

Evaluation of the Uncertainties used to Perform Flow Security Assessment: A Real Case Study

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Abstract— In this paper, a validation framework is proposed to evaluate the quality of uncertainty forecasts, when used to perform branch flow security assessment. The consistency between probabilistic forecasts and observations and the sharpness of the uncertainty forecasts is verified with advanced metrics widely used in weather prediction. The evaluation is completed by assessing the added value of exploiting uncertainty forecasts over the TSO current practices of using deterministic forecasts. For electric power industry, this proposed validation framework provides a way to compare the performance among alternative uncertainty models, when used to perform security assessment in power systems. The quality of the proposed metrics is illustrated and validated on historical data of the French transmission system.

Index Terms— Forecast uncertainty, Performance evaluation, Power system security, Probabilistic forecast, Systems operation.

I. INTRODUCTION

An essential task of Transmission System Operators (TSO) is to perform online security assessment. This involves checking ahead the impact of plausible contingencies on system operational limits and then, conditioned by the result of a risk-based assessment defined by the probability and potential impacts of these contingencies, operators decide on the necessary actions to keep the system in a secure state [1]. Usually, this analysis is performed at least for N-1 situations, over time horizons including two-days ahead, one-day ahead and intraday operation, based on deterministic forecasts of the grid state. However, online operation is nowadays being affected by an increased amount of uncertainty, which may impact the forecasts of system security. Major sources of uncertainties in power systems are the large penetration of intermittent renewable energy sources, the operation of the liberalized electricity market, the normal load evolution and, last but not least, the online control actions taken by operators to ensure system security (like topological actions and generation rescheduling and redispatch).

This rationale triggered several research works aiming to perform online security assessment by taking into account the uncertainty of forecasts, beyond the traditionally consideration of deterministic forecasts [2]-[8]. In particular, the R&D

iTesla project (<http://www.itesla-project.eu>), co-funded by the European Commission 7th Framework Program (EC FP7), targeted the development of an online dynamic security analysis platform for European-wide grid models, able to account for uncertainties in security margin evaluation and to handle curative remedial actions to face contingencies. The iTesla security assessment approach was successfully tested for overload situations on the French network, as described in [6]. The presented results show that the consideration of forecasted uncertainties is of the utmost importance, since from apparently secure deterministic forecasted network states, it is possible to arise unsecure situations that need to be tackled in advance by the system operator.

After the end of the iTesla project, research works aiming to improve the created advanced platform were further developed. The current described work is one of such researches. Namely, this paper addresses the validation of probabilistic uncertainty models used by the online security assessment platform to characterize plausible future states around each expected deterministic state, when used to forecast the flow on the French transmission lines. Besides evaluating the quality of the generated flow uncertainties with advanced metrics usually used in weather prediction, this work also evaluates the added value of using probabilistic forecasts of the power system security for the overload problem. To this end, a validation framework was implemented through the development of scripts written in the R programming language [9]. These scripts are open source and available on the iTesla Power System Tools (iPST) repository (on <https://github.com/itesla/ipst/tree/master/mcla-evaluation>). To the best of authors knowledge, the proposed validation framework is new, being of paramount importance for enabling the electric power industry to compare the performance among alternative uncertainty models, when used to perform security assessment in power systems.

The paper is organized as follows: first, an overview of the evaluated uncertainty models is presented in Section II; then Section III describes the metrics used to evaluate flow uncertainty forecasts; Section IV explains how the methodology was extended to evaluate the added value from using the uncertainty model to assess security for the overload

problem; Section V presents the obtained results from applying the described validation framework for the analyzed case study and, finally, section VI presents the general conclusions obtained from this research.

II. UNCERTAINTY MODEL

The used platform builds a model of the forecast errors of the stochastic inputs (e.g. renewable energy source injections, load power absorptions) conditioned to the forecasted values of the inputs. This task is carried out by a Monte Carlo Like Approach (MCLA) module fully described in [5] and available on iPSST repository (on .../ipst/tree/master/mcla). An overview of the evaluated uncertainty models is described next.

