STATE ESTIMATION IN DMS - COMBINING FUZZY INFORMATION AND MEASUREMENTS

Vladimiro Miranda
INESC -Macau (Inst. de Eng. de Sist. e Computadores de Macau)
Univ. Macau - Taipa - MACAU
Fax: 853.831694 email: vmiranda@umac.mo

Jorge Pereira J. Tomé Saraiva
INESC (Inst. de Eng. de Sistemas e Computadores)
P. República 93 - 4000 Porto - PORTUGAL
Fax: 351.2.2084172 email: jpereira@duque.inescn.pt

Abstract - In this paper we describe an algorithm to solve the state estimation problem specially intended to be used in DMS - Distribution Management Systems. In the paper we emphasise the most important difficulties in solving this problem applied to distribution networks and we propose a methodology to integrate qualitative information expressed by fuzzy numbers. A case study is also be included in order to evaluate the impact of the number of measurements affected by uncertainty and the amplitude of that uncertainty on the results obtained with the proposed methodology.

Keywords - State Estimation, DMS, Fuzzy Models

I. INTRODUCTION

In a DMS environment it is necessary to estimate values of voltages and currents in the network to evaluate the system security and the operation quality. However, differently from the typical situation for higher voltage networks, in the distribution networks there is not redundancy on the measures done and available in a SCADA system installed in a control centre as, in fact, there is not measurement devices and the operator is blind regarding the actual operating conditions.

This situation explains why direct state estimation methods in use in EMS can be used in DMS. However, it is possible to use qualitative descriptions modelled by fuzzy sets regarding the loads (based on typical load patterns) to build a fuzzy state estimator that combines this knowledge with classical data available in a SCADA system.

This approach will make it possible to obtain an image of the state of the network, possibly having a narrower degree of uncertainty when compared with the one obtained using estimated loads and to establish a coherency between measured data and estimated loads. This is a necessary and most valuable result to implement DMS in control centres. This paper describes a model to perform such a function and examples that clarify the interest of this approach.

In order to obtain these results, one should strength the role of fuzzy set theory. Fuzzy numbers will be used to adequately model uncertainties in data and fuzzy set operations will be the framework to perform numerical calculations with values related to qualitative or inexact data.

II. DMS

A. Characteristics

Some time ago and in a simplified way, one admitted that a DMS would integrate the same set of basic functions that today typically are part of EMS dedicated to large generation/transmission systems.

However, the development of such systems showed the contrary. The distribution systems have a very particular degree of complexity to which one should not forget to add a problem of dimension that is not present in the generation/transmission systems.

Today the distribution networks also have generation capacity directly linked to them. However, they keep the radial topologies or, at least, operated like that using sectionalises or interrupters to break loops. This, by itself, introduces binary variables, non convexities in the mathematical modelizations of several distributions problems. These characteristics contribute to rise the complexity level of these problems to a higher degree when compared to the traditional problems.

B. State Estimation

Till recently, the need for a state estimation model in the distribution networks was not completely understood. But as the network automation level increased and, mostly, with the expansion, sometimes very rapid, of disperse generation directly connected to networks till 30 kV, the need for state estimation rose. The distribution systems are longer passive networks but, rather, they have all the characteristics of large generation/transmission systems, without losing the aspects that turn their mathematical modelization very complex.

Unfortunately, given the territorial scale in which the distribution systems developed, it is today irrealistic to adopt a strategy of measuring in a redundant way a large number of variables in order to characterise the state of the system. Therefore, there is an unavoidable deficit of data that prevents the traditional state estimation models from being directly used.

This way, a state estimator to be used in distribution networks should integrate two basic properties that are not conveniently available in the models and applications that

the market and software houses offer to utilities. These properties are:

- in the absence of measures, there should be the possibility of integrating in an estimator, qualitative information or knowledge about the types of consumers;
- integrated treatment of binary variables corresponding to the state (opened or closed) of sectionalises or interrupters, together with the available measures and the knowledge of the classes of loads to be supplied;

Apart from this aspects, one should not forget that measures of current are the ones that are usually available in distribution networks, namely on the feeders coming from substations. This is a new feature that should be integrated in the distribution networks DMS models when compared to the measures usually available in generation/transmission networks. The treatment of measures of current is more difficult from a mathematical point of view. Therefore, this is a new challenge to be addressed.

