EXPERIENCES IN STATE ESTIMATION MODELS FOR DISTRIBUTION SYSTEMS INCLUDING FUZZY MEASURES

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Abstract - This paper describes how State Estimation can be done even if some measurements are given as fuzzy data. The fuzzyfied model presented belongs to the Least Squares family. The text includes the theoretical developments and examples, demonstrating the interest of the new approach. A specific model for Distribution Networks is presented, because these are a particularly promising field of application of the technique.

I. INTRODUCTION

In recent years the technology of the control centers in distribution systems went through some drastic changes. Evolving from simple SCADA systems, the concept of DMS - Distribution Management Systems gained growing acceptance. In fact the complexity of the operations to be taken care of, at distribution control centers, increased notoriously due to, among other factors:

- increasingly large degree of penetration of small scale generation directly branched on the distribution networks;
- adoption of active policies in load management by utilities;
- adoption of new protection technologies, namely coordinated digital relaying and computer relaying;
- new operation challenges to be met (including wheeling through the distribution network, or co-existence with third party or competitive generation at distribution level).

As a consequence, the distribution control centers began requiring the use of functions that were in the past typical or exclusive of higher level systems; among these, we find the need for State Estimation. State estimation for distribution networks has some special characteristics, that require the development of models different from those used in generation-transmission systems:

a) Not may power measures are found - instead, current measures are quite common, but these are a source of very incomplete information, because they relate only with the magnitude of the value being considered; therefore, some problems may arise when solving mathematical models, due to the singularity of some matrices.
b) It is not economically feasible to admit that a policy, similar to the one adopted in transmission systems, can be adopted for distribution networks, as far as a widespread use of metering devices is concerned; therefore, for many years, lack of on line information will be characteristic, due to the lack of measurements.
c) Connected to the distribution grid, there is nowadays an increasing number of small generating units (hydro, wind, etc.) and many of them are private owned.

However, in distribution systems one has access to qualitative information on the type and relative importance of consumers connected to the grid. This information could relate to the class of consumption or even be as detailed as having defined some type of load curve characteristic of such consumption. The same could be said for dispersed generation: information about wind, for instance, could give an indication on the amount of generation expected from wind turbines or wind farms, even if no direct measurements are available.

At INESC we conducted experiences to assess the feasibility of state estimation models, specific for distribution networks, that would "marry" the characteristics of traditional state estimators with qualitative information, expressed by fuzzy sets. We thought of using fuzzy numbers to express unknown or uncertain measure values for the following reasons:

- fuzzy numbers are able to translate to a numerical form qualitative linguistic declarations, of the uncertain number type ("around 3") or the uncertain interval type ("roughly between 2 and 4");
- one may translate into a fuzzy number information from billing files, or from clustering exercises on typical load curves, relative to one consumer or a group of consumers, in a part of a network where no measurements are available;
- this information is not, in general, of the probabilistic type or, even if probabilistic models were conceivable, the cost and effort to develop and validate them would be excessive in most cases;
- we wanted to develop and test a model that could be seen as an extension of proven knowledge, so that it could be understood and accepted by those familiar with practice in utilities;
- and finally, this type of fuzzyfication of power system models has demonstrated that the fuzzy output obtained also gives information about the sensitivity of the results regarding the uncertainty in the data, which seemed interesting in the State Estimation environment.

This paper reports the promising results of those experiments, and is organized as follows: we will refer to some basic principles of calculus with fuzzy numbers; we will describe the basic theory of the fuzzyfication of a State Estimation model

based on the Least Squares minimization; we will illustrate its consequences on a single line system; we will derive the Fuzzy State Estimator model to evaluate branch flows and currents; and we will give some selected results from tests in larger systems (the IEEE 39 bus system and the Madeira Island reduced system).

The model presented in the paper refers specifically to three phase (admitted balanced) networks, such as commonly used in Europe. Works preliminary to the one presented here and that are relevant to understand its context or contents are: fuzzy set modeling of power flows [1-3] and state estimation models for distribution networks [4,5].

