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A Hybrid Biased Random Key Genetic Algorithm for a Production and Cutting Problem

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Abstract:

This paper deals with a very common problem in the home-textile industry. Given a set of orders of small rectangles of fabric the problem consists of determining the lengths and widths of a set of large rectangles of fabric to be produced and the corresponding cutting patterns. The objective is to minimize the total quantity of fabric necessary to satisfy all orders. The approach proposed uses a biased random-key genetic algorithm for generating sets of cutting patterns which are the input to a sequential heuristic procedure which generates a solution. Experimental tests based on a set of 100 random generated problems with known optimal solution validate quality of the approach.

Keywords: Biased random-key genetic algorithm, Cutting pattern, Cutting problem, Sequential heuristic procedure, random-keys.

1. INTRODUCTION

In this paper, we consider a real-life 2-dimensional cutting stock problem arising in a home textile make-to-order company specializing in producing large rectangles of fabric stock sheets (cloth produced by weaving or knitting textile fibres) and cutting them into smaller rectangular pieces (see Fig. 1).

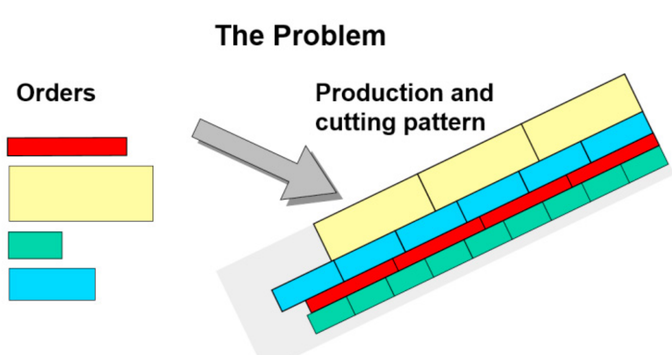

Fig. 1. Example of a problem.

According to the typology by Wäscher et al. (2007) the problem presented falls into the input minimization assignment kind and belongs to the problem type 2D Rectangular Open Dimension – MHLOPP or MHKP.

Johnson (1979) has shown that the two-dimensional cutting stock problems are NP-hard. Therefore, solving exactly practical problems of the type considered in this paper it is quite time consuming.

Metaheuristic techniques are a frequently used tool for finding approximate solutions of hard combinatorial optimization problems. Several authors have used methaherusitics to find practical solution to problems which are too complex to be solved efficiently with other heuristics (Dowsland (1993), Jakobs(1996), Lodi et al. (1998, 1999a, b), and Faero et al. (2003))

The paper is organized as follows: In Section 2, we formulate the problem. In Section 3, a novel biased random-key based genetic algorithm approach is presented. Then in Section 4 some computation experiments and results are provided.

2. THE PROBLEM

Formally the problem can be formulated as an integer program over all the efficient patterns as follows:

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$$\text{Minimize Total Used Area of Fabric} = \sum_{j=1}^{P} W_j \times X_j$$

Subject to:

$$\sum_{j=1}^{P} a_{i,j} \frac{X_j}{L_i} \geq D_i \qquad i = 1, \ldots, N$$

$$X_j \geq 0 \text{ and integer} \qquad j = 1, \ldots, P$$

where,

i = Index of product i ($i=1, \ldots, N$)

j = Index of pattern j ($j=1, \ldots, P$)

D_i = Demand of product i

L_i = Length of product i

W_j = Width of pattern j

X_j = Length of fabric produced with pattern j

$a_{i,j}$ = Number of products i included along the width of pattern j

A product is a small rectangle of fabric with width and length specified in the order placed by the customers. A pattern is a combination of product along the width of the fabric.

Gilmore and Gomory (1961) have given a solution with the Linear Programming relaxation and a column generation technique. However, for practical applications, their approach has the drawback that many different patterns are generated that are not easily managed in the production process and that leads to high setup costs. A review of the solutions techniques is given by Haessler and Sweeny (1991).

In the next section we propose a novel hybrid biased random-key genetic algorithm (BRKGA) which solves the problem extremely well even for large instances.

3. THE NEW APPROACH

We begin this section with an overview of the proposed solution approach. This is followed by a discussion of the biased random-key genetic algorithm (Gonçalves and Resende, 2011), including detailed descriptions of the solution encoding, the sequential heuristic procedure solution and the evolutionary process.

3.1 Overview

The new approach is based on a constructive sequential heuristic procedure (SHP) which, given a set of ordered cutting patterns, finds a solution to the problem.

The role of the genetic algorithm is to evolve the set of cutting patterns used by the SHP. The following phases are applied to each chromosome:

1. Pattern generation: This first phase decodes the chromosome into a set of cutting patterns, i.e., the width of the patterns and the corresponding products included on each of them.