On the following points, we will present the basics ideas regarding a model that addresses the first of the two described concerns - the integration of qualitative knowledge in numerical calculations. The secret of this model relies on the use of concepts of Fuzzy Set Theory.

III. BASIC FUZZY NUMBERS OPERATIONS

A normalised fuzzy set A is characterised by a membership function $\mu_A \to [0,1]$ associating each element x_1 to its compatibility degree with a Universe X_1 [6]. In certain circumstances, the membership function μ_A of a fuzzy set A may be interpreted as a possibility distribution.

$$A = \{ (x_1, \mu_A(x_1)),\ x_1 \in X_1 \} \tag{1}$$

An α-level set or an α-core, for each $\alpha \in [0,1]$, of a fuzzy set A defined in X_1 is the hard set A_α obtained from A such that

$$A_\alpha = \{x_1 \in X_1 : \mu_A(x_1) \in \alpha \} \tag{2}$$

A fuzzy set A is a **fuzzy number** if it is convex and its membership function is piecewise continuous. Any operation $*$ with real numbers may be extended to fuzzy numbers, according to the Extension Principle; given two fuzzy numbers A and B, a fuzzy number Z may be generated by

$$Z = A * B \tag{3}$$

$$\mu_{A*B}(z) = \sup\{\min(\mu_A(a), \mu_B(b)) \mid \forall\ a * b = z\} \tag{4}$$

Triangular and trapezoidal fuzzy numbers are commonly used as models for fuzzy data. Fig. 1 shows a trapezoidal fuzzy number P, that can be represented by its break points

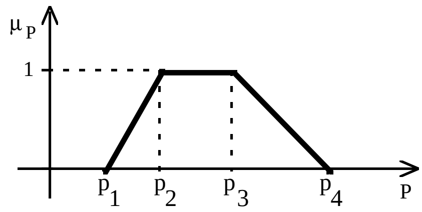


Fig.1 - Trapezoidal fuzzy number

(p_1, p_2, p_3, p_4). The interpretation of P, as a fuzzy load, may derive from the linguistic declaration: *load may occur between p_1 and p_4 but it is likely to occur in [p_2, p_3].*

No strict ordering can be associated to fuzzy numbers. The Centre of Mass technique is widely used in control applications. In state estimation application to be described, we will partially order fuzzy numbers following the hierarchical procedure from [8], using the concepts of Amplitude, Central Value and Removal. For the fuzzy number in Fig. 1, the CV is given by $(p_2+p_3)/2$, the amplitude corresponds to length of the interval [p_1, p_4], and its Removal is given by

$$\mathrm{Rem}(P) = \frac{p_1 + p_2 + p_3 + p_4}{2} \tag{5}$$

III. CLASSICAL CRISP ALGORITHM

A. General State Estimation Model

State estimation aims to identify the values of a set of variables - state variables - that explain a set of measurements according to some criterium. Let us consider that m measurements are available and there are n state variables to estimate. Let us assume that:

- Z is the measurement vector, with m elements;
- X is the **state vector**, with n elements;
- h(.) is the function vector that relates the state variables and the measurements (m functions);
- ε is the **measurement noise vector** (m noises).

A general state estimation model is then given by:

$$Z = h(X) + \varepsilon \tag{6}$$

The elements of the measurement vector, or the measurement variables, can be:

- bus power injection measurements ($S_i=P_i+jQ_i$);
- branch power flow measurements ($S_{ij}=P_{ij}+jQ_{ij}$);
- bus voltage measurements (V_i);
- branch current measurements (I_{ij}).