A. The “original” uncertainty model

In the offline workflow, the raw data (snapshots and deterministic forecasts of the stochastic variables) are pre-treated and clustered by the k-means clustering technique. After applying the Principal Component Analysis for dimensionality reduction, and a pair copula decomposition with C-vines to simulate higher order dependencies among Principal Components, these are sampled and back-projected onto the original variable space, getting the unconditioned samples of snapshots (SN) and forecasts (FO). In the online platform, these samples are conditioned to the specific forecast power system state (here called online base case) using a conditional sampling based on Nataf Transformation according to (1), where $\tilde{sn}'_j$ and $\tilde{y}'_j$ are the sample vectors of the j -th normalized Nataf-transformed snapshot (sn) and forecast (y) variable, $y^{(0)}$ is the vector of normalized Nataf-transformed specific forecasts coming from the online base case, and $\Sigma_{\tilde{sn}'\tilde{y}'}$, $\Sigma_{\tilde{y}'\tilde{y}'}$ are the relevant covariance matrices.

$$\tilde{sn}'_{j|y^{(0)}} = \tilde{sn}'_j + \Sigma_{\tilde{sn}'\tilde{y}'} \cdot \Sigma_{\tilde{y}'\tilde{y}'}^{-1} (y^{(0)} - \tilde{y}'_j) \quad (1)$$

B. The “adapted” uncertainty model

Applying the conditional sampling techniques in the online environment has highlighted two issues [10]: (1) the original variables may be multimodal, with a non-Gaussian probability distribution, which reduces the consistency of the conditioned samples - extracted using Nataf transformation - with actual observations; (2) preliminary results highlight the occurrence of overfitting of the conditioning technique. Therefore, an “adapted” uncertainty model was developed by performing some upgrades to the earlier described “original” uncertainty model, to increase the algorithm prediction capability at the expense of the spread of the relevant samples, namely:

1) Reduce matrices complexity: forecast covariance matrix $\Sigma_{\tilde{y}'\tilde{y}'}^{-1}$ is set diagonal (neglecting correlations among forecasts), and only the correlation between each snapshot and its forecast is retained in $\Sigma_{\tilde{sn}'\tilde{y}'}$. As a result, the conditional sampling for the j -th sn and y variable depends only on the covariance matrix of these 2 variables.

2) Sample separately the multimodal and the unimodal variables: the pair (SN, FO) of each multimodal injection is

fitted using a Gaussian mixture with the lowest Akaike Information Criterion, as detailed in [10], while the unimodal variable set is treated as a whole with the Nataf transformation-based conditional sampling method, thus neglecting correlations between multimodal and unimodal variables, and among multimodal variables.

III. EVALUATION OF FLOW UNCERTAINTY FORECASTS

The aim of this analysis is to measure the quality of uncertainty models, when used to estimate the flows in transmission lines. With this goal, the analyzed variables are the rms value of electric currents in transmission lines (I , in A). In order to have equivalent metrics between variables, these are previously normalized by dividing them by the line maximum permanent limit (I_{max} , in A) associated to one point in time (timestamp) of the evaluated time period. In fact, although line limits may change over time due to seasonality, in this analysis a unique base value is mandatory for each variable, for proper computation of multi-temporal metrics. For each considered normalized electric current (I , in pu A of I_{max}), a distinct statistical analysis is performed for the cases with opposite directions of active power flow, assuming these two situations define very distinct operating conditions in the power system.

In this research, for each analyzed timestamp, a forecast is formed by a set of prediction states comprising the deterministic forecast and a finite number of associated plausible future states generated by the MCLA, henceforth named ensemble. Assuming that the real observed behavior is provided by recorded SN's, the statistical consistency between the ensembles and observations and the ensembles sharpness is checked with advanced metrics widely used in weather prediction [11] that are described next.

A. Univariate Rank Histogram

As referred in [12], the goal of probabilistic forecasting is to maximize the sharpness of predictive distributions subjected to calibration, with calibration being the previously referred statistical consistency between the probabilistic forecasts and observations. Sharpness measures the concentration of the predictive distributions and, therefore, characterizes the uncertainty by estimating the range of forecast errors. The rank histogram, also known as Talagrand Diagram [13], is a powerful tool for calibration check, since it summarizes the rank position of the verified observations with respect to its ensemble for a testing time period. Assuming an ensemble with m members, for each timestamp the member values are ordered and the position (i.e. the rank) of the SN in this ordering is recorded. For instance, the rank will be 1 if the SN is below all the ensemble members and will be $m+1$ if the SN is above all the m ensemble members. For optimal result, each bin of the rank histogram should have the same frequency of observations. Therefore, the rank histogram measures how well the spread of the ensembles represents the true uncertainty of the observations (SN's) and checks the bias of the ensembles. Fig. 1 illustrates some possible forms of the rank histogram as well as its interpretation for the analyzed

