variable Neighbourhood Search; N/A: Not applicable/not provided. **Sources of uncertainty (U)** – TT: Travel times; ST: Service times. **Sources of dynamism (D)** – T: Tasks.

4.3 Routing with transfers or cross-docking requirements

The literature review also shows that synchronisation is common in routing problems that acknowledge cross-docking requirements or transfers of merchandise or people. In these types of problems, transfer locations exist where it is necessary to ensure operation synchronisation between the drop-off and collection of the request at the transfer/cross-docking location by different vehicles. Unlike problems from the previous category, routing problems with transfers or cross-docking are characterised as less scheduling-centric since synchronisation is typically confined to fewer tasks (transfer or cross-docking tasks), and more typical routing aspects are considered, such as capacity constraints.

Table 6 supports this discussion by classifying each reference framed in this problem category.

Predominant applications Although routing problems with transfers or cross-docking requirements are not new, the incorporation of operation synchronisation in these problems seems to be increasingly relevant. In fact, more than half of the collected references date from 2019 onwards. Last-mile delivery is the most predominant and natural application of routing problems with transfers or cross-docking. Within the context of transfers, the PDP with Transfers (e.g., Masson, Lehuédé, & Péton, 2013; Peng, Al Chami, Manier, & Manier, 2019) and the VRP with Cross-Docking (e.g., Grangier, Gendreau, Lehuédé, & Rousseau, 2017, 2019; Yin & Chuang, 2016) are the most representative problem variants. In the PDP with Transfers, vehicles must perform pickup and delivery requests, but the pickup task may not necessarily be performed by the same vehicle that will deliver it. As for the VRP with Cross-Docking, cross-docking locations are used to consolidate and transfer load between vehicles. To satisfy these problem requirements, it is necessary to ensure that the drop-off of merchandise at transfer/cross-docking locations occurs before their collection by another vehicle. Similarly to what happens with the VRP with Cross-Docking, multiple-echelon VRPs are also a predominant variant (e.g., Dellaert, Woensel, Crainic, & Saridarq, 2021; Grangier, Gendreau, Lehuédé, & Rousseau, 2016), which also involve operation synchronisation in order to take into account the temporal precedence relationships between routes of consecutive echelons of the routing network. Recent literature considers more diverse problem applications where transfers are motivated by the usage of different types of vehicles or types of transportation. For example, Anderluh, Hemmelmayr, and Nolz (2017); Anderluh, Nolz, Hemmelmayr, and Crainic (2019) consider a routing problem where outer-city transportation is performed by vans and inner-city distribution is performed by cargo bikes. For this purpose, load transfer must be synchronised between vehicles at specific locations. Although the transportation of merchandise is the most significant business context, transfer requirements can also be found in routing problems targeted for people transportation (e.g., Masson, Lehuédé, & Péton, 2014).

Objective functions The objective function components found in these problems are more traditional and in line with the ones typically found in standard routing problems. In comparison with routing problems considering the synchronisation of schedules, the variety of optimisation objectives is significantly less diverse. Distance costs are the most prevalent objective being minimised (e.g., Nolz, Absi, Cattaruzza, & Feillet, 2020), followed by the minimisation of vehicle fixed costs (e.g., Qu & Bard, 2012). These objectives are frequently minimised simultaneously (e.g., Mirhedayatian, Crainic, Guajardo, & Wallace, 2019). Other alternative objectives can still be found, such as the minimisation of the total route duration (e.g., Brandstätter, 2019; Brandstätter & Reimann, 2018), the minimisation of waiting times (e.g., H. Li, Wang, Chen, & Bai, 2021) or even the minimisation of the makespan (e.g., Salazar-Aguilar, Langevin, & Laporte, 2013). The predominance of traditional objective functions (e.g., distance costs, fixed costs) in this problem category to the detriment of time-related objective components (e.g., delays, travel time) is probably a consequence of synchronisation aspects not being as predominant as the previous problem category.

Predominant problem aspects Within this category of problems, operation synchronisation is used to establish temporal precedence between the unloading and loading tasks of different vehicles. This is necessary to ensure that a vehicle does not collect goods at a transfer location before the other vehicle arrives. Besides synchronisation, these problems are typically subject to several local and global aspects. Regarding local aspects, capacity constraints are frequently considered (e.g., Medina, Hewitt, Lehuédé, & Péton, 2019; Rais, Alvelos, & Carvalho, 2014). In addition to capacity constraints, time window constraints are also commonly found in routing problems with transfers or cross-docking requirements (e.g., Brandstätter & Reimann, 2018; Qu & Bard, 2012).

Solution approaches In terms of approximate methods, Large Neighbourhood Search and its adaptive version are commonly used (e.g., Anderluh, Larsen, et al., 2019). This is in part because many of these problems are a natural generalisation of more standard routing problem variants, such as the Pickup and Delivery Problem, for which ALNS and LNS have been successfully applied. Other commonly used approximate methods encompass population-based heuristics such as ant colony optimisation (e.g., Huang, Blazquez, Huang, Paredes-Belmar, & Latorre-Nuñez, 2018).

The overall structure of these problems also favours the adoption of decomposition approaches and multi-phase heuristic methods. For example, solving one distribution echelon at a time and handling the synchronisation afterwards is a possible approach to reduce complexity (e.g., Nolz et al., 2020). These methods tackle problems in an incremental way, breaking them down into smaller subproblems at the expense, however, of increasing the risk of suboptimality.

Although less common, exact methods can also be found, most of them consisting of column generation approaches such as branch-and-price and branch-and-cut (e.g., Marques, Sadykov, Deschamps, & Dupas, 2020).

The problem instances adopted for this category of problems differ greatly. While some references use real-life instances (e.g., Masson et al., 2013, 2014), a significant number of works generate and adapt instances from Solomon (1987), initially destined to the VRP with Time Windows (e.g., Anderluh et al., 2017; Grangier et al., 2016). Other works resort to the random generation of new instances (e.g., Dellaert et al., 2021; Medina et al., 2019).

Routing under uncertainty and/or dynamism This problem category lacks references that consider uncertain parameters or unknown information. Only uncertainty in travel times has so far been acknowledged. In the context of city logistics, Anderluh, Larsen, et al. (2019) use a Monte Carlo simulation with an optimisation approach for a 2-echelon VRP involving the synchronisation of vans and bicycles at satellite locations considering uncertain travel times. Medbøen, Holm, Msakni, Fagerholt, and Schütz (2020) consider uncertain travel times triggered by harsh weather conditions in a maritime transportation problem, also resorting to a simulation-optimisation framework. These observations suggest there are opportunities to explore alternative methodologies to tackle the issues of uncertainty in these problems.

Table 6: References pertaining to routing problems with transfers or cross-docking requirements

Reference	Application	Objective function	Problem aspects	Solution method	Uncertainty and/or dynamism
Anderluh et al. (2017)	Last-mile delivery; Bike delivery	min DC, FC, RD	L-C, L-RL, S-O, G-T, G-O	GRASP	–
Anderluh, Larsen, et al. (2019)	Last-mile delivery; Bike delivery	min DC, FC, RD	L-C, L-RL, S-O, G-T, G-O	MCS	U-TT
Anderluh, Nolz, et al. (2019)	Last-mile delivery; Bike delivery	min DC, FC, RD	L-C, L-RL, S-O, G-T, G-O	LNS	–
Brandstätter and Reimann (2018)	Last-mile delivery	min DC, FC, RD	L-O, L-TW, S-O, G-T, G-O	H	–
Brandstätter (2019)	Last-mile delivery	min DC, RD	L-C, L-RL, S-O, G-T, G-O	ACO, MH	–
Dellaert et al. (2021)	City logistics	min DC, FC	L-TW, S-O, G-T, G-O, L-O	BP	–
Grangier et al. (2016)	City logistics	min DC, FC	L-O, S-O, G-T, G-O	ALNS	–
Grangier et al. (2017)	Cross-docking	min DC	L-O, S-O, G-T, G-O	LNS	–
Grangier et al. (2019)	Cross-docking	O	L-O, S-O, G-T, G-O, G-R	MH	–
Huang et al. (2018)	Not specified	min DC, FC	L-O, L-C, L-RL, S-O, G-T, G-O	ACO	–
H. Li et al. (2021)	Last-mile delivery	min DC, FC, WT	L-C, L-TW, S-O, G-T, L-O	MS, ALNS	–
Marques et al. (2020)	Not specified	min DC, FC	L-C, S-O, G-T, G-O, G-R	BC, BP	–
Masson et al. (2013)	People transportation	min DC	L-C, S-O, G-T, G-O	ALNS	–
Masson et al. (2014)	People transportation	min DC	L-O, L-C, L-RL, S-O, G-T, G-O	ALNS	–
Medbøen et al. (2020)	Ship routing	min D, O	L-RL, S-O, G-T, G-O	MS	U-TT
Medina et al. (2019)	Long-haul and local transportation	min FC, O	L-O, L-C, L-TW, S-O, G-T, G-O	MH	–
Mirhedayatian et al. (2019)	Courier transportation	min DC, FC, O	L-O, S-O, G-T, G-O, L-O	DA	–
Nolz et al. (2020)	City logistics	min DC	L-C, L-TW, S-O, G-T, G-O, L-O	MS	–
Peng et al. (2019)	Not specified	min DC, O	L-O, L-C, S-O, G-T, G-O	PSO	–
Qu and Bard (2012)	Air transportation	min FC	L-C, L-TW, S-O, G-T	GRASP, ALNS	–
Rais et al. (2014)	Not specified	min DC	L-O, L-C, L-TW, S-O, G-T, G-O	MS	–
Salazar-Aguilar et al. (2013)	Road marking	min M	L-C, S-O, G-T, G-O	ALNS	–
Yin and Chuang (2016)	Not specified	min DC, FC	L-O, L-C, S-O, G-T, G-O	ABC	–

