Integrated planning of inbound and outbound logistics with a Rich Vehicle Routing Problem with Backhauls

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Abstract

This paper addresses the integration of the planning decisions concerning inbound logistics in an industrial setting (from the suppliers to the mill) and outbound logistics (from the mill to customers). The goal is to find the minimum cost routing plan, which includes the cost-effective outbound and inbound daily routes (OIRs), consisting of a sequence of deliveries of customer orders, pickup of a full truck-load at a supplier, and its delivery to the mill. This study distinguishes between three planning strategies: opportunistic backhauling planning (OBP), integrated inbound and outbound planning (IIOP) and decoupled planning (DIOP), the latter being the commonly used, particularly in the case of the wood-based panel industry under study. From the point of view of process integration, OBP can be considered as an intermediate stage from DIOP to IIOP. The problem is modelled as a Vehicle Routing Problem with Backhauls, enriched with case-specific rules for visiting the backhaul, split deliveries to customers and the use of a heterogeneous fleet. A new fix-and-optimise matheuristic is proposed for this problem, seeking to obtain good quality solutions within a reasonable computational time. The results from its application to the wood-based panel industry in Portugal show that IIOP can help to reduce total costs in about 2.7%, when compared with DIOP, due to better use of the delivery truck and a reduction of the number of dedicated inbound routes. Regarding OBP, fostering the use of OIRs does not necessarily lead to better routing plans than DIOP, as it depends upon a favourable geographical configuration of the set of customers to be visited in a day, specifically, the relative distance between a linehaul that can be visited last in a route, a neighboring backhaul, and a mill. The paper further provides valuable managerial insights on how the routing plan is impacted by the values of business-related model parameters which are set by the planner with some degree of uncertainty. Results suggest that increasing the maximum length of the route will likely have the largest impact in reducing transportation costs. Moreover, increasing the value of a reward paid for visiting a backhaul can foster the percentage of OIR in the optimal routing plan.

Keywords: logistics planning, vehicle routing with backhauls, rich vehicle routing, forest industry

Highlights:

- Studying different planning strategies of inbound and outbound logistics processes
- Mathematical VRPB-based model for addressing different integration strategies
- Adapting fix-and-optimise matheuristic to solve the problem
- Application in a case-study from the wood-based panel industry
- Providing managerial insights on the impacts of relevant planning parameters

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1 Introduction

The optimisation of the logistics processes has a whopping effect on improving the cost-efficiency of supply chains. Specifically, in forest-based supply chains, the inbound logistics bringing the wood from the forest to the mill can represent up to 30% of the total costs (Audy, Lehoux, D'Amours, & Rönnqvist, 2010), while the outbound logistics bringing the wood-based products from the mill to the consumers can be equally high.

Despite recent studies showing that integrated planning of supply chain operations can lead to better results than decoupled planning (e.g., Amorim, Günther, & Almada-Lobo, 2012), inbound and outbound logistics planning are still dealt separately in most forest industries, as well as in other sectors. The complexity of the logistics operations, specificities of the transportation fleet and customer service levels are frequent justifications for this fact. In the wood-panel based industry, outbound logistics planning establishes the minimum-cost daily routes, henceforth called outbound routes (ORs), for delivering the ordered amounts of finished products to customers. This process accrues from the mill's production plan and impacts on the customer order lead time. Inbound logistics establishes the inbound routes (IRs), usually of a dedicated log-truck, consisting of a sequence of full truck-load trips between a wood sourcing location and the mill. The process is affected by wood procurement planning, ultimately impacting on the upstream forest harvest scheduling decisions. Similar transportation planning settings appear in the retail industry. Namely, in cases in which the retailer has the option to pick-up products at suppliers besides just simply distributing to stores (Yano et al., 1987).

This paper studies the integration of inbound and outbound logistics in the context of the wood-based panel industry. The case study is driven from a real-life industrial application that operates on a multi-mill setting. The production strategy of the wood-based panels at each mill is Make-to-Order. The finished products are shipped to the customers in the day after its production. The stock of raw materials should be at least one week to overcome fluctuations in wood supply. The outbound logistics are planned locally, in the transportation department of each mill, while the inbound logistics are planned centrally, considering the bulk demand for all the mills. The goal here is to find daily minimum-cost outbound and inbound routes (OIRs) where the vehicle departing from each mill firstly performs a sequence of deliveries of the amounts ordered by the customers, and secondly, whenever is cost-effective, picks up a full truck-load of raw materials at a nearby supplier, and delivers it at the closest company's mill. OIRs allow better use of the delivery truck, when compared with ORs and further avoid dedicated IRs. This is possible because the driver can easily adapt the same truck that transported the wood boards with reinforcements in its structure so it can transport a full truck-load of wood chips. For wood-based supply chains, it is common that the inbound transport is carried in full truck-loads (e.g., Carlsson & Rönnqvist, 2007; Derigs et al., 2012; Hirsch, 2011).

In this paper, the problem of finding OIRs is modelled as a Vehicle Routing Problem with Backhauls (VRPB). The VRPB is a variant of the well-known Vehicle Routing Problem (VRP) where the route visits several customers, in some performing deliveries (referred as linehauls) and in others pickups (the backhauls), but all deliveries must be made before any pickups (Goetschalckx & Jacobs-Blecha, 1989). In this study, we use the VRPB as a mean to tackle Integrated Vehicle Routing Problems, as outlined by Bektaş, Laporte, and Vigo (2015), since the routing decisions related with the process of inbound logistics and those of the outbound logistics are dealt jointly. Moreover, there are essential business-related rules arising from our application to the wood-based panel industry that determine route feasibility, which are not yet fully covered in the VRPB literature and justify the formulation of a new variant of a rich VRPB, in line with the taxonomy proposed by Lahyani, Khemakhem, and Semet (2015).

The first set of business-related rules addressed in this study relate to the conditions determining the visit to a backhaul: i) the backhaul can only be visited after all deliveries are performed, here called a precedence constraint, because the reinforcement of the truck for transporting the wood chips can only occur after the last delivery of the wood-based panels; ii) there is at most one backhaul visited per route because the amount picked up is always a full truck-load since there are no wood availability constraints at suppliers; iii) if there is a pickup at a backhaul it is mandatory that the same route includes its delivery at a mill. This is another type of precedence constraint ensuring that a mill is visited after a backhaul. However, operational practice indicates that the unloading mill may or may not be the mill of origin, because the company owns several mills geographically dispersed, and the truck can end the route in any of these mills, as long as the compatibility requirements between the types of raw materials available at the backhaul and accepted at the mill are accounted for; iv) a backhaul may or may not be visited, which is known in the literature as selective backhauling; v) routes without a backhaul are also feasible, and in this case, the route ends in the last linehaul visited, similarly to what occurs in an Open VRP (see Figure 1). There are other studies on VRPB that work with precedence constraints and selectiveness. However, the possibility to optimise the decisions about visiting or not a backhaul and further choosing the delivering mill in order to minimise total logistics costs are new and important features of the problem under study. Another important case-specific rule determines that each customer may be visited more than once by different vehicles, known in the VRP literature as *split deliveries*. The bundle of panels to be delivered at the linehaul customer is of variable size and weight. Therefore, several smaller bundles can be transported by the same truck, but larger bundles may need multiple trucks serving the same customer. Lastly, the available fleet is composed of trucks which are *heterogeneous* in terms of the transportation capacity. The transport is entirely outsourced to third-party carriers and paid based on a fixed daily use cost and a variable cost depending on the travelling distances of the 'for-hire' vehicles. We further emphasize that these business rules are also applicable in other industries besides the wood-panel one, such as in grocery retail.

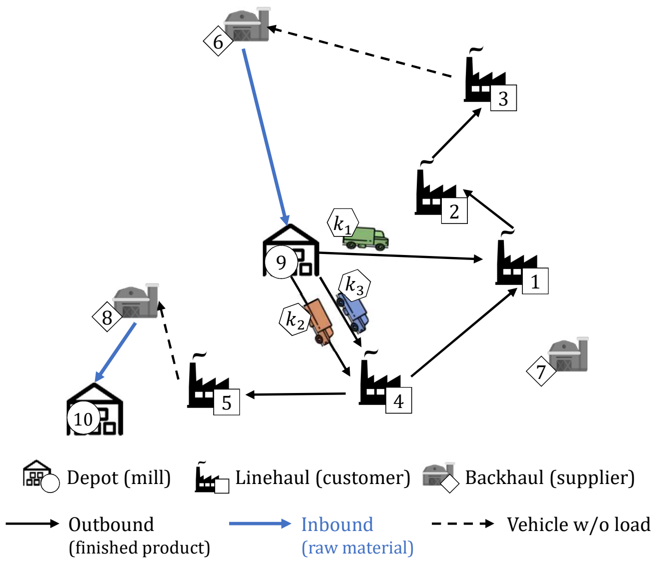


Figure 1: Problem representation

The complexity of this real-world problem motivates a study about the main strengths and shortcomings of different inbound and outbound planning strategies, with greater or fewer degrees of integration. Furthermore, given the considerable size that these problems can achieve, it becomes relevant to envisage a scalable solution method, able to cope with the operational reality.

This research builds on a literature review on VRPB and other rich VRP variants with similarities to our problem. The first contribution of this paper is to develop a mathematical formulation to address a rich VRP that is primarily used to solve different planning strategies for obtaining OIRs. We apply it to a case study in the wood-based panel industry in Portugal and draw conclusions by comparing the routing plans obtained with those alternative planning strategies. Another contribution is to provide valuable managerial insights for planners about the impact of business-related model parameters over the optimal routing plan. Another contribution is to adapt the fix-and-optimise matheuristic presented by Sahling, Buschkühl, Tempelmeier, and Helber (2009) for obtaining good quality solutions for larger instances of this problem within a reasonable computational time.

The remainder of this paper is organised as follows. Section 2 provides a critical review of the literature regarding integrated transportation planning with a particular connection to the VRPB. This review covers extensions of VRPBs and solution methods developed to solve both artificial and real instances, and allows us to place our work in context. Section 3 presents the mathematical formulation of the three logistics planning strategies investigated in this work, namely the opportunistic backhauling, the integrated and the decoupled inbound-outbound transportation planning. Section 4 describes the solution approach developed, which is based on a fix-and-optimise algorithm. Section 5 presents the computational experiments performed with close-to-reality instances from a wood-based industry in Portugal. The routing plans obtained for the three planning strategies are compared, and relevant managerial insights are envisaged. The main conclusions are presented in Section 6.

2 Critical review of the state of the art

In the literature on logistics and transportation, the term integrated planning is broadly used to refer to situations where the routing decisions are tackled jointly with other decisions (Speranza, 2018). In some situations, the integration is between transportation decisions of different planning levels, for example, strategic decisions concerning the design of the transportation network and the tactical decisions related with the routes and assignment of the transport vehicles (e.g., Bouchard, D'Amours, Rönnqvist, Azouzi, & Gunn, 2017). In other situations, the integration is between the routing decisions and the decisions concerning other processes of the supply chain. The special issue by Bektaş et al. (2015) on the integrated VRP shows examples of cases where vehicle routing is interlinked with decisions related to loading, production (or inventory), location, and speed optimisation. As an example, production-routing problems integrate production, products delivery (i.e., outbound logistics), and usually also inventory decisions (e.g., Adulyasak, Cordeau, & Jans, 2015). There are several examples in the forest literature where wood transportation to the mill (i.e., inbound logistics) and the upstream process of forest harvesting are planned jointly (e.g., Marques, Audy, D'Amours, & Rönnqvist, 2014).

As indicated by Speranza (2018), a common feature of the studies on integrated transportation planning is that dealing with those decisions separately or hierarchically by solving the problems independently, leads to a sub-optimal solution for the integrated problem. In fact, integrated planning potentiates global efficiency gains, usually translated into cost savings. As an example, Archetti and Speranza (2015) present significant savings of around 9.5% in terms of total cost and 9.0% in terms of the number of vehicles used when using a heuristic solution for an inventory-routing problem, in comparison with the solution obtained by sequentially and optimally solving the inventory management and the routing problems.

The main particularity of our study, not yet fully covered in the literature, is that the integration is between two processes of the supply chain – inbound and outbound logistics – wherein both processes the relevant decisions are related with the optimal vehicle routes. In fact, in our problem, it is the same vehicle that may perform both processes. There are significant differences in respect to the modelling approach because, in the other cases of integrated VRPs, such as production-routing, there are at least two types of decision variables, one for each process, and the correspondent linking constraints. While in ours, there are only the decision variables related to routing. The linkage between the two processes accrues from the way the routes are built.

The problem class that mostly resembles our problem is the VRPB, firstly introduced by Deif and Bodin (1984). Since then, there are several VRPB variants being studied in the framework of practical applications, as shown in the recent review of Koç and Laporte (2018). In general terms, the VRPB consists in finding the minimum cost routes, which start and end at the depot and visit a set of customers partitioned into linehauls (customers who require deliveries), and backhauls (customers who require pickups), all must be visited contiguously (e.g., Wade & Salhi, 2002).