II. BASIC FUZZY NUMBER OPERATIONS

A normalized fuzzy set A is characterized by a membership function $\mu_A \rightarrow [0,1]$ associating each element x_1 to its compatibility degree with a Universe X_1 [6]. In certain circumstances, the membership function μ_A of a fuzzy set A may be interpreted as a possibility distribution.

$$A = \{ (x_1, \mu_A(x_1)),\ x_1 \in X_1 \} \tag{1}$$

An α-level set or an α-core of a fuzzy set A defined in X_1 is the hard set A_α obtained from A for each $\alpha \in [0,1]$ such that

$$A_\alpha = \{ x_1 \in X_1 : \mu_A(x_1) \geq \alpha \} \tag{2}$$

A fuzzy set A is said to be a **fuzzy number** if it is convex over the real line R such that its membership function is piecewise continuous. Any operation $*$ with real numbers may be extended to fuzzy numbers, according to the Extension Principle; given two fuzzy numbers A and B, a fuzzy number Z may be generated by

$$Z = A * B \tag{3}$$

$$\mu_{A*B}(z) = \sup\left\{ \min\left(\mu_A(a), \mu_B(b)\right) \mid \forall\ a*b = z \right\} \tag{4}$$

This applies namely to the most usual operations, such as the sum + or the product x . In particular, one must retain that the subtraction A - B must be dealt with as a sum A + (-B).

Triangular and trapezoidal fuzzy numbers are commonly used as models for fuzzy data. Fig. 1 shows a trapezoidal fuzzy number P, which is sometimes represented by its break points using (p_1, p_2, p_3, p_4). The representation of a sum and a subtraction of two fuzzy numbers A, B becomes very simple:

$$\begin{aligned} A + B &= (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4) \\ &= (a_1+b_1,\ a_2+b_2,\ a_3+b_3,\ a_4+b_4) \end{aligned} \tag{5}$$

$$\begin{aligned} A - B &= (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4) \\ &= (a_1-b_4,\ a_2-b_3,\ a_3-b_2,\ a_4-b_1) \end{aligned} \tag{6}$$

The interpretation of the fuzzy number P, in Fig. 1, as a fuzzy load, may derive from the linguistic declaration: *load may occur between p_1 and p_4 but it is likely to occur in $[p_2, p_3]$*. It may be also said that this represents a fuzzy interval, or an interval with fuzzy boundaries.

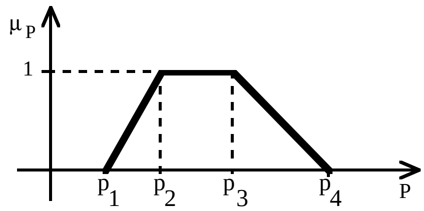

Fig.1 - Trapezoidal fuzzy number

No strict ordering can be associated to fuzzy numbers. The Center of Mass technique is widely used in control applications. But in this paper, we will partially order fuzzy numbers following the hierarchical procedure from [7], using the concepts of Removal, Central Value and Amplitude. For the fuzzy number in Fig. 1, its Removal is given by

$$\mathrm{Rem}(P) = \frac{p_1 + p_2 + p_3 + p_4}{2} \tag{7}$$

The central value CV of a fuzzy number is defined as the mean value of its 1.0-cut. For the fuzzy number in Fig. 1, the CV is given by $(p_2+p_3)/2$. The amplitude corresponds to length of the interval $[p_1, p_4]$.

III. CLASSICAL CRISP ALGORITHM

A. General State Estimation Model

State estimation aims to identify the values of a set of variables - state variables - that explain a set of measurements according to some criterium. Let us consider that m measurements are available and that n state variables were selected. Assume that:

- Z is the measurement vector (mx1);
- X is the **state vector** (nx1);
- h(.) is the function vector that relates the state variables and the measurements (mx1);
- ε is the **measurement noise vector** (mx1).

A general state estimation model is then given by

$$Z = h(X) + \varepsilon \tag{8}$$

The elements of the measurement vector, or the measurement variables, may be:

- bus power injection measurements ($S_i=P_i+jQ_i$);
- branch power flow measurements ($S_{ij}=P_{ij}+jQ_{ij}$);
- bus voltage measurements (V_i);
- branch current measurements (I_{ij}).