variable (I , in pu A), provided that the ensemble members are independent and identically distributed, as ensured by the MCLA. Together with each univariate rank histogram, a discrepancy index, named Δ index, is also calculated:

$$\Delta = \sum_{i=1}^{m+1} |f_i - 1/(m+1)| \times (m+1)/(2m) \quad (2)$$

where f_i is the observed relative frequency of each bin i . This metric measures the deviation from uniformity in the rank histogram. It computes the sum of the deviations from uniformity between the $m+1$ bins and the optimal result for the relative frequency in each bin (i.e. $1/(m+1)$) and is normalized to obtain $\Delta \in [0,1]$. Lower Δ values means that the bins are closer to represent frequencies associated with a perfectly reliable forecast.

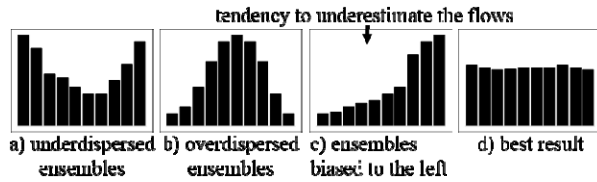


Figure 1. Interpretation of rank histograms for flow forecasts.

B. Frequency of observations falling outside the ensemble

The number of situations where the SN falls outside the ensemble can be obtained from the rank histogram, namely from the frequency of observations in bin 1 and $m+1$. However, these values do not remove the influence of outliers. To avoid this influence, the ensemble range of values is firstly defined to be inside quantiles p and $1-p$ (Q_p and Q_{1-p} , respectively). These quantiles are calculated individually for each point in time, being therefore named marginal quantiles [14]. In the multi-temporal analysis of each variable, by comparing the observed values with their marginal quantiles, the relative frequency is computed for the following situations: (1) SN not in $[Q_p, Q_{1-p}]$; (2) SN exceeding Q_{1-p} , quantifying therefore underestimated flow situations, which for branch flows closer to their limits are prone to provide missed alarm (MA) classification errors.

C. Continuous Ranked Probability Score (CRPS)

The Continuous Ranked Probability Score (CRPS) consists of a single metric that aims to evaluate the overall performance of the uncertainty model. The minimal value of this metric aims to identify the model that maximizes ensemble sharpness without deteriorating the Euclidean distance between the observation and the ensemble [12], [15]. For each timestamp, considering the observed value (SN) and the ensemble empirical cumulative distribution, the CRPS is defined by the integral of the squared heights of the shaded region that is illustrated in Fig. 2 for three distinct situations. Lower CRPS values are preferred since it means that the ensembles present lower dispersion and, at the same time, are mostly concentrated around the SN value (like illustrated in the last situation of Fig. 2). The CRPS reduces to the mean absolute error if the forecast is deterministic. For the multi-temporal analysis, the mean value of the CRPS metric was

computed for each analyzed variable, here named $\overline{CRPS}$. This metric results from the combination of the following two metrics: (1) $\overline{d}_{SN \leftrightarrow ensemble}$: mean value of the Euclidean distance of the SN to ensemble members; (2) $\overline{std}_{ensemble}$: mean value of the ensemble standard deviation.

D. Distance of observations falling outside the ensemble

Besides the two earlier described metrics embraced by the CRPS, it is also important to measure how distant the observed values are when falling outside the ensemble. In particular, for branch flow security assessment it is important to evaluate the mean magnitude of the obtained underestimated flows. In fact, when using the ensembles to detect overload situations, the magnitude of underestimated flows is closely related with the possibility of having MA situations. Given that, and assuming that during online operation the transmission line flows are estimated by the upper value of their marginal quantiles (i.e. by Q_{1-p}), the following metrics (illustrated in Fig. 3) are also computed for each analyzed variable: (1) $\overline{d}(Q_p)$: mean value of the distances between the SN and quantile p only for situations where the SN does not reach this quantile; (2) $\overline{d}(Q_{1-p}) = \overline{d}_{underestimate}$: mean value of underestimated distances, namely, of the distances between the SN and quantile $1-p$ only for situations where the SN exceeds this quantile.