The VRPB is not usually considered as an example of integrated vehicle routing planning. In fact, many of the industrial applications of the VRPB focus on the outbound logistics process, for example, in retail companies (e.g., Eguia, Racero, Molina, & Guerrero, 2013; Goetschalckx & Jacobs-Blecha, 1989). In these cases, the route prioritises first all the products deliveries, and only afterwards the pickups, in order to attain a high vehicle utilisation. The customers are all of the same type (e.g., stores), but with different requirements (i.e., pickup or delivery) and the picked up material can be of a different type that cannot be mixed with the delivered products, such as empty boxes, damaged products or post-consumption material in reverse logistics. In other applications, such as the distribution of equipment to rentals (e.g., Dominguez, Guimarans, Juan, & de la Nuez, 2016), or package delivery over a distribution network (e.g., Yu & Qi, 2014), the inbound and outbound material is the same, and it is all planned together as a unique logistic distribution process.

Contrarily, we argue that our case study can be considered integrated transportation planning because the inbound and outbound logistics are two separate processes that nowadays are planned independently, involving different types of customers – i.e., suppliers of raw materials vs. consumers of finished products – sharing in common the depot/mill. Yano et al. (1987) study a case resembling ours, in a retail chain with one centralized distribution centre, 40 stores and nearby vendors, where the route includes the delivery of goods to stores and the pickup of goods in nearby vendors. Planning includes dedicated routes for the vendors whenever it is not cost-efficient to include them in the delivery routes. The results of this work allowed savings in the order of a half-million dollars. With a similar strategy, Paraphantakul, Miller-Hooks, and Opasanon (2012) report a case-study in a cement industry, where cement customers are linehaul customers, and lignite mines are backhaul customers. The problem was solved using an ant colony optimisation method, and the company was able to save about 12% in the average tour duration.

The literature review on VRPB reveals examples of mathematical models, exact and heuristic methods for solving distinct problem variants. A general integer linear programming formulation and set partitioning formulation for the VRPB are presented in Koç and Laporte (2018). Among the most common extensions of VRPB found in the literature are the incorporation of time windows (Gutiérrez-Jarpa, Desaulniers, Laporte, & Marianov, 2010; Küçükoğlu & Öztürk, 2013; Nguyen, Crainic, & Toulouse, 2016; Ropke & Pisinger, 2006), multi-periods (Davis, Sengul, Ivy, Brock, & Miles, 2014; Nguyen et al., 2016), multi-depots (Chávez, Escobar, Echeverri, & Meneses, 2015), heterogeneous fleet (Lai, Crainic, Francesco, & Zuddas, 2013; Salhi, Wassan, & Hajarat, 2013) and split deliveries (Gutiérrez-Jarpa et al., 2010; Lai, Battarra, Francesco, & Zuddas, 2015; Nguyen et al., 2016; Wassan, Wassan, Nagy, & Salhi, 2017). There are also variants on the nature of the backhauling, such as the mixed VRPB that also allows deliveries to linehauls after pickups in backhauls (e.g., Yazgitutuncu, Carreto, & Baker, 2009).

As the research on transportation planning advances more and more towards its practical application, several extensions of VRPs that consider real-life aspects of the logistics problems have emerged in the literature. The VRPs that cover such aspects, namely the integration of different logistics operations (e.g., inbound and outbound transport), the consideration of uncertainty or dynamism, or the inclusion of real constraints (e.g., time windows and multi-periodicity), fall into the vast class of Rich VRPs (Caceres-Cruz, Arias, Guimarans, Riera, & Juan, 2014; Lahyani et al., 2015). As our problem concerns a VRP with selective backhauls, heterogeneous fleet, and split deliveries, we can classify it as a rich VRPB. Table

1 presents a description of other VRPBs found in the literature that relate to our work, including the real-life aspects addressed in the problem and the respective types of solution methods used to solve the VRPB.

Table 1: Characteristics of the Rich VRPB under study and related works in the literature

Reference	VRPB features							Solution method		
	TW	HF	SD	MD	MP	SB	MB	Exact	Metaheuristic	Matheuristic
Yano et al. (1987)						•		•		
Ropke and Pisinger (2006)	•								•	
Gribkovskaia, Laporte, and Shyshou (2008)						•			•	
Gutiérrez-Jarpa, Marianov, and Obreque (2009)	•		•			•		•		
Paraphantakul et al. (2012)	•		•						•	
Küçükoğlu and Öztürk (2013)	•	•							•	
Salhi et al. (2013)		•							•	
Lai et al. (2013)		•	•						•	
Davis et al. (2014)					•			•		
Chávez et al. (2015)				•					•	
Nguyen et al. (2016)	•		•		•				•	
Oesterle and Bauernhansl (2016)	•	•					•	•		
Wassan et al. (2017)			•						•	
Our problem		•	•			•				•

Legend: TW (time-windows), HF (heterogeneous fleet), SD (split deliveries), MD (multi-depot), MP (multi-periodic), SB (selective backhauls), MB (mixed backhauls)

From Table 1, it is possible to observe that metaheuristics are the most popular methods used to solve VRPBs. This results from the fact that the VRPB is an NP-hard problem and, as such, very few exact methods are known to be efficient for large scale problems. Yano et al. (1987) describe the problem using a set-covering formulation and then solve it using a procedure based on a Branch-and-Bound that starts from an initial solution obtained with simple heuristics. Gutiérrez-Jarpa et al. (2009) introduce a Branch-and-Cut algorithm to solve a VRPB with split deliveries and test it in new problem instances adapted from the VRP instances with up to 100 customers, but only those instances with 50 customers or less can be solved to optimality. Davis et al. (2014) use a commercial solver to find optimal transportation schedules that allow food banks to collect food donations from local sources and to deliver food to charitable agencies, through food delivery points. The problem is solved in two phases: first, the problem is formulated as a set-covering model to assign charitable agencies to food delivery points, and then, the problem is formulated as a VRPB enriched with constraints related to food safety, operator workday and collection frequency, also using the optimal solution of the first phase as an input. Oesterle and Bauernhansl (2016) also study a logistic problem of a food company but considering a mixed VRPB with time windows, heterogeneous fleet, manufacturing capacity and driving time limits. The problem is formulated as a mixed integer programming model and also solved with a commercial solver in two phases. The first phase creates clusters of customers to visit, and at the second phase, the routes in each cluster are optimised.

With respect to metaheuristics, both local search and population-based methods have proved to be very efficient to deal with VRPB and its extensions. Examples of local search metaheuristics include tabu search (Gribkovskaia et al., 2008; Nguyen et al., 2016), adaptive large neighborhood search (Ropke & Pisinger, 2006), and variable neighborhood search (Wassan et al., 2017). Examples of population-based metaheuristics developed for the VRPB include ant colony optimisation (Chávez et al., 2015; Paraphantakul et al., 2012) and evolutionary algorithms (Küçükoğlu & Öztürk, 2013). Moreover, two-phase heuristics are also investigated in the works of Salhi et al. (2013) and Lai et al. (2013).

Regarding matheuristic approaches, no references related to its adaptation to the VRPB were found.

However, the literature accounts for several matheuristic approaches for various solving VRP variants. For example, the fix-and-optimise approach was initially proposed by Sahling et al. (2009) for a lot-sizing problem, but it has been gaining recent interest in the literature for solving several rich routing problems with real-life aspects (e.g., Neves-Moreira, Almada-Lobo, Cordeau, Guimarães, & Jans, 2019). This matheuristic consists in iteratively fixing different sets of binary variables from a mathematical model, thus allowing a commercial solver to only solve smaller parts of the global problem. Depending on the problem, the selection of the variables to be fixed or released needs to be carefully designed. Most references frame this approach in a variable neighbourhood decomposition search (Hansen, Mladenović, & Perez-Britos, 2001), where the number of variables to be released is progressively increased as a way to increase the neighbourhood sizes being explored (e.g., Darvish, Archetti, Coelho, & Speranza, 2019; Soares, Marques, Amorim, & Rasinmäki, 2019). Other research works use distinct heuristic concepts, such as tabu search (e.g., Rieck, Ehrenberg, & Zimmermann, 2014) by using a tabu list for the variables being fixed.

Our work is distinct from the ones revisited in this section. It contributes to the literature because it not only describes a new formulation for a rich VRPB that can be used to address different transportation planning strategies but also investigates a fix-and-optimise method to solve the problem, which was not yet addressed in VRPB literature.

3 Problem formulation

This section outlines the main planning strategies for the integration of inbound and outbound logistics processes, which will be addressed in this paper. For each one of these planning strategies, mathematical formulations will be provided, which will be the basis for the sections that follow.

3.1 Logistics planning strategies

The integration of inbound and outbound logistics by finding the optimal OIRs can be staged in two distinct planning strategies, in opposition to a simpler strategy of decoupled planning, similar to what is used today by the company:

- Opportunistic backhauling planning (OBP): In this strategy, the primary process to be considered is the outbound logistics. The outbound transportation plan encompasses ORs and cost-effective OIRs, but another plan exists for IRs. There is an underlying idea that OIRs can provide only a residual amount of the raw materials demanded and IRs assure the vast majority of the demand.
- Integrated Inbound and Outbound Planning (IIOP): In this strategy, both processes of inbound and outbound logistics are planned jointly. The transportation plan encompasses all types of routes – ORs, OIRs and IRs.
- Decoupled Inbound and Outbound Planning (DIOP): This strategy implies that both processes of inbound and outbound logistics are planned independently. The outbound transportation plan (or delivery plan) encompasses the ORs, while the inbound plan (or supply plan) encompasses IRs, there are no OIRs. In the current situation of the case study, logistics planning occurs in separate company departments. IRs are planned centrally and ORs are planned in a department at each mill.

From the point of view of process integration, OBP can be considered an "intermediate" stage, from DIOP towards IIOP, as well as from the point of view of the level of organisational changes needed for its adoption. In fact, OBP impacts mostly on the planners of the outbound logistics in each mill and on the truck drivers while IIOP implies a major restructuring from merging (and possibly centralising) the inbound and outbound logistics planning departments. From a modelling point of view, the mathematical formulation for OBP and IIOP are similar. For the purpose of simplification, this section focuses on OBP, making the necessary adjustments to IIOP afterwards. The section ends with the description of DIOP.

3.2 Opportunistic backhauling planning (OBP)

OBP can be modelled as a rich, capacitated Vehicle Routing Problem with selective backhauls and split deliveries. Considering a set of mills M, a set of linehaul customers L whose demand needs to be fulfilled, and a set of suppliers backhauls B with raw materials available for the mills that may or may not be visited. The problem consists in finding the optimal daily minimum-cost routes for a set of trucks K, starting at the mill, encompassing one or many deliveries to linehauls, and including at the most one pickup of a full truck-load of a given type of raw materials at a backhaul, which is selected based on the best fit with one of the possible destination mills. The set of types of raw materials to be collected at a backhaul is represented by set P. Hence, the problem components include:

- the fleet of $|K|$ trucks, where each truck $k \in K$ has a given capacity (Q_k) and can perform both deliveries and pickups. There is a fixed cost for the daily usage of a vehicle $\left(f^k\right)$ and a variable cost $\left(c_{ij}^k\right)$ proportional to the travelled distances;
- the $|M|$ mills owned by the company that are geographically dispersed. Each mill $m \in M$ receives wood chips and produces wood-based panels on a make-to-order basis. The fleet is assigned to a specific mill or origin (or depot), from where the routes start. According to operational practice, in case of a route with a backhaul, the truck can unload the raw materials in any of the company's mills, which may or may not be the mill of origin. There is a minimum amount of raw materials to be backhauled to all mills (β);
- the $|L|$ linehaul customers that are characterized by a given demand of a finished product, which must be fulfilled (q_l) at each linehaul $l \in L$. Split deliveries can occur, meaning that each customer may be visited more than once (each visit consisting in at least a ψ amount), but each truck may visit a customer at most once;
- the $|B|$ backhaul suppliers that are also geographically dispersed. Also, according to the operational practice, it is assumed that all have unlimited availability, hence pickups correspond to full-truck loads. The type of raw materials that are available may also vary amongst them;
- the $|P|$ types of raw materials consisting of wood chips of variable size and moisture content, sawdust and recycled wood. Some types of raw materials are more desirable to the mills than others. There are also compatibility issues with respect to the types of raw materials available and demanded at the different locations.

Contrarily to other VRPs found in the literature, the time window constraints related to the earliest or latest time to arrive at each location are not of importance. However, the maximum distance travelled in a route is limited by a parameter α. It is noteworthy that the route length can be constrained in terms of travelling time, to account for driving time regulations stating maximum driving or working times. However, in this case, the value of the maximum distance travelled was set with the planner as an average of the actual routes length, already implicitly considering all the necessary stops, hence simplifying problem modelling. In summary, the characteristics of the feasible routes are: i) start at a home depot with the truck loaded up to its maximum capacity, with the products ordered by the linehaul customers; ii) perform a sequence of deliveries to the linehauls; iii) if it is cost-effective and doable during the maximum route length, the vehicle travels empty to a nearby backhaul supplier to pick up a full truck-load of raw materials to be delivered at any of the company's mill, where the route ends (specific to OIRs); and iv) if a backhaul is not visited, the route is ended when the truck is empty after visiting the last linehaul of the route (specific to ORs), as the company does not pay for trips where the truck does not transport merchandise.

