# COMBINING FUZZY AND PROBABILISTIC DATA IN POWER SYSTEM STATE ESTIMATION

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***Abstract*** - In this paper we present a state estimation model that integrates two types of data: telemetered data available from SCADA systems and qualitative information stored in data bases or provided by users. This information is modeled by Fuzzy Set Theory concepts. As a result, the state of the system will be characterized by fuzzy numbers that reflect the crisp data and specified uncertainty. The paper also presents a new model for integrating topology information in a State Estimation process.

## 1. Introduction

State Estimation (SE) is certainly one fundamental technique in Energy Management Systems (EMS). Its roots are grounded on probabilistic assumptions about errors in measurements, either resulting from malfunctioning of equipment and transmission lines or from non simultaneous acquisition of data. It is in a way a statistical method and it requires redundancy of measurements in order to be able to give a reliable picture of a power system state [1].

When the technology of EMS started to be adopted at distribution level, constituting the embryo of what is nowadays called Distribution Management Systems (DMS), one was immediately aware that the simple transposition of models and software was not possible in general. A few characteristics of the distribution systems are responsible for this, among which:

– in distribution systems, switching plays a major role in the operation of the networks, a fact which is not usually accounted for in EMS environments;
– distribution systems are not equipped (it would require prohibitive investments) with measuring devices in such a scale that redundancy in measurements would allow a classical approach to state monitoring.

At INESC we have devoted some research effort to deal with these two questions in a DMS environment. This paper presents, for the first time, a simultaneous answer to these two points. It follows the publication of [2] and [3], where some combination of measurements with random errors and loads or injections with uncertainty described by fuzzy numbers was presented.

In the temporary absence of some measurements, classical SE models introduced the concept of "pseudo-measurements" to allow the mathematical models to give an answer as realistic as possible. These pseudo-values were generated from stored data in situations similar to the current one, and were then treated "exactly" as if they were values obtained from measuring devices.

In distribution, we can only find typically measurements at the substations; lines and loads are not usually monitored, not even low voltage substations. So, there is at any moment a generalized uncertainty about the power demand conditions and therefore about line loading.

In many countries and utilities, distribution loads have been characterized through the adoption of "typical load curves", in many cases clustered by types of uses such as industrial, domestic, etc. It happens, for instance in France [4] or in Portugal [5]; in this latter case, the work developed by INESC for EDP-Electricidade de Portugal, SA is quite innovative because of the successful use of neural networks to model distribution consumptions.

The concept of "typical" is vague, as translated by that linguistic label. A typical load curve has somehow associated an idea of a general characteristic shape together with some allowed band of variation. In order not to mask this vagueness (which however has a precise semantic meaning), we have decided to represent distribution loads as *fuzzy loads*, and so applications as Fuzzy Power Flow [6] and Fuzzy State Estimation (FSE) are born. The consequence of this attitude is that the uncertainty in data is transported to the results obtained: instead of single values in a SE, we get from a FSE fuzzy descriptions of node voltages or line flows.

We can think of it (to simplify) as having "bands of possibility" for the voltages or line flows, as a consequence of not having a precise knowledge on the actual loads (a lack of knowledge which is not probabilistic).

One interesting consequence, as we will see, is that combining uncertain (fuzzy) node data with some measurements (even if affected with random errors) allow, in many cases, to reduce the range of uncertainty of the data themselves (as if some uncertainty would be incompatible with the partial picture of the system given by the set of available measurements).

This approach allows one to define, at distribution operation, levels of security for the distribution branches based on a risk-and-regret approach.

This paper shows that this technique can be combined with an innovative way of introducing switch state variables in a SE model, in which their binary character is modeled through the

use of a continuous differentiable quadratic equation. This allows the use of all classical analytical tools adopted in SE in a straightforward way; the tests so far conducted have been giving very promising results and in the present moment we feel enthusiastic about the technique.

## 2. A Model for Fuzzy Data Combined with Random Errors

### 2.1. Uncertainty Representation

Let's admit a most common situation in distribution networks, that the load at a certain node is only known through its qualitative characteristics (such as the composition of clients from a LV substation in terms of percentage of domestic, industrial and commercial usages). From some "typical" load curve defining a band of possible power consumption values, based possibly on past history, let's admit that a fuzzy prediction for the actual load is possible, for instance under the form of a trapezoidal fuzzy number (see figure 1).

