Impact of Load Uncertainties in Spot Prices

J. Tomé Saraiva
jsaraiva@inescn.pt
FEUP/DEEC - Faculdade de Engenharia da Univ. do Porto
INESC - Instituto de Engenharia de Sistemas e Computadores
Largo Mompilher, 22, 4050 – 392 Porto, PORTUGAL, Fax: +351.2.2084172

Abstract: In the scope of the move towards the implementantion of market mechanisms, the electricity transmission sector has a central role providing the physical interconnection between several agents. Since avoiding cross subsidies is a general objective and that legal or, at least contabilistic, umbundling is required new tariffs schemes are necessary in order to remunerate the transmission service. Several tariffs in use include marginal based terms that, however, may be very dependent on load levels or load uncertainties. In this paper we present a fuzzy set based approach to deal with load uncertainties in order to compute local marginal prices. This approach is based on a DC Fuzzy Optimal Power Flow algorithm integrating an estimate of active losses. This algorithm is finally integrated in a Monte Carlo simulation tool in order to obtain estimates of average nodal prices considering component outages.

Keywords: Electricity markets, transmission tariffs, marginal prices, volatility.

I. INTRODUCTION

The development of market mechanisms in power systems is responsible for important changes in the way the electricity industry is organized. Market forces, new legislation, conservation and environmental concerns, new technological advances enabling new organizational solutions are usually referred as the main driving forces to this move to the market. In any case, the implementation of market mechanisms is already well established in several countries and is spreading to several others in America, Europe and Australia. In this move several actions were taken in order to create competition in the generation side by designing new mechanisms to provide interaction between generation companies and distributors, retailing companies or eligible consumers either by centralized pool markets or by admitting a direct bilateral relationship. In several approaches these two frameworks coexist together with several types of financial instruments turning it more necessary the coordination of transmission users in order to guarantee system security.

In this move to the market the traditional vertical utilities typically went through a reorganization process in order to turn, at least from a contabilistic point of view, the generation, transmission and distribution sectors independent. This separation or umbundling can be legally required leading to new and more numerous agents or can only be required from a contability point of view, as it is accepted in the EU Directive 96/92 on the common electricity market. In any case this separation aims at identifying and assigning costs to each activity in order to avoid cross subsidies and to implement to each sector the most adequate regulatory policy so that the companies, either on the transmission, or in the distribution areas can be adequately remunerated.

The regulatory frameworks [1] can vary from typical cost of service policies, to price cap methodologies or to several types of incentive based regulations. On the transmission sector, regulatory bodies traditionally adopt cost of service policies considering that transmission companies are usually operating with high standard efficiency levels and that the contribution of the cost of transmission to the overall cost structure is low as well as the share of transmission investments in total investments in the electricity sector. Therefore, once operation, administrative and investment costs, or others, are approved for a given regulatory period it is necessary to design tariff schemes in order to recover those costs.

Regarding transmission tariffs, the solutions adopted by several countries are quite different. In general, they should be transparent, easy to apply, understandable, accountable and provide a non discriminatory treatment to all agents. The methodologies used to design these tariffs may be based on embedded methods [2] as the Postage Stamp, MW-Mile, Use or Modulus or Zero-Counter Flow Methods. These methods are, in general, criticized since they fail at providing good economic signals to agents in order to induce more efficient uses of the transmission network. Some of them correspond to assign a value to each user proportional to "the use of the network" without taking into account the actual operation conditions of the network.

In order to overcome some of these deficiencies some transmission tariff schemes include marginal terms, for instance to assign the cost of losses or to try to deal with congestion costs. These terms are based on the calculation of nodal marginal terms frequently for some periods of the day or, in some cases, as average values obtained for several load scenarios and some specified outage conditions. In purely theoretical terms, local marginal prices vary from instant to instant since they are dependent on the load level, on the generation dispatch and on the available components. Therefore, the short term marginal price at instant t on node k, also known as spot price at node k, can be defined as the variation of the global cost function considering that, at instant t, the demand at node k is increased of 1 unit [3,4]. The integration of marginal terms in tariff schemes is very appealing given that they

Paper BPT99–355–21 accepted for presentation at the IEEE Power Tech '99 Conference, Budapest, Hungary, Aug 29 Sept 2, 1999.

ultimately lead to a more efficient use of the system considering the economic signals they inherently transmit [5,6]. However, this procedure has some drawbacks:

- firstly, spot prices are typically related to operation costs and so they do not integrate any information related to the required investments on the network. This means that, if tariffs were purely marginalist, the network company would not be able to recover all its costs. This is why, in several countries, marginal tariff terms are supplemented by terms based on embedded methods aiming at ensuring that the whole remuneration is obtained;
- secondly, spot prices are very volatile [7]. In fact they depend on the load level, the availability of components and on the dispatch policy.