3.2.1 Modelling approach

The rVRPB under study is modelled as a graph $G = (V, A)$ where V is the set of all vertices, $V = \{0\} \cup L \cup B \cup M$ and A is the set of all possible arcs. We adopt a standard flow VRP formulation with 3-index decision variables x^k_{ij} equal to 1 if vehicle $k \in K$ travels from customer $i \in V$ to $j \in V$ and zero otherwise. Like in the standard VRPB formulation proposed by Parragh, Doerner, and Hartl (2008), we distinguish the vertices in linehauls and backhauls, in order to model the precedence constraints.

However, the typical VRPB constraints assuring that each vertex is visited exactly once do not apply, due to the possibility of selective backhauls (i.e., backhauls may or may not be visited) and the split deliveries at the linehauls (i.e., linehauls are visited more than once).

To avoid the complexity of a multi-depot and open VRP, we propose a *2-echelon backhauls network*, starting and ending at the same fictitious depot 0. In fact, when the route starts, the fictitious depot corresponds to the mill of origin from where the customers' orders will be delivered. Since there is a fleet dedicated to each mill when the route starts, routing planning for each mill can be done separately as a single depot. When the route ends, the fictitious depot corresponds to a fictitious location whose distance from the last vertex visited in the route is equal to zero. Hence, the 2-echelon backhauls network is composed by the first echelon of backhauls corresponding to the suppliers and the second echelon of backhauls corresponding to the mills to be supplied by the backhauled amounts. Additional constraints are needed to assure that a mill can only be visited after a backhaul (see Figure 2).

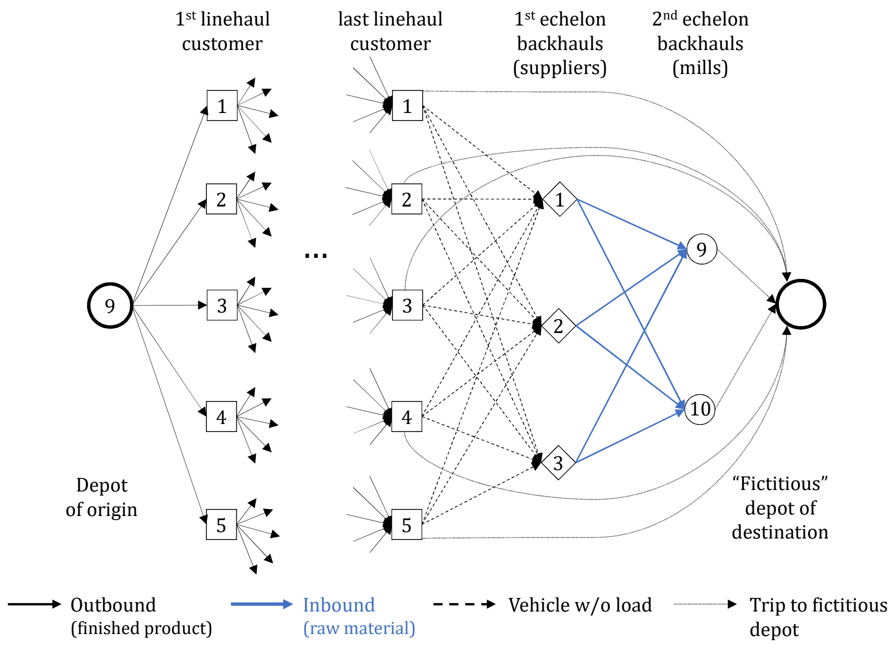
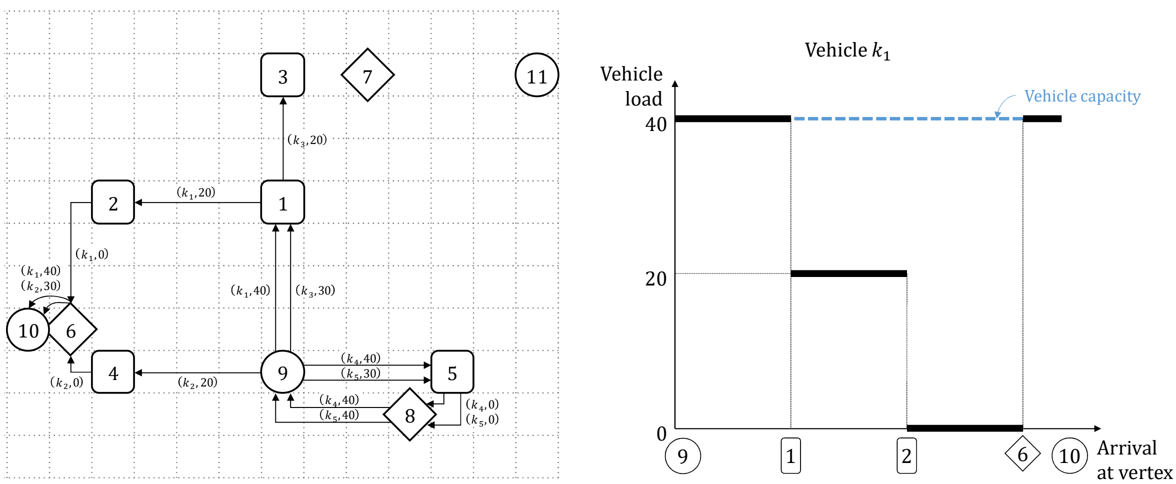


Figure 2: Network representation of the problem

The decisions whether a backhaul is visited in a route or not, and if so, to which mill to go next, are based on a new parameter related with the *reward paid for visiting that backhaul and a mill next* (δ_{bm}). Like in previous studies of VRP with selective pickups (e.g., Gribkovskaia et al., 2008) and other formulations of VRP with profits (e.g., Aras, Aksen, & Tekin, 2011), the reward is used to make an arc linehaul to backhaul more or less attractive. The reward corresponds to a payment per each ton of raw materials picked-up in a backhaul and delivered in a neighbouring mill. If the route ends after visiting the last linehaul, then there is no positive reward associated with that route. Hence, the reward parameter is used in the objective function, which trades-off between the sum of the travelling costs for visiting the backhaul after the last linehaul and moving from there to a mill, and the reward gained for visiting that backhaul. The reward parameter is also used to address compatibility issues related to the type of raw material p to be transported from a given backhaul b to a given mill m. In fact, if p is not available in b or not accepted in m then $\delta_{bm} = 0$. On the contrary, if there are several types of raw materials that can be transported from b to m, the value of δ_{bm} corresponds to the value of the most profitable material because there are no other aspects determining the choice between them. Consequently, the set P does not need to be considered in this model. However, in other real-life applications where the availability at the backhauls and or demand at the mills is limited and varies per type of product, the set P should be properly incorporated in the model, leading to a four-index decision variable x.

A new decision variable is needed to assure that, despite the possibility of *splitting the deliveries* to a linehaul, each delivery cannot exceed the truck capacity and that the total amount delivered in the several routes that visit it meets the expected demand. Previous studies used continuous variables w_i^k representing the quantity transported by vehicle $k \in K$ to/from customer $i \in V$ for a similar purpose (e.g., Nikolakopoulos, 2014). However, in the rVRPB under study, without time windows, these variables are insufficient for sub-tour elimination. In this context, a new set of continuous variables u_{ij}^k represent the load of vehicle $k \in K$ when traversing arc $(i, j) \in A$. Variables u_{ij}^k are a natural adaptation of variables u_i (Bektaş et al., 2015; Toth & Vigo, 2014) to a multi-route and split delivery situation. Additional constraints are needed to account for the routes with backhauls. In this case, the truck-load is higher before visiting the first linehaul, then progressively decreases until reaching zero after visiting the last linehaul. If a backhaul is visited, the pickup corresponds to a full truck-load. As an example, for a given route k, encompassing $\{0, i, i', i'', j, 0\}$, where $i, i', i'' \in L$ and $j \in B$, then the following rules apply: $u_{0i}^k \leq u_{ii'}^k \leq u_{i'i''}^k, u_{i''j}^k = 0, u_{j'0}^k = Q_k$.

Figure 3 exemplifies a feasible solution for the OBP starting in the node 9, in a network composed by 5 linehauls (numbered 1 to 5), 3 backhauls (numbered 6 to 8) and 3 mills (numbered 9 to 11). For simplification purposes, only the arcs used in the solution are represented in Figure 3a. The demand (in ton) at the linehauls is $q_1 = 30, q_2 = 20, q_3 = 20, q_4 = 20, q_5 = 70$. The reward for visiting a backhaul is 0.1€/ton in all cases. The available fleet is composed of 5 trucks, with capacity (in ton) $Q_1 = 40, Q_2 = 30, Q_3 = 30, Q_4 = 40, Q_5 = 40$. The linear distances between vertices (d_{ij}) are computed in reference to the background grid with 1km by 1 km, for example, $d_{13} = 2$ km. The fixed cost for using a vehicle is zero, and the variable cost is 1 €/km.



(a) feasible solution for vehicles k_1, k_2, k_3, k_4, k_5 (b) variation of the load of vehicle k_1 along the route

Figure 3: Example of a feasible solution for a rVRPB

The routing plan foresees the use of all five vehicles: k_1, k_2, k_4 and k_5 are OIRs while k_3 is an OR ending after visiting linehaul 3. There are split deliveries in linehauls 1 and 5. Total costs are 29€ and total revenues are 15€. The values of u_{ij}^k for truck 1 are shown in Figure 3b.

This example is instrumental in showing the impact of the reward value over the final routing solution. In fact, the route visiting linehaul 4 will always visit backhaul 6, and then mill 10, because the extra cost for visiting this pair backhaul-mill is 1€ $\left(d_{4,6} = 1e, d_{6,10} = 0 \implies c^k_{4,10} = 1\right)$ and the minimum revenue is 3€ ($\delta_{bm} = 0.1$€/ton, $\min\{Q_k\} = 30$ ton $\implies u^k_{6,10} \geq 30, \forall k \in K, \delta_{6,10} = 0.1$. Applying a similar logic, it is expected that the route visiting linehaul 3 will visit backhaul 7 if $\delta_{7,11} \geq 0.4$, since the extra cost for visiting the backhaul and mill is 4€ and $u^k_{7,11} \geq 30, \forall k \in K$.

3.2.2 Mathematical formulation

For the sake of convenience, before presenting the mathematical formulation, we resume the necessary decision variables, sets and parameters.

Decision variables:

$$x^k_{ij} \quad \begin{cases} 1 & \text{if vehicle } k \text{ travels from location } i \text{ to } j; \\ 0 & \text{otherwise.} \end{cases}$$

u^k_{ij} load of vehicle $k \in K$ when traversing arc $(i,j) \in A$

Sets:

L set of linehauls (customers where finished products are delivered)

B set of backhauls (suppliers where raw materials can be picked up)

M set of mills (where raw materials are delivered if a backhaul is visited)

V set of vertices; $V = \{0\} \cup L \cup B \cup M$

K set of vehicles

Parameters:

q_i quantity to be delivered to customer $i \in L$ (ton)

c^k_{ij} cost of transportation with vehicle $k \in K$ from $i \in V$ to $j \in V$ (€)

f^k fixed cost of using vehicle $k \in K$ in a daily route (€)

Q_k transportation capacity of vehicle $k \in K$ (ton)

d_{ij} travelling distance from $i \in V$ to $j \in V$ (km)

α maximum distance travelled in a route (km)

β minimum amount of raw materials to be backhauled (ton)

δ_{bm} reward for picking up one unit of raw material at backhaul $b \in B$ and delivering it to mill $m \in M$ (€)

ψ minimum amount of order delivered to a linehaul (ton)

Model [P0]

$$\min \sum_{k \in K} \sum_{j \in V \setminus \{0\}} f^k x^k_{0j} + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c^k_{ij} x^k_{ij} - \sum_{k \in K} \sum_{i \in B} \sum_{j \in M} \delta_{ij} u^k_{ij} \tag{1}$$

subjected to:

$$\sum_{i \in V} x^k_{ij} \leq 1 \qquad \forall j \in L \cup B, \forall k \in K \tag{2}$$

$$\sum_{i\in B}\sum_{j\in B}\sum_{k\in K} x_{ij}^k = 0 \tag{3}$$

$$\sum_{i\in V\setminus B}\sum_{j\in M}\sum_{k\in K} x_{ij}^k = 0 \tag{4}$$

$$\sum_{i\in M}\sum_{j\in V\setminus\{0\}}\sum_{k\in K} x_{ij}^k = 0 \tag{5}$$

$$\sum_{i\in B}\sum_{j\in L\cup\{0\}}\sum_{k\in K} x_{ij}^k = 0 \tag{6}$$

$$\sum_{j\in B}\sum_{k\in K} x_{0j}^k = 0 \tag{7}$$

$$\sum_{j\in L} x_{0j}^k = \sum_{i\in L\cup M} x_{i0}^k \qquad \forall k\in K \tag{8}$$

$$\sum_{i\in V} x_{ij}^k = \sum_{i\in V} x_{ji}^k \qquad \forall j\in V, \forall k\in K \tag{9}$$

$$u_{ij}^k \leq Q_k x_{ij}^k \qquad \forall (i,j)\in A, \forall k\in K \tag{10}$$

$$\sum_{i\in V} u_{ij}^k - \sum_{i\in V} u_{ji}^k \geq \psi \sum_{i\in V} x_{ij}^k \qquad \forall j\in L, \forall k\in K \tag{11}$$

$$\sum_{i\in L}\sum_{j\in B\cup\{0\}}\sum_{k\in K} u_{ij}^k = 0 \tag{12}$$

$$\sum_{i\in V}\sum_{k\in K} \left(u_{ij}^k - u_{ji}^k\right) = q_j \qquad \forall j\in L \tag{13}$$

$$\sum_{i\in V}\sum_{j\in V} d_{ij} x_{ji}^k \leq \alpha \qquad \forall k\in K \tag{14}$$

$$\sum_{i\in B}\sum_{j\in M}\sum_{k\in K} u_{ij}^k \geq \beta \tag{15}$$

$$x_{ij}^k \in \{0,1\}, u_{ij}^k \geq 0 \qquad \forall (i,j)\in A, \forall k\in K \tag{16}$$