As an alternative process, we can admit that an experienced operator may produce a statement such as *load is likely to be between 0.3 and 0.4 MW, and it will not be below 0.2 nor above 0.5 MW.* This is represented in figure 1, too. When at least one measurement is represented by a fuzzy value, we are in the presence of a Fuzzy State Estimation process.

Uncertainty can affect voltage values, active and reactive flows, generations or injected powers. The knowledge about these values will be represented by fuzzy set theory concepts [7]. Among other types of fuzzy sets, we will use trapezoidal fuzzy numbers as the one sketched in figure 1 to represent the uncertainty affecting the interval of values that can be assigned to a variable.

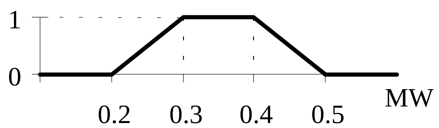


Fig. 1 - Trapezoidal fuzzy number describing an fuzzy assessment of an active load value.

According to this figure, values in [0.3;0.4] - 1.0 $\alpha$ cut - have maximum degree of membership in the sense that they have an high degree of compatibility with the knowledge related to this variable. Values smaller than 0.2 or higher than 0.5 have no compatibility with that knowledge and, therefore, their membership degree is zero.

### 2.2. Fuzzy State Estimation Model

Power system fuzzy state estimation aims at identifying the values of a set of fuzzy variables - **state variables** - that explain a set of measurements according to some criteria. Let us consider that m measurements are available, including uncertain and qualitative information represented using trapezoidal fuzzy numbers - fuzzy measurements - and there are n state variables to estimate. Let us assume that:

- Z is the **measurement vector**, with m elements;
- X is the **fuzzy state vector**, with n elements;
- h(.) is the function vector that relates the state variables and the measurements (m functions);
- $\varepsilon$ is the **measurement noise vector** (m noises).

A general fuzzy state estimation model is then given by:

$$Z = h(X) + \varepsilon \qquad (1)$$

The elements of the measurement vector, or the measurement variables, can be:

- bus power injection measurements ($S_i$=$P_i$+j$Q_i$);
- branch power flow measurements ($S_{ij}$=$P_{ij}$+j$Q_{ij}$);
- bus voltage measurements ($V_i$);
- branch current measurements ($I_{ij}$).

In traditional State Estimation models the elements of the state vector usually are bus voltages and phases. For this model, no assumptions are required on the type of distributions associated with the random errors in vector $\varepsilon$; estimations on individual error variances are possible, giving place to a variance and covariance matrix R whose inverse may be used as a weight matrix. If the errors are assumed to be independent, R turns into a diagonal matrix.

Power system state estimation problems can be solved using an algorithm that aims at identifying the values of the state variables that minimize the weighted sum of the square errors, according to expression (2).

$$\min \varepsilon^T R^{-1} \varepsilon \qquad (2)$$

Although the weights may correspond in principle to the inverse of the measurement variances, giving more credibility to more precise devices, flexible weight allocation procedures may be adopted to represent several other characteristics of the real world problem and the mathematical modelling adopted..

Equation (2) represents a Weighted Least Square - WLS - problem whose solution algorithm is well known and obtained by replacing $\varepsilon$ by an expression obtained from (1). This minimization problem is then solved by formulating a set of equations expressing the stationary conditions of this function, called normal equations. In expression (3) H(X) represents the measurement Jacobian matrix.

$$H(X)^T R^{-1} [Z - h(X)] = 0 \qquad (3)$$

This model relates to the estimation of the **X** variables, and the result obtained represents the mean of a sample distribution (formulas for the estimation of an unbiassed variance of these estimators are available also).

Let us consider that a fuzzy vector of measurements Z is available, that is, a vector that integrates, at least, one measurement modeled by a fuzzy number. This fuzzy vector can be decomposed in a crisp vector of measurements $Z_1$ and in a fuzzy vector of deviations. Vector $Z_1$ can be obtained from Z by replacing the fuzzy measurements in Z by their Central Values while the vector of deviations can be obtained

by performing the subtraction between Z and $Z_1$ using the rules of fuzzy arithmetic.