Legend: Objective function – DC: Distance costs, FC: Vehicle fixed costs; RD: Route duration; M: Makespan; D: Delays; VCP: Vehicle-customer preferences; TT: Travel time; OT: Operation time; PT: No. of performed tasks; WT: Wait time; O: Other. **Problem aspects** – *Local aspects (L-):* C: Capacity constraints; TW: Time Window constraints; RL: Route Length constraints; *Synchronisation aspects (S-):* O: Operation constraints; M: Movement constraints; *Global aspects (G-):* T: Task constraints; O: Operation constraints; R: Resource constraints. **Solution method** – ABC: Artificial Bee Colony; ACO: Ant Colony Optimisation; ALNS: Adaptive Large Neighbourhood Search; BC: Branch-and-Cut; BP: Branch-and-Price; DA: Decomposition Approach; DP: Dynamic Programming; DW: Dantzig-Wolfe Decomposition; GA: Genetic Algorithm; GRASP: Greedy Randomised Adaptive Search Procedure; H: Heuristic approach; ILS: Iterated Local Search; LNS: Large Neighbourhood Search; MA: Memetic Algorithm; MCS: Monte-Carlo Simulation; MH: Matheuristic; MS: Mathematical Solver (MIP/CP); PSO: Particle Swarm Optimisation; SA: Simulated Annealing; TS: Tabu Search; VNS: Variable Neighbourhood Search; N/A: Not applicable/not provided. **Sources of uncertainty (U)** – TT: Travel times; ST: Service times. **Sources of dynamism (D)** – T: Tasks.

4.4 Routing with autonomous vehicles

The review also showed that a significant subset of references pertains to routing problems involving autonomous vehicles. These problems typically involve two different types of vehicles – regular vehicles and autonomous vehicles – the latter is an auxiliary vehicle to the former. The limited autonomy of autonomous vehicles makes them only able to travel up to certain distances, and they may need to return to regular vehicles for charging or picking up new requests to be delivered.

Table 7 supports this discussion by presenting each reference framed in this problem category.

Predominant applications The emergence of routing problems with synchronisation considering autonomous vehicles is fairly recent. This can be sustained by the fact that the overwhelming majority of the collected references date from 2018 onwards.

The literature identifies several advantages in considering autonomous vehicles in routing problems. Especially in unmanned aerial vehicles (e.g., drones), their ability to fly allows them to bypass traffic congestion and physical obstacles, thus leading to faster and more efficient delivery times. Another advantage consists in the smaller environmental footprint that autonomous vehicles have when compared to traditional vehicles.

Last-mile delivery is the main application where this category of problems arises (e.g., Agatz, Bouman, & Schmidt, 2018; Murray & Chu, 2015). In this context, a customer is visited by either a truck or an autonomous vehicle. Autonomous vehicles use trucks as mobile intermediate depots, where they can retrieve the packages to be delivered, thus attenuating their limited autonomy and capacity.

Objective functions The objective functions considered in these types of problems tend to lean towards the minimisation of time-related components, such as travel times (e.g., Simoni, Kutanoglu, & Claudel, 2020), total route duration (e.g., Kitjacharoenchai et al., 2019) or the overall makespan (e.g., Es Yurek & Ozmutlu, 2018). In fact, problems acknowledging the classical objective of minimising the distance costs are seldom found (e.g., Coindreau, Gallay, & Zufferey, 2021). This may possibly be attributed to the fact that these problems are typically motivated by the need to increase service level rather than the cost decrease. Other less common objectives concern the minimisation of delays (e.g., Boysen, Schwerdfeger, & Weidinger, 2018) or the minimisation of waiting times (e.g,. H. Li, Wang, Chen, & Bai, 2020).

Predominant problem aspects Different synchronisation aspects in this category of problems depend on the underlying assumptions. Although they are autonomous, the limited autonomy of drones requires that they may need to be "parked" on the truck for charging while the truck is performing its route. In these situations, the truck and drone are subject to movement synchronisation, as they are moving simultaneously (e.g., Es Yurek & Ozmutlu, 2018). Traditional vehicles may also support the service of autonomous vehicles, namely in terms of replenishment of goods, thus triggering the need for operation synchronisation between these two types of vehicles (e.g., Mourad, Puchinger, & Woensel, 2021).

Aside from synchronisation, this category of problems is characterised by having a significant number of local aspects. One of the most important local aspects to consider in VRPs involving autonomous vehicles is operation constraints. These constraints are necessary to ensure that the autonomous vehicles respect certain sequences in their routes, which is especially relevant when they can only perform one delivery at a time before having to return to the support vehicle (e.g., Dell'Amico, Montemanni, & Novellani, 2019). Another local aspect commonly found is route length constraints. For example, an autonomous vehicle may only be able to travel a certain distance before needing to recharge its battery or return to a depot. This can significantly impact the feasibility of certain routes and must be taken into account when solving VRPs involving autonomous vehicles (e.g., Coindreau et al., 2021). In addition to the above local aspects, capacity constraints may also impact VRPs with autonomous vehicles, especially if the vehicles can perform multiple deliveries before reloading (e.g., Kitjacharoenchai & Lee, 2019). The possibility of this depends on load size and the capacity of the autonomous vehicle.

As far as global aspects are concerned, the involvement of multiple delivery vehicles may trigger the need to account for task constraints where each customer must be served by its appropriate type of vehicle. This requirement is vital for applications where only one type of vehicle can perform a certain task. At the same time, the coordination requirements between vehicles, namely in terms of synchronisation, may also require specific operation constraints that ensure that synchronised operations (e.g., truck and drone meetup) be performed by different vehicles.

Solution approaches In terms of approximate methods, Adaptive Large Neighbourhood Search (ALNS) continues to be the most common approach (e.g., Kitjacharoenchai, Min, & Lee, 2020). It should be noted that very few population-based methods were found. Kitjacharoenchai et al. (2019) present a genetic algorithm for its constructive heuristic. Exact approaches, on the other hand, are mostly confined to branch-and-cut (Schermer, Moeini, & Wendt, 2020; Tamke & Buscher, 2021) or the use of general-purpose commercial solvers (e.g., Reed, Campbell, & Thomas, 2022). Other approaches like dynamic programming (e.g., Simoni et al., 2020) or decomposition approaches (e.g., Es Yurek & Ozmutlu, 2018) can also be found with the purpose of reducing complexity and dividing the problem into smaller sub-problems.

In terms of problem instances, a significant portion of the references use randomly generated instances for their computational experiments (e.g., Agatz et al., 2018; Coindreau et al., 2021; Es Yurek & Ozmutlu, 2018). Some works adapt existing instances from the literature. For example, Kitjacharoenchai and Lee (2019) adapt existing instances from Augerat (1995), which focus on the Capacitated VRP. One can also find instances adapted from Solomon (1987), which tackle the VRP with Time Windows (e.g., H. Li et al., 2020).