The objective function (1) minimizes the total costs, decomposed into fixed costs (proportional to the number of vehicles used) and the variable costs (proportional to the total travelled distance), decreased by the revenue obtained for visiting backhauls and mills in the course of the OIR. Constraints (2) assure that any location can be visited at most once by each truck. Regardless, each linehaul and backhaul can be visited by several routes. Constraints (3)–(7) deal with route precedence rules, resulting from the specificities of this rVRPB for the wood-based panel industry. Specifically, constraints (3) state that the transport from a backhaul to another backhaul is not possible. Constraints (4) assure that the mill can only be visited after a backhaul. Constraints (5) assure that after visiting a mill, the only possibility is to go to the ending depot. Constraints (6) state that after visiting a backhaul, the next visit cannot be to a linehaul nor to the depot. Constraints (7) assure that the route cannot visit a backhaul after the depot. Constraints (8) and (9) are the typical VRP flow conservation constraints, at the depot and at each vertex, respectively. Constraints (10) are linking constraints, assuring that there is only a given amount transported to/from the customer if the customer is visited. Constraints (11) to (13) assure the elimination of sub-tours. Specifically, constraints (11) assure that the load of trucks progressively decreases as it visits the linehauls, and the amount delivered should be higher than a minimum amount. By considering the lower bound of the minimum amount, the model avoids undesirable solutions where $x_{ij}^k = 1$ and $u_{ji}^k - u_{ij}^k = 0$, which may occur for example if a linehaul (i') is visited in the course of a route from i to j, i.e., $x_{ii'}^k = x_{i'j}^k = 1$ (instead of $x_{ij}^k = 1$) but the amount delivered in i' is zero $\left(u_{i'i}^k - u_{ii'}^k = 0\right)$ due to the fact that the distance matrix does not obey to the triangular inequality (i.e., $\exists\, d_{ij} : d_{ij} > d_{ii'} + d_{i'j}$). Constraints (12) state that the truck leaves empty after visiting the last linehaul and constraints (13) assure that the demand at the linehauls is completely fulfilled. Constraints (13) together with constraints (2) account for the possibility of split deliveries at the linehauls. Constraints (14) assure that the maximum allowable distance of the daily route cannot be exceeded. It is noteworthy that if the maximum route length is constrained by the time travelled, then, this would require another type of auxiliary variables to count the route duration and consequent changes in these constraints, with similarities with other VRPs with time windows (e.g., Toth & Vigo, 2014). Constraints (15) set a minimum amount of raw materials to be backhauled to mills. Finally, constraints (16) determine the domain of the decision variables.

3.2.3 Special situation in which the rVRPB is simplified to a rich capacitated VRP

A problem variant of the rVRPB consists in removing constraints (14) and (15). In this situation, where there is no limitation to the route length and there is no minimum backhauling amount, a backhaul will be visited whenever it is cost-effective, according to the trade-off between the extra transportation cost (from travelling from the last linehaul, to that backhaul and to its closest mill) and the revenue (associated with delivering the load from the backhaul to the closest mill). From a modelling perspective, this means that, knowing which is the last visited linehaul in a route, it is possible to compute beforehand if and which backhaul and mill should be visited to minimize total costs. Consequently, the mathematical model can be simplified to a Rich Capacitated VRP (rCVRP) with split deliveries. This problem will only consist in sequencing the linehauls to be visited in each route, thus determining which linehaul will be last in each route.

This adaptation relies on a data pre-processing procedure (described in Algorithm 1) which consists in computing the minimum cost of having a given linehaul visited last in a vehicle route. If the cost of visiting a backhaul at the end of the route is lower than finishing the route at the depot (line 5), the cost associated with the arc heading to the depot is updated to the summed costs of pickup at the backhaul, delivering to the mill and returning to the depot, subtracted by the corresponding reward for performing the delivery to that mill (line 6). All combinations of vehicles, linehauls, backhauls, and mills are tested in this pre-processing stage, therefore ensuring that the vehicle arcs heading to the depot account for the minimum possible cost, which either corresponds to performing backhauling at the most advantageous locations or finishing its route after visiting the last linehaul. Finally, the sets of backhauls and mills are removed from the problem.

Algorithm 1: Data pre-processing for adapting the rVRPB to a rCVRP

1 **foreach** *vehicle* k **in** K **do**
2   **foreach** *linehaul customer* j **in** L **do**
3   **foreach** *backhaul customer* i **in** B **do**
4   **foreach** *mill customer* m **in** M **do**
5   **if** $c^k_{ji} + c^k_{im} + c^k_{m0} - \delta_{im} \cdot Q_k < c^k_{j0}$ **then**
6                     $c^k_{j0} := c^k_{ji} + c^k_{im} + c^k_{m0} - \delta_{im} \cdot Q_k$;
7 $V := V \setminus (B \cup M)$; $B := \varnothing$; $M := \varnothing$;

Afterwards, the new model for the rCVRP can be built upon [P0] by changing the objective function and removing constraints related with the sets of backhauls and mills, as shown in model [P1].

Model [P1]

$$\min \sum_{k \in K} \sum_{j \in V \setminus \{0\}} f^k x^k_{0j} + \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c^k_{ij} x^k_{ij} \tag{1b}$$

subjected to (2), (8)–(13) and (16) of model [P0]

3.3 Integrated Inbound and Outbound Planning (IIOP)

As stated before, the IIOP strategy consists in jointly planning all types of routes, including OIRs, ORs only for delivery of finished products and IRs for pickup of raw materials. Model [P2] for IIOP can be built upon adaptations of [P0], that account for the IRs, as follows. Constraints (7) are removed to allow dedicated routes from the depot to a backhaul. A new parameter δ^D_{bm} represents the reward for picking up one unit of raw material at backhaul $b \in B$ and delivering it to mill $m \in M$ (€) in the course of the dedicated route. A new set of auxiliary continuous variables y^k_{ij} is needed to represent the amount picked up in $b \in B$ and delivered in mill $m \in M$ by vehicle $k \in K$ in a direct route. The objective function (1c) is adapted accordingly. A new set of constraints (17) defines variables y^k_{ij} and constraints (18) set its bounds. Considering an arc $(i, j), i \in B, j \in M$, with $x^k_{ij} = 1$, if $x^k_{0i} = 1, i \in B$, then k is in a dedicated route, and according to the conjugation of constraints (17) and (18), $y^k_{ij} = u^k_{ij}$. If $x^k_{0i} = 0, i \in B$, then k is in an OIR, and $y^k_{ij} = 0$.

Model [P2]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x^k_{0j} + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c^k_{ij} x^k_{ij} - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta^D_{ij} y^k_{ij} - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta_{ij}(u^k_{ij} - y^k_{ij}) \tag{1c}$$

subjected to (2)-(6), (8)–(16) of model [P0] and

$$y^k_{ij} \le Q_k x^k_{ij} \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{17}$$

$$y^k_{ij} \le u^k_{ij} - (1 - x^k_{0i})Q_k \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{18}$$

$$y^k_{ij} \ge 0 \qquad \forall i \in B, \forall j \in M, \forall k \in K \tag{19}$$

3.4 Decoupled Inbound and Outbound Planning (DIOP)

As stated before, DIOP corresponds to the planning strategy currently used, where the ORs and IRs are planned independently and there are no OIRs. For the outbound logistics planning, the optimal ORs can be obtained by solving model [P3] that is an adaptation of model [P0], considering the nonexistence of backhauls and mills. For the inbound planning, the optimal IRs can be obtained by solving a model [P4], also an adaptation of model [P0], acknowledging only the routes from the depot/mill of origin to the backhauls.

Model [P3]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x^k_{0j} + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c^k_{ij} x^k_{ij} \tag{1d}$$

subjected to (2), (8)–(11), (13)–(14) and (16) of model [P0]

Model [P4]

$$\min \sum_{k\in K} \sum_{j\in V\setminus\{0\}} f^k x^k_{0j} + \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c^k_{ij} x^k_{ij} - \sum_{k\in K} \sum_{i\in B} \sum_{j\in M} \delta^D_{ij} u^k_{ij} \tag{1e}$$

subjected to (2)–(6), (8)–(10) and (14)–(16) of model [P0]

4 Solution approach

4.1 Fix-and-optimise approach

As stated in the literature review, the complexity of the VRP problems in real-life applications justifies the use of matheuristics. In this study, all the different models presented before are solved with a fix-and-optimise (F&O) approach in case of the large instances (i.e., more than 30 customers). This solution method was firstly presented by Sahling et al. (2009) for lot-sizing problems, but has been successively used for solving complex routing problems with promising results (e.g., Larrain, Coelho, & Cataldo, 2017; Neves-Moreira, Amorim, Guimarães, & Almada-Lobo, 2016).

The F&O matheuristic approach consists in iteratively solving several smaller mixed integer programming (MIP) sub-problems of the original model. The design of each sub-problem is problem-dependent and the obtained results highly depend on its adequate design. In this approach, we define a sub-problem as a set of decision variables to be either released or fixed in the original MIP model. Fixing a variable consists in setting its lower and upper bounds to the current solution value, thus precluding it from being changed in a solver iteration. On the other hand, releasing a variable consists in restoring a fixed variable to its original lower and upper bound values. For the problem at hand, two distinct sub-problem types were conceived, named `RouteRelease` and `LocationRelease`.

The `RouteRelease` sub-problem releases all decision variables associated with a given set of routes in the incumbent solution, based on proximity criteria of these routes. Route proximity is defined by the centroids of each route, which are computed as the non-weighted averages of the location coordinates that are visited. The outline of the `RouteRelease` sub-problem construction procedure is illustrated in Algorithm 2. The procedure starts by computing the centroid of each route in the incumbent solution (lines 2–4). For unused vehicles, the route's centroid is given by the depot's coordinates. A pivot route is selected at random from the incumbent solution (line 5), after which all other routes are ordered by its centroid's distance to the pivot route (line 6). The n routes with the lowest distance to centroid of the pivot route are then released in the sub-problem (lines 7–12).

Algorithm 2: Route Release sub-problem construction

```
   input: vars (MIP model routing decision variables),
          sol (incumbent solution),
          n (number of routes to be released in the subproblem)
1  released_routes = ∅; centroid_list = ∅;
2  foreach route in sol do
3      compute centroid of route;
4      append centroid of route to centroid_list ;
5  rt ← random route; cnt ← centroid of rt ;
6  order centroid_list by descending order of their distance to cnt ;
7  released_routes ← n first routes in centroid_list ;
8  foreach var in vars do
9      if var is associated with a vehicle in released_routes then
10         release var;
11     else
12         fix var ;
```

The `LocationRelease` sub-problem consists in releasing a given set of linehaul locations based on its geographical proximity. The procedure is described in Algorithm 3, and it starts by selecting a pivot linehaul (line 1), after which we retrieve all routes in the incumbent solution where the pivot linehaul is visited. Afterwards, we retrieve all the additional linehauls that are visited in these routes (lines 4–5).

Finally, the n closest linehauls to the pivot linehaul that were previously selected are released (lines 8–14).

Algorithm 3: Location Release sub problem construction

```
   input: vars (MIP model routing decision variables),
          sol (incumbent solution),
          n (number of locations to be released in the subproblem)
1  released_locations = ∅; candidates = ∅; loc ← random linehaul ;
2  foreach route in sol do
3      if route traverses loc then
4          foreach linehaul in route do
5              append linehaul to candidates ;
6  order candidates by descending order of their distance to loc;
7  released_locations ← n first locations in candidates;
8  foreach var in vars do
9      if var is associated with a linehaul in released_locations then
10         release var;
11     else if var is associated with a mill then
12         release var;
13     else
14         fix var ;
```

The overall structure of the matheuristic is shown in Algorithm 4.