In this paper we describe a methodology to deal with load changes and component outages in the scope of the computation of nodal marginal prices. To do this we model load uncertainties using fuzzy set concepts that are integrated in a Fuzzy Optimal Power Flow algorithm [8] aiming at building membership functions of nodal prices. This methodology can be incorporated in a Monte Carlo simulation process in order to obtain estimates of average values. Section II includes a brief presentation of some Fuzzy Set Concepts and section III describes the global simulation algorithm including the Fuzzy Optimal Power Flow, the active losses estimation and local marginal prices modules. Section IV includes results for some sampling states and conclusions are presented in Section V.

II. FUZZY SET BASIC CONCEPTS

A Fuzzy Set $\tilde{A}$ has a membership function $\mu_{\tilde{A}}(x)$ relating each element x to its compatibility degree in $\tilde{A}$ (1). In normalized fuzzy sets this degree ranges from 0.0 to 1.0 leading to a gradual transition between the extreme complete belonging of x to $\tilde{A}$ - degree 1.0 - and no belonging of x to $\tilde{A}$ - degree 0.0. An α-cut of $\tilde{A}$ is the hard set A_α defined for each $\alpha \in [0.0,1.0]$ according to (2).

$$\tilde{A} = \{(x_1, \mu_{\tilde{A}}(x_1)), x_1 \in X\} \quad (1)$$

$$A_\alpha = \{x_1 \in X : \mu_{\tilde{A}}(x_1) \geq \alpha\} \quad (2)$$

Fuzzy numbers are a special class of fuzzy sets. They are convex fuzzy sets defined on the real line such that their membership function is normalized and piecewise continuous. Trapezoidal fuzzy numbers, as the one in Figure 1, can be used to represent the uncertainty affecting the interval [a2,a3] while not discarding values in [a1,a2] and in [a3,a4]. The central value of a trapezoidal fuzzy number is the average value of its 1.0 cut. Regarding Figure 1, it corresponds to the mean of a_2 and a_3.

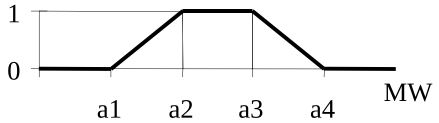

Figure 1 - Trapezoidal fuzzy number.

Some applications require a defuzification step to translate a fuzzy set into a real number. One available technique is the Centroid of a fuzzy set $\tilde{A}$ given by (3).

$$\text{Centroid}(\tilde{A}) = \frac{\sum x.\mu_{\tilde{A}}(x)}{\sum \mu_{\tilde{A}}(x)} \quad (3)$$

III. MONTE CARLO SIMULATION APPROACH

A. General description

The Monte Carlo simulation allows us to integrate two types of uncertainties: the ones related to load deviations - modelled by fuzzy set concepts - and the ones related to component outages - represented by probabilistic models. As results it provides estimates for average values of local marginal prices reflecting the specified uncertainties.

The algorithm uses a non-chronological sampling strategy by using the Forced Outage Rate of each component to declare it in operation or in failure in each state. The sampled states are then analysed by running a Fuzzy Optimal Power Flow to obtain the membership function of each node marginal price. The centroids of these functions are finally used to evaluate the convergence of the simulation by calculating for each node k the β coefficient given by (4). In this expression V represents the variance of the sample of the centroids of the marginal price of node k, ρk, N is the number of sampled states and E is the current estimate of the average value of the referred sample. The process converges if, for all nodes, the calculated β are smaller than a specified threshold.

$$\beta_k^2 = \frac{V(\rho_k^{centroid})}{N.(E(\rho_k^{centroid}))^2} \quad (4)$$

B. Basic DC Fuzzy Optimal Power Flow Algorithm

A Fuzzy Optimal Power Flow - FOPF - is an optimization study aiming at identifying the best generation strategy according to some criterium if at least one load is modelled by a fuzzy number. The FOPF approach adopts the DC model to represent the operation conditions of the network.