2. Solution construction: The second phase uses the cutting patterns produced by the BRKGA in the previous phase and produces a solution using the sequential heuristic procedure, SHP.

3. 3. Fitness evaluation: The final phase computes the fitness of the solution obtained in phase 2 (a measure of quality of solution, i.e. the total area of the fabric used).

Figure 2 illustrates the sequence of steps applied to each chromosome generated by the BRKGA.

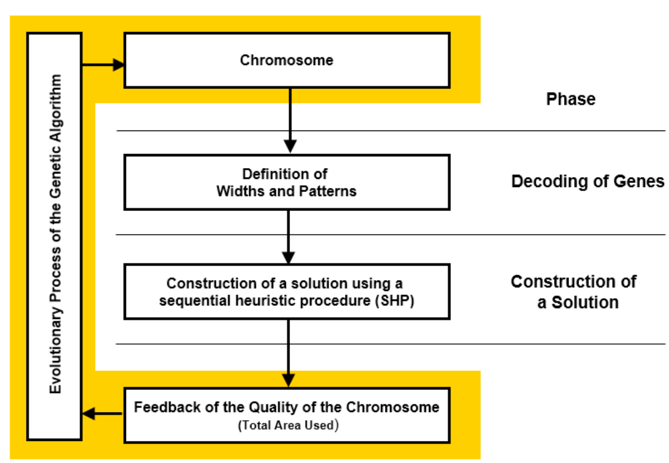

Fig. 2. Architecture of the approach.

3.2 Solution Encoding

A solution is represented by a vector of random keys (Bean, 1994). Since the maximum number of patterns in an optimal solution will equal the number of different products we will encode the solution as a set of N patterns. Let NPW be the maximum number of products that can be included in a pattern width then a chromosome will have the structure depicted in Figure 3.

$$
\begin{array}{l}
(r_1, \dots , r_{NPW} , \\
\qquad \dots \quad , \\
r_{(N-1) \times NPW + 1}, \dots , r_{N \times NPW})
\end{array}
$$

Fig. 3. Chromosome structure.

3.3 Solution Construction

A solution is constructed using the patterns generated by the BRKGA. These patterns are used sequentially by the SHP to determine the length to be produced using each pattern. The basic idea of the SHP is to use each pattern until an order runs out. Figure 4 presents a flowchart for the SHP.

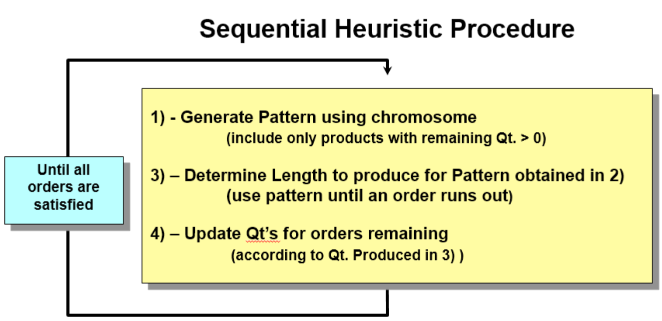

Fig. 4. Sequential heuristic procedure.

3.4 Evolutionary Process

The evolutionary process used follows the evolutionary strategy proposed for BRKGA´s by Gonçalves and Resende (2011) and is summarized in Figure 5.

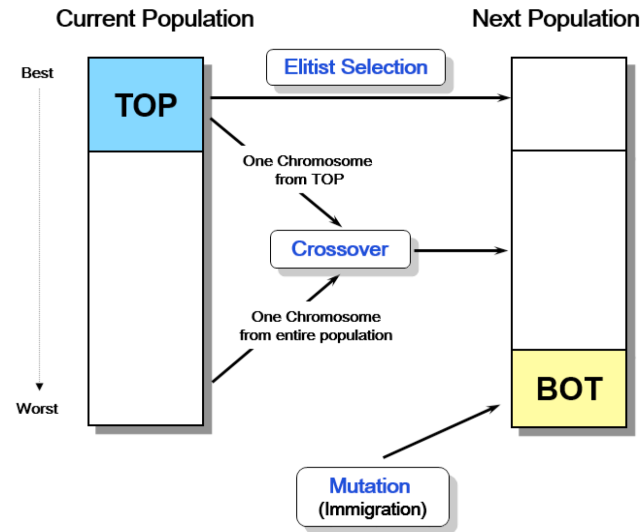

Fig. 5. Evolutionary strategy used by BRKGA's.