Routing under uncertainty and/or dynamism Very few problems are set within dynamic or uncertain settings. Within the scope of dynamic routing, Dayarian, Savelsbergh, and Clarke (2020) present a VRP with drone resupply, where orders arrive dynamically. In this problem, the number of locations to serve may not be known when the vehicle departs from the depot, and therefore the routes need to be periodically re-optimised. Considering problems subject to uncertainty, Mourad et al. (2021) present a pickup and delivery problem where autonomous robot vehicles may take advantage of public transportation services to extend their geographical reach. However, the robots depend on the passenger's stochastic demand for the service since passengers are prioritised over robots. The scarcity of literature on uncertainty and dynamism supports the thesis that this topic is still largely unexplored.

Table 7: References pertaining to routing problems with autonomous vehicles

Reference	Application	Objective function	Problem aspects	Solution method	Uncertainty and/or dynamism
Agatz et al. (2018)	Last-mile delivery	min DC	L-O, L-RL, S-O, S-M, G-T, G-O	DP	–
Coindreau et al. (2021)	Unmanned aerial vehicles	min DC, TT, M	L-O, L-TW, L-RL, S-O, S-M, G-T	ALNS	–
Dayarian et al. (2020)	Unmanned aerial vehicles	max PT	L-O, L-TW, S-O	MH	D-T
Dell'Amico et al. (2019)	Last-mile delivery; Unmanned aerial vehicles	min RD	L-O, L-RL, S-O, S-M, G-T, G-O	MS	–
Es Yurek and Ozmutlu (2018)	Last-mile delivery; Unmanned aerial vehicles	min M	L-O, L-RL, S-O, S-M, G-T, G-O	MH	–
Kitjacharoenchai et al. (2019)	Last-mile delivery; Unmanned aerial vehicles	min RD	L-O, L-RL, S-O, S-M, G-T, G-O	H	–
Kitjacharoenchai and Lee (2019)	Last-mile delivery; Unmanned aerial vehicles	min TT	L-O, L-C, S-M, G-T	MS, LNS	–
Kitjacharoenchai et al. (2020)	Last-mile delivery; Unmanned aerial vehicles	min TT	L-O, L-C, S-M, G-T	MS, ALNS	–
H. Li et al. (2020)	Last-mile delivery; Unmanned aerial vehicles	min DC, OT, WT	L-C, L-RL, S-O, S-M, G-T, L-O	MS, ALNS	–
Mourad et al. (2021)	People transportation; Autonomous robots	O	L-C, L-O, S-O, G-T, G-O	ALNS	U-D
Murray and Chu (2015)	Last-mile delivery; Unmanned aerial vehicles	min M	L-O, L-RL, S-O, S-M, G-T, G-O	H	–
Reed et al. (2022)	Last-mile delivery	min RD	L-O, S-O, S-M, G-T, G-O	MS	–
Schermer et al. (2020)	Last-mile delivery; Unmanned aerial vehicles	min M	S-O, S-M, G-T, G-O	BC	–
Simoni et al. (2020)	City logistics; Autonomous robots	min TT	L-C, S-O, G-T, G-O	DP	–
Tamke and Buscher (2021)	Last-mile delivery; Unmanned aerial vehicles	min M	L-O, L-RL, S-O, S-M, G-T, G-O	BC	–

Legend: Objective function – DC: Distance costs, FC: Vehicle fixed costs; RD: Route duration; M: Makespan; D: Delays; VCP: Vehicle-customer preferences; TT: Travel time; OT: Operation time; PT: No. of performed tasks; WT: Wait time; O: Other. **Problem aspects** – *Local aspects (L-):* C: Capacity constraints; TW: Time Window constraints; RL: Route Length constraints; *Synchronisation aspects (S-):* O: Operation constraints; M: Movement constraints; *Global aspects (G-):* T: Task constraints; O: Operation constraints; R: Resource constraints. **Solution method** – ABC: Artificial Bee Colony; ACO: Ant Colony Optimisation; ALNS: Adaptive Large Neighbourhood Search; BC: Branch-and-Cut; BP: Branch-and-Price; DA: Decomposition Approach; DP: Dynamic Programming; DW: Dantzig-Wolfe Decomposition; GA: Genetic Algorithm; GRASP: Greedy Randomised Adaptive Search Procedure; H: Heuristic approach; ILS: Iterated Local Search; LNS: Large Neighbourhood Search; MA: Memetic Algorithm; MCS: Monte-Carlo Simulation; MH: Matheuristic; MS: Mathematical Solver (MIP/CP); PSO: Particle Swarm Optimisation; SA: Simulated Annealing; TS: Tabu Search; VNS: Variable Neighbourhood Search; N/A: Not applicable/not provided. **Sources of uncertainty (U)** – TT: Travel times; ST: Service times. **Sources of dynamism (D)** – T: Tasks.

4.5 Routing with trailers or passive vehicles

The fourth and final major category considers problems with the involvement of trailers or other passive vehicles. These problems are characterised by the dependence relationships between vehicles: one vehicle may need to traverse an arc only if another vehicle is able to traverse another arc. Table 8 supports this discussion by presenting each reference framed in this problem category.

Predominant applications The most representative application of this problem category consists of routing problems involving vehicles that cannot move autonomously between locations. These restrictions are most typically found in problems involving the routing of trailers that require an auxiliary vehicle for them to move. The most representative application is the Truck and Trailer Routing Problem (TTRP) (Chao, 2002). The TTRP consists in determining the routes for a set of trucks that may have a trailer coupled to them. Trailers allow a reduction of the number of routes needed to perform deliveries to customers by increasing transportation capacity; however, due to site-dependent constraints, some customers are unable to be visited by trailers, so trucks must first visit a transhipment node, transfer load between the truck and trailer and uncouple. Afterwards, the truck can perform sub-tours to trailer-incompatible customers. Another problem variant is the Active-Passive VRP (APVRP) (Meisel & Kopfer, 2012). One of the applications of this problem is in drayage operations, where active vehicles (trucks) are responsible for transporting passive vehicles (containerised cargo) from a pickup to a delivery location, taking into account that there is no pre-defined assignment of trailers to trucks, unlike the TTRP. Other applications involving passive vehicles arise, e.g., in airport ground handling. The Driver and Vehicle Routing Problem (e.g., Domínguez-Martín, Rodríguez-Martín, & Salazar-González, 2018b) intends to not only schedule tasks but also assign drivers to the vehicles throughout the day.

Objective functions The minimisation of distance costs is the predominant objective function component being optimised in problems with trailers or passive vehicles (e.g., Derigs, Pullmann, & Vogel, 2013), being also sometimes combined with the minimisation of vehicle fixed costs (e.g., Drexl, 2014). This observation is in line with what would be expected from typical problems in freight transportation, where a balance between these two components is critical. Other objectives concern the minimisation of total route duration (e.g., Soares, Marques, Amorim, & Rasinmäki, 2019) is one example of them, as well as the minimisation of the makespan (e.g., Salazar-Aguilar, Langevin, & Laporte, 2012) or the minimisation of delays (e.g., Fink et al., 2019; Furian et al., 2018).

Predominant problem aspects For this problem category, movement synchronisation is crucial for correctly establishing the active-passive dependencies between operations. At the same time, operation synchronisation is also typically required for simultaneous movement synchronisation between vehicles.

Concerning local aspects, capacity constraints are usually required in problems concerning freight transportation (e.g., Drexl, 2014). However, for applications involving the movement synchronisation of drivers and vehicles, capacity issues are mostly discarded. Operation constraints are also necessary to account for successor/predecessor relationships within each route.

On global aspects, task constraints ensure that each task is performed by compatible vehicles. Operation constraints are also necessary for ensuring that given operations are performed by the same vehicle or by different vehicles. However, it should be noted that in certain applications, certain tasks and operations may have to be considered optional (e.g., Derigs et al., 2013). For example, the TTRP needs to account for alternatives on how trailer-compatible customers are served – either by a truck with a trailer or only by a truck – as well as potential transhipment. In other problems, like the APVRP, tasks must be considered optional such that a "best-effort" solution can be achieved (e.g., Tilk et al., 2018).

Solution approaches With regards to exact methods, the literature shows a tendency for the use of column-generation methods (e.g., Rothenbächer, Drexl, & Irnich, 2018; Tilk et al., 2018). Standalone branch-and-cut approaches are also commonly found (e.g., Domínguez-Martín, Rodríguez-Martín, & Salazar-González, 2018a). The predominance of these approaches in this problem category can probably be explained by the large size that these problems and resulting models can take, allied with the potential that synchronisation constraints can have in the generation of cutting planes and the elimination of large portions of the branching tree.