As elements of the state vector, one usually chooses bus voltages and phases. The components of vector ε are usually considered to be random variables having Gaussian distribution with zero mean and covariance R. If these are assumed to be independent, R is a diagonal matrix. Its diagonal ii element corresponds to the variance of the i-th measurement σ_i^2.

B. State Estimation Crisp Algorithms Review

Power system state estimation problems are usually solved using two major types of algorithms. In the algorithms of the first type, one tries to minimize the sum of the absolute values

of the errors. With the second type of algorithms one tries to identify the values of the state variables that minimize the weighted sum of the square errors, according to expression (9).

$$\min\ \varepsilon^T R^{-1} \varepsilon \tag{9}$$

The values in R^{-1}, or σ_i^{-2}, are used to apply different weights to the measurements. A smaller σ may be assigned to measurements of higher quality, while measurements obtained from poor quality equipment will have higher σ values. In some situations, one can add pseudo-measurements to reach an observability condition for the network. In this case, one can also assign higher σ values to these measurements in order to translate a lower confidence level on these measurements.

Equation (9) represents a Weighted Least Square -WLS- problem whose solution is well known and obtained by replacing ε obtained from (8) in (9). This minimization problem is then solved by formulating a set of equations expressing the stationary conditions of this function. This set of equations can be obtained differentiating equation (10) regarding the state variables X and making these derivatives equal to zero (11). In this expression H represents the measurement Jacobian matrix.

$$\min\ [Z - h(X)]^T R^{-1} [Z - h(X)] \tag{10}$$

$$H(X)^T R^{-1} [Z - h(X)] = 0 \tag{11}$$

This set of equations can be solved iteratively using the Newton-Raphson´s method. At the (k+1)th iteration the refreshed values of the state variables can be obtained from their values in the k-th iteration by (12) and G is the gain matrix given by (13). Equation (12) is known as *normal equation* of the WLS problem. The iterative process ends when the difference of the values of the state variables in iterations k and k+1 are smaller than a specified tolerance.

$$X^{k+1} = X^k + (G^k)^{-1} \left(H^K\right)^T R^{-1} \left[Z - h(X^K)\right] \tag{12}$$

$$G^k = \left(H^K\right)^T R^{-1} \left(H^K\right) \tag{13}$$

Several techniques [4] are described in the literature to solve this problem. The most common and well-known are the fully coupled version of the normal equation method and its decoupled formulation. Some other more specialized techniques are also described in the literature. For instance, in reference [5] describes a technique appropriate to solve distribution network state estimation problems. In that paper the measurements are converted into currents and the state estimation algorithm is adapted accordingly.

IV. STATE ESTIMATION WITH FUZZY MEASURES

A. General Considerations

The state estimation problem of a distribution network considering that at least one measurement is modeled as a fuzzy number will now be addressed. Such a state estimation problem can be efficiently solved linearizing the functions h(X) but the quality of the results will depend on the selected linearization point. Therefore, the developed algorithm integrates two major steps. In the first one, a state estimation problem considering only crisp measures will be solved aiming at identifying a linearization point. This point will be used in the second phase to linearize the functions h(X) and to reflect the uncertainty of data in the results.

B. The Linearization Point

Let us consider that a Z vector of measures is available. This vector integrates at least one measure modelled by a fuzzy number. A crisp vector of measures Z_1 can be obtained from Z if the fuzzy measures in Z are replaced by their Central Values. This crisp vector will now be used to run a crisp state estimation problem and compute the state vector X_1 to be used as a linearization point. In this phase, we use the iterative process defined for the normal equation crisp algorithm (12).

C. Dealing with Deviations on the Measures

Let us now consider that the value of at least one measure changed so that a new measurement vector Z´ is available. Variations ΔZ (14) can be reflected on the results of the state estimation using the gain matrix G given by (13) obtained in the last iteration of the previously run deterministic exercise. Therefore, estimates of the variations in the state variables can be obtained by (15) and the new values of these variables are given approximately by (16).

$$\Delta Z = Z' - h(X_1) \tag{14}$$

$$\Delta X = (G^{-1} H^T R^{-1}) \Delta Z \tag{15}$$

$$X = X_1 + \Delta X \tag{16}$$

D. Fuzzy State Estimation Algorithm

If some elements of the measurement vector are modeled by fuzzy numbers, expressions (14-16) must be fuzzified. Therefore, in the positions corresponding to the fuzzy measures, vector ΔZ includes fuzzy numbers representing the deviations of the fuzzy measures regarding the $h(X_1)$ value computed previously, using their central values.