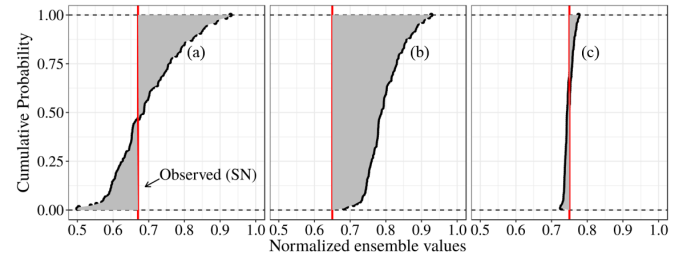


Figure 2. Illustration of the CRPS, the integral of the squared heights in the shaded region. For convenience, the figures shade the absolute values, not the squared heights. Three cases are presented: (a) SN inside an ensemble with some spread; (b) SN < ensemble; (c) SN inside a very sharp ensemble.

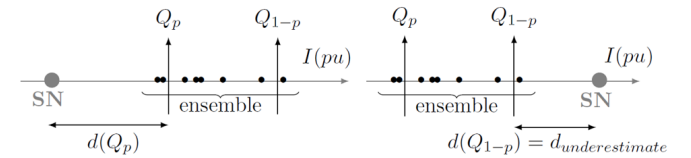


Figure 3. Graphical representation of distance metrics.

IV. EVALUATION OF FLOW UNCERTAINTY FORECASTS

The purpose of this analysis is to evaluate the capability of probabilistic uncertainty models in assessing the security of each contingency for the overload problem and, in particular, assessing the added value of exploiting uncertainty forecasts (UF) over deterministic forecasts (DF). Like in previous analyses, the examined variables are the rms value of electric currents in transmission lines (I , in A). However, since now the computed metrics mainly aim at checking if the steady-

state flows in each transmission line are not exceeding the maximum permanent limit associated to each specific timestamp ($I_{max}(t)$, in A), a variable normalization is not mandatory for this analysis. The metrics used in the analysis are described next.

A. Security classification

For each timestamp (t), each observed flow ($I_{SN}(t)$) is assumed overloaded if exceeding its associated $I_{max}(t)$ value. For deterministic forecasts ($I_{DF}(t)$), a security margin (SM) is assumed by identifying an overload if $I_{DF}(t) \cdot (1+SM)$ exceeds $I_{max}(t)$. For uncertainty forecasts, assuming that transmission line flows are estimated by the upper value of their marginal quantiles (Q_{1-p}), an overload situation is identified if Q_{1-p} exceeds $I_{max}(t)$. For each instant in time, the power system is assumed unsecure if at least one transmission line flow presents overload problems. Being the true security classification provided by the observed flows (i.e. by $I_{SN}(t)$), for a testing period the following classification situations are checked for DF and UF (for each flow and also for the power system): (1) a true unsecure situation wrongly classified as secure, i.e. a Missed Alarm (MA); (2) a true secure situation wrongly classified as unsecure, i.e. a False Alarm (FA).

B. Trade-off analysis between classification errors

Intuitively, when using a higher value of SM (for DF) and of $1-p$ (for UF), the applied classification decision rules may decrease the number of missed alarms overloads at the expense of increasing the number of false alarms. Therefore, a trade-off analysis was performed to evaluate the impacts of the used $1-p$ and SM values on the relative rate of MA and FA for the testing period. The purpose of this analysis is to identify cutoff values (for SM and $1-p$) being able to decrease the MA rate provided by the traditionally applied deterministic forecasts ($I_{DF}(t)$) without an excessive increase of FA situations. To provide confidence intervals describing the uncertainty for the calculated percentages of MA and FA, the bootstrap method [16] is applied with the following approach: (1) with random resampling (by assuming no seasonality dependency in the time series of classification errors for each transmission line); (2) with the studentized bootstrap to infer confidence intervals, where the percentage of MA (and FA) are seen as the mean value of a binary sample.