Within the realm of approximate methods, ALNS continues to be highly used (e.g., Parragh & Cordeau, 2017). Some works involving movement synchronisation also use other metaheuristic concepts such as Variable Neighbourhood Search (e.g., Coindreau, Gallay, & Zufferey, 2019; Ritzinger, Hu, Koller, & Dragaschnig, 2017) or Tabu Search (e.g., Xue, Zhang, Lin, Miao, & Yang, 2014).

These solutions approaches are used in instances with different characteristics. For the TTRP, Chao (2002) proposed a set of benchmark instances, which were generally adopted by subsequent research (e.g., Derigs et al., 2013). However, most works either adapt existing benchmark instances from other problems (e.g., Parragh & Cordeau, 2017) or rely on the random generation of new instances (e.g., Hu & Wei, 2018; Meisel & Kopfer, 2012). Other works perform computational experiments with real-life instances (e.g., Coindreau et al., 2021; Xue et al., 2014).

Routing under uncertainty and/or dynamism Uncertainty and dynamism issues in this problem category are practically non-existent, which sustains the pattern observed in the other problem categories that were discussed and analysed.

The literature does not exhibit any problems attending to uncertainty issues. As for dynamic routing, only Fikar, Juan, Martinez, and Hirsch (2016a) present a discrete-event-driven metaheuristic for a routing problem with combined trip sharing and walking within the home health care application. Home carers may move between locations by foot (if the destination is within walking distance) or be transported by a vehicle that can transport multiple carers. The solution algorithm iteratively generates new solutions based on the occurrence of events over time, such as the pickup or delivery of carers by the vehicles or the assignment of the next customer to a home carer.

Table 8: References pertaining to routing problems with trailers or passive vehicles

Reference	Application	Objective function	Problem aspects	Solution method	Uncertainty and/or dynamism
Chao (2002)	Not specified	min DC	L-C, L-O, S-O, S-M, G-T, G-O, G-R	TS	–
Coindreau et al. (2019)	Energy provider company	min DC	L-RL, S-O, S-M, G-T, G-O	VNS	–
Derigs et al. (2013)	Not specified	min DC	L-C, L-O, S-O, S-M, G-T, G-O, G-R	LS, LNS	–
Domínguez-Martín et al. (2018a)	Airline flight scheduling	min DC	S-O, S-M, G-T, G-O	BC	–
Domínguez-Martín et al. (2018b)	Airline flight scheduling	O	L-RL, S-O, S-M, G-T, L-O	BC	–
Drexl (2014)	Not specified	min DC, FC	L-C, S-O, S-M, G-T, G-O	BC	–
Fikar, Juan, Martinez, and Hirsch (2016b)	Home Health Care	min TT, WT	S-O, S-M, G-T, G-O	H	D-T
Fink et al. (2019)	Airport ground handling	min RD, D	S-O, S-M, G-T, G-O	BP	–
Furian et al. (2018)	Patient transportation	min D	L-O, S-O, S-M, G-T	H	–
Hu and Wei (2018)	Big-size cargo transportation	min M	S-O, S-M, G-T, G-O	ACO	–
Meisel and Kopfer (2012)	Container transportation	min DC, RD, max PT	L-O, L-TW, S-O, S-M, G-T, G-O	ALNS	–
Parragh and Cordeau (2017)	Infrastructure service providers	min DC	L-C, L-O, S-O, S-M, G-T, G-O, G-R	BP, ALNS	–
Ritzinger et al. (2017)	Container drayage	O	L-O, S-O, S-M, G-T, G-O	VNS	–
Rothenbächer et al. (2018)	Raw milk collection	O	L-O, S-O, S-M, G-T, G-O, G-R	BC, BP	–
Salazar-Aguilar et al. (2012)	Snow plowing	min M	S-O, S-M, G-O	ALNS	–
Soares et al. (2019)	Biomass residues production and delivery	min DC, FC, RD	L-O, S-O, S-M, G-T, G-O	MH	–
Tilk et al. (2018)	Not specified	min DC, RD, max PT	L-O, L-TW, S-O, S-M, G-T, G-O	BP, BC	–
Xue et al. (2014)	Container drayage	min DC, FC	L-O, L-TW, S-O, S-M, G-T, G-O	TS	–
Yakıcı, Dell, Hartman, and McLemore (2018)	Military	min DC, O	L-RL, S-O, S-M, G-T, G-O	ACO	–

Legend: Objective function – DC: Distance costs, FC: Vehicle fixed costs; RD: Route duration; M: Makespan; D: Delays; VCP: Vehicle-customer preferences; TT: Travel time; OT: Operation time; PT: No. of performed tasks; WT: Wait time; O: Other. **Problem aspects** – *Local aspects (L-):* C: Capacity constraints; TW: Time Window constraints; RL: Route Length constraints; *Synchronisation aspects (S-):* O: Operation constraints; M: Movement constraints; *Global aspects (G-):* T: Task constraints; O: Operation constraints; R: Resource constraints. **Solution method** – ABC: Artificial Bee Colony; ACO: Ant Colony Optimisation; ALNS: Adaptive Large Neighbourhood Search; BC: Branch-and-Cut; BP: Branch-and-Price; DA: Decomposition Approach; DP: Dynamic Programming; DW: Dantzig-Wolfe Decomposition; GA: Genetic Algorithm; GRASP: Greedy Randomised Adaptive Search Procedure; H: Heuristic approach; ILS: Iterated Local Search; LNS: Large Neighbourhood Search; MA: Memetic Algorithm; MCS: Monte-Carlo Simulation; MH: Matheuristic; MS: Mathematical Solver (MIP/CP); PSO: Particle Swarm Optimisation; SA: Simulated Annealing; TS: Tabu Search; VNS: Variable Neighbourhood Search; N/A: Not applicable/not provided. **Sources of uncertainty (U)** – TT: Travel times; ST: Service times. **Sources of dynamism (D)** – T: Tasks.

5 Discussion and conclusions

Synchronisation is becoming an ever more interesting topic when solving real-world transportation problems. This paper provided a systematisation of knowledge on routing problems with synchronisation in an attempt to organise the topic and unveil opportunities for future research.

Consequently, this research provided a clarification of the synchronisation concept. We adopt and suggest a narrower scope on the concept of synchronisation, using it only when it results from interdependencies between routes. With this criterion in mind, we establish that synchronisation can be summarised into two main types, consisting of either temporally synchronising tasks or making a vehicle's movement depend on another vehicle's movement.

The rationale behind this approach was supported by a classification schema based on existing schemas in the literature. The schema classifies different problem aspects in terms of the interdependencies they trigger in the routing problem: whether it is within a single route, multiple routes, among the entire routing plan or between different processes of the supply chain.

To further clarify the concept of synchronisation, a general class of the Vehicle Routing Problem with Synchronisation was defined by means of a mathematical formulation, which models both types of synchronisation. An extended modelling framework was also devised, which contains other common routing aspects that can be frequently found along with synchronisation. This formulation, along with the classification schema, served as a basis for classifying and organising the literature into four major categories of problems.

It has been found that routing problems focused on the synchronisation of schedules (e.g., Mankowska et al., 2013) constitute a large portion of the reviewed references. However, given the number of recent papers on routing problems with autonomous vehicles (e.g., Reed et al., 2022), one can probably state that this class of problems has been the subject of more active research in recent years.

Routing problems with synchronisation of schedules are typically characterised by having synchronisation with precedence or simultaneity as its most predominant problem aspect. Besides time window constraints, traditional routing aspects, such as capacity or route length constraints, are seldom found. Routing problems with trailers or passive vehicles (e.g., Meisel & Kopfer, 2012) is the problem category where movement synchronisation is most predominant. In the remaining problem categories, synchronisation is viewed as a complement to routing problems with more traditional routing aspects, such as capacity or route length. For example, although problems with transfers and cross-docking requirements (e.g., Masson et al., 2013) typically still acknowledge operation synchronisation, these requirements are found in more specific tasks of the problem. In a similar fashion, problems with autonomous vehicles typically use both operation and movement synchronisation to complement typical routing requirements. The current research trend observed in the topic of autonomous vehicles suggests a possible research trend towards problems where synchronisation complements more traditional routing constraints to the detriment of scheduling and synchronisation-focused problems.