The solution method requires an initial solution s_0 with objective function f_0, which is obtained through a greedy nearest neighbour heuristic (line 1): we select a random vehicle and construct its route by visiting the nearest unsatisfied linehaul until vehicle capacity is exhausted. The process is repeated until all linehaul demand is satisfied. No routes to backhauls are considered in the constructive phase.

After obtaining an initial solution, the matheuristic is then started. To that effect, sub-problem construction is initiated, whose size is controlled through the general principles of a Variable Neighbourhood Decomposition Search (VNDS), similar to what is presented in Hansen et al. (2001). Sub-problems are constructed in line 7, after which the MIP model is fed the incumbent solution s_{cur} and the sub-problem is solved by a MIP solver (lines 8–9).

After each solver iteration, the obtained solution s_{solve} is evaluated against the incumbent solution (lines 10–15). If the obtained solution did not yield an improvement of at least `imp` (line 10), we consider this a non-improvement iteration and increment the non-improvement counter. Nevertheless, we will accept the obtained solution even if it is not significantly better than the previous one (line 15). After a given number of consecutive non-improvements, the VNDS framework takes place either by increasing sub-problem size or switching the sub-problem type, if the current sub-problem size has been maxed out (lines 16–24). In the occurrence of a significant improvement of the problem's objective function, sub-problem type is re-set to `RouteRelease` and its initial size (line 11).

The matheuristic approach always initializes with the `RouteRelease` sub-problem type and the `LocationRelease` sub-problem is used after a significant number of non-improvements of the `RouteRelease` sub-problem. This algorithmic structure was conceived by bearing in mind that `RouteRelease` would be used as a more disruptive sub-problem, which would explore more disperse sections of the solution space, while the `LocationRelease` sub-problem focuses more on intensification.

4.2 Data pre-processing

Pre-processing the instance data related to the network generation is a common procedure in VRPs (e.g., Parragh et al., 2008; Soares et al., 2019) to simplify the mathematical formulation and achieve better

Algorithm 4: Matheuristic outline

```
input: MIPmodel (mixed integer programming model),
       P (list of possible sub-problems to be used),
       n_p (initial neighbourhood size for sub-problem p),
       N_p (maximum neighbourhood size for sub-problem p),
       I_{p,n} (limit of consecutive non-improvement iterations for sub-problem p of size n),
       TL_{p,n} (time limit for solver iterations of sub-problem p of size n),
       imp (minimum solution improvement to reset the no-improvement counter i)
s_0, f_0 = nearest_neighbour();
s_cur = s_0; f_cur = f_0; i = 0;
p = "RouteRelease";
while termination criteria not met do
    n = n_p;
    while n ≤ N_p do
        construct sub-problem of type p with size n ;
        feed MIPmodel with initial solution s_cur;
        s_solve, f_solve = MIPsolve(MIPmodel, TL_{p,n});
        if f_solve < f_cur − imp then
            s_cur = s_solve; f_cur = f_solve; i = 0; p = "RouteRelease";
            break
        else
            if f_solve < f_cur then
                s_cur = s_solve; f_cur = f_solve;
            i = i + 1;
            if i > I_{p,n} then
                i = 0;
                if n = N_p then
                    if p = "LocationRelease" then
                        p = "RouteRelease";
                    else p = "LocationRelease";
                    break
                else increase n;
output s_cur, f_cur
```

performance in the optimisation solver. The pre-processing procedure used prior to solving the models is threefold. The sub-set of arcs to be considered is presented in Table 2.

First, we remove all the arcs that lead to an unfeasible route, i.e., arcs that violate the precedence constraints (3) to (7). Second, we eliminate all arcs from linehauls to backhauls where its visit is not economically worthwhile, according to the given reward for visiting a backhaul. These arcs are only generated if they respect the condition exhibited in line 5 of Algorithm 1.

Third, arcs from backhauls are only generated to its closest mill, as delivering merchandise to more distant mills will only induce an increase of the problem's objective function.

It should be noted that this data pre-processing procedure does not cut off optimal solutions only if we do not impose a minimum inbound quantity to be collected from backhauls via constraints (15). If this is not the case, this procedure may induce sub-optimality or even turn the model infeasible because there are no cost-effective backhauls to visit. Therefore, in these situations, a trade-off between optimality and simplicity must be taken into account.

Table 2: Pre-processing the problem network

Origin \ Destination	{0} depot or mill of origin	Linehaul l	Linehaul $l' \neq l$	Backhaul b	Mill m	{0} *fictitious depot*
{0} depot or mill of origin		•	•	(*)		
Linehaul l			•	(**)		•
Linehaul $l' \neq l$		•		(**)		•
Backhaul b					(+)	
Mill m						•
{0} *fictitious depot*						

Legend:
(*) is generated in IIOP but not in OBP;
(**) for each linehaul, generate the arcs to all the backhauls that lead to a cost-effective solution (i.e. satisfy line 5 of Algorithm 1);
(+) for each backhaul, only generate the arc to the minimum cost mill.

5 Computational experiments

The proposed approach was applied in a case study in a wood-based panel company in Portugal. The mathematical model was implemented in Gurobi 7.5 commercial solver. The solution method was developed in Python 3.6. The mathematical models were subject to the data pre-processing procedure described earlier and used to compare the gains of the IIOP strategy with the DIOP one, which is currently done by the company. A set of experiments were also done to provide valuable managerial insights for planners. Lastly, the performance of the proposed solution method was compared with a commercial MIP solver for problem instances of increasing size, which were based on real routing plans executed by the company.

5.1 Case study

This study was motivated by a real-life application in the wood-based panel company, firstly presented in Amorim, Marques, and Oliveira (2014). The focal company owns several mills, each producing a specific portfolio of wood-based panels mainly for furniture, construction and decoration. The case study is at one of the mills in Portugal that produces around 1.2 thousand tons of wood-based panels per day, in a make-to-order basis, and assures its delivery to an average of 30 customers distributed over the entire Iberian Peninsula. The average daily consumption of raw materials is 1,750 m^3. The study uses real data regarding the customers' orders in two of the most representative operational days. There are 30 customers to be visited, whose ordered amounts are in average 35.5 ton/customer, varying between 0.05 and 399 ton. The values of the model parameters are an approximation of those provided by the planners. The distances between locations were computed by resorting to the Google Maps routing engine.

Nowadays, the outbound routes are planned to start in the morning of the next day at the opening hour of the mill of origin. It is assumed that all routes can start at the same time, and there are no time windows conditioning the time of arrival to customers, suppliers or mills. The responsible for outbound logistics determines the exact number of trucks needed for the next day and groups the customers to be visited in each route according to empirical rules that rely on the customers' geographical location. Then, the routes are assigned to the third-party logistics operators (3PL) with whom there are valid outsourced contracts. The generic contractual conditions are a fixed cost of 70€ per truck used and a variable cost of 1.7€ per km travelled. The fleet available at the mill of origin in each day encompassed 100 trucks, of which 20 trucks have capacity up to 10 tons, 40 trucks have a capacity of 20 tons, and 40 trucks have a capacity of 40 tons, summing up a total transportation capacity of 2,600 ton. Each vehicle must deliver at least 0.5 ton to each customer visited (i.e., $\psi = 0.5$), except when its demand is lower than this parameter. All trucks are prepared to do IRs, if needed.

Overall, the current logistics process results in a low rate of inbound-outbound flow integration, and the logistics planner has very little visibility about the arrival time to the customers and the time and characteristics of the inbound loads.

The 3PL assigns a truck driver to each route. Then, the driver is responsible for establishing the sequence for visiting all outbound customers, the path and schedules, which may or may not be optimal. The decision of either to visit a supplier (backhaul) or not is often taken by the driver, based on the extra cost for visiting a known supplier in the vicinity of the last costumer (linehaul) visited in the route, in case it is doable within the route maximum duration length set by the 3PL business rules and conditioned by transportation legislation. There are 75 possible suppliers of wood-chips for IRs. The raw materials may be delivered back to the mill of origin or alternatively to any of the other three mills owned by the company in the Iberian Peninsula. Currently, there is no minimum amount of backhauling required. According to the experience of the planners, the reward for each backhauling can go up to 10 €/ton.

The computational results were obtained with two major groups of instances (A and B) built upon the previous case study description, each one of them corresponding to a representative operational day. Instances A differ from instances B with respect to the average distance between the linehauls. The linehauls in instances A are more geographically dispersed, with an average distance between linehauls and depot of 461 km, while in instances B it is 197 km. Baseline instances A30 and B30 correspond to the situation described, with 30 linehauls, a total demand of 1,853 ton, 75 backhauls, 4 mills and 100 available trucks. Instances among the same instance group differ in the number of linehauls to visit (10, 30 or 50 linehauls) and the number of possible backhauls (0, 25, 50, 75 or 100 backhauls). The instances were generated in a cumulative manner, i.e., the largest instances contain all locations considered in smaller instances. The selection of the locations to be included/removed in the instances was performed randomly from the dataset of the case study.

5.2 Comparison among distinct planning strategies

Instances A10 and B10 were used in these experiments to compare and quantify the benefits of adopting an OBP or IIOP strategy versus DIOP because it is possible to solve the model quickly to optimality while larger instances require the proposed matheuristic whose gaps to optimality could bias the results. Furthermore, the resulting routing plan can be easily visualized.

To perform this comparison, two different reward values were considered (1€/ton and 7€/ton). In order to avoid results biased by different reward values for IRs and OIRs, the backhauling reward was set regardless of the type of route (whether it was a dedicated backhaul route or an opportunistic one) and is generically called reward instead of backhauling reward. The inbound quantity to be satisfied was set to 160 ton of raw materials, which corresponds to approximately twice the outbound quantities of finished products in these instances, taking into account the mills' productive efficiency. The remaining parameters remained unchanged throughout the instances, with $\alpha = 1,200$ km and $\psi = 0.5$ ton.

For the IIOP strategy, model [P2] was solved to optimality and a given backhauling amount was set. The DIOP models [P3] and [P4] were also solved to optimality with this same backhauling amount to allow a fair comparison. In respect to the OBP strategy, the rationale to allow its comparison with the remaining strategies consisted in: (i) solving the OBP model [P0] to optimality, replacing constraints (15) by a similar set of constraints where a maximum (instead of minimum) backhauling amount of 160 ton is set; (ii) solving model [P4] to obtain the IRs for the differential amount between 160 ton and the already backhauled amount via OBP; (iii) computing the total costs for these two models.

The obtained results are presented in Table 3. In these instances, the matheuristic was not required, since the computational time for proving optimality in the solver was very short (less than 5 min on average). In these experiments, the number of binary decision variables ranged from 10,000 to 17,000.

Table 3: Comparison between alternative Inbound and Outbound Planning strategies

Reward (€/ton)	Instance	Planning strategy	Objective Function	Costs (€)			No. routes				Backhauled amount (ton)	No. trucks used	Runtime (s)
				Total	Fixed	Transport	Total	OIR	OR	IR			
7.00	A10	Integrated	2,536	3,656	350	3,306	7	0	3	4	160	5	821
		Opportunistic	2,714	3,834	420	3,414	7	1	3	3	160	6	45
		Decoupled	2,536	3,656	350	3,306	7	0	3	4	160	5	37
	B10	Integrated	768	1,888	280	1,608	4	3	0	1	160	4	36
		Opportunistic	771	1,891	280	1,611	4	4	0	0	160	4	32
		Decoupled	896	2,016	350	1,666	7	0	3	4	160	5	517
1.00	A10	Integrated	3,496	3,656	350	3,306	7	0	3	4	160	5	60
		Opportunistic	3,496	3,656	350	3,306	7	0	3	4	160	5	47
		Decoupled	3,496	3,656	350	3,306	7	0	3	4	160	5	34
	B10	Integrated	1,802	1,962	350	1,612	5	4	1	0	160	5	21
		Opportunistic	1,811	1,971	350	1,621	6	1	2	3	160	5	332
		Decoupled	1,856	2,016	350	1,666	7	0	3	4	160	5	243

The analysis of these results shows that the logistics planning strategy leading to the lowest cost is IIOP in all the experiments. In some cases, the strategy OBP performs better than DIOP, as intuitively expected, but in others, it does not. This is because the OBP model is myopic in the sense that it includes all OIRs that are cost-effective for a given backhauling reward value, but does not trade-off between OIRs and IRs as it happens with the IIOP model.

Instance B10 with a reward value of 7€/ton exemplifies a case where the IIOP strategy is better than OBP and better than DIOP strategies. The total costs of the resulting logistics plans are 1,888€, 1,891€, and 2,016€ respectively. The optimal IIOP routing plan consists of three OIRs (for trucks k_1, k_2 and k_5) and one IR (for truck k_4) (Figure 4a). While, the optimal plan for OBP encompasses three OIRs identical to the later plan, and one extra OIR (k_3) (Figure 4b). The OIR k_3 is still cost-effective for that reward value, but it is costlier than doing the alternative IR k_4 as in the IIOP plan. No IRs are foreseen in the OBP strategy because the routing plan obtained by solving model [P0] already fulfils the whole demanded backhauled amount; therefore there is no stimulus for finding IRs with model [P4] afterwards.