V. CASE STUDY

The proposed validation framework is here illustrated and validated by computing the earlier described metrics for the French transmission lines. In the French transmission network model that was used, the boundary nodes are assumed to be connected to fictitious loads in lower voltage levels (20 kV and below). Foreign grids are represented by equivalents. The stochastic variables comprise the active and reactive power injection of loads and renewable energy sources, totaling thousands of variables (around 8000). The used historical data comprise Day-Ahead Congestion Forecasts (DACF's) for deterministic forecasts and SN files for the observed network states. In this study, the DACF's were previously modified by mixing the topology of the SN's with the injections of the forecasts. This procedure eliminated the topological discrepancies between DACF and SN network states from the

analysis: a major source of inconsistency between SN's and DACF's of the French transmission system that was not treated by the evaluated uncertainty models. To avoid making inferences with insufficient statistical data, a minimum number of 100 timestamps was assumed to include each variable in the analysis. In all the studies, ensembles with 50 members (including the DACF) were considered. All the metrics results were obtained from the developed scripts written in the R programming language that are available on the iPSST repository (on [.../ipst/tree/master/mcla-evaluation](#)).

A. Evaluation of the "original" uncertainty model

Aiming to test the proposed metrics to evaluate flow uncertainty forecasts, the "original" uncertainty model (characterized in section II) was trained with the French historical data from February to March 2017 (with the available 890 hourly timestamps). This model was then used, by the MCLA referred in section II, to estimate the uncertainty of pre-contingency power flows from February to April 2017 (1354 timestamps). Finally, these results were used to compute the univariate metrics earlier described in section III to evaluate the quality of flow uncertainty forecasts for the flow in all the 380 kV French lines (for 695 lines providing 1241 variables). For this analysis, marginal quantiles were defined by $p=0.05$ (i.e. by $Q_{0.05}$ and $Q_{0.95}$). Some of the most relevant results obtained are described next. Fig. 4 presents the obtained rank histogram for two analyzed flows (i.e. two electric currents associated with a constant direction of active power flow). The rank histogram for "flow A" diagnoses that the ensembles characterizing the uncertainty for this flow are underdispersed and biased to the left, presenting a large Δ index of 0.4. The rank histogram for "flow B" is much more calibrated, having a smaller Δ index of 0.12. This example illustrates how well the Δ index accurately summarizes the quality of the rank histogram.

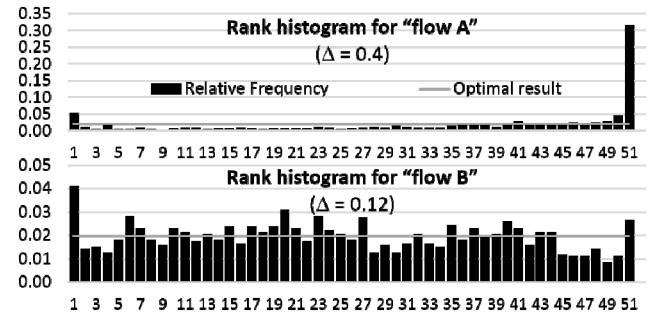


Figure 4. Some obtained rank histograms with the associated Δ index value.

In the scatter plots in Fig. 5, each point shows the value of the computed metrics for each analyzed flow. As expected, the first scatter plot describes a strong relationship between the quality of the rank histogram (summarized by the Δ index) and the relative frequency of SN's falling outside the ensemble. However, the next two plots indicate that there are no relevant relationships between the rank histogram and distance metrics (here illustrated only for the $\overline{d}_{SN \leftrightarrow ensemble}$) and between the rank histogram and the $\overline{CRPS}$. On the contrary, the last plot describes a strong relationship between the $\overline{CRPS}$

and distance metrics. These results show that when evaluating the quality of ensembles, to obtain a clear picture we must combine the evaluation provided by the rank histograms and the CRPS metric. For this specific tested case, the metric based on distance computation ($\bar{d}_{SN \leftrightarrow ensemble}$) suggests higher quality results than the ones associated to the rank histogram. In fact, as presented in Fig. 5, for many of the analyzed flows the relative frequency of SN's falling outside the ensemble is very high (e.g., a 0.5 value means that half of the SN's are falling outside). By listing the flows with a Δ value higher than 0.8, the least calibrated transmission lines were identified to be power plant evacuation lines, suggesting that these flow forecasts may be improved by including the unscheduled operating conditions of conventional power plants in the uncertainty model.