In the first step, the FOPF runs a deterministic DC-OPF (5 to 10) study for a crisp version of the fuzzy load vector obtained by substituting each fuzzy load by the corresponding central value, Pl_k^{ctr}. For this crisp load vector the following crisp DC-OPF can be formulated:

$$\min f = \sum c_k.Pg_k + G.\sum PNS_k \quad (5)$$

$$\text{subj.} \quad \sum Pg_k + \sum PNS_k = \sum Pl_k^{ctr} \quad (6)$$

$$Pg_k^{min} \leq Pg_k \leq Pg_k^{max} \quad (7)$$

$$PNS_k \leq Pl_k^{ctr} \quad (8)$$

$$P_b^{min} \leq \sum a_{bk}.(Pg_k + PNS_k - Pl_k^{ctr}) \leq P_b^{max} \quad (9)$$

This model corresponds to a traditional DC-OPF where one aims at minimizing the generation cost given that Pg_k is the generation in node k, c_k is the corresponding cost and PNS_k is the power not supplied in node k. Pg_k^{min}, Pg_k^{max}, P_b^{min} and P_b^{max} are the minimum and maximum generation and branch flow limits and a_{bk} is the DC sensibility coefficient of the flow in branch b regarding the injected power in node k.

After performing this DC-OPF study and having identified a feasible and optimal solution for it, the next step corresponds to deal with load uncertainties. They can be represented by fuzzy numbers regarding the respective central values (10). At a given uncertainty level, for instance, at the 0-cut the uncertainty specified for load k can be modeled by a parameter Δ_k in [Δ_{k1}, Δ_{k4}].

$$\tilde{Pl}_k = Pl_k^{ctr} + (\Delta_{k1}; \Delta_{k2}; \Delta_{k3}; \Delta_{k4}) \qquad (10)$$

This representation can be extended to all loads affected by uncertainties. The corresponding parameters can then be integrated in the previous optimization problem wherever loads appear in that model. This means that new terms will be added to constraints (6) and (9). This process ultimately leads to a multiparametric formulation given by (11) to (13) where b and b' are vectors integrating the right hand side terms of the constraints considering that some of them are independent and some others depend on Δ_k. Using the inverse of the base matrix of the crisp DC-OPF problem one can integrate terms depending on Δ_k on the initial feasible and crisp solution (14).

$$\min f = c^t.X \qquad (11)$$

$$\text{subj.} \quad A.X = b + b'(\Delta_k) \qquad (12)$$

$$\Delta_{k1} \le \Delta_k \le \Delta_{k4} \qquad (13)$$

$$X^{opt} = B^{-1}.(b + b'(\Delta)) \qquad (14)$$

The presence of these parameters can turn the crisp solution non feasible in some zones of the hypervolume defined by (13). This is true for combinations of values of the parameters leading to a negative value of one variable. If that is case, the algorithm proceeds by identifying vertices of that hypervolume according to some rules detailed in [8]. Once these vertices are identified, a set of parametric analysis are run each of them leading to partial membership functions of generations, branch flows or PNS. The final results are obtained applying the Fuzzy Union Operator on those partial results.

To better explain this parametric analysis let us consider a two fuzzy load system, $\tilde{Pl}_1$ and $\tilde{Pl}_2$. In Figure 2 we represent the 0.0 and 1.0 cuts of their conjoint membership function. Let us still consider that vertex Y has to be analysed. The FOPF algorithm departs from the optimal and feasible solution of problem (5 to 9) corresponding to point O and uses a parameter δ in order to vary the two loads so that one goes from O to X, in the first place, and from X to Y, in the second one.

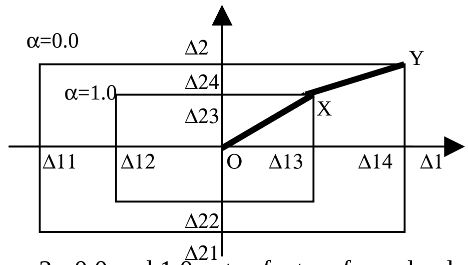

Figure 2 - 0.0 and 1.0 cuts of a two fuzzy load system.

In the first problem δ starts at 1.0 - in point O - and is reduced till 0.0 - in point X. Once X is reached, δ is reset to 1.0 and is reduced till 0.0 to get to point Y. When going from O to X the membership degree of the solutions is 1.0 - since the analysed load combinations are located in the 1.0 cut - while varies from 1.0 to 0.0 when going from X to Y. In this sense, the value of δ gives information about the membership value of the partial solutions. The generation, branch flows and PNS partial results obtained for each set of parametric studies are aggregated using the fuzzy union operator in order to build global membership functions [8].