The decoupled planning strategy for instance B10 leads to a 6.8% increase of the total costs when compared with the previous, due to the increase of the transportation costs and also the use of five vehicles instead of four (Figure 4c). The overall routing plan encompasses four IRs (obtained with model [P4]), plus three ORs (obtained with model [P3]). The IRs are similar to the ones of vehicle k_4 in the IIOP strategy but the ORs are not. This is because the linehauls are re-distributed in the routes in a different way when the visit to backhauls is not considered in the same model. For example, linehaul 7 was split deliveries according to the IIOP and OBP plans due to its geographical proximity to the backhaul 13. This no longer happens in the decoupled planning, and this linehaul is visited only once in the course of a longer route that extends up to linehaul 10.

Conversely, instance A10 with a reward value of 7€/ton, exemplifies a case where the performance of the IIOP strategy is the same as the DIOP strategy (3,656€), and the OBP performs worse than the other planning strategies (3,834€, 4.9% worse). The optimal IIOP routing solution consists of three ORs and four IRs (Figure 4e). In this setting, with the linehauls more geographically dispersed and farther from the suppliers and neighbouring mills, it is cheaper to visit several times supplier 16 in dedicated IRs than considering OIRs. However, the solution of the OBP model, which is myopic with respect to this possibility, encompasses one OIR that visits the cost-effective backhaul 22 (Figure 4f).

The analysis of these results also shows that the backhauling reward value has a significant impact on the routing plans and can lead to different conclusions with respect to the comparison between the alternative planning strategies. For example, when the reward value is lowered to 1€/ton, the results for instance B10 show that the visit to backhauls 15 and 16 are no longer cost-effective in the IIOP strategy. Hence, the routing plan consists of four OIRs, all visiting backhaul 13 (Figure 4d) and 1 OR. The total costs are 3.8% higher than in the experiment with a reward of 7€/ton, due to an increase in the total transportation distance and in the use of five vehicles instead of four.

It is noteworthy that lowering the value of the reward for visiting the backhauls has a negative effect on revenue and consequently, increasing the value of the objective function (134% higher than with previous experiment with 7€). For this case, the IIOP strategy still performs better than the OBP and DIOP. However, the total cost savings are reduced to 2.7% and 2.2%, respectively. This is due to the fact that with a lower reward value, the use of OIRs is less attractive, and the inbound demand must, therefore, be satisfied with dedicated backhaul routes.

In instance A10, when the reward value is lowered to 1€ per ton, there is no visit to a backhaul that is cost-effective. Hence the optimal plan for the OBP strategy does not consider any OIR, and it is identical to the IIOP and DIOP strategies described before.

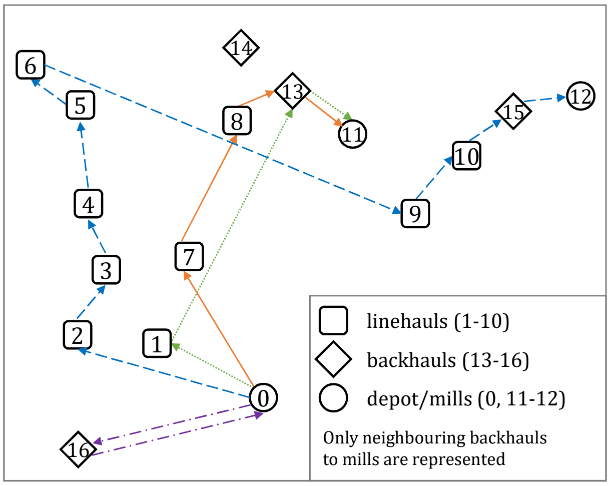
These findings suggest that IIOP is the strategy that allows the optimisation of the combination between backhauling and inbound routing, but under specific circumstances that favour the supply of raw materials through cost-effective OIRs instead of direct IRs, OBP can perform better than DIOP. As shown in these experiments, these circumstances depend on the backhauling reward value for visiting a backhaul in an OIR and on the geographical configuration of the logistics network of the planning day, especially the relative distance between a linehaul that can be visited last in a route, and a neighbouring backhaul and mill.

As stated before, the opportunistic planning strategy can be considered an "intermediate" stage from DIOP towards IIOP. The transition from DIOP to opportunistic planning is smoother since it is restricted to organisational changes within the local outbound logistics offices in each mill, while the IIOP also impacts in the central office currently responsible for inbound logistics planning. During this intermediate stage using the OBP strategy, the planners need to compare the optimal routing plan with the outcome of the DIOP strategy in order to establish if backhauling is favourable for the set of customers visited in each day.

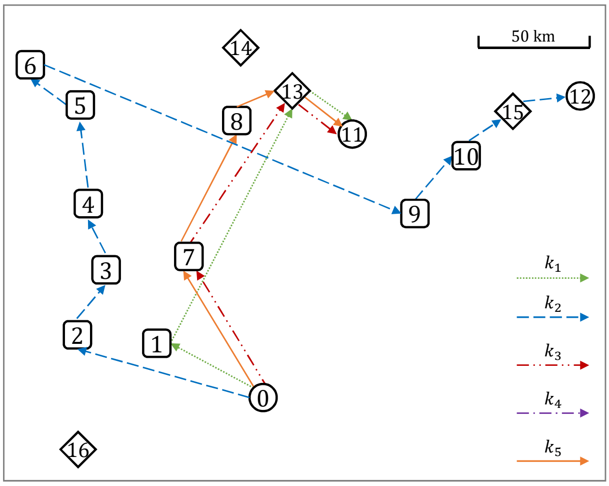
5.3 Managerial insights

Focusing on OBP, which is the strategy likelier to be adopted by the planners in this case study, additional experiments were designed for instances with 10 linehauls (A10, B10), to provide managerial insights on how the values of key parameters of model [P0] set by the planners with some degree of uncertainty, may actually impact on the routing plan. The parameters under study are:

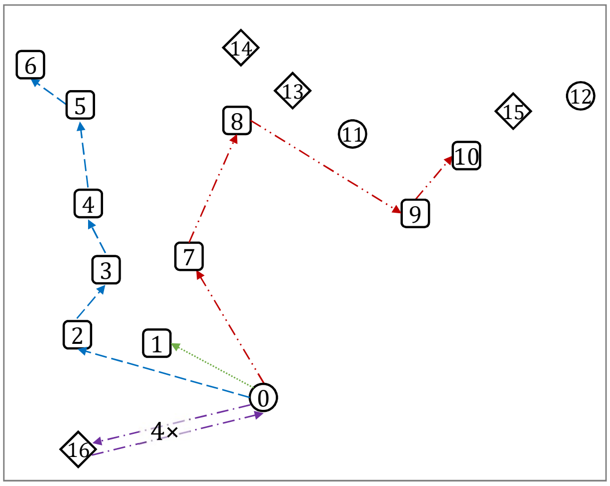
- backhauling reward (δ), i.e., incentive for picking up one unit of raw material in a backhaul $b \in B$ and delivering it to a mill $m \in M$. For simplification purposes, it is assumed that the reward is the same for all backhauls and mills that accept compatible types of raw materials. Based on experts opinions, the reward can vary in a range from 1 to 10 euros per ton;
- minimum backhauled amount (β), i.e., amount of raw materials to be backhauled in OIR. If parameter $\beta = 0$ then no visits to backhauls are required; if $\beta \geq 1$ ton, then, at least, one backhaul must be included; and if $\beta \geq 41$ ton, then, at least, two backhauls must be included), since the maximum truck capacity is 40 ton;
- maximum length of the route (α), i.e., maximum distance travelled in a route. The values tested are $1,200$ km (corresponding to the longest distance from the mill to a linehaul in instance A10)



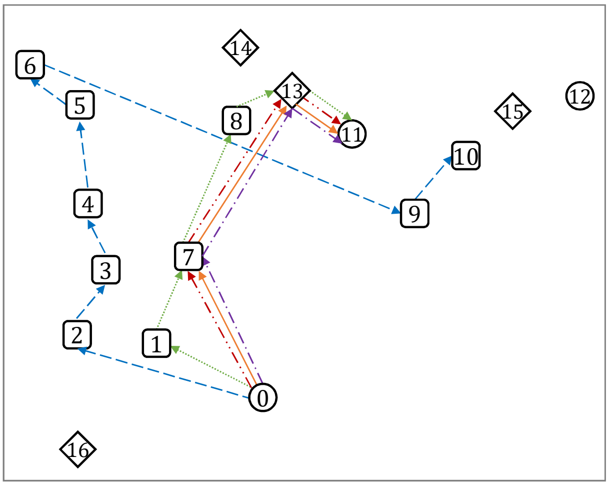
(a) B10, integrated strategy (7€/ton)



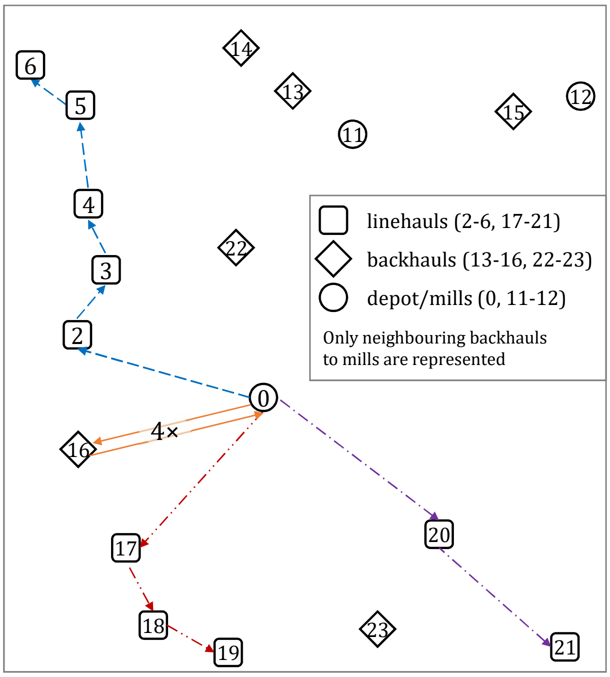
(b) B10, opportunistic strategy (7€/ton)



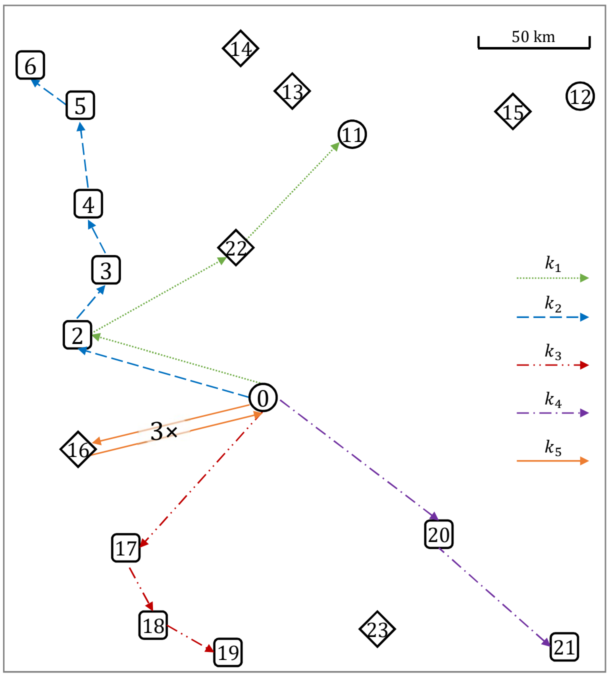
(c) B10, decoupled strategies (1 and 7€/ton)



(d) B10, integrated strategy (1€/ton)



(e) A10, integrated, decoupled (1 and 7€/ton) and opportunistic strategies (1€/ton)



(f) A10, opportunistic (7€/ton)

Figure 4: Graphical representation of the planning strategies for instances A10 and B10

and $1,500$ km;

- minimum delivery (ψ), i.e., the minimum allowable amount of order delivered to a linehaul, conditioning the possibility of splitting the order of a linehaul into more than one deliveries done by different trucks. The values tested were $\psi = 0.5$ ton, meaning that many split deliveries are allowed, $\psi = 5$ ton and $\psi = 10$ ton (corresponding to the capacity of the smallest truck), meaning that split deliveries are more restricted.

Let us state the baseline conditions for this analysis $\delta = 6$ €/ton for instance A10 and $\delta = 1$ €/ton for instance B10, $\alpha = 1200$ km for A10 and $\alpha = 400$ km for B10, $\beta = 0$ and $\psi = 0.5$ ton. The results of the experiments presented in Table 4 and in the Appendix are the basis for managerial insights that can be valuable for route planning.

Impact of the variation of the value of reward for visiting a backhaul (δ)

Results generically confirm a positive effect in the objective function of increasing the value of δ, because more OIRs are performed, often with the same number of trucks. The total transportation costs increase, due to the increase in the total distances travelled, but these are compensated with a higher total reward collected. The first managerial insight that can be formulated is that planners wishing to foster an increase of OIRs should set the reward value at least equal to the extra travelling costs for visiting the most cost-effective backhaul (i.e. the costs for travelling from the last linehaul to the backhaul and from there to the closest mill).