The different problem categories exhibit distinct problem applications. With regard to problems with transfers, cross-docking or autonomous vehicles, last-mile delivery is the prime application of these problems. Nevertheless, in the case of routing problems with transfers, people transportation is also a significant application to be highlighted. Problems with trailers or passive vehicles typically appear in the context of freight transportation involving trucks and trailers. However, some references leverage the concepts of active and passive vehicles to tackle other routing applications, such as the simultaneous scheduling and routing of staff with vehicles or other applications requiring a passive resource to be transported by a vehicle. Considering the category of problems with synchronisation of schedules, it has mostly been driven by its application to home health care. However, these problems also appear in the context of routing challenges that require that a shared resource cannot be used simultaneously by more than one vehicle (e.g., charging stations and the charging of electric vehicles). This problem category probably exhibits the most potential to be researched in the context of new problem applications that require operation synchronisation.

The review has shown that a variety of objective functions are used, depending on the specific problem category being analysed. Problems with synchronisation of schedules and problems with autonomous vehicles trend toward more scheduling-focused objectives, such as the minimisation of travel times, makespan, waiting times or delays. This observation is a reflection of the motivations behind these problems, which typically are guided towards the improvement of service level. As for problems with transfers or cross docking and problems with trailers or passive vehicles, the literature trends toward more classical objective functions, such as distance costs and vehicle fixed costs. This trend reflects the focus of these problems on operational efficiency. Therefore, it is expected that these objective functions will continue to be addressed in future research, given their relevance in each of their respective problem categories.

Although there has been an effort in the development of solution methods for VRPs with synchronisation, it is the opinion of the authors that additional work must still be done to overcome the underlying complexity and combinatorial nature of these routing problems. The ALNS heuristic is commonly used in every problem category due to its general popularity in the routing field. However, it appears that problem categories where synchronisation is more critical to solution feasibility – routing problems with synchronisation of schedules and problems with trailers or passive vehicles – tend towards the adoption of column-generation and/or branch-and-cut approaches. On the other hand, problems with transfers, cross-docking or autonomous vehicles appear to favour approaches based on dynamic programming concepts or matheuristics.

For some problem categories, the process of benchmarking and comparing different solution approaches is still hampered by the fact that there is still no commonly adopted set of instances for routing problems with synchronisation. Computational experiments typically resort to randomly generated instances or instances specially targeted towards tackling a real-life problem. Nevertheless, in the case of problems with synchronisation of schedules and in truck-and-trailer routing, the literature appears to be converging towards the adoption of a set of common benchmarking instances. Therefore, this issue is expected to be tackled as future research work emerges.

It is also possible to conclude from this review that very few problems considering uncertainty or dynamism exist. The approaches so far adopted in the realm of routing with uncertainty sources have so far limited themselves to simulation, with little adoption of alternative approaches. This constitutes one of the possible research avenues for future work. With regards to dynamic planning, it is expected that the continuous development of more efficient and effective solution methods for deterministic and static problems will also ease the barriers to entry into the topic of dynamic planning.

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A Extended modelling framework for the Vehicle Routing Problem with Synchronisation

This section provides an extended and more complete version of the mathematical formulation for the VRP with synchronisation constraints, according to the previously described conceptual and classification schema. This complete version provides a flexible mathematical formulation to model synchronisation aspects together with other common problem requirements. The formulation that is presented will not focus on efficiency or scalability but rather on systematisation and consistency.

A.1 Assumptions

Similarly to the simplified version, we consider a generic vehicle routing problem base formulation, whose purpose is to perform a set of n tasks, geographically dispersed through m locations, by means of a set of routes being performed by a set of K vehicles. The routes start and end in a depot through tasks 0 and $n+1$, respectively.

Whenever possible, this modelling framework takes advantage of the notation and equations already presented in the simplified formulation. Therefore, the formulation will be presented in an incremental fashion.

In the simplified mathematical formulation, it is assumed that each vehicle performs only one route, and therefore these concepts may be used interchangeably. In this extended modelling framework, vehicles can perform multiple trips during the planning horizon by assuming that a vehicle route can have multiple legs that start and end in the depot. In this case, the depot location can have optional tasks for each vehicle, which are specifically designed for finishing and starting trips. For each of these tasks, the vehicle will be able to enter it to finish a trip, after which it will exit that task in order to start a new trip. The number of tasks that need to be generated for each vehicle under these conditions depends on the problem size, being $n-1$ a possible upper bound.

A.2 Sets, parameters, decision variables and objective function

Most of the sets and parameters of the problem have already been presented in Table 2. However, in this formulation, some notation needs to be added or extended. Table 9 presents the sets and parameters to be added or redefined.

The transportation network $\mathcal{G} = (\mathcal{N}_0, \mathcal{A})$ remains a directed and incomplete graph of tasks that need to be performed, whose set of arcs is defined as in Equation (1).

Let $\psi(i)$ be an auxiliary function that returns the location that is associated with task i.

It is assumed that travel costs respect the triangle inequality, meaning that $c^k_{ij}+c^k_{jl} \geq c^k_{il}, \forall i,j,l \in \mathcal{N}_0, k \in \mathcal{K}$. This assumption is equally applicable to travel distances d_{ij} and travel times t_{ij}. If multiple trips are acknowledged, it can be inferred that, for each vehicle k and each task i contained in $\mathcal{O}^k$, parameter r^k_i will take value 1, and value 0 in all other instances.

A.3 Decision variables

For this extended modelling framework, we may need to consider these additional decision variables.

u^k_{ij} Load of vehicle $k \in \mathcal{K}$ when traversing arc $(i,j) \in \mathcal{A}$

Variables u^k_{ij} are necessary for capacitated VRPs, i.e. with interdependencies triggered by vehicle capacity and customer/task demand.

Table 9: Additional sets and parameters of the extended modelling framework

Sets		
$\mathcal{O}^k$	Set of depot tasks for finishing and starting possible trips of route $k \in \mathcal{K}$	$\mathcal{O}^k = \{o_1^k, \ldots, o_O^k\}$
$\mathcal{O}$	Set of all depot tasks for finishing and starting possible trips	$\mathcal{O} = \bigcup_{k \in \mathcal{K}} \mathcal{O}^k$
$\mathcal{E}_i$	Set of tasks that can be performed immediately after task $i \in \mathcal{N}_0$	
$\mathcal{R}^\bullet$	Set of operations (i, j) where tasks i and j can be performed by the same vehicle	$\mathcal{R}^\bullet \subseteq \mathcal{R}$
$\mathcal{R}^\star$	Set of operations (i, j) where tasks i and j can be performed by different vehicles	$\mathcal{R}^\star \subseteq \mathcal{R}$
$\mathcal{R}^\delta$	Set of all operations (i, j)	$\mathcal{R}^\delta \subseteq \mathcal{R}^\bullet$
$\mathcal{R}^\Delta$	Set of operations (i, j) where tasks i and j can be performed by different vehicles	$\mathcal{R}^\Delta \subseteq \mathcal{R}^\bullet$
$\mathcal{R}^\lambda$	Set of operations (i, j) subject to lower-bounding synchronisation constraints	$\mathcal{R}^\lambda \subseteq \mathcal{R}^\star$
$\mathcal{R}^\mu$	Set of operations (i', j') subject to upper-bounding synchronisation constraints	$\mathcal{R}^\mu \subseteq \mathcal{R}^\star$
Parameters		
a_i, b_i	Earliest, latest possible time to begin performing task $i \in \mathcal{N}_0$	
d_{ij}	Travel distance from location $\psi(i)$ to location $\psi(j)$	
q_i	Demand to be satisfied for task $i \in \mathcal{N}_0$	
e_i	takes value 1, iff task $i \in \mathcal{N}$ must be performed, i.e., is mandatory; 0, otherwise	
e_{ij}	takes value 1, iff operation $(i, j) \in \mathcal{R}$ must be performed, i.e., is mandatory; 0, otherwise	
r_i^k	takes value 1, iff task $i \in \mathcal{N}_0$ can be performed by vehicle $k \in \mathcal{K}$; 0, otherwise	
Q^k	Capacity of vehicle $k \in \mathcal{K}$	
D^k	Maximum route length for route $k \in \mathcal{K}$	
δ_{ij}	Min. time offset between the arrival times of operation $(i, j) \in \mathcal{R}^\delta$	
Δ_{ij}	Max. time offset between the arrival times of operation $(i, j) \in \mathcal{R}^\Delta$	

The domain and nature of the decision variables used in this model are presented at the end of this modelling framework in constraints (??)–(18).