C. Integration of estimates of active losses

Active losses are one of the causes leading to a geographic differentiation of nodal marginal prices. Therefore, if adopting a DC based algorithm it is crucial to integrate an estimate of these losses. Active losses in branch b from node i to node j are approximately calculated by (15) considering that voltage magnitudes are 1.0 pu. In this expression θ_i, θ_j and g_{ij} are the phase angles in nodes i and j and the conductance in branch ij.

$$Loss_{ij} \approx 2.g_{ij}.(1 - \cos\theta_{ij}) \qquad (15)$$

Apart from other more involving processes, one simple way of computing an estimate of branch losses consists of running a set of DC-OPF crisp studies in a sort of iterative scheme. At the end of each DC-OPF run, the phase angles and branch losses are computed and half of the losses in each branch is added to the load in each extreme bus. This change on loads leads to a change in generation and thus in phase angles. The process converges when, in two successive iterations, the differences between phase angles, in each node, are inferior than a specified level.

When running the DC-OPF parametric studies, loads are changed leading to changes in phase angles and in branch losses. In order to accommodate these changes, one can estimate, for each identified vertex, how active losses in each system branch are altered regarding the estimate already considered for point O. If Δ_{kp} represents for vertex p the deviation of load in node k, the changes in branch ij active losses are approximately given by (16). The derivative in this expression is given by (17) where Z_{mn} represents the element of line m/column n of the inverse of the DC model admittance matrix. Finally, half of $\Delta Loss_{ij}$ is added to the load deviation in bus i and the other half is added to the deviation in bus j.

$$\Delta Loss_{ij} = \sum_{nodek} \frac{\partial Loss_{ij}}{\partial Pl_k}.\Delta_{kp} \qquad (16)$$

$$\frac{\partial Loss_{ij}}{\partial Pl_k} = 2.g_{ij}.\sin\theta_{ij}.(-Z_{ik} + Z_{jk}) \qquad (17)$$

D. Evaluation of Local Marginal Prices

Local marginal prices can be obtained using the values of dual variables of the related optimization problem. For problem (5 to 9) expression (18) should be used to obtain a set of crisp nodal marginal prices related to the crisp load scenario integrating the central values of fuzzy loads. In this expression γ represents the Lagrange multiplier of constraint (6) while the second term measures how much the cost function varies due to a change in branch losses caused by an increased of 1 unit of the load in bus k.

$$\rho_k = \gamma + \gamma.\frac{\partial Loss}{\partial Pl_k} - \sum \eta.\frac{\partial P_{ij}}{\partial Pl_k} + \sigma_k \qquad (18)$$

$$\frac{\partial f}{\partial Pl_k}(losses) = \frac{\partial f}{\partial Loss}.\frac{\partial Loss}{\partial Pl_k} \qquad (19)$$

The second term can be obtained using (19) considering that, once losses are estimated, they are treated as loads. Therefore, the first derivative in (19) corresponds to γ. The second derivative in (19) is given by (20). In this expression, the derivative of the losses in a branch ij regarding the load in node k is given by (17).

$$\frac{\partial Loss}{\partial Pl_k} = \sum_{\text{all branches}} \frac{\partial Loss_{ij}}{\partial Pl_k} \qquad (20)$$

The third term in (18) measures the variation of the cost function if a branch flow constraint is active. In this case, the corresponding dual variable, η, is non zero. It should be noted that, differently from constraint (6), an increase of 1 unit in the load in node k is not directly reflected on a change on the right term of branch flow constraints. In fact, the load in node k is multiplied by the symmetric of the sensitivity of the flow in branch ij regarding the load in node k. In (18) the derivative of P_{ij} regarding P_{lk} corresponds to symmetric of this sensibility coefficient. Finally, the fourth term corresponds to the dual variables of constraint (8). These dual variables will only be non zero if the network is already very stressed so that a new load unit in node k directly increases PNS.

The values given by (18) for the crisp load scenario have a membership degree 1.0. When running the parametric DC-OPF studies nodal marginal prices are altered since load changes can turn the initial optimal basis infeasible. In this case, a dual simplex algorithm is used to recover feasibility. This process leads to changes in the basic variables and thus in the nodal marginal costs. Therefore, for each selected vertex the two parametric studies are run performing the dual simplex iterations if feasibility is lost. Whenever this occurs, expression (18) is used to compute nodal marginal prices. The membership degree assigned to a new set of nodal marginal prices is 1.0 if the dual analysis is performed within the first parametric study (to go from O to X, in the case of the example of Figure 2). If this dual analysis is performed for the second parametric study (to go from X to Y in the referred example), the membership degree coincides with the value of parameter δ. Once all vertices are analysed, the final nodal marginal price membership functions are obtained by aggregating partial results using the fuzzy union operator.