In the first phase of the algorithm, the crisp vector $Z_1$ will be used to run a crisp state estimation problem to compute the state vector $X_1$. In this phase, the elements of the noise vector associated to the fuzzy measurements, are considered to have zero mean. This can, in fact, be interpreted as the central value of a fuzzy representation of the noises of those measurements At this point, it should be stressed that all measurement errors are represented by a probability distribution even though the mean values of some of them are affected by uncertainty represented by fuzzy numbers.

The computation of the crisp state vector corresponding to the measurement vector $Z_1$ is performed by solving the set of non linear equations (3). This set of equations can be solved iteratively using the Newton-Raphson´s method. At the (k+1)th iteration the refreshed values of the state variables can be obtained from their values in the last iteration by (4). In this equation G is the gain matrix given by (5).

$$X_1^{k+1} = X_1^k + (G^k)^{-1}\left(H^K\right)^T R^{-1}\left[Z_1 - h(X_1^k)\right] \qquad (4)$$

$$G^k = \left(H^K\right)^T R^{-1}\left(H^K\right) \qquad (5)$$

In the second phase, deviations in measurements are reflected on the results of the state estimation using the gain matrix G computed in the last iteration of the previously run crisp state estimation. Therefore, estimates of the variations for the fuzzy state variables can be obtained using equation (6).

$$\Delta X = \left(G^{-1}H^T R^{-1}\right)\left(Z - h\left(X_1\right)\right) \qquad (6)$$

$$X = X_1 + \Delta X \qquad (7)$$

According to expression (6), we evaluate the fuzzy deviation of a measurement by the difference between each fuzzy measurement Z and the crisp value obtained for that measurement by the corresponding h() function. This vector of fuzzy numbers will now be multiplied by $G^{-1}H^TR^{-1}$, using the rules of fuzzy arithmetic, in order to compute the fuzzy vector $\Delta X$ that represents the fuzzy deviations of the state variables. The final fuzzy state vector is obtained by performing the fuzzy addition of $\Delta X$ with $X_1$, that is, with the crisp state vector computed in the first phase of the algorithm (7).

With the described algorithm one is able to characterize the state estimation results in terms of membership functions for the state variables. These variables have a probabilistic nature since in traditional state estimation models the error measurements are assigned a probabilistic interpretation. However, they also have a fuzzy nature since the mean of the errors are represented by fuzzy numbers having a zero central value. Accordingly, this Fuzzy State Estimation model represents a novel way of combining two theoretical frameworks to model uncertainty: probabilistic and fuzzy models. Fuzzy Sets and Probabilities are not competing againt each other, instead they must be seen as complementary.

### 2.3. Computation of other variables

The DMS operators may also be interested in analyzing the possible behavior of other variables, such as power flows and currents. However, the evaluation of the membership functions of these variables should not be performed using the fuzzy values of the state variables computed with the previous algorithm. The necessary arithmetic operations are possible, but the results should display too large uncertainties.

These membership functions must be obtained directly from the values of the original fuzzy measurements. To do this we can linearize a generic function $F_{ij}$ representing either branch active and reactive power flows or currents, taking the first terms of their Taylor's series computed around $X_1$. In expression (8), $V_i$, $V_j$, $\theta_i$, $\theta_j$ are the voltages and phases in buses i and j.

$$\Delta F_{ij} \cong \left.\frac{\partial F_{ij}}{\partial \theta_i}\right|_{X1} \Delta\theta_i + \left.\frac{\partial F_{ij}}{\partial \theta_j}\right|_{X1} \Delta\theta_j + \left.\frac{\partial F_{ij}}{\partial V_i}\right|_{X1} \Delta V_i + \left.\frac{\partial F_{ij}}{\partial V_j}\right|_{X1} \Delta V_j \qquad (8)$$

The derivatives of $P_{ij}$, $Q_{ij}$ and $I_{ij}$ in expression (8)can be organized in the matrix $J_{FL}(X)$. Each element of this matrix corresponds to the derivatives of the active and reactive flows and currents regarding the elements of the state vector $X_1$. Defining $\Delta FL$ as the vector of the fuzzy deviations of these variables, we can rewrite equations (8) in the form of (9).

$$\Delta FL = J_{FL}(X_1)\ \Delta X \qquad (9)$$

This expression can be used to evaluate the fuzzy deviations of $P_{ij}$, $Q_{ij}$ and $I_{ij}$ directly from the fuzzy measurement data. The final membership functions (10) are obtained adding their fuzzy deviations to $FL(X_1)$, using the rules of fuzzy arithmetic.

$$FL = FL(X1) + \left(J_{FL}(X_1)\left(G^{-1}H^T R^{-1}\right)\right)\left(Z - h\left(X1\right)\right) \qquad (10)$$

Vector FL includes the values of fuzzy power flows and fuzzy magnitude currents, associated with the fuzzy vector Z.