This ΔZ fuzzy vector will now be multiplied by $G^{-1}H^TR^{-1}$ using the rules of fuzzy arithmetic in order to compute the fuzzy vector ΔX that will represent the fuzzy deviations of the state variables. The final fuzzy state vector is obtained by performing the fuzzy addition of ΔX with X_1, that is, with the crisp state vector computed in the first phase of the algorithm.

E. Evaluation of Fuzzy Power Flows and Currents

We have so far presented a linearized algorithm to build the membership functions of a set of state variables. But the user may also be interested in analysing the possible behavior of other variables, such as power flows and currents. The evaluation of these membership functions, however, cannot be performed using the fuzzy values of the sate variables computed with the previous algorithm (the arithmetical operations are possible, but the results are erroneous, namely displaying too large uncertitudes).

These membership functions must be obtained directly from the values of the original measurements. To do this we

linearized the generic function F_{ij} representing either the branch active and reactive power flows and the currents, taking the first terms of their Taylor's series around X_1. In expression (17), V_i, V_j, θ_i, θ_j are the voltages and phases in buses i and j.

$$\Delta F_{ij} \cong \left.\frac{\partial F_{ij}}{\partial \theta_i}\right|_{X1} \Delta\theta_i + \left.\frac{\partial F_{ij}}{\partial \theta_j}\right|_{X1} \Delta\theta_j + \left.\frac{\partial F_{ij}}{\partial V_i}\right|_{X1} \Delta V_i + \left.\frac{\partial F_{ij}}{\partial V_j}\right|_{X1} \Delta V_j \quad (17)$$

The derivatives of P_{ij}, Q_{ij} and I_{ij} can be organized in the matrix $J_{FL}(X)$. Each element of this matrix corresponds to the derivatives of the active and reactive flows and currents regarding the elements of the state vector X_1. Defining ΔFL as the vector of the fuzzy deviations of these variables, we can rewrite equations (17) in the form of (18).

$$J_{FL}(X) = \begin{bmatrix} \left.\frac{\partial P_{ij}}{\partial \theta_k}\right|_X & \left.\frac{\partial P_{ij}}{\partial V_k}\right|_X \\ \left.\frac{\partial Q_{ij}}{\partial \theta_k}\right|_X & \left.\frac{\partial Q_{ij}}{\partial V_k}\right|_X \\ \left.\frac{\partial I_{ij}}{\partial \theta_k}\right|_X & \left.\frac{\partial I_{ij}}{\partial V_k}\right|_X \end{bmatrix} \qquad \Delta FL = \begin{bmatrix} \Delta P_{ij} \\ \Delta Q_{ij} \\ \Delta I_{ij} \end{bmatrix}$$

$$\Delta FL = J_{FL}(X_1)\ \Delta X \quad (18)$$

Using (13) in (18) we obtain (19). This expression can be used to evaluate the fuzzy deviations of P_{ij}, Q_{ij} and I_{ij} directly from the fuzzy measurement data. The final membership functions are obtained adding their fuzzy deviations to FL, as in (20). Vector FL includes the values of power flows and the magnitude currents, associated with the fuzzy vector Z'.

$$\Delta FL = \left(J_{FL}(X_1)\left(G^{-1}H^T R^{-1}\right)\right)\Delta Z \quad (19)$$

$$FL = FL(X_1) + \Delta FL \quad (20)$$

F. Illustrative Toy Example

We now present, for didactic purposes, the results obtained with the fuzzy state estimation algorithm in a small toy network. We used four crisp data: the injected reactive powers Q_1 and Q_2 (4.45 pu and -4.0 pu), voltage in bus 2 (0.97 pu) and active flow P_{12} (6.05 pu). The voltage at bus 1 was specified as the trapezoidal fuzzy number given by

$$V_1 = (1.0,\ 1.01,\ 1.02,\ 1.03)\ \text{pu} \quad (21)$$

The fuzzy results are

$$V_1 = (1.007,\ 1.011,\ 1.015,\ 1.019)\ \text{pu} \quad (22)$$
$$V_2 = (0.967,\ 0.971,\ 0.975,\ 0.979)\ \text{pu} \quad (23)$$
$$\theta_2 = (-0.048,\ -0.047,\ -0.047,\ -0.046)\ \text{pu} \quad (24)$$