As elements of the state vector - **state variables** - one usually chooses bus voltages and phases. The components of vector ε are usually considered to be random variables having Gaussian distribution with zero mean and covariance R. If these are assumed to be independent, R is

a diagonal matrix, and the diagonal elements correspond to the variances of measurements.

B. State Estimation Classical Algorithms

Power system state estimation problems are usually solved using two major types of algorithms. In the algorithms of the first type, one tries to minimize the sum of the absolute values of the errors. With the second type of algorithms one tries to identify the values of the state variables that minimize the weighted sum of the square errors, according to expression (7), where R is the covariance matrix of the measurements.

$$\min\ \varepsilon^T R^{-1} \varepsilon \qquad (7)$$

In the second type of algorithms the weights are defined by the inverse of the measurements variances. Therefore, the measurements with smaller variances have higher quality, and its weights have higher values. While measurements with poor quality have higher variances values, and its weights are smaller. In some situations, some authors referred the possibility of add pseudo-measurements to reach an observability condition for the network. In this case, the variances values of these measurements should have greatest values.

Equation (7) represents a Weighted Least Square - WLS - problem whose solution is well known and obtained by replacing ε obtained from (6). This minimization problem is then solved by formulating a set of equations expressing the stationary conditions of this function. In expression (8) H represents the measurement Jacobian matrix.

$$H(X)^T R^{-1} [Z - h(X)] = 0 \qquad (8)$$

This set of equations can be solved iteratively using the Newton-Raphson´s method. At the (k+1)th iteration the refreshed values of the state variables can be obtained from their values in the last iteration by (9). In this equation G is the gain matrix given by (10).

$$X^{k+1} = X^k + (G^k)^{-1} \left(H^K\right)^T R^{-1} \left[Z - h(X^K)\right] \qquad (9)$$

$$G^k = \left(H^K\right)^T R^{-1} \left(H^K\right) \qquad (10)$$

There are several techniques [4] described in the literature to solve this problem. The most common and well-known are the fully coupled version of the normal equation method and its decoupled formulation. Some other more specialised techniques are also described in the literature. For instance, reference [6] describes a technique adequate to solve distribution network state estimation problems. In that paper the measurements are converted into currents and the state estimation algorithm is adapted accordingly.

IV. STATE ESTIMATION WITH FUZZY MEASURES

A. General Considerations

The state estimation problem of a distribution network considering that at least one measurement is modelled as a fuzzy number will now be addressed. Such a state estimation problem can be efficiently solved linearizing the functions h(X) but the quality of the results will depend on the selected linearization point. Therefore, the developed algorithm integrates two major steps. In the first one, a state estimation problem considering only crisp measures will be solved aiming at identifying a linearization point. This point will be used in the second phase to linearize the functions h(X) and to reflect the uncertainty of data in the results.

Let us consider that a vector of measures Z is available, and this vector integrates at least one measure modelled by a fuzzy number. A crisp vector of measures Z_1 can be obtained from Z if the fuzzy measures in Z are replaced by their Central Values. This crisp vector will now be used to run a crisp state estimation problem and compute the state vector X_1 to be used as a linearization point. In this phase, we use the iterative process defined for the normal equation crisp algorithm (9).

B. Fuzzy State Estimation Algorithm

Let us now consider that the value of at least one measure changed so that a new measurement vector Z' is available. Variations in the measurements can be reflected on the results of the state estimation using the gain matrix G obtained in the last iteration of the previously run deterministic exercise (III.B). Therefore, estimates of the variations in the state variables can be obtained and the new values of these variables are given (11).

$$X = X_1 + \left(G^{-1} H^T R^{-1}\right)\left(Z' - h\left(X_1\right)\right) \qquad (11)$$

If some elements of the measurement vector are modelled by fuzzy numbers, expression (11) can be applied. Therefore, we include in the positions corresponding to these measures in the vector (Z'-h(X_1)) fuzzy numbers representing the deviations of the fuzzy measures Z' regarding the h(X_1) values. This fuzzy numbers vector will now be multiplied by $G^{-1}H^TR^{-1}$, using the rules of fuzzy arithmetic, in order to compute the fuzzy vector ΔX that will represent the fuzzy deviations of the state variables. The final fuzzy state vector is obtained by performing the fuzzy addition of ΔX with X_1, that is, with the crisp state vector computed in the first phase of the algorithm.