## 3. A Model for Switch Status Variables

In distribution networks the system in operation usually changes more frequently than in transmission systems where the topology in operation is well established and steady. Apart from this distinctive aspect, in some situations due to problems with the communication links between substations and Control Centers or due to the absence of digital measurements regarding the status of switching devices the operator may not know what the topology in operation is. In [8] and [9], topological issues were treated using a least absolute value state estimation and by considering power flows as state variables in branches where the status of the corresponding switching devices are unknown.

In INESC we started some experiences to introduce topology variables in power system models, namely in state estimation models. In this sense, we though that, in most situations, the pattern defined by voltage, power and current measurement should be enough to estimate the topology of the network from two points of view:

- we would like to estimate the value of topology variables considering that we have no digital measurement for a switching device status;
- we may even have some information for the status of a device but that data may be wrong.

In any case, the binary nature of topology variables makes the process difficult. Binary variables are usually a nuisance, because they give a combinatorial nature to problems and do not allow the use of well behaved analytical methods based on derivatives. It must also be said that replacing a variable $x \in \{0,1\}$ for a relaxed version $x \in [0,1]$ has not been satisfactory, namely because rounding the non integer results does not usually guarantee a correct result.

In the experiences performed so far we included in the models equations of the type represented in (11) to circumvent these problems, that is, to keep the binary nature of the admissible values of those variables while representing them as solutions of a continuous function

$$x^2 - x = 0 \qquad (11)$$

Using this approach, the two mentioned situations can be dealt with in the State Estimation model by considering equations (12) and (13) below. With equation (12) we model situations for which in the SCADA data base there is information about the status of a device (0 or 1) but this value can be wrong due to problems in data acquisition of transmission. In other cases, there may not be any value for this variable. Therefore, its value should be constrained by (12).

$$D_{ij}^{m} = D_{ij}^{2} + \varepsilon_k \qquad (12)$$

$$0 = D_{ij} - D_{ij}^{2} + \varepsilon_k \qquad (13)$$

In these expressions, $D_{ij}^{m}$ represents the measured value of a topology variable related to the status of branch ij, $D_{ij}$ is the corresponding state variable and $\varepsilon_k$ is the related error. Using this approach the h(X) functions representing power or current measurements and injected powers should be affected by the corresponding topology variables in case the status of that branch is not known or is affected by errors. In expressions (14) to (18) $P_{ij}$, $Q_{ij}$ and $I_{ij}$ represent the usual expressions for active and reactive power flow and current in branch ij.

$$P_{ij}^{m} = P_{ij}\, D_{ij} \qquad (14)$$

$$Q_{ij}^{m} = Q_{ij}\, D_{ij} \qquad (15)$$

$$I_{ij}^{m} = I_{ij}\, D_{ij} \qquad (16)$$

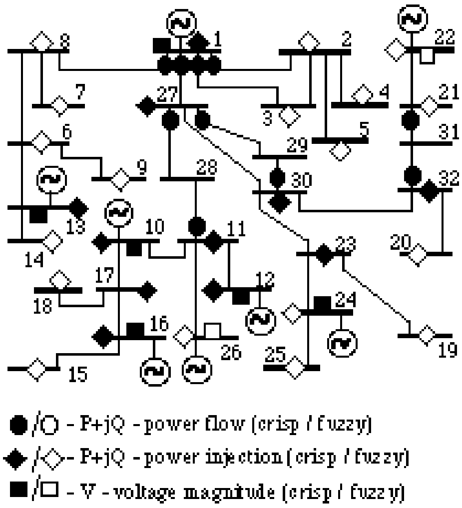


Fig. 2 - Simplified network from Madeira Island.

$$P_{i}^{m} = \sum_{j} P_{ij}\, D_{ij} \qquad (17)$$

$$Q_{i}^{m} = \sum_{j} Q_{ij}\, D_{ij} \qquad (18)$$

In many test cases studied this model always gave correct results and very good convergence characteristics, even if several status variable data were made wrong on purpuse. Although the objective of this paper is not to describe in detail this model, we can reveal that convergence characteristics are enhanced with an adequate weight control strategy.