For instance A10 the minimum δ should be 7€/ton. Below that value, there is no backhaul that is cost-effective, hence, no OIRs are included in the optimal routing plan. The number of trucks needed increases for four to five. Increasing δ to 8 €/ton improve the value of the objective function but do not change the costs, because the number of trucks and the routing plan remains the same. However, very high values of δ are not beneficial as it leads to the use of a large truck fleet. Hence, the percentual increase of total costs is much higher than the gains in the value of the objective function, and the resulting routing plan is hardly adopted in practice. For example, a δ equal to 10€/ton in instance A10 leads to costs 243% higher than in the baseline, corresponding to the highest number of 34 OIRs out of the 36 routes that compose the optimal routing plan.

Regarding instance B10, the linehauls are less geographically dispersed than in A10; thus, a slight increase in the δ leads to significant changes in the number of OIRs and the improvement in the objective function value. In fact, the baseline experiment with a δ equal to 1€/ton already leads to 3 OIRs and one for each of the trucks used. For δ equal or higher than 5€/ton the routing plan changes drastically to 39 OIRs requiring 39 extra trucks.

Impact of the variation of the required backhauled amount (β)

Experiments suggest that increasing β has a negative impact on the value of the objective function because it increases the transportation costs for the mandatory visit to backhaul. However, in some instances, such as A10, it leads to an increase in OIRs, while in others, such as B10, it leads to an increase of the number of IRs. A second managerial insight for planners relates to the fact that the geographical dispersion between the linehauls, backhauls and mills is the determining factor for finding the optimal routing plan, as discussed in Section 5.2. It is also noteworthy that, under some circumstances (e.g. for $\delta \leq 2$ and $\beta > 0$ for A10), the solution turns infeasible because the pre-processing algorithm guarantees that only cost-efficient integrated routes can be created.

Impact of the variation of the delivery amount at a linehaul (ψ)

Experiments indicate that increasing ψ has a slightly negative effect on the value of the objective function. Although this may imply the use of fewer vehicles, this also decreases the possibility of creating integrated routes and, as such, the possibility of collecting a higher total reward.

There is a complementary relation between the key parameters ψ and δ in fostering the number of OIRs in the optimal routing plan. In practice, if ψ is low, means that more visits to the linehauls are allowed, and so, there is more flexibility in the routing plan to include OIR, especially if the reward for visiting a backhaul is high. In fact, the number of OIRs is maximized (34 out of 35 routes) if ψ is very low (e.g., 0.5 ton) and δ is very high (e.g., 10€). However, these high number of integrated routes (e.g. 34 out of 36 routes) can hardly represent the common practice (Figure 5). Hence, another managerial insight for planners relates to the importance of properly addressing the trade-off between the offered reward and the maximum number of visits allowed to a linehaul, which is specific for each case.

Impact of the variation of the maximum length of the route (α)

Experiments show that increasing α tends to improve the objective function, due to the decrease in the number of required vehicles and the possibility to visit a larger number of linehauls is the same route. However, without a direct impact on the number of OIR. As an example for instance A10, increasing α from $1,200$ km to $1,500$ km, all other parameters remaining the same as in the baseline scenario, lead to a decrease of 32% in the value of the objective function, related with the use of 3 vehicles instead of 4. In instance B10, the increase from 400 km to 800 km, leads to a decrease of 22% in the value of the objective function due to longer routes, using the same fleet of 3 trucks.

In summary, results show that there are several trade-offs that need to be analysed by planners to balance the increase of OIRs and the increase in transportation costs. In particular, results suggest that α is the parameter that impacts the most in improving the value of the objective function and costs (improvements of 32% in instance A10 e 22% in B10, because it enables to use fewer vehicles, and fewer distances travelled, however, do not necessarily foster OIRs.

Moreover, the main parameter to be taken into account for planners willing to improve OIRs is δ. As discussed before, OIRs tend to be included in the routing plan when the reward value is above a threshold, corresponding to visiting the first cost-effective backhaul. The value of this threshold depends on the geographical dispersion of nodes in the transportation network and particularly the distance between the last visited linehaul, the closest backhaul and its neighbouring mill.

5.4 Performance of the solution approach

Despite the fact that the solver is able to obtain optimal solutions within a few minutes for problem instances of 10 linehauls, this is not the case for larger instances. In these cases, the use of the matheuristic is justified in order to obtain good quality solutions in a shorter computational time. A set of computational experiments was envisaged to validate the proposed solution approach. Instances of group A and B were solved using the standalone MIP solver approach and the fix-and-optimise matheuristic. These experiments were performed in an Intel Xeon E5-2450 @ 2.10GHz CPU with capacity for 16 simultaneous processing threads.

Both approaches were run for 3,600s, with $\alpha = 1,200$, $\beta = 0$ and $\psi = 0.5$. The MIP solver was executed once for each instance using Gurobi's default parameters and the fix-and-optimise approach was run 10 times for each instance, using the parameters described in Table 5.

Table 4: Summary of the experiments on the impact of the values of model parameters

Inst.	Parameter				Objective Function	Costs (€)			Routes			Runtime (s)	MIP Gap
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
A10	**1,200**	**0**	**0.5**	**6**	**4,703**	**4,703**	**280**	**4,423**	**4**	**0**	**4**	**19**	**1.8%**
	–	–	–	7	-1%	+5%	+25%	+4%	+1	+1	0	105	0.6%
	–	–	–	8	-2%	+5%	+25%	+4%	+1	+1	0	112	0.3%
	–	–	–	10	-46%	+243%	+800%	+208%	+32	+34	-2	272	1.9%
	–	1	–	–	0%	+5%	+25%	+4%	+1	+1	0	19	1.0%
	–	41	–	–	+2%	+12%	+50%	+10%	+2	+2	0	190	0.0%
	1,500	–	–	–	-32%	-32%	-25%	-32%	-1	0	-1	26	1.6%
	–	–	5	–	+1%	+6%	+25%	+4%	+1	+1	0	28	2.0%
	–	–	5	10	-7%	+35%	+75%	+33%	+3	+5	-2	304	0.4%
	–	–	10	–	+1%	+6%	+25%	+4%	+1	+1	0	130	1.9%
	–	–	10	10	-5%	+21%	+25%	+21%	+1	+3	-2	308	0.7%
B10	**400**	**0**	**0.5**	**1**	**2,002**	**2,002**	**210**	**1,792**	**3**	**3**	**0**	**16**	**0.0%**
	–	–	–	2	0%	+4%	+33%	+1%	+1	-2	+3	28	0.0%
	–	–	–	5	-105%	+284%	+1,300%	+165%	+39	+36	+3	32	0.0%
	–	1	–	–	+2%	+4%	+33%	+1%	+1	-2	+3	39	0.0%
	–	41	–	–	+7%	+11%	+67%	+4%	+2	-1	+3	41	0.0%
	800	–	–	–	-22%	-20%	0%	-22%	0	-2	+2	72	0.0%
	–	–	5	–	0%	0%	0%	0%	0	-3	+3	25	0.0%
	–	–	5	5	-23%	+57%	+267%	+32%	+8	+5	+3	20	0.0%
	–	–	10	–	0%	0%	0%	0%	0	-3	+3	29	0.0%
	–	–	10	5	-15%	+25%	+133%	+13%	+4	+1	+3	18	0.0%

Legend: α: maximum length of the route (km); β: minimum backhauled amount (ton); ψ: minimum delivery amount (ton); δ: backhauling reward (€/ton); Runtime: computational time after which no better solution was obtained (seconds); MIP Gap: percentual difference obtained by Gurobi between the upper and lower bound of the branch-and-bound method. All models were run with a maximum time limit of 3,600s. The first row of results for each instance (highlighted in bold) contains the baseline values, and the rows that follow exhibit either the absolute or the percentual variation compared with the baseline values (except for Runtime and MIP Gap values).

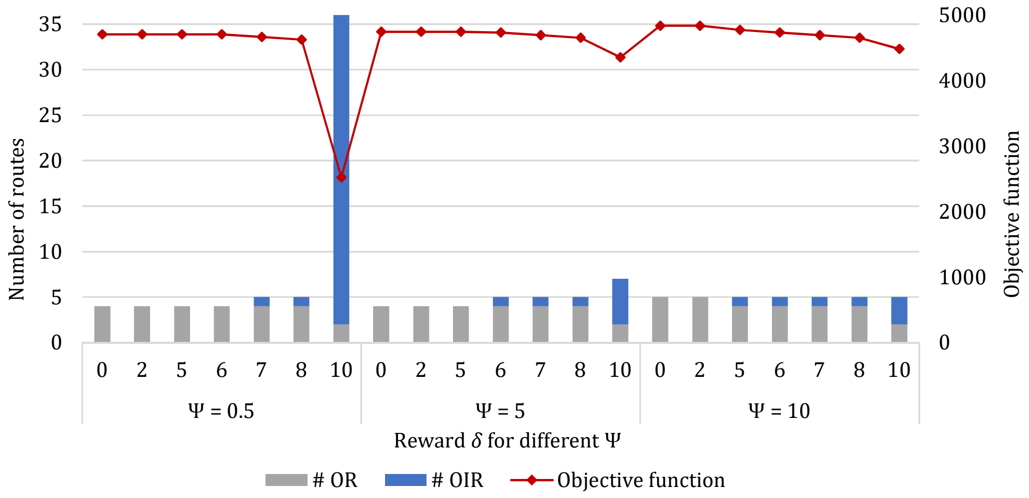


Figure 5: Impact of the variation of the reward (δ) and minimum delivery (ψ) ($\alpha = 1,200, \beta = 0$)

Table 5: Used parameters for the matheuristic approach

Parameters		**Value**
Termination criteria	Time limit	3,600s
No-improvement criterion	Improvement between consecutive iterations lower than	100
`RouteRelease` sub-problem	Subproblem sizes	4, 6, 8, 16 routes
	No-improvement limit to change subproblem size	2 iterations
	MIP solver iteration time limit	Multiples of 15s (according to sub-problem size)
`LocationRelease` sub-problem	Subproblem sizes	2, 4, 6, 8 linehauls
	No-improvement limit to change subproblem size	2 iterations
	MIP solver iteration time limit	Multiples of 15s (according to sub-problem size)

Table 6 summarizes the computational results of the MIP solver and matheuristic approaches for the 30 problem instances.

The results demonstrate that both the MIP solver and the matheuristic are adequate for solving instances up to 10 linehauls (groups A10 and B10), as the solver is able to prove optimality for most instances and the matheuristic easily reaches the same solution as the MIP solver. For larger instances, the MIP solver yields optimality gaps up to 32% for instances of groups A30, A50, B30 and B50. Specifically to instances of group A, it is possible to observe the increase in the number of routes that perform backhauling as the number of backhaul locations progressively increases. In instances of group B30, a single OIR is used when backhauling is allowed, and in group B50 no opportunistic backhauling is performed. However, the obtained solutions by the MIP solver when the number of backhauls increases do not necessarily improve, contrarily to what would be expected. Furthermore, for instances of group B50, the solver is unable to find a single feasible solution within the 1-hour limit for 4 out of the 5 instances. This fact is probably due to the increase in model size and complexity when more backhaul locations are being considered, thus requiring more time for Gurobi to reach identical solutions when exploring the branch-and-bound tree.

The matheuristic approach exhibits small standard deviation values for the 10 repetitions performed for each instance, thus suggesting that the obtained results are robust. The negative percentual difference values between the solver and the matheuristic suggest that the matheuristic is able to converge correctly to better solutions, as opposed to the solver, which exhibits very high optimality gaps. This negative percentual difference tends to be increasingly more expressive with the increase in instance size. Furthermore, results also suggest that the matheuristic also takes better advantage of the increase in the number of backhaul locations, as the objective function values generally decrease when the number of backhaul locations increases.

From these results, we can say that the proposed matheuristic approach is adequate for solving the problem at hand. For instances of considerable size, the MIP solver starts to struggle in finding feasible solutions in an acceptable time limit, and apparently also has greater difficulties taking advantage of backhauling, while the matheuristic is able to decrease the overall logistics costs with an increase in the number of backhaul locations, therefore yielding more consistent results.