C. Evaluation of Fuzzy Power Flows and Currents

In the previous sections we have presented an algorithm where the results are the membership functions of a set of state variables. But the DMS operator may also be interested in analysing the possible behaviour of other variables, such as power flows and currents. The evaluation of these membership functions, however, cannot be performed using the fuzzy values of the state variables computed with the previous algorithm (the arithmetic operations are possible, but the results should display too large uncertainties).

These membership functions must be obtained directly from the values of the original measurements. To do this we linearized the generic function F_{ij} representing either the branch active and reactive power flows and the currents, taking the first terms of their Taylor's series around X_1. In expression (12), V_i, V_j, θ_i, θ_j are the voltages and phases in buses i and j.

$$\Delta F_{ij} \cong \left.\frac{\partial F_{ij}}{\partial \theta_i}\right|_{X1} \Delta\theta_i + \left.\frac{\partial F_{ij}}{\partial \theta_j}\right|_{X1} \Delta\theta_j + \left.\frac{\partial F_{ij}}{\partial V_i}\right|_{X1} \Delta V_i + \left.\frac{\partial F_{ij}}{\partial V_j}\right|_{X1} \Delta V_j \quad (12)$$

The derivatives of P_{ij}, Q_{ij} and I_{ij} can be organised in the matrix $J_{FL}(X)$. Each element of this matrix corresponds to the derivatives of the active and reactive flows and currents regarding the elements of the state vector X_1. Defining ΔFL as the vector of the fuzzy deviations of these variables, we can rewrite equations (12) in the form of (13).

$$\Delta FL = J_{FL}(X_1)\ \Delta X \quad (13)$$

Using (13) in (14) we obtain (15). This expression can be used to evaluate the fuzzy deviations of P_{ij}, Q_{ij} and I_{ij} directly from the fuzzy measurement data. The final membership functions are obtained adding their fuzzy deviations to FL, as in (20). Vector FL includes the values of power flows and the magnitude currents, associated with the fuzzy vector Z'.

$$FL = FL(X1) + \left(J_{FL}(X_1)\left(G^{-1}H^T R^{-1}\right)\right)\left(Z' - h(X1)\right) \quad (14)$$

D. Illustrative Toy Example

We now present, for didactic purposes, the results obtained with the fuzzy state estimation algorithm in a small toy network. We used four crisp data: the injected reactive powers Q_1 and Q_2 (4.45 pu and -4.0 pu), voltage in bus 2 (0.97 pu) and active flow P_{12} (6.05 pu). The voltage at bus 1 was specified as the trapezoidal fuzzy number given by

$$V_1 = (1.0, 1.01, 1.02, 1.03) \text{ pu} \quad (15)$$

The fuzzy results are given by (16) to (18).

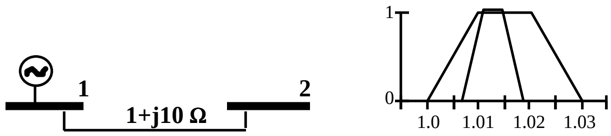


Fig. 2. Toy network and comparison between fuzzy data and results for V_1 - the larger distribution corresponds to the declared uncertain data

$$V_1 = (1.007, 1.011, 1.015, 1.019) \text{ pu} \quad (16)$$

$$V_2 = (0.967, 0.971, 0.975, 0.979) \text{ pu} \quad (17)$$

$$\theta_2 = (-0.048, -0.047, -0.047, -0.046) \text{ pu} \quad (18)$$

Comparing the fuzzy number specified for V1 and the result obtained, we can see that there was a reduction of the uncertainty. This reduction can be explained considering that the specified V1 value integrates incoherencies regarding other specified measures. The state estimation process eliminates those incoherencies thus leading to a reduction of the uncertainty of the estimated value.