## 4. Examples

### 4.1. System data

In this section we present results obtained for the Madeira Island (Portugal, North Atlantic) MV reduced network sketched in figure 2, using the algorithm detailed in the previous sections.

Figure 2 indicates the places where telemetered measurements are available - symbols in black - and places for which there is qualitative information - symbols in white - that will be integrated in the State Estimation model. Data of this system - branch and nodal data - can be obtained from the authors.

A state estimation simulation study was run for this system, specifying trapezoidal fuzzy numbers for measurements affected by uncertainty, whose 0.0 and 1.0 $\alpha$ cut correspond to $\pm 10\%$ and $\pm 5\%$ around the corresponding central value. These are quite large uncertainty ranges; nevertheless, this specification constitutes a good robustness test to the FSE model.

We also simulated that for six of the system branches the corresponding state was not known for sure or simply there was no information in the SCADA data base about it. These branches are represented by doted lines in figure 3.7

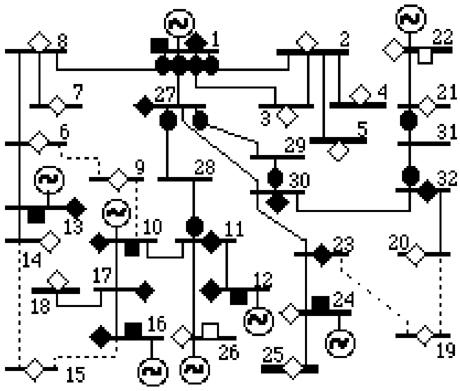


Fig. 3 - Simplified network from Madeira Island with 6 switching devices in 6 different lines.

### 4.2. Results

The State Estimation result *correctly* indicated that branches 15-16, 6-9 and 19-25 are in operation while branches 14-15, 9-10 and 19-20 are not. These results are consistent with the measurement specified data and are an example of how powerful the presented technique can be.

After identifying the topology in operation, the obtained crisp state vector was used to perform the linearization that is inherent to the second phase of the Fuzzy State Estimation algorithm as detailed in section 2.2 and 2.3.

In figures 4, 5 and 6 one presents membership functions for several variables.Figure 4 depicts them for two state variables - the voltages in buses 22 and 26. In figure 6 one presents the computed membership functions for variables for which there was no measurement or fuzzy estimate available.

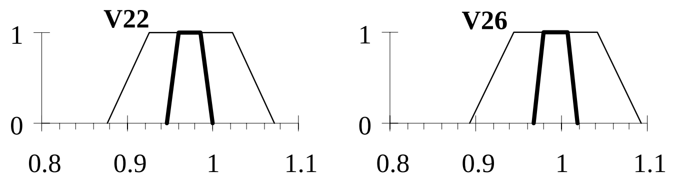


Fig. 4 - Membership functions for specified (thin) and calculated (thick) voltages in the buses 22 and 26.

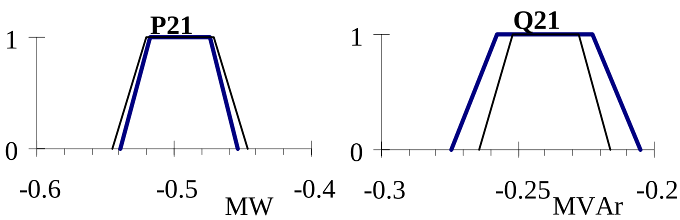


Fig. 5 - Membership functions for specified (thin) and calculated (thick) active and reactive load in the bus 21.

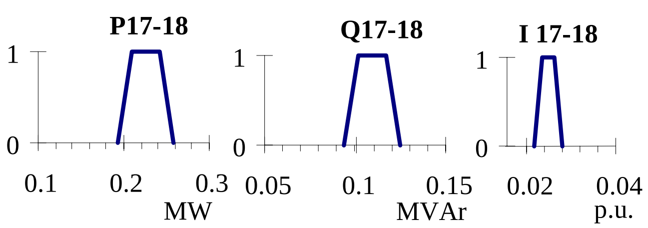


Fig. 6 - Membership functions for calculated active and reactive power flow and current in the line 17-18.