E. Results

The described algorithm gives for each sampled state the membership functions of generations, branch flows, PNS and nodal marginal prices. It should be noted that these results are essentially different since the first three ones correspond to continuous membership functions reflecting the continuous uncertain behavior specified for loads. On the contrary, the membership functions of nodal marginal prices integrate a set of discrete points associated to changes in the basis of the optimization problem required in order to recover feasibility. Regarding the Monte Carlo simulation, it outputs estimates of maginal values of the nodal marginal prices centroids given by (21).

$$E(\rho_k^{centroid}) = \frac{1}{N}.\sum_{n=1}^{N} \rho_{k,n}^{centroid} \qquad (21)$$

IV. CASE STUDY

A. System data

The previous methodology is illustrated with results from a case study using the IEEE 24 bus RTS system. Tables I, II and III include the characteristics of generators, branches and the central values of loads. The fuzzy load membership functions were obtained multiplying the central values by (0.9; 0.95; 1.05; 1.1). Bus 23 was selected for reference and 10000 \$/MWh was adopted for the PNS cost.

B. Crisp spot price computation

Nodal prices were evaluated considering that all components are in service, that line 2-6 is out of service and finally that an overlapping outage of generators 7/3, 13/1

Table I - Generator characteristics.

bus	gen no.	Pgmax MW	cost \$/MWh	bus	gen no.	Pgmax MW	cost \$/MWh
1	1	40.0	3.0	15	3	24.0	2.0
1	2	40.0	3.0	15	4	24.0	2.0
1	3	152.0	4.0	15	5	24.0	2.0
1	4	152.0	4.0	15	6	310.0	6.0
2	1	40.0	3.0	16	1	310.0	5.5
2	2	40.0	3.0	18	1	800.0	9.0
2	3	152.0	4.0	21	1	800.0	8.0
2	4	152.0	4.0	22	1	100.0	2.0
7	1	200.0	5.0	22	2	100.0	2.0
7	2	200.0	5.0	22	3	100.0	2.0
7	2	200.0	5.0	22	4	100.0	2.0
13	1	394.0	6.0	22	5	100.0	2.0
13	2	394.0	6.0	22	6	100.0	2.0
13	3	394.0	6.0	23	1	310.0	5.0
15	1	24.0	2.0	23	2	700.0	7.0
15	2	24.0	2.0	23	3	700.0	7.0

and 23/2 occurred. In each of these three states, evaluations integrating and not considering losses were performed. Table IV presents crisp nodal spot prices for these three states considering and not considering branch losses.

Table II - Branch characteristics.

extreme buses	r (pu)	x (pu)	Pmax (MW)	extreme buses	r (pu)	x (pu)	Pmax (MW)
1 2	.0026	.0139	175	12 13	.0061	.0476	500
1 3	.0546	.2112	175	12 23	.0124	.0966	500
1 5	.0218	.0845	175	13 23	.0111	.0865	500
2 4	.0328	.1267	175	14 16	.0050	.0389	500
2 6	.0497	.1920	175	15 16	.0022	.0073	500
3 9	.0308	.1190	175	15 21	.0063	.0490	500
3 24	.0023	.0839	400	15 21	.0063	.0490	500
4 9	.0268	.1037	175	15 24	.0067	.0519	500
5 10	.0228	.0883	175	16 17	.0033	.0259	500
6 10	.0139	.0605	175	16 19	.0030	.0231	500
7 8	.0159	.0614	175	17 18	.0018	.0144	500
8 9	.0427	.1651	175	17 22	.0135	.1053	500
8 10	.0427	.1651	175	18 21	.0033	.0259	500
9 11	.0023	.0839	400	18 21	.0033	.0259	500
9 12	.0023	.0839	400	19 20	.0051	.0396	500
10 11	.0023	.0839	400	19 20	.0051	.0396	500
10 12	.0023	.0839	400	20 23	.0028	.0216	500
11 13	.0061	.0476	500	20 23	.0028	.0216	500
11 14	.0054	.0418	500	21 22	.0087	.0678	500

Table III - Load central values.

bus	load (MW)	bus	load (MW)
1	194.4	13	477.0
2	174.6	14	349.2
3	324.0	15	570.6
4	133.8	16	180.0
5	127.8	17	0.0
6	244.8	18	599.4
7	225.0	19	325.8
8	307.8	20	230.4
9	315.0	21	0.0
10	351.0	22	0.0
11	0.0	23	0.0
12	0.0	24	0.0

Table IV - Crisp values of nodal marginal prices.