E. Measures of Current

Measures of current are very usual in distribution networks. That is why these measures must be integrated in the state estimation model. The described model is able to integrate such measures as described in C. However, due to the linearization process the estimated values for currents can be affected by errors specially if the current values are close to zero. In reference [5] the interested reader will find a corrective process to reduce these errors.

V. CASE STUDY AND RESULTS

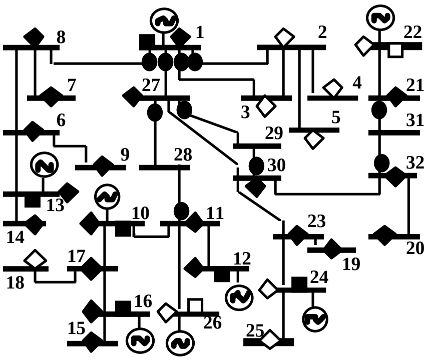
A. Simplified Network From Madeira Island

As illustrative case study of the developed state estimation model we will present results obtained for a network based on the distribution network of Madeira Island (Portugal). This network includes 32 buses and 33 branches. The networks is a typical distribution network and also includes two wind farms connected to buses 22 and 26.

We admitted that, due to some communication problems, some information available on-line on some buses was lost. In other cases, there will not even be measurement devices installed in some buses. In both situations, qualitative or historical information was used to model the knowledge about voltages and injected powers to build fuzzy numbers.

For simulation purposes we considered two sets of measurements. The first is sketched in the network presented in figure 3 and integrates fuzzy data in 9 buses (9 injected power values and 2 voltages). The second is sketched in figure 4 and includes fuzzy data in 18 buses (18 injected power values and 2 voltages). In these figures the crisp measures are represented by symbols with dark background and the fuzzy ones have white background.

The uncertainty related to the fuzzy data is modelled by two sets of fuzzy numbers. The first one corresponds to



● - **P+jQ - power flow**
◆ - **P+jQ - power injection**
■ - **V - voltage magnitude**

Fig. 3 - Simplified network from Madeira Island (configuration of the first measurements system).

multiply the central value of the measure by (0.9;0.95;1.05;1.1). The second one is obtained multiplying the central value of the measure by (0.8;0.9;1.1;1.2). With this simulation we aim at evaluating the performance of the algorithm regarding two issues: the level of the specified uncertainty and the number of measures affected by uncertainty.

To assess the quality of the results obtained, we developed an algorithm based on the gradient optimisation algorithm. One can build an accurate sketch of the membership function of a variable by minimising and maximising its value for selected α-levels. This objective function should be subject to equations (10) and to the inequality constraints related to the ranges of the fuzzy measures for each α-level considered.

B. Results

Tables I to III display the average and maximum errors of the removal, central value and amplitudes of voltages and phases (V and θ), active and reactive flows (P and Q) and currents (I). These errors relate the membership functions built using the fuzzy state estimation algorithm described in the paper and the ones obtained with the gradient method. These membership functions are considered to be the exact ones.

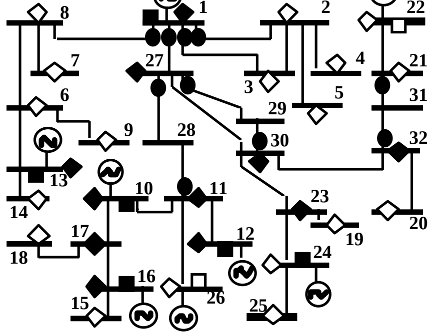


Fig. 3 - Simplified network from Madeira Island (configuration of the second measurements system).

Figures 5 and 6 present the membership functions of the voltage in bus 26 for four situations. In these figures the thick line corresponds to result obtained at the end of the fuzzy state estimation exercise while the one in a thin line is the specified fuzzy measure. The membership function at the left side correspond to narrower fuzzy measures while the ones at the right side correspond to data and results obtained using as data central values multiplied by (0.8, 0.9, 1.1, 1.2).