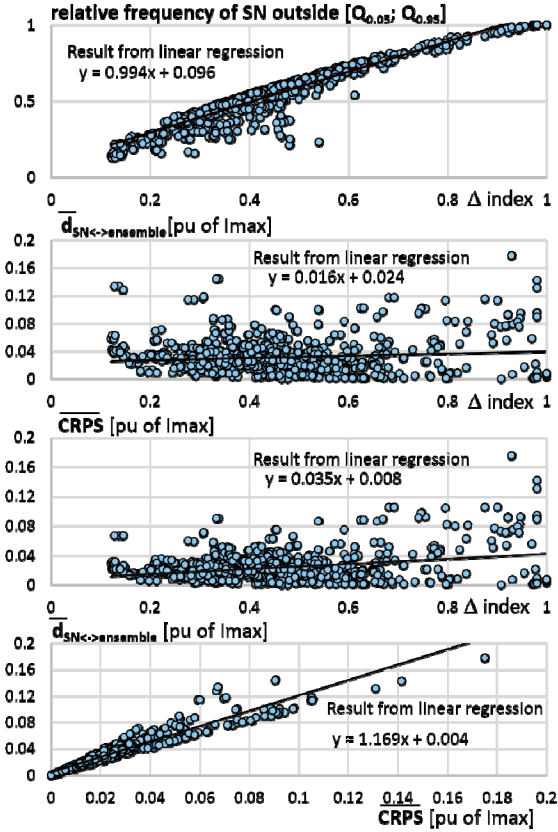


Figure 5. Metrics behavior for the “original” uncertainty model.

B. Comparing uncertainty models

Aiming to compare the “adapted” and “original” uncertainty models (both characterized in section II) each model was trained with the French historical data from February to March 2017. Both models were then used by the MCLA to estimate the uncertainty of pre-contingency flows in all the 380 kV French lines for April 2017 (464 timestamps). Finally, the univariate metrics described in section III were computed assuming that marginal quantiles are defined by $Q_{0.05}$ and $Q_{0.95}$. The most relevant results are presented in Fig. 6, showing the difference between the “adapted” and “original” metrics values for each analyzed flow variable. Since the best result aims minimizing all the metrics, in the

plots of Fig. 6, a negative value means the metric is being improved by the “adapted” approach (and vice-versa). The first plot of Fig. 6 indicates that, for most of the flows, the quality of the uncertainty model was improved (from the “original” to the “adapted” approach) in reducing the relative frequency of having observed flow values falling outside the ensembles. From the second plot, it is visible that this improvement was obtained at the expense of increasing the ensembles spread and the distance between the observed values (in the SN's) and the ensemble members.

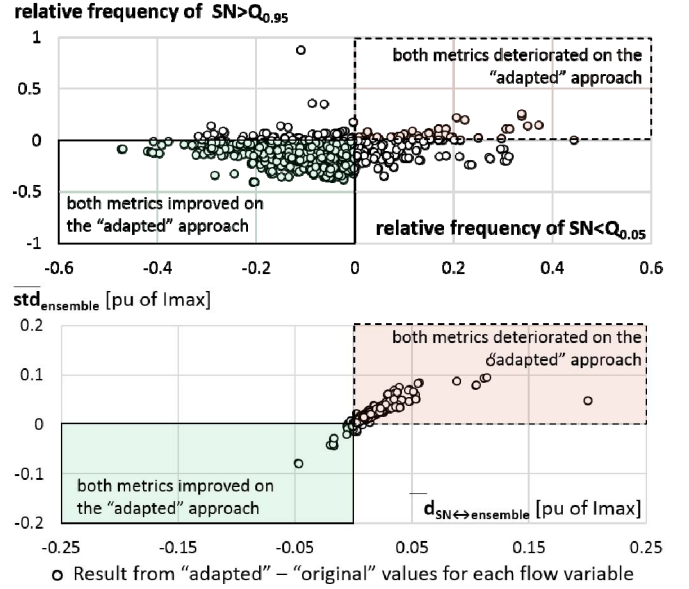


Figure 6. Comparing the “adapted” with the “original” uncertainty model.