bus	no outages		line 2-6 outage		gen. outage	
	no loss	loss	no loss	loss	no loss	loss
1	8.12	8.18	8.11	8.13	9.00	9.05
2	8.12	8.18	8.11	8.12	9.00	9.05
3	8.08	8.25	8.08	8.24	9.00	9.13
4	8.12	8.27	8.12	8.25	9.00	9.17
5	8.12	8.26	8.12	8.24	9.00	9.16
6	8.13	8.33	10000.00	10000.00	9.00	9.23
7	5.00	5.14	5.00	5.13	9.00	9.16
8	8.13	8.34	8.13	8.34	9.00	9.26
9	8.12	8.25	8.12	8.25	9.00	9.15
10	8.13	8.27	8.13	8.27	9.00	9.17
11	9.13	9.26	9.13	9.26	9.00	9.15
12	7.15	7.26	7.15	7.25	9.00	9.13
13	6.00	6.05	6.00	6.04	9.00	9.08
14	8.54	8.69	8.54	8.69	9.00	9.12
15	8.00	8.12	8.00	8.13	9.00	9.03
16	8.00	8.10	8.00	8.11	9.00	9.04
17	8.00	8.10	8.00	8.11	9.00	9.01
18	8.00	8.12	8.00	8.12	9.00	9.00
19	7.66	7.74	7.66	7.74	9.00	9.05
20	7.37	7.40	7.37	7.40	9.00	9.02
21	8.00	8.10	8.00	8.11	9.00	8.97
22	8.00	8.01	8.00	8.02	9.00	8.87
23	7.21	7.21	7.21	7.21	9.00	9.00
24	8.03	8.19	8.03	8.19	9.00	9.11

C. Fuzzy local marginal price evaluation

The FOPF was run in order to obtain the membership functions of local marginal prices for the three system states already referred. This analysis was performed estimating and not including estimates of branch losses. Remember that branch losses are, together with congestion terms, the responsible for the geographic differentiation of marginal prices. The membership functions of some nodal marginal prices are indicated below. In bold, one indicates the crisp value as it would be obtained from the crisp study, that is not considering load uncertainties.

<u>All components available - no losses</u>

$\tilde{\rho}_1 = \{(7.00;1.0),(7.75;1.0),(\mathbf{8.12};1.0)\}$

$\tilde{\rho}_3 = \{(7.00;1.0),(7.72;1.0),(\mathbf{8.08};1.0)\}$

$\tilde{\rho}_6 = \{(7.00;1.0),(7.76;1.0),(\mathbf{8.13};1.0)\}$

$\tilde{\rho}_7 = \{(\mathbf{5.00};1.0)\}$

$\tilde{\rho}_{11} = \{(7.00;1.0),(8.00;0.46),(8.59;1.0),(\mathbf{9.13};1.0)\}$

$\tilde{\rho}_{17} = \{(7.00;1.0),(7.65;1.0),(7.99;1.0),(\mathbf{8.00};1.00)\}$

$\tilde{\rho}_{20} = \{(7.00;1.0),(7.13;1.0),(\mathbf{7.37};1.0),(8.00;0.46)\}$

<u>All components available - with loss computation</u>

$\tilde{\rho}_1 = \{(7.04;1.0),(7.80;1.0),(8.09;0.99);(\mathbf{8.18};1.0),(8.19;1.0)\}$

$\tilde{\rho}_3 = \{(7.16;1.0),(7.89;1.0);(\mathbf{8.25};1.0)\}$

$\tilde{\rho}_6 = \{(7.17;1.0),(7.94;1.0),(8.23;0.99);(8.32;1.0),(\mathbf{8.34};1.0)\}$

$\tilde{\rho}_7 = \{(5.11;1.0),(5.12;1.0),(5.13;1.0),(\mathbf{5.15};1.0),(5.17;0.99)\}$

$\tilde{\rho}_{11} = \{(7.11;1.0),(8.15;0.99),(8.71;1.0),(\mathbf{9.26};1.0)\}$

$\tilde{\rho}_{17} = \{(7.11;1.0),(7.77;1.0),(\mathbf{8.10};1.0)\}$

$\tilde{\rho}_{20} = \{(7.03;1.0),(7.17;1.0),(\mathbf{7.40};1.0),(8.04;0.99)\}$

<u>Line 2-6 outage - no losses</u>

$\tilde{\rho}_1 = \{(4.00;1.0),(7.00;1.0),(7.75;1.0),(\mathbf{8.11};1.0)\}$

$\tilde{\rho}_3 = \{(6.08;1.0),(7.00;1.0),(7.72;1.0),(\mathbf{8.08};1.0)\}$

$\tilde{\rho}_6 = \{(\mathbf{10000.0};1.0)\}$

$\tilde{\rho}_7 = \{(\mathbf{5.00};1.0)\}$

$\tilde{\rho}_{11} = \{(7.00;1.0),(7.10;1.0),(8.00;0.70),(8.59;1.0),(\mathbf{9.13};1.0)\}$