C. Analysis of the Results

The results obtained demonstrate that the fuzzy state estimation algorithm described can be used to evaluate the state of a network where a significative number of measurements are affected by uncertainty. One can also conclude that the relative average and maximum errors of the removal, central value and amplitude are not significantly affected in the numbers of fuzzy measures is increased or even if the level of uncertainty grows. In some situations one can even see that for the same variables the uncertainty of the results is larger than the one specified data. This can be explained considering that the fuzzy and

TABLE I
RELATIVE ERRORS FOR MEASURES AFFECTED BY ±10% OF UNCERTAINTY FOR THE FIRST SYSTEM OF MEASURES

	Removal		Central Value		Amplitude	
	average (%)	max. (%)	average (%)	max. (%)	average (%)	max. (%)
V	0.003	0.005	0.001	0.002	0.009	0.020
θ	0.004	0.009	0.073	0.347	0.306	4.167
P	0.001	0.007	0.024	0.498	0.032	0.302
Q	0.002	0.010	0.026	0.627	0.136	1.389
I	0.153	1.609	0.129	2.183	0.187	2.381
average	0.033		0.051		0.132	

TABLE II
RELATIVE ERRORS FOR MEASURES AFFECTED BY ±20% OF UNCERTAINTY FOR THE FIRST SYSTEM OF MEASURES

	Removal		Central Value		Amplitude	
	average (%)	max. (%)	average (%)	max. (%)	average (%)	max. (%)
V	0.011	0.019	0.005	0.008	0.030	0.048
θ	0.015	0.037	0.363	1.422	0.615	1.183
P	0.005	0.029	0.089	1.478	0.044	0.212
Q	0.007	0.038	0.120	1.911	0.154	0.870
I	0.907	7.462	0.306	2.657	0.711	5.714
average	0.192		0.175		0.309	

TABLE III
RELATIVE ERRORS FOR MEASURES AFFECTED BY ±10% OF UNCERTAINTY FOR THE SECOND SYSTEM OF MEASURES

	Removal		Central Value		Amplitude	
	average (%)	max. (%)	average (%)	max. (%)	average (%)	max. (%)
V	0.003	0.007	0.001	0.003	0.006	0.020
θ	0.005	0.013	0.113	0.543	0.304	4.167
P	0.002	0.009	0.025	0.498	0.026	0.178
Q	0.002	0.011	0.058	0.990	0.044	0.248
I	0.241	1.970	0.107	1.124	0.243	2.665
average	0.051		0.061		0.123	

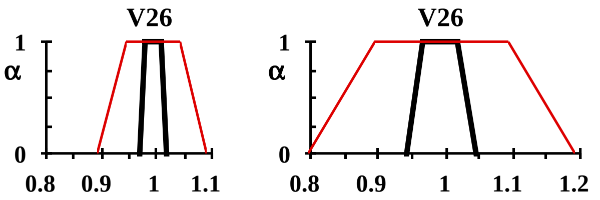


Fig. 5 - Membership functions for specified (thin) and calculated (thick) voltages in the bus 26 for the first measures system.

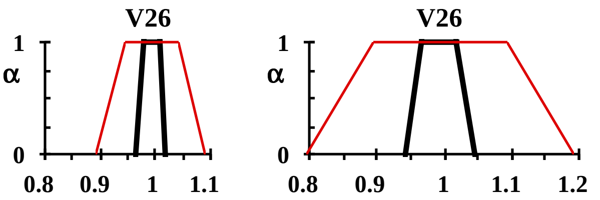


Fig. 6 - Membership functions for specified (thin) and calculated (thick) voltages in the bus 26 for the second measures system.

crisp specified measurements can be affected by incoherence that are eliminated during the fuzzy state estimation process leading to a set of coherent results.