C. Evaluation of the “adapted” uncertainty model in assessing system security for overload situations

The ultimate goal of the evaluated uncertainty models is to perform security assessment for the overload problem. Therefore, this evaluation is only completed after assessing the added value of having uncertainty forecasts over the TSO current practices of using deterministic forecasts. This was performed by applying the evaluation methodology described in section IV for a severe N-2 contingency situation impacting an electric peninsula with very low level of centralized generation – a test case adapted to the limits of the current uncertainty model – during the first months of 2018 (a particularly loaded period for the French transmission system). With this aim, the “adapted” uncertainty model was trained with historical data from January and February 2018. This model was then used, by the MCLA, to estimate the uncertainty of the post-contingency flows in all the 380 kV and 225 kV French lines for March 2018. Finally, these results were used to compute the metrics described in section IV, namely to produce the trade-off analysis presented in Fig. 7. This figure evaluates the impact of using different cutoff values (of SM for DF and $1-p$ for UF) on the relative rate of MA and FA for the French EHV/HV transmission system, over the testing period (March 2018). It also presents the misclassification results obtained from the traditionally used deterministic forecasts (provided by DACF's with no SM). In this analysis, a 95% confidence level is assumed for the

bootstrap confidence intervals (presented on the magnitude of the cross lines and on $CI.inf$ and $CI.sup$ values). A maximum value of 20% load was assumed for the DACF's security margin (i.e. a maximum of 1.2DACF).

The results in Fig. 7 show that, at a 95% confidence level, there is no SM value that enables deterministic forecasts to decrease the MA rate provided by the traditionally used DACF's. On the contrary, this is achieved by the uncertainty forecasts if using a proper quantile value ($Q_{0.8}$ for the analyzed case). This presents an added value of using uncertainty forecasts over deterministic forecasts. According to the obtained results, for each 1% decrease in missed alarms, false alarms increase around 3.3% (see results on the table in Fig. 7). Starting to use this uncertainty model over the traditionally used forecasts for the analyzed contingency is a choice of the TSO, according to their adopted risk criteria.

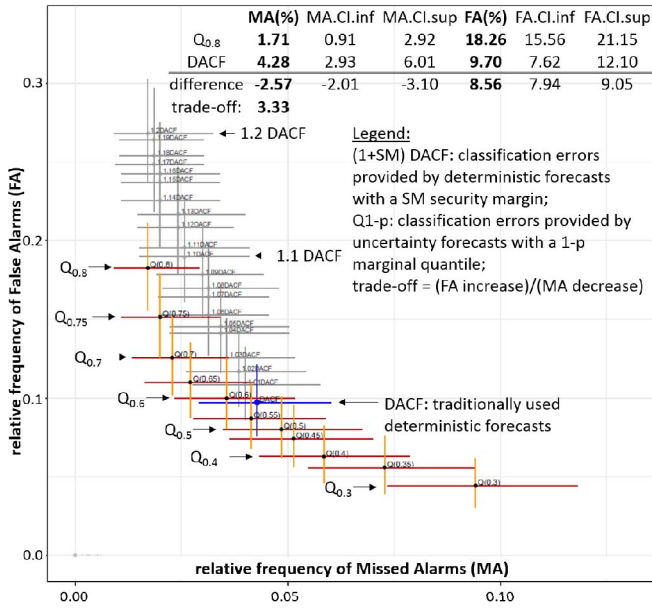


Figure 7. Trade-off analysis of the obtained classification errors from the flow security assessment of the French EHV/HV transmission system.

VI. CONCLUSIONS

In this work a general methodology is proposed to evaluate the quality of uncertainty models to perform branch flow security assessment in power systems. Besides evaluating the quality of the generated flow uncertainties with advanced metrics usually used in weather prediction, this research also evaluates the added value of using probabilistic forecasts over deterministic forecasts. The quality of the proposed validation framework is, in this paper, illustrated on historical data of the French transmission system. In particular, it is shown that in order to obtain a clear picture of the quality of probabilistic forecasts, the evaluations provided by the rank histogram and the CRPS advanced metrics must be combined. The obtained results also illustrate how the interpretability of these advanced metrics can be complemented by using simpler associated metrics. Furthermore, the added value of using uncertainty forecasts over deterministic forecasts is

demonstrated for a severe N-2 contingency situation. Although being developed to address branch overload problems, with proper adaptations, this work can be extended to other power system security problems.

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