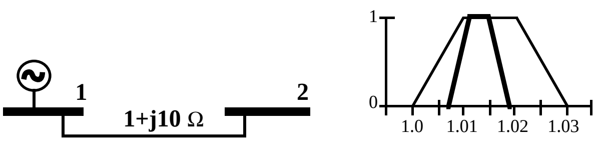

Fig. 2. Toy network and comparison between fuzzy data and results for V_1 - the larger distribution corresponds to the declared uncertain data

Comparing the fuzzy number specified and obtained for V_1, we notice an uncertainty reduction! This can be explained in two ways: 1 - the resulting V_1 includes more information (from the other measures) than the original V_1, and therefore it is only natural that its uncertainty may be reduced; 2 - the original V_1 may be seen as a description of possible mean values + errors, and the resulting V_1 is the distribution of just the mean values, as proposed by the Fuzzy State Estimation Algorithm.

G. Current Measurements

According to [5], current measurements can be incorporated in the state estimation model provided that all branch power flows and bus injection measurements are converted into equivalent current values. For the current magnitude this procedure must be adapted since no phase information is available. Therefore the magnitudes of the equivalent currents are equal to the measured ones while their phases correspond to the phases of the calculated currents.

Based on the equivalent currents, the state estimation crisp algorithm (12) and its fuzzified version (15) can still be used, with the function h(X) relating the current magnitudes to the state variables.

H. Current Corrective Procedure

Similarly to the situation described in [1], due to linearization, errors may occur when building current membership functions, namely for very small current values. In these cases, the current magnitude may appear with negative values. Corrected values may however be obtained if their real and imaginary parts are also calculated. In this case the derivatives of these real and imaginary parts regarding the state variables must be integrated in the $J_{FL}(X)$ matrix and their deviations calculated using (19).

The real and imaginary parts of a current, at a level α, define a rectangle in the complex plane. This happens namely at the 0 and 1 levels. If one remembers that, for every instantiation of a real an imaginary part, the magnitude of the current must be given by $I_m = \sqrt{I_r^2 + I_i^2}$, we just have to check which pairs of I_r,I_i give the calculated I_m, obtained from (19) and (20), in the feasible region (positive values of I_m).

This gives an indication on the trajectory followed by I_m, within the rectangular region mentioned above; then the calculation of corrected I_m values is straightforward. Of course, if the same magnitude value I_m occurs at different levels α_1 and α_2, we take I_m with a possibility level α = Max $\{\alpha_1, \alpha_2\}$.

V. APPLICATION EXAMPLES

A. IEEE 39-bus System

The method was tested using the IEEE 39-bus, 46-branch system in Fig. 3. In tables I and II we present the values of the specified measures (injected active and reactive powers, voltages and active and reactive flows). Buses with uncertain data are identified with the symbol $*$ in the tables (which show their central values) and by white rectangles in Fig. 3.

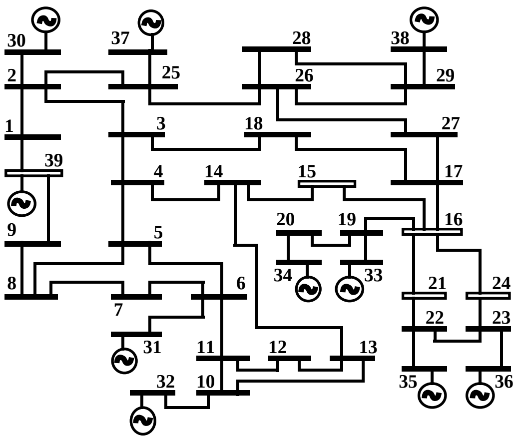

Fig. 3. IEEE 39-bus network.