The objective function of the base VRP problem remains the same that is defined in Equation (2).

A.4 Constraints

The extended modelling framework is subject to the following constraints from the simplified formulation:

- Constraints (4), which establish the inflow and outflow conservation;
- Constraints (5), which ensure that every vehicle starts and ends its route at the depot;
- Constraints (6), which establish that a vehicle can only leave the depot once;
- Constraints (7)–(8), which correctly establish the values that variables w_i^k must take;
- Constraints (9)–(11), which are linking constraints that set the values of decision variables $y_{ij}^{kk'}$ based on the values of variables x_{ij}^k;
- Constraints (15)–(16), which establish operation synchronisation constraints;
- Constraints (17), which establish movement synchronisation constraints;
- Constraints (12)–(14), which establish the domain and nature of decision variables x_{ij}^k, $y_{ij}^{kk'}$ and w_i^k.

Along with these constraints from the simplified formulation, inequalities (18) establish the domain and nature of decision variables u_{ij}^k.

$$0 \leq u_{ij}^k \leq Q^k \qquad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K} \tag{18}$$

Constraints (18) define variables u_{ij}^k as continuous, whose values cannot exceed the maximum capacity of the vehicle associated with each variable.

The modelling framework can now be extended by the global and local aspects presented in the subsections that follow.

A.5 Global aspects

Global task constraints Task constraints constitute a fundamental component of any routing problem, since they establish the requirements for performing each task. In the simplified formulation, we assume that there is only one type of task constraint among all tasks. However, a VRP with multiple operational constraints may require differentiated task constraints for different types of tasks, in which case different types of constraints will apply for each task. In this mathematical formulation, it is assumed that each task may be performed at most once, and there may exist tasks that are not mandatory. This may be the case for multi-trip VRPs, for example, where it may be necessary to account for intermediate optional tasks located at the depot in order to have a vehicle performing multiple legs from the depot in the course of its route.

$$\sum_{k \in \mathcal{K}} \sum_{i:(i,j) \in \mathcal{A}} x_{ij}^k \leq 1 \qquad \forall j \in \mathcal{N} \tag{19}$$

$$\sum_{k \in \mathcal{K}} \sum_{i:(i,j) \in \mathcal{A}} x_{ij}^k \geq e_j \qquad \forall j \in \mathcal{N} \tag{20}$$

$$x_{ij}^k \leq r_i^k r_j^k \qquad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K} \tag{21}$$

Constraints (19) ensure that tasks can be performed at most once. Constraints (20) establish whether a given task is mandatory or not, depending on the value of parameter e_i. Constraints (21) are vehicle-task compatibility constraints: they ensure that travelling from task i to task j is only allowed if vehicle k is allowed to perform both tasks.

Global operation constraints The purpose of operation constraints is to ensure that operations are performed under their correct conditions. These conditions may consist in ensuring that a given operation is mandatory or optional or even establishing that a given operation must/can be performed by the same or different vehicles. The constraints that follow are able to ensure these conditions.

$$\sum_{k\in\mathcal{K}}\sum_{k'\in\mathcal{K}} y_{ij}^{kk'} \leq 1 \qquad \forall (i,j)\in\mathcal{R} \tag{22}$$

$$\sum_{k\in\mathcal{K}}\sum_{k'\in\mathcal{K}} y_{ij}^{kk'} \geq e_{ij} \qquad \forall (i,j)\in\mathcal{R} \tag{23}$$

$$\sum_{k\in\mathcal{K}} y_{ij}^{kk} = 0 \qquad \forall (i,j)\in\mathcal{R}\setminus\mathcal{R}^{\bullet} \tag{24}$$

$$\sum_{k\in\mathcal{K}}\sum_{k\in\mathcal{K}:k\neq k'} y_{ij}^{kk'} = 0 \qquad \forall (i,j)\in\mathcal{R}\setminus\mathcal{R}^{\star} \tag{25}$$

Constraints (22) ensure that each operation can be performed at most once. Constraints (23) establish whether a given operation is mandatory or not, depending on the value of parameter e_{ij}. Constraints (24) are applied to an operation that cannot be performed by the same vehicle: in these situations, the operation, if performed, must forcefully be performed by different vehicles. Constraints (25) are applied to an operation that cannot be performed by different vehicles: in these situations, the operation, if performed, must forcefully be performed by the same vehicle.

Global resource constraints The implementation of global resource constraints in a mathematical formulation is highly dependent on the nature of the resource being controlled. As an example, we will present the implementation of demand constraints for a VRP with Split Deliveries. In this problem variant, locations typically have more than one delivery task, and it is necessary to ensure that the total load that is left at each task is equal to the location's demand.

The nature of the split deliveries problem aspect is basically the same as any other type of resource constraints: we have a bounded/limited resource (in this case, the demand of a location) that requires vehicles to coordinate the amount of load between themselves, so that the global constraint is satisfied (in this case, that the total load from vehicles equals the location's demand), thus triggering inter-vehicle competition and trade-offs.

For modelling the demand requirements, we assume that each task can be performed at most once, although it is not required to be performed.

Parameter q_i becomes associated with locations instead of tasks. Therefore, parameter q_l will now represent the total demand to be satisfied by location $l \in \mathcal{L}$. Analogously to what was previously stated for q_i, we assume that $q_l < 0$ if load is to be delivered at l and $q_l > 0$ if load is to be picked up at l.

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}:\psi(j)=l}\sum_{i:(j,i)\in\mathcal{A}} u_{ji}^{k} - \sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}:\psi(j)=l}\sum_{i:(i,j)\in\mathcal{A}} u_{ij}^{k} = q_l \qquad \forall l\in\mathcal{L} \tag{26}$$

Constraints (26) reflect the demand constraints for each location. They impose that the difference between the sum of vehicle loads entering tasks of location l (first summand) and the sum of vehicle loads exiting these same tasks (second summand) must be equal to the total demand of location l.

A.6 Local aspects

Capacity constraints The inclusion of capacity constraints can be achieved by adding the constraints that follow and considering decision variables u_{ij}^k and parameters q_i and Q^k.

$$u_{ij}^k \leq Q^k x_{ij}^k \qquad \forall (i,j) \in \mathcal{A}, k \in \mathcal{K} \tag{27}$$

$$\sum_{i:(i,j)\in\mathcal{A}} u_{ij}^k + q_j x_{ij}^k = \sum_{i:(j,i)\in\mathcal{A}} u_{ji}^k \qquad \forall j \in \mathcal{N} \setminus \mathcal{O}, k \in \mathcal{K} \tag{28}$$

Constraints (27) are linking constraints between variables x_{ij}^k and u_{ij}^k; they impose that vehicle k cannot transport load from i to j if the vehicle is not traversing that arc $\left(x_{ij}^k = 0 \implies u_{ij}^k = 0\right)$. Constraints (28) state that the difference between the vehicle load when entering and leaving a customer must be equal to the demanded quantity, except for depot tasks for finishing and starting trips, if applicable. For the sake of generality, we assume that $q_i < 0$ if load is to be delivered at customer i and $q_i > 0$ if load is to be collected. These constraints, together with constraints (18), establish the capacity limits that must be imposed on each vehicle.

When multiple trips are acknowledged, additional capacity constraints are typically required to ensure that load variables u_{ij}^k are reset before starting eventual new trips. In these situations, new constraints (29) and (30) are introduced.

$$\sum_{o:(i,o)\in\mathcal{A}} u_{io}^k = 0 \qquad \forall k \in \mathcal{K}, o \in \mathcal{O}^k \tag{29}$$

$$\sum_{j:(o,j)\in\mathcal{A}} u_{oj}^k = 0 \qquad \forall k \in \mathcal{K}, o \in \mathcal{O}^k \tag{30}$$

Constraints (29) establish that the total vehicle load is equal to zero when finishing a trip. Constraints (30) establish that the total vehicle load is equal to zero when starting a new trip. Depending on the specific problem being tackled, at least one of these constraints is required. For routing problems whose purpose is to deliver load from the depot location to customers, constraints (29) must be applied. For routing problems whose purpose is to collect load from customers to the depot location, constraints (30) must be applied instead. Instead, if the problem consists in picking up and delivering load between tasks in a similar fashion to a Pickup and Delivery Problem, then both constraints must be applied.