VI. CONCLUSIONS

From the results obtained during this research (some of them presented in the paper), we should like to strength some ideas:

- the algorithm works (!), which is remarkable considering that power system engineers are more used to crisp classical algorithms. It should be stressed that this algorithm combines qualitative and numerical information;
- the algorithm can be used to estimate values and credibility levels of electric variables even in places where the uncertainty is large, due to lack of observability of the network (lack of measurements);
- the algorithm reduces the uncertainty on variables characterising the behaviour of the network where the uncertainty was initially large. This occurs because we integrate qualitative or historical information in the process.

We believe that the fuzzy state estimation algorithm described will be integrate in a near future in DMS software because it answers to one of the most important problems that distribution networks present to the modern automation and control processes in distribution centres: the uncertainty spread through the whole network regarding the desegregated instantaneous values of loads.

VII. ACKNOWLEDGEMENT

The authors wish to thank the cooperation of EEM - Empresa de Electricidade da Madeira S.A.

The research described in this paper was partially financed by contract PRAXIS 43/94.

VIII. REFERENCES

[1] V. Miranda, M. Matos, and J.T. Saraiva, "Fuzzy Load Flow - New Algorithms Incorporating Uncertain Generation and Load Representation", 10th PSCC, Graz, August 1990; in Proceedings of the 10th PSCC, Butterworths, London.

[2] J.T. Saraiva, V. Miranda, and L.M.V.G. Pinto, "Impact on Some Planning Decisions From a Fuzzy Modelling of Power Systems", IEEE Trans. on Power Systems, vol. 9, no.2, May 1994; also in Proceedings of 18th PICA Conference, Scottsdale, Phoenix, Arizona, May 1993.

[3] J.T. Saraiva, V. Miranda, M.A. Matos, "Generation and Load Uncertainties Incorporated in Load Flow Studies", Proceedings of MELECON 91, Ljubljana, 1991.

[4] M.B. Do Coutto Filho, A.M. Leite da Silva, and D.M. Falcao, "Bibliography on power system state estimation (1968-1989)", IEEE Trans. on Power Systems, Vol. 5, No.3, Aug. 1990.

[5] V. Miranda, J. Pereira, and J.T. Saraiva, "Experiences in State Estimation Models for Distribution Systems Including Fuzzy Systems", Proceedings of Stocholm Power Tech, Stocholm, Sweden, June 1995.

[6] C.N. Lu, J.H. Teng, and W.-H.E. Liu, "Distribution System State Estimation", Proceedings of IEEE/PES 1994 Winter Meeting, New York, 1994.

[7] L.A. Zadeh, "Fuzzy Sets", Information and Control, pp. 338 - 353, August 1965.

[8] A. Kaufmann, M.M. Gupta, Fuzzy Mathematical Models in Engineering and Management Science, North Holland ed., 1988.

Vladimiro Miranda was born in Oporto, Portugal, on March 11th, 1955. He received his Licenciado, Ph.D. and Agregado degrees from the Faculty of Eng. of Oporto University (FEUP) in 1977, 1982 and 1991, in Electrical Engineering. In 1981 he joined FEUP and currently holds the position of Professor Associado. In 1985 he joined also INESC where he held until 1996 the position of Project Manager, Head of IDEIA - Information and Decision in Energy Systems. He is now Visiting Full Professor in the University of Macau and Chairman of the Executive Board of INESC-Macau.

Jorge Pereira was born in Viseu, Portugal, on April 22th, 1969. He received his Licenciado degree from the Faculty of Science of Oporto University in 1991 in Applied Mathematics to Computer Science. In 1995 he got the master degree in Electrical Engineering from Faculty of Eng. of the Oporto University. Since 1991 he is a researcher at INESC. Since 1995 he is Assistente at Faculty of Economics of Oporto University.

João Tomé Saraiva was born in Oporto, Portugal, on August 18th, 1962. He received his Licenciado, M.S. equivalent and Ph.D. degrees from the Faculty of Eng. of Oporto University (FEUP) in 1985, 1988 and 1993, respectively, all in Electrical Engineering. Since 1993 he is Professor Auxiliar at FEUP. Since 1989 he is a researcher at INESC.