TABLE I
VALUES OF BUS MEASUREMENTS.

bus	(pu)	bus	(pu)	bus	(pu)
P 3	-3.220	Q 3	-0.024	Q 39	2.686
P 4	-5.000	Q 4	-1.840	V 30	1.040
P 7	-2.338	Q 7	-0.840	V 31	0.980
P 8	-5.220	Q 8	-1.760	V 32	0.980
P 11	0.000	Q 11	0.000	V 33	0.990
P 13	0.000	Q 13	0.000	V 34	1.010
P 14	0.000	Q 14	0.000	V 35	1.040
P 18	-1.580	Q 18	-0.300	V 36	1.060
P 26	-1.390	Q 26	-0.170	V 37	1.020
P 27	-2.810	Q 27	-0.755	V 38	1.020
P 28	-2.060	Q 28	-0.276		
P 29	-2.830	Q 29	-0.269	P* 15	-3.200
P 30	2.500	Q 30	3.000	P* 16	-3.294
P 31	5.728	Q 31	2.579	P* 21	-2.740
P 32	6.500	Q 32	2.685	P* 24	-3.086
P 33	6.320	Q 33	1.491	P* 39	-1.040
P 34	5.080	Q 34	2.028	Q* 15	-1.530
P 35	6.500	Q 35	3.710	Q* 16	-0.323
P 36	5.600	Q 36	3.016	Q* 21	-1.150
P 37	5.400	Q 37	1.218	Q* 24	-0.922
P 38	8.300	Q 38	2.358	V* 39	1.030

TABLE II
VALUES OF BRANCH MEASUREMENTS.

extreme buses	P (pu)	Q (pu)	extreme buses	P (pu)	Q (pu)
39 9	0.169	1.910	5 8	3.164	0.405
39 1	-1.209	0.776	5 6	-4.866	-0.743
30 2	2.500	3.000	7 6	-4.247	-0.650
31 6	5.728	2.579	7 8	1.909	-0.190
32 10	6.500	2.685	12 11	-0.024	-0.438
33 19	6.320	1.491	12 13	-0.061	-0.422
34 20	5.080	2.028	22 21	6.076	2.707
35 22	6.500	3.710	17 16	-2.067	0.365
36 23	5.600	3.016	19 16	4.541	1.303
37 25	5.400	1.218	20 19	-1.746	0.470
38 29	8.300	2.358	25 2	2.396	-0.991
5 4	1.701	0.339	25 26	0.747	1.053

The uncertainty affecting those buses was given by trapezoidal numbers whose extreme values of 0.0 and 1.0 cuts are given by (0.9, 0.95, 1.05, 1.1) times the central values.

To assess the quality of the results obtained, we developed an algorithm based on the gradient optimization algorithm.

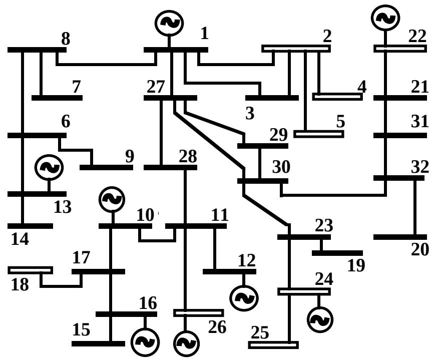

Fig 4 - Madeira Island reduced network.

One can build an accurate sketch of the membership function of a variable by minimizing and maximizing its value for selected α-levels. This objective function should be subject to equations (11) and to the inequality constraints related to the ranges of the fuzzy measures for each α-level considered.

In table III we present the average and maximum errors of the fuzzy results affecting the Removal, the Central Value and the Amplitude of voltages, phase angles, active and reactive flows and currents, relative to the results obtained with the gradient algorithm, assumed as "exact".