$\tilde{\rho}_{17} = \{(6.84;1.0),(7.00;1.0),(7.65;1.0),(7.99;1.0),(\mathbf{8.00};1.0)\}$

$\tilde{\rho}_{20} = \{(6.97;1.0),(7.00;1.0),(7.13;1.0),(\mathbf{7.37};1.0),(8.00;0.70)\}$

<u>Line 2-6 outage - with loss computation</u>

$\tilde{\rho}_1 = \{(4.00;1.0),(7.00;1.0),(7.75;1.0),(\mathbf{8.13};1.0)\}$

$\tilde{\rho}_3 = \{(6.23;1.0),(7.16;1.0),(7.88;1.0),(\mathbf{8.24};1.0)\}$

$\tilde{\rho}_6 = \{(\mathbf{10000.0};1.0)\}$

$\tilde{\rho}_7 = \{(5.11;1.0),(\mathbf{5.12};1.0),(5.16;1.0)\}$

$\tilde{\rho}_{11} = \{(7.11;1.0),(7.21;1.0),(8.14;0.70),(8.71;1.0),(\mathbf{9.26};1.0)\}$

$\tilde{\rho}_{17} = \{(6.95;1.0),(7.11;1.0),(7.77;1.0),(8.10;1.0),(\mathbf{8.11};1.0)\}$

$\tilde{\rho}_{20} = \{(7.01;1.0),(7.03;1.0),(7.17;1.0),(\mathbf{7.40};1.0),(8.04;1.0)\}$

<u>Generators overlapping outage - no losses</u>

$\tilde{\rho}_1 = \{(8.00;1.0),(\mathbf{9.00};1.0),(110.26;1.0),(4917.60;1.0)\}$

$\tilde{\rho}_3 = \{(8.00;1.0),(\mathbf{9.00};1.0),(77.70;1.0),(3374.49;1.0)\}$

$\tilde{\rho}_6 = \{(8.00;1.0),(\mathbf{9.00};1.0),(121.02;1.0),(5427.49;1.0)\}$

$\tilde{\rho}_7 = \{(5.00;1.0),(\mathbf{9.00};1.0),(120.40;1.0),(5397.91;1.0)\}$

$\tilde{\rho}_{11} = \{(8.00;1.0),(\mathbf{9.00};1.0),(153.03;1.0),(6936.75;1.0)\}$

$\tilde{\rho}_{17} = \{(-82.92;1.0),(7.86;1.0),(8.00;1.0),(\mathbf{9.00};1.0)\}$

$\tilde{\rho}_{20} = \{(8.00;1.0),(\mathbf{9.00};1.0),(51.58;1.0);(2169.51;1.0)\}$

Generators overlapping outage - with loss computation

$\tilde{\rho}_1 = \{(8.04;1.0),(\mathbf{9.05};1.0),(110.69;1.0);(4938.99;1.0)\}$

$\tilde{\rho}_3 = \{(8.12;1.0),(\mathbf{9.13};1.0),(78.62;1.0);(3412.26;1.0)\}$

$\tilde{\rho}_6 = \{(8.20;1.0),(\mathbf{9.23};1.0),(122.71;1.0);(5502.66;1.0)\}$

$\tilde{\rho}_7 = \{(5.14;1.0),(\mathbf{9.16};1.0),(121.62;1.0);(5456.52;1.0)\}$

$\tilde{\rho}_{11} = \{(8.12;1.0),(\mathbf{9.15};1.0),(154.07;1.0);(6982.01;1.0)\}$

$\tilde{\rho}_{17} = \{(-91.92;1.0),(7.83;1.0),(8.02;1.0),(\mathbf{9.01};1.0)\}$

$\tilde{\rho}_{20} = \{(8.02;1.0),(\mathbf{9.02};1.0),(51.75;1.0);(2176.65;1.0)\}$

It should also be stressed that these membership functions do not correspond to continuous ones, but rather to a set of discrete points each one associated to a membership degree. These points correspond to the crisp value obtained for the crisp load vector and some others related to the changes in the basis of the optimization problem reflecting the iterations of the Dual Simplex algorithm required to recover the feasibility of the solution.

Regarding these values it is important to note that marginal prices are strongly dependent on the load level since, for each of the six sets of results, they are, in some cases drastically altered only as a result of load variations. Apart from that, marginal prices are also very dependent on the reliability of system components. In fact, if some components are in outage, the system operates in a more stressed condition leading to heavy transmission congestion and, ultimately, to Power Not Supplied. This volatility is also present if one computes the centroids of those membership functions, as presented in Table V.