Local operation constraints The interdependencies triggered by operations within a route have two major types of constraints, which can be applied depending on the specific application.

Type 1 constraints. These constraints establish the sets of tasks that a route can perform immediately after a given task is performed. In other words, these constraints allow for the definition of the possible sequences of tasks (or arcs) that vehicles may perform (or traverse) in the course of their routes.

These interdependencies can be modelled by adding the following constraints and considering set $\mathcal{E}_i$.

$$\sum_{l:(l,i)\in\mathcal{A}} x_{li}^k - \sum_{j\in\mathcal{E}_i} x_{ij}^k = 0 \qquad \forall i \in \mathcal{N}_0 : |\mathcal{E}_i| > 0, k \in \mathcal{K} \tag{31}$$

Constraints (31) establish that a task j from set $\mathcal{E}_i$ will need to be performed immediately after task i. In practice, and as a consequence of the effect of these constraints, these operation constraints provide a preprocessing of the transportation network. For each task i subject to these operation constraints, each potential subsequent task $j \notin \mathcal{E}_i$ will necessarily form an arc (i, j) that is infeasible and, therefore, can be removed from $\mathcal{A}$.

Type 2 constraints. These constraints, on the other hand, have the purpose of imposing time offsets between tasks of an operation that is being performed by the same route. Therefore, these constraints will only be binding if the operation is performed in the same route.

These interdependencies can be modelled by adding the following constraints and considering set $\mathcal{R}$ and parameters δ_{ij} and Δ_{ij}.

$$w_i^k + \delta_{ij} \leq w_j^k + T\left(1 - y_{ij}^{kk}\right) \qquad \forall (i,j) \in \mathcal{R}^\delta, k \in \mathcal{K} \tag{32}$$

$$w_i^k + \Delta_{ij} + T\left(1 - y_{ij}^{kk}\right) \geq w_j^k \qquad \forall (i,j) \in \mathcal{R}^\Delta, k \in \mathcal{K} \tag{33}$$

Constraints (32) and (33) are the type 2 constraints that impose time offsets between tasks of a given operation (i, j). Specifically, constraints (32) set a minimum time offset between performing task i and performing task j. They ensure that task j can only start being performed δ_{ij} time units after starting to perform task i. Analogously, constraints (33) set a maximum time offset between performing task i and performing task j. They ensure that task j must start being performed up to Δ_{ij} time units after starting to perform task i.

Time window constraints Time windows are easily modelled through the addition of the following constraints and by considering parameters a_i and b_i.

$$\sum_{i:(i,j)\in\mathcal{A}} a_j x_{ij}^k \leq w_j^k \leq \sum_{i:(i,j)\in\mathcal{A}} b_j x_{ij}^k \qquad \forall j \in \mathcal{N}, k \in \mathcal{K} \tag{34}$$

These constraints (34) establish lower and upper bounds to the arrival time of a vehicle to a given task, according to its desired time windows.

Route length constraints Limiting the route length of a vehicle can either be performed through the total duration of the route or through the total travelled distance.

If parameter D^k refers to a route's total travelled distance, then the problem aspect is modelled as follows.

$$\sum_{(i,j)\in\mathcal{A}} d_{ij} x_{ij}^k \leq D^k \qquad \forall k \in \mathcal{K} \tag{35}$$

Equations (35) limit the total travelled distance of each route up to parameter D^k.

Instead, if parameter D^k refers to a route's total duration, then different constraints must be introduced.

$$w^k_{N+1} \leq D^k \qquad \forall k \in \mathcal{K} \tag{36}$$

Equations (36) limit the total duration of each route up to parameter D^k.

For routing problems acknowledging multiple trips, besides limiting the duration of the complete route, it is also common to limit the duration of each trip. In these cases, a new parameter D'_k will designate the maximum duration of each trip in route $k \in \mathcal{K}$. For each vehicle $k \in \mathcal{K}$, we consider set $\mathcal{O}^k = \{o_1, ..., o_O\}$, of cardinality O, and assume that these depot intermediate tasks must be performed in a pre-established order. With this in mind, let auxiliary set $\mathcal{O}'_k = \{(o_1, o_2), (o_2, o_3), ..., (o_{O-1}, o_O)\}$ be the ordered pairs of intermediate tasks that establish this order. Under these conditions, the following constraints apply.

$$\sum_{i:(i,o)\in\mathcal{A}} x^k_{io} \geq \sum_{i:(i,o')\in\mathcal{A}} x^k_{io'} \qquad \forall k \in \mathcal{K}, (o, o') \in \mathcal{O}'_k \tag{37}$$

$$w^k_{o_1} \leq w^k_0 + D'_k \qquad \forall k \in \mathcal{K} \tag{38}$$

$$w^k_{o'} \leq w^k_o + D'_k \qquad \forall k \in \mathcal{K}, (o, o') \in \mathcal{O}'_k \tag{39}$$

$$w^k_{N+1} \leq w^k_{o_{O-n}} + nD'_k \qquad \forall k \in \mathcal{K}, n = 0, ..., O-1 \tag{40}$$

Constraints (37) establish that if multiple trips are performed in a route, the depot intermediate tasks must be performed in a pre-established order. Constraints (38)–(40) limit the total duration of each trip up to parameter D'_k. To that effect, Constraints (38) limit the duration of the first trip and constraints (39) limit the duration of the trips that follow. Finally, constraints (40) limit the duration of the last trip that is performed.

A.7 Instantiation and validation of the modelling framework

This section outlines the main adaptations to the proposed modelling framework that are required to successfully model several VRP variants. The first problem variant being instantiated is the VRP with Time Windows. Although this problem does not include synchronisation aspects, the following problems will build upon this initial instantiation in an incremental fashion.

A.7.1 VRP with Time Windows

The VRP with Time Windows (VRPTW) consists in a Capacitated VRP (CVRP) where each customer is visited exactly once, and visits to customers must occur between given time limits (Savelsbergh, 1992). For each of the customers to be visited, there will be a single task being performed, and therefore $|\mathcal{N}| = |\mathcal{L}|$. Modelling the VRPTW with the proposed modelling framework is trivial since it builds on the instantiation of a CVRP. To instantiate a CVRP with the proposed modelling framework, it is necessary to acknowledge the extended modelling framework along with global task constraints (19)–(20) with $e_j = 1, \forall j$, and capacity constraints (27)–(28). The VRPTW is then instantiated by adding time window constraints (34) for each customer. The demand of task i, q_i, will correspond to the total demand of its corresponding location/customer, $\psi(i)$. In this case, since the routing problem only includes delivery tasks, $q_i < 0$.

A.7.2 Home Health Care Routing and Scheduling Problem

The Home Health Care Routing and Scheduling Problem (HHCRSP) is a routing problem whose purpose is to obtain a set of routes for home health care staff in order to visit a set of patients that require certain services to be executed at their locations (e.g., Mankowska et al., 2013). The problem needs to acknowledge different staff qualifications for different services and have some synchronisation requirements at patient locations. Certain services at customers must be synchronised, i.e., they must be performed simultaneously by more than one staff member or within a given temporal offset.

The HHCRSP can easily be applied to the proposed formulation by considering the following adaptations:

- *Staff members and qualifications*: in this case, a staff member corresponds to a given vehicle, which in turn corresponds to a route. Each staff member is only able to perform certain tasks according to their qualifications. These qualifications require the addition of constraints (21), which will establish if a given staff member k is able to perform task/service i.
- *Home Health Care services*: in this particular problem, each service corresponds to a task that needs to be satisfied by a staff member at the patient's corresponding location $\psi(i)$. The HHCRSP also assumes there are some services that must be performed by multiple staff members. In these specific cases, synchronised services are split into two or more tasks (depending on the number of staff members required), being these tasks later intertwined with operation synchronisation constraints.
- *Simultaneous services*: a simultaneous service is characterised by having staff members start it exactly at the same time. The proposed modelling framework is able to handle this situation graciously. For an operation (i, j) corresponding to the tasks of a simultaneous service, we are able to guarantee that $w_i^k = w_j^k$ by applying exact synchronisation (constraints (15) and (16)) to (i, j), with $\lambda_{ij} = \mu_{ij} = 0$.
- *Services with precedence*: a service that must be performed before another is easily modelled through lower-bounded operation synchronisation for a given operation (i, j), with $\lambda_{ij} > 0$. In specific cases, it may be required to ensure that service j cannot be performed until service i is finished (i.e. no service overlap), which can be achieved by setting $\lambda_{ij} = s_i$.