TABLE III
RELATIVE ERRORS FOR MEASURES AFFECTED WITH ±10% OF UNCERTAINTY

	Removal		Central Value		Amplitude	
	average	max.	average	max.	average	max.
V	0.01%	0.06%	0.00%	0.03%	0.08%	0.17%
θ	0.01%	0.03%	0.22%	2.46%	0.49%	2.05%
P	0.02%	0.28%	0.06%	0.63%	1.17%	8.70%
Q	0.06%	0.85%	0.23%	4.66%	0.12%	0.56%
I	0.30%	8.89%	0.15%	4.75%	1.14%	17.71%

B. Madeira Island Reduced Network

The Madeira Island Reduced Network sketched in fig. 4 was also used to test the described algorithm. It has characteristics more close to distribution networks and includes wind generation connected to buses 22 and 26. We have simulated some communication problems leading to the loss of on-line information from such buses plus from a region in the island. Buses with uncertain data are represented by white rectangles.. Some measurements, namely voltage and bus injected powers in buses 22 and 26, where specified as fuzzy numbers.

The uncertainty affecting those measurements was given by trapezoidal numbers of the form (0.8, 0.9, 1.1, 1.2) times the central values specified. In figure 5 we present the results obtained for the specified and calculated V_{22} and V_{26}, at both wind generation sites.

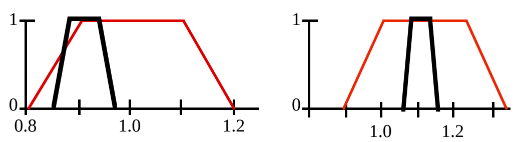

Fig 5 - Membership functions for specified and calculated voltages V_{22} (right) and V_{26} (left) - the larger distributions correspond to the declared uncertain data.

Table IV gives the average and maximum relative errors of the Removal, the Central Value and the Amplitude of the results, compared to "exact" values obtained by the same gradient approach as before.

TABLE IV
RELATIVE ERRORS FOR MEASURES AFFECTED BY ±20% OF UNCERTAINTY

	Removal		Central Value		Amplitude	
	average	max.	average	max.	average	max.
V	0.02%	0.03%	0.01%	0.01%	3.12%	3.65%
θ	0.02%	0.07%	0.24%	0.92%	1.86%	2.96%
P	0.01%	0.05%	0.07%	1.01%	1.03%	3.94%
Q	0.01%	0.10%	0.18%	3.33%	2.66%	7.72%
I	0.77%	8.22%	0.33%	3.32%	2.37%	14.11%

C. Analysis of the Results

The relative errors presented in tables III and IV for the Removal and the CVs indicate that the fuzzy state estimation model is giving accurate results, close enough to the "exact" ones given by the gradient technique. The largest relative errors for the Removal were obtained for currents. As stated in section IV.H, the current magnitudes can be corrected, namely when their possible value range is partially negative, due to linearization errors. In Fig. 6 we present the membership function for the current magnitude of branch 16-24 in the IEEE network, before and after the necessary corrective phase. The corrected curve is quite close to the "exact" curve, and they would not be distinguishable from each other in the figure.

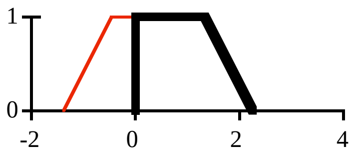

Fig. 6 - Current magnitude in line 16-24 of the IEEE 39-bus system, before - thin line - and after applying a corrective procedure (the thin line coincides with the thick line on the right side of the distributions).

One can also notice that, in some cases, the amplitude relative errors are the ones that assume the larger values. These error values must be interpreted considering that the Removal and Central Value take into account the relative position in the real axis of the membership function to be compared. On the contrary the Amplitude does not consider this aspect and so even small amplitude differences can produce a large relative amplitude error.

VI. CONCLUSIONS

In this paper we described a first approach to the state estimation problem integrating fuzzy numbers as measurements. It is important to refer that the membership functions of the state variables are very efficiently computed since the method is in general non iterative. In all the cases studied, the accuracy of the approximate results thus obtained was very good, as assessed by an independent method, impractical for operation purposes but giving "exact" results.

The examples presented illustrate that adding qualitative information to a state estimation procedure may improve the quality of the results, and also that the fuzzy state estimation procedure may contribute to "narrow" the uncertainty on some values in the system.

For a system lacking vital information (even losing observability), the fuzzy approach may constitute a valuable help to system operators, giving them margins of possibility for network values, based on accessible measurements and qualitative information on the past behavior of the system or on the type of consumption or generation at selected buses.

VII. ACKNOWLEDGEMENT

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VIII. REFERENCES

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