In these figures, a thin line represents the specified membership function for each variable, and a thick line represents the computed membership function.

### 4.3. Interpretation

The analysis of these membership functions deserves some comments. One of the most interesting features of the developed Fuzzy State Estimation model is that the specified fuzziness is in some cases - as the ones represented in figure 4 - larger than the uncertainty resulting for the same variables from the membership functions obtained at the end of the study.

This can be explained if we remember that State Estimation models are a way to eliminate incoherencies in data relative to a process, in this case a power system running in a steady state. Therefore, one can think that the uncertainty ranges defined as data integrate values that are incoherent with the pattern defined by the global set of measurements. The Fuzzy State Estimation model acts as a filter that eliminates values in uncertain data not compatible with the rest of the measurements.

We also can see in Fig. 5 that the uncertainty attributed to reactive load in bus 21 is narrower than the range of values compatible with the other available measurements.

### 4.4. Systematic Evaluation of the Quality of the Results

Some insight on the evaluation of the quality of the computed membership functions was obtained by comparing them with the ones built using an alternative method, based on a gradient optimization method and withthe following steps:

- the membership functions were discretized in several $\alpha$ cuts ;
- for each variable to be computed and for each of those cuts two optimization problems were solved, to minimize and maximize the value of that variablewithin the ranges defined by the $\alpha$ cuts and with the constraints imposed by the state estimation model.

A set of fuzzy numbers can be compared by using in a hierarchical order the following three criteria [10]:

- Removal - the removal of $\tilde{A}$ regarding a crisp number k is the average value of the areas $A_1$ and $A_2$. $A_1$ is determined by the left side of $\tilde{A}$, by the number k and by the horizontal lines corresponding to the horizontal axes and to 1.0 membership degree. $A_2$ is defined in a similar way by replacing the left function of the membership function of $\tilde{A}$ by the right function.
- Central Value - corresponds to the average value of the 1.0 $\alpha$ cut;
- Amplitude - corresponds to the amplitude of the 0.0 $\alpha$ cut of $\tilde{A}$.

| | Removal average (%) | Removal max. (%) | Central Value average (%) | Central Value max. (%) | Amplitude average (%) | Amplitude max. (%) |
|---|---|---|---|---|---|---|
| V | 0.003 | 0.007 | 0.001 | 0.003 | 0.006 | 0.020 |
| θ | 0.005 | 0.013 | 0.113 | 0.543 | 0.304 | 4.167 |
| P | 0.002 | 0.009 | 0.025 | 0.498 | 0.026 | 0.178 |
| Q | 0.002 | 0.011 | 0.058 | 0.990 | 0.044 | 0.248 |
| I | 0.241 | 1.970 | 0.107 | 1.124 | 0.243 | 2.665 |
| average | 0.051 | | 0.061 | | 0.123 | |

Tab. 1 - Comparison of methods - average and maximum relative deviations.

In table 1, one presents the average and maximum relative deviations obtained for several variables using the FSE and the gradient model. The results directly obtained by the two methods are remarkably close. That is, we may be confident that the FSE results have good quality and a good accuracy. Of course, the gradient model was developed only for the purpose of assessing this characteristic - it is unthinkable to use it in any real world application.

## 5. Conclusions

The development of specific models for DMS environments has motivated the research for a successful match between probabilistic and fuzzy set concepts. In a State Estimation process for a distribution network, where the lack of availability of measurements is a dominant characteristic, this would be most advantageous.

The work reported in this paper brings together two innovations directly useful in DMS - a straightforward way of handling topology definition problems, through a clever modeling of switch or breaker status variables, and a generalized way of handling data represented by fuzzy values.

The Fuzzy State Estimation model implies some extra computing effort, compared with a crisp State Estimation model; however, this is not really significative and it must be balanced against the extra amount of information obtained - one cannot gain more information at zero cost, usually.

Some remarkable results can be seen in the examples presented, namely that the Fuzzy State Estimation model even allows in some cases to narrow the range of uncertainty about fuzzy data. This may represent really *a new way* of building a history file for loads that are not directly monitored, which could be of great interest in managing distribution systems.

It must finally be said that the Fuzzy State Estimation model is not specific of distribution systems, and can be applied directly to transmission systems and included in EMS environments, adding either its switch status variable handling ability or its fuzzy data representation capacity.

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