A.7.3 Pickup and Delivery Problem with Transfers

The Pickup and Delivery Problem with Transfers (PDPT) is a generalisation of the standard Pickup and Delivery Problem (PDP), which, in turn, is a generalisation of the CVRP.

The PDP acknowledges two main types of locations: pickup and delivery customers, which are previously paired (e.g., Parragh, Doerner, & Hartl, 2008). The main additional requirements of a PDP compared to a CVRP consist in the fact that pickup and delivery of a given request must be performed by the same vehicle and that pickup must be performed before delivery.

The PDP with Transfers extends upon the assumptions of the PDP by allowing pickup and delivery tasks to be performed by different vehicles (e.g., Masson et al., 2013). To that effect, transfer locations exist where the picked-up load can be transferred from one vehicle to another.

In these circumstances, the PDPT is modelled by considering the following aspects of the instantiation of the CVRP, described previously:

- *Pickup and delivery tasks*: each pickup and delivery request, composed of pickup task i and delivery task j, each one of them located at its corresponding customers, is considered a mandatory operation $(i, j) \in \mathcal{R}$, which must be performed by either the same or different vehicles.

- *Transfer tasks*: for each transfer location and pickup and delivery request (i, j) present in the problem, two additional tasks i' and j' must be considered, where task i' represents the drop-off of the load of (i, j) at the transfer location and task j' represents the pickup of that transfered load.
- *Local constraints*: besides the traditional capacity constraints, it is also necessary to ensure that if a pickup and delivery request is performed by only one vehicle (i.e., the request is fulfilled without transfer), the pickup task i must be performed before its corresponding delivery task j. Furthermore, it is also necessary to ensure temporal precedence for operations (i, i') and (j', j) if they are performed. For the request (i, j), this is achieved by applying local operation constraints (32) to each request (i, j), with $\delta_{ij} = 0$, or, alternatively, $\delta_{ij} = s_i$, if tighter constraints are preferred. For operations (i, i') and (j', j), local operation constraints (32) are also applied with $\delta_{ii'} = 0$ and $\delta_{j'j} = 0$, respectively, or $\delta_{ii'} = s_i$ and $\delta_{j'j} = s_{j'}$, if tighter constraints are preferred.
- *Optional tasks*: all tasks present at transfer locations are not mandatory, and therefore, for these tasks, parameters e_j will be equal to zero.
- *Operations concerning transfers*: taking into account the problem requirements, pairs (i, i') and (j', j) constitute optional operations, which, if performed, must be performed by the same vehicles. Therefore, in these cases, global operation constraints (23) and (25) apply. Furthermore, pairs (i', j') also constitute optional operations that, if performed, must be performed by different vehicles. In these instances, global operation constraints (23) and (24) apply.
- *Synchronising transfer*: for operations (i', j'), being performed in a transfer, it is necessary to ensure that a vehicle visiting task j' is not being performed before task i' is performed by a different vehicle. Taking this requirement into account, operations (i', j') are subject to operation synchronisation, ensuring a minimum difference between the arrival times of both vehicles. Therefore, lower-bounding synchronisation constraints (15) are applied to operations (i', j'), with $\lambda_{i'j'} \geq 0$ defining the minimal difference between the arrival times.

A.7.4 Truck and Trailer Routing Problem

The Truck and Trailer Routing Problem (TTRP) is a generalisation of the VRP, which is characterised by acknowledging two different types of vehicles – trucks (active vehicles) and trailers (passive vehicles) (e.g., Chao, 2002). Trailers cannot move between locations without a truck transporting them. In this problem, there are site-dependency constraints related to trailers only visiting certain customers, which splits the customer set into truck-only customers and vehicle customers, which allow trailers in their locations. This additional practical constraint leads to a truck leaving the trailer at a vehicle customer and performing a sub-tour to other customers. In sum, trucks may perform three types of routes: routes with no trailer attached, routes with a trailer coupled to it at all times – in which case it can only visit vehicle customers –, and routes where trailers are temporarily left at a vehicle customer to serve truck-only customers.

The TTRP can also be modelled using the proposed mathematical formulation by performing some adjustments to it as follows:

- *Tasks involved in the problem*: because of the additional complexity provided by the TTRP, vehicle customer locations will have more than one task in order to account for the arrival of multiple vehicles to locations, as well as the multiple visits being performed by the same vehicle. With this in mind, there will be only one task for each truck-only location, which translates the delivery of the load required by the customer. As for vehicle customer locations, there will be four major types of tasks: (i) tasks representing the arrival of the truck ("truck arrival tasks"); (ii) tasks representing the (eventual) arrival of a trailer coupled to the truck ("trailer arrival tasks"); (iii) tasks representing the return of the truck to the customer after decoupling from the trailer and performing a truck-only subtour ("truck return tasks"); (iv) tasks for having the trailer recouple to the truck ("trailer coupling tasks"). Depending on the type of task, it should be performed by either a truck or a trailer. Therefore, constraints (21) must be applied and parameters r_i^k should also be adjusted to 0 for incompatible task and vehicle combinations.

- *Optional tasks*: for tasks at truck-only locations, all tasks must be considered mandatory. However, in vehicle customer locations, due to the multitude of situations that can be verified, only truck arrival tasks are mandatory. Because trailers may not be required to visit vehicle customers if the truck is able to satisfy the customer's demand, trailer arrival tasks are optional, and therefore do not require being satisfied. Consequently, because truck return tasks and trailer coupling tasks only make sense to be performed if trailer delivery tasks are also performed, these too are considered optional, which implies that, for these tasks, the value of parameter e_j must be set to zero.
- *Demand satisfaction*: in the TTRP, the demand of vehicle customers can be satisfied by any of the vehicles that arrive at its location – i.e., the truck or the trailer. Therefore, since the allocation of demand from each of the vehicles becomes unknown *a priori*, the problem requires the addition of new decision variables. This problem requirement is successfully modelled by considering global resource constraints (26) along with the new underlying decision variables.
- *Task precedences within the same route*: due to the problem's intrinsic requirements, there are some route sequences in the TTRP that cannot happen. For example, a truck delivery task cannot be performed without either performing a trailer delivery task or another truck delivery task immediately before. Another example consists in trailer coupling tasks, which must be immediately followed by a trailer delivery task. These task precedence requirements are achieved through local operation constraints. To successfully acknowledge this problem requirement, we add all the possible task sequences (i, j) that may occur within a route to set $\mathcal{R}$ and apply constraints (31). Alternatively, preprocessing procedures can be devised so that the problem's graph incorporates only feasible task sequences.
- *Truck and trailer simultaneous movement*: modelling the simultaneous movement between trucks and trailers is relatively painless by using the operation and movement synchronisation constraints previously presented. Considering a passive operation (i, j), which corresponds to a possible task sequence/arc being performed by a trailer, and also considering all active operations $(i', j') \in \mathcal{R}_{ij}^{\alpha}$ that can be synchronised with operation $(i, j) \in \mathcal{R}^{\rho}$, movement synchronisation is therefore applied for every operation (i, j) of a trailer where it needs to be transported by a truck, i.e., where $\psi(i) \neq \psi(j)$. Constraints (17) are added considering these mathematical abstractions. Additionally, it is also necessary to ensure that these task sequences are performed at the same time, which is why it is also necessary to add operation synchronisation constraints between these task sequences. For modelling this simultaneous movement, it is only necessary to ensure that the arrival time of the trailer at task j is exactly equal to the arrival time of the truck at either j', which corresponds to adding two operations (j, j') and (j, j'') to sets $\mathcal{R}^{\lambda}$ and $\mathcal{R}^{\mu}$, with $\lambda_{jj'} = \lambda_{jj''} = \mu_{jj'} = \mu_{jj''} = 0$ and adding constraints (15) and (16).
- *Global operation constraints*: one of the requirements of the TTRP is that if a truck leaves a trailer at a vehicle customer to perform a truck-only subtour, then it must be the same truck performing the delivery at that vehicle customer that must also recouple the trailer

that was left there. This problem requirement is accomplished by applying global operation constraints (25) to each pair of truck delivery tasks and truck return tasks within the same vehicle customer location.