Table 6: Computational results of the MIP and matheuristic approaches for 30 problem instances

Problem instance						MIP Solver					Fix-and-optimise					% diff.
Group	$\|L\|$	$\|B\|$	$\|K\|$	$\sum q_i$	$\sum Q_k$	OF	MIP Gap	Runtime	No. routes	No. OIRs	Objective Function: Average	Objective Function: Standard Deviation	Runtime	No. routes	No. OIRs	
A10	10	0	100	82	2,600	3,215	0.0%	50	3	0	3,215	0	31	3	0	0.0%
	10	25	100	82	2,600	3,215	0.0%	205	3	0	3,215	0	37	3	0	0.0%
	10	50	100	82	2,600	3,215	0.0%	60	3	0	3,215	0	28	3	0	0.0%
	10	75	100	82	2,600	3,215	0.0%	55	3	0	3,215	0	30	3	0	0.0%
	10	100	100	82	2,600	3,215	0.0%	31	3	0	3,215	0	33	3	0	0.0%
A30	30	0	100	1,853	2,600	13,745	18.4%	3,110	53	0	13,627	19	2,957	53	0	-0.9%
	30	25	100	1,853	2,600	13,696	19.6%	3,444	54	26	13,404	20	2,885	53	25	-2.1%
	30	50	100	1,853	2,600	13,701	20.1%	2,758	54	32	13,374	21	2,879	53	31	-2.4%
	30	75	100	1,853	2,600	13,833	20.9%	3,187	55	33	13,366	19	2,576	53	31	-3.4%
	30	100	100	1,853	2,600	13,702	20.1%	1,877	56	34	13,369	20	2,731	53	31	-2.4%
A50	50	0	100	2,061	2,600	20,986	29.9%	2,968	65	0	19,465	363	3,463	64	0	-7.2%
	50	25	100	2,061	2,600	19,717	26.5%	1,902	64	30	19,019	194	3,527	63	26	-3.5%
	50	50	100	2,061	2,600	19,907	27.3%	3,469	65	37	19,079	313	3,397	64	31	-4.2%
	50	75	100	2,061	2,600	20,829	30.5%	3,019	65	36	19,167	423	3,464	64	32	-8.0%
	50	100	100	2,061	2,600	21,365	32.3%	2,653	66	33	19,116	327	3,513	65	36	-10.5%
B10	10	0	100	82	2,600	1,575	1.9%	129	3	0	1,575	0	39	3	0	0.0%
	10	25	100	82	2,600	1,575	2.7%	117	3	0	1,575	0	39	3	0	0.0%
	10	50	100	82	2,600	1,575	0.0%	53	3	0	1,575	0	39	3	0	0.0%
	10	75	100	82	2,600	1,575	0.0%	51	3	0	1,575	0	44	3	0	0.0%
	10	100	100	82	2,600	1,575	0.0%	51	3	0	1,575	0	61	3	0	0.0%
B30	30	0	100	818	2,600	8,695	20.1%	3,032	23	0	8,210	9	2,267	22	0	-5.6%
	30	25	100	818	2,600	8,626	19.4%	3,354	23	1	8,212	15	2,730	21	1	-4.8%
	30	50	100	818	2,600	9,162	30.5%	2,473	23	0	8,206	12	2,292	22	1	-10.4%
	30	75	100	818	2,600	8,350	16.8%	3,262	21	1	8,206	10	2,876	21	1	-1.7%
	30	100	100	818	2,600	9,003	32.1%	441	23	1	8,198	2	2,474	22	1	-8.9%
B50	50	0	100	2,054	2,600	–	–	–	–	–	45,393	956	3,419	68	0	–
	50	25	100	2,054	2,600	–	–	–	–	–	45,325	793	3,420	69	0	–
	50	50	100	2,054	2,600	–	–	–	–	–	45,759	526	3,388	64	0	–
	50	75	100	2,054	2,600	46,991	13.8%	2,830	69	0	45,592	655	3,359	65	0	-3.0%
	50	100	100	2,054	2,600	–	–	–	–	–	45,768	646	3,412	67	0	–
															Average	**-3.0%**

Legend: $|L|$: number of linehauls to be visited; $|B|$: number of possible backhauls; $|K|$ total number of vehicles available; $\sum q_i$: total quantity to be delivered to linehauls (ton); $\sum Q_k$: total vehicle transportation capacity (ton); OF: Final value of the objective function (€); Runtime: average computational time after which no better solution was obtained (seconds); MIP Gap: Percentual difference obtained by Gurobi between the upper and lower bounds of the branch-and-bound method; No. routes: total number of routes; No. OIR: number of routes that include visit to a backhaul; % diff.: percentual difference of the fix-and-optimise average objective function towards Gurobi's objective function (a negative difference favours the matheuristic). For the fix-and-optimise approach, the route indicators correspond to the repetition whose objective function value was closest to the obtained average.

6 Conclusions and future work

Integrating planning processes requires a thorough assessment of both quantitative benefits pertaining to the expected decrease in the related costs and qualitative impacts related to the usual need of breaking functional silos. This work explored, mainly, the quantitative aspect of integrating outbound and inbound logistics routes. We used as a background a case-study from the wood-based panel industry, but the results and approaches developed are generalisable for other settings in which this integration can be modelled as an rVRPB (e.g., grocery retail, cement distribution). Besides modelling three possible planning strategies (i.e., OBP, IIOP, and DIOP), we have also developed a matheuristic to tackle real-world instances of this problem.

Three key conclusions emerge from our computational study. Firstly, the intuitive idea that intermediate levels of integration would always result in better planning outcomes was not verified. DIOP outperforms OBP in certain geographical contexts where the distribution network is more dispersed. In our studies, this happened in instance A10 when the average distance between linehaul customers and the depot of origin is 197 km. In this case, it was actually cheaper to assure the supply of raw material through dedicated inbound routes (i.e. going to and from the nearest supplier) than including a visit to a supplier at the end of the outbound route (i.e. after visiting all customers). The IIOP model does this trade-off, but the OBP model is myopic to the possibility of doing direct inbound routes, hence, leading to worse results than DIOP.

Secondly, we confirm that there are important parameters dealt by the planners with some degree of uncertainty that actually can have a great influence on the total costs of the routing plan. This study analysed four of these parameters - backhauling reward, minimum backhauled amount, maximum length of the route and minimum delivery amount allowed. Results suggest that increasing the maximum length of the route leads to the largest impact in the performance of the routing plan but including a quantitative reward for each supplier visited will likelier increase the proportion of integrated inbound and outbound routes in the overall routing plan. In fact, the total reward (€/ton) should be equal or higher than the extra transportation costs for the most cost-effective supplier. Meaning that the extra distance travelled empty from the last customer to the nearest supplier and then full from there to the neighbouring mill is minimized.

Finally, the developed matheuristic proved to be a suitable approach to tackle this problem and this fact reiterated the interest of fix-and-optimise to solve routing problems.

Future work could be devoted to merging the qualitative and quantitative assessments related to the integration of planning processes. In particular, the study of integrated inbound and outbound routes is of interest due to its potential in improving the ever-relevant sustainability dimension.

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Supplementary material

Table 7: Experiments on the impact of the values of model parameters

Inst.	Parameter				Objective Function	Costs (€)			Routes			Runtime (s)	MIP Gap
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
A10	1200	0	0.5	6	4703	4703	280	4423	4	0	4	19	1.8%
A10	1200	0	0.5	7	4665	4945	350	4595	5	1	4	105	0.6%
A10	1200	0	0.5	8	4625	4945	350	4595	5	1	4	112	0.3%
A10	1200	0	0.5	10	2523	16123	2520	13603	36	34	2	272	1.9%
A10	1200	0	5	6	4732	4972	350	4622	5	1	4	28	2.0%
A10	1200	0	5	7	4692	4972	350	4622	5	1	4	219	0.6%
A10	1200	0	5	8	4652	4972	350	4622	5	1	4	123	0.3%
A10	1200	0	5	10	4356	6356	490	5866	7	5	2	304	0.4%
A10	1200	0	10	6	4732	4972	350	4622	5	1	4	130	1.9%
A10	1200	0	10	7	4692	4972	350	4622	5	1	4	175	1.1%
A10	1200	0	10	8	4652	4972	350	4622	5	1	4	237	0.5%
A10	1200	0	10	10	4482	5682	350	5332	5	3	2	308	0.7%
A10	1200	1	0.5	6	4705	4945	350	4595	5	1	4	19	1.0%
A10	1200	1	0.5	7	4665	4945	350	4595	5	1	4	109	0.8%
A10	1200	1	0.5	8	4625	4945	350	4595	5	1	4	153	0.2%
A10	1200	1	0.5	10	2523	16123	2520	13603	36	34	2	2523	0.7%
A10	1200	1	5	6	4732	4972	350	4622	5	1	4	137	0.5%
A10	1200	1	5	7	4692	4972	350	4622	5	1	4	357	0.7%
A10	1200	1	5	8	4652	4972	350	4622	5	1	4	241	0.6%
A10	1200	1	5	10	4356	6356	490	5866	7	5	2	165	0.2%
A10	1200	1	10	6	4732	4972	350	4622	5	1	4	96	0.2%
A10	1200	1	10	7	4692	4972	350	4622	5	1	4	57	0.4%
A10	1200	1	10	8	4652	4972	350	4622	5	1	4	200	0.1%
A10	1200	1	10	10	4482	5682	350	5332	5	3	2	94	0.5%
A10	1200	41	0.5	6	4802	5282	420	4862	6	2	4	190	0.0%
A10	1200	41	0.5	7	4722	5282	420	4862	6	2	4	226	0.1%
A10	1200	41	0.5	8	4642	5282	420	4862	6	2	4	126	0.2%
A10	1200	41	0.5	10	2523	16123	2520	13603	36	34	2	185	0.8%
A10	1200	41	5	6	4828	5308	420	4888	6	2	4	234	0.5%
A10	1200	41	5	7	4748	5308	420	4888	6	2	4	155	0.5%
A10	1200	41	5	8	4668	5308	420	4888	6	2	4	180	0.6%
A10	1200	41	5	10	4356	6356	490	5866	7	5	2	107	0.6%
A10	1200	41	10	6	4830	5310	350	4960	5	2	3	166	0.3%
A10	1200	41	10	7	4750	5310	350	4960	5	2	3	57	0.4%
A10	1200	41	10	8	4670	5310	350	4960	5	2	3	89	0.7%
A10	1200	41	10	10	4482	5682	350	5332	5	3	2	67	0.1%

Continued on next page

Table 7 – *Continued from previous page*

Inst.	**Parameter**				**Objective Function**	**Costs (€)**			**Routes**			**Runtime (s)**	**MIP Gap**
	α	β	ψ	δ		Total	Fixed	Transport	Total	OIR	OR		
A10	1500	0	0.5	6	3215	3215	210	3005	3	0	3	26	1.6%
A10	1500	0	0.5	7	3177	3457	280	3177	4	1	3	357	1.0%
A10	1500	0	0.5	8	3137	3457	280	3177	4	1	3	123	2.3%
A10	1500	0	0.5	10	1034	14634	2450	12184	35	34	1	504	0.3%
B10	400	0	0.5	1	2002	2002	210	1792	3	0	3	16	0.0%
B10	400	0	5	1	2002	2002	210	1792	3	0	3	25	0.0%
B10	400	0	10	1	2002	2002	210	1792	3	0	3	29	0.0%
B10	400	0	0.5	2	2001	2081	280	1801	4	1	3	28	0.0%
B10	400	0	5	2	2001	2081	280	1801	4	1	3	21	0.0%
B10	400	0	10	2	2002	2002	210	1792	3	0	3	18	0.0%
B10	400	0	0.5	5	-108	7692	2940	4752	42	39	3	32	0.0%
B10	400	0	5	5	1534	3134	770	2364	11	8	3	20	0.0%
B10	400	0	10	5	1710	2510	490	2020	7	4	3	18	0.0%
B10	400	1	0.5	1	2041	2081	280	1801	4	1	3	39	0.0%
B10	400	1	5	1	2041	2081	280	1801	4	1	3	37	0.0%
B10	400	1	10	1	2059	2099	280	1819	4	1	3	21	0.0%
B10	400	1	0.5	2	2001	2081	280	1801	4	1	3	33	0.0%
B10	400	1	5	2	2001	2081	280	1801	4	1	3	36	0.0%
B10	400	1	10	2	2019	2099	280	1819	4	1	3	30	0.0%
B10	400	1	0.5	5	-108	7692	2940	4752	42	39	3	27	0.0%
B10	400	1	5	5	1534	3134	770	2364	11	8	3	19	0.0%
B10	400	1	10	5	1710	2510	490	2020	7	4	3	16	0.0%
B10	400	41	0.5	1	2133	2213	350	1863	5	2	3	41	0.0%
B10	400	41	5	1	2133	2213	350	1863	5	2	3	32	0.0%
B10	400	41	10	1	2133	2213	350	1863	5	2	3	20	0.0%
B10	400	41	0.5	2	2053	2213	350	1863	5	2	3	27	0.0%
B10	400	41	5	2	2053	2213	350	1863	5	2	3	20	0.0%
B10	400	41	10	2	2053	2213	350	1863	5	2	3	10	0.0%
B10	400	41	0.5	5	-108	7692	2940	4752	42	39	3	38	0.0%
B10	400	41	5	5	1534	3134	770	2364	11	8	3	40	0.0%
B10	400	41	10	5	1710	2510	490	2020	7	4	3	14	0.0%
B10	800	0	0.5	1	1566	1606	210	1396	3	1	2	72	0.0%
B10	800	0	0.5	2	1503	1743	210	1533	3	3	0	23	0.0%
B10	800	0	0.5	5	-778	7222	2800	4422	40	40	0	54	0.0%

Legend: α: maximum length of the route (km); β: minimum backhauled amount (ton); ψ: minimum delivery amount (ton); δ: backhauling reward (€/ton); Runtime: computational time after which no better solution was obtained (seconds); MIP Gap: percentual difference obtained by Gurobi between the upper and lower bound of the branch-and-bound method. All models were run with a maximum time limit of 3,600s.