4. COMPUTATIONAL EXPERIMENTS

To evaluate the performance and the capabilities of the BRKGA approach presented in this paper we performed a series of computational experiments. The numerical experiments were conducted on a computer with a Intel Xeon E5-2630 @2.30GHz CPU and 16 GB of physical memory running the Linux operating system with Fedora release 18. The BRKGA approach was coded using the C++ programming language a single-thread version of the executable was used.

4.1 BRKGA configuration

The BRKGA configuration used for all tests was the one presented in Figure 6.

Population Size:	**3** × the number of products.
Crossover:	The probability of tossing heads was made equal to **0.7**.
Selection:	Copies to the next generation the top **10%** of the previous population chromosomes.
Mutation:	Substitutes with randomly generated chromosomes the bottom **20%** of the population chromosomes.
Fitness:	**Total Area Used** (to minimize)
N° of Populations:	**3**
Exchange Frequency:	Every **15** generations
N° of Seeds:	**1**
Stopping Criterion:	Stops after **250** generations.

Fig. 6. BRKGA configuration.

The configuration presented in Figure 6 was the best configuration obtained using a small group of instances (instances =1, 11, 21, …, 91). The intervals used for the parameters are given in Table 2.

Table 2. Parameters Interval.

Parameter	**Interval**
Population Size	1, 2, 3, 5, 10 times the number of products
Selection	0.05, 0.10, 0.15, 0.20
Crossover	0.7, 0.8, 0.9
Mutation	0.10, 0.20, 0.30

4.2 Benchmark Instances

Since there are no benchmark instances available for this type of problem we generated 100 random instances with known optimal solution (with zero waste).

4.3 Results

We compare the performance of the BRKGA approach against the optimal solutions of the bench mark instances.

Table 1 presents the number of instances within a % interval deviation from the optimal solution.

Table 1. BRKGA Results.

% Deviation from Optimal	Number of instances
0 - 0.1	57
0.1 - 0.5	34
0.5 - 1.0	8
1.0 - 1.5	1
> 1.5	0

As can be seen the performance of the BRKGA is excellent since all the instances have less than 1.5% waste and for 91% of the instances the waste was less than 0.5 %.

Figure 7 depicts a detailed solution found by the BRKGA approach for an instance with 26 products.

Pattern Nº	Width	Pattern	Waste	Length
1	48	1 x 6x63 - 6 x 6x39 - 1 x 6x70	0	78
2	68	1 x 6x105 - 3 x 17x39 - 1 x 7x9	4	819
3	88	1 x 47x182 - 3 x 9x42 - 1 x 8x66 - 1 x 6x33	0	4368
4	83	1 x 14x21 - 1 x 6x105 - 6 x 9x42 - 1 x 9x33	0	825
5	98	1 x 29x10 - 1 x 6x70 - 7 x 9x11	0	980
6	48	1 x 38x77 - 1 x 9x11	1	2156
7	73	1 x 53x35 - 1 x 14x21 - 1 x 6x105	0	2415
8	53	1 x 6x105 - 1 x 38x77 - 1 x 8x77	1	11550
9	78	2 x 21x91 - 6 x 6x33	0	4290
10	68	1 x 6x105 - 1 x 38x77 - 3 x 8x33	0	1309
11	48	6 x 8x33	0	165
12	88	8 x 9x42 - 2 x 8x66	0	1386
13	48	1 x 47x50	1	1050
14	63	1 x 6x105 - 3 x 17x11 - 1 x 6x63	0	5005
15	98	1 x 55x75 - 2 x 21x91	1	5733
16	63	1 x 29x10 - 2 x 17x39	0	7098
17	78	1 x 48x78 - 1 x 6x105 - 4 x 6x63	0	2730
18	103	1 x 40x42 - 1 x 51x65 - 1 x 6x105 - 1 x 6x63	0	30030
19	48	6 x 8x66	0	594
20	63	1 x 29x10 - 1 x 14x21 - 2 x 6x105 - 1 x 8x33	0	3480
21	63	1 x 51x65 - 1 x 6x105 - 1 x 6x63	0	15015
22	83	1 x 55x75 - 2 x 14x21	0	6675
23	48	1 x 14x21 - 3 x 6x105 - 2 x 8x33	0	4095
24	78	1 x 6x63 - 9 x 8x33	0	2376
25	73	3 x 14x21 - 5 x 6x63	1	630
26	78	13 x 6x63	0	4851

Fig. 7. Example of a solution found by the BRKGA.

5. CONCLUSION

In this paper we presented a hybrid biased random key genetic algorithm to solve a problem very common in the home-textile industry. Patterns are generated by the BRKGA and a sequential heuristic procedure is used to construct a solution. The approach quality was validated by experimental tests on a set of 100 random generated problems with known optimal solution.

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