Finally, the membership function obtained for bus 17 for the overlapping outage of the three generators also deserves an explanation. In the first place, it should be noted that the membership function integrates four ordered pairs. Three of them, correspond to marginal prices ranging from 7.00 to 9.00 and they reflect load variations in an operation condition in which PNS is still zero. The fourth pair corresponds to the maximum bus loads that, together with the outages, lead to non zero PNS. In this case, the marginal price is negative. This means that in this case an increase of 1 MW in the load of bus 17 leads to a decrease in the total generation cost. In fact, the load increase alleviates some branch flow constraints making room to increase generations while decreasing power not supplied.

Table V - Centroids of the membership functions.

bus	no outages	line 2-6 outage	gen. outage
1	7.86	6.72	1266.69
2	7.86	6.75	1279.24
3	7.77	7.37	877.03
4	7.95	7.21	1324.28
5	7.94	8.21	1356.41
6	7.99	10000.00	1410.70
7	5.14	5.13	1398.11
8	8.01	7.72	1405.67
9	7.93	7.49	1351.01
10	7.94	7.76	1442.15
11	8.31	8.09	1788.34
12	7.47	7.47	1246.50
13	7.07	7.06	1388.86
14	8.06	7.87	2568.29
15	7.67	7.61	76.95
16	7.76	7.61	9.78
17	7.66	7.60	-31.36
18	7.67	7.49	3.93
19	7.58	7.47	308.05
20	7.40	7.33	561.36
21	7.66	7.48	25.03
22	7.57	7.39	-5.13
23	7.40	7.40	698.62
24	7.73	7.46	385.55

V. CONCLUSIONS

In this paper we presented an integrated algorithm to deal with uncertainties in loads and in system component availability in the computation of local marginal prices. The algorithm addresses the availability of system components using probabilistic concepts while it integrates fuzzy models to represent load uncertainties. This way, it does not perform any kind of load sampling, but rather it treats loads in an holistic way. As results, for each sampled state the algorithm outputs, apart from other information, the membership functions of local marginal prices and the corresponding centroids. The sampling process organized in terms of a Monte Carlo simulation allows us to obtain estimates of average values of those centroids in each bus. These values can be used to build more robust transmission tariff schemes since they take in account the main reasons explaining the volatility of those local marginal prices.

VI. ACKNOWLEDGMENT

This work was partially financed by contract PRAXIS 2/2.1/TIT/1634/95.

VII. REFERENCES

[1] D. Shirmohammadi, X. V. Filho, B. Gorenstin, M. V. Pereira, "Some Fundamental Technical Concepts About Cost Based Transmission Pricing", IEEE Trans. on Power Systems, vol. 11, no. 2, May 1996.

[2] Maragon Lima, J. W., "Allocation of Transmission Fixed Charges: An Economic Interpretation", Proceedings of Stokcholm Power Tech' 95, Stokcholm, 1995.

[3] Caramanis, M. C., Bohn, R. E., Schweppe, F. C., "Optimal Spot Pricing: Practice and Theory", IEEE Transaction on PAS, Vol. 101, nº 9, September 1982.

[4] Schweppe, F. C., Caramanis, M. C., Tabors, R. D., Bohn, R. E., Spot Pricing of Electricity, Kluwer Academic Publishers, 1988.

[5] M. Rivier, I. Perez Arriaga, G. Luengo, "JUANAC: A model for Computation of Spot Prices in Interconnected Power System", Proceedings of PSCC' 90, Graz, August 1990.

[6] M. Rivier, I. Perez-Arriaga, "Computation and Decomposition of Spot Prices for Transmission Pricing", Proceedings of PSCC' 93, Avignon, 1993.

[7] J. Finney, H. Othman, W. Rutz, "Evaluating Transmission Congestion Constraints in System Planning", IEEE Trans. on Power Systems, vol. 12, no. 3, August 1997.

[8] J.T. Saraiva, V. Miranda, L.M.V.G. Pinto, "Impact on Some Planning Decisions From a Fuzzy Modelling of Power Systems", IEEE Trans. on Power Systems, vol. 9, no. 2, May 1994.

VIII. BIOGRAPHIES

João Tomé Saraiva was born in Porto, Portugal, on August 18, 1962. He received his licenciate, M.S. equivalent and Ph.D. degrees from the Fac. of Eng. of Porto University (FEUP) in 1985, 1988 and 1993, respectively, all in Electrical Engineering. In 1985 he joined FEUP where he is Auxiliar Professor. In 1989 he joined also INESC as a researcher.