OPTIMAL EMPLACEMENT OF SWITCHING DEVICES IN RADIAL DISTRIBUTION NETWORKS

Vladimiro Miranda, Almeida do Vale

Dep. Eng. Electrotécnica - Faculdade de Engenharia Universidade do Porto

Alexandre Cerveira

Departamento de Informática Universidade Nova de Lisboa

PORTUGAL

1. SUMMARY

Planning a radial distribution network requires, among several options, decisions to be taken about to or not to include switching devices in the network branches.

The limitation of failure consequences and the decrease in consumer supply unavailability, in areas not directly involved in equipment failures, count for their inclusion. Nevertheless one must keep in mind that these devices cost money and that they themselves are liable to fail, increasing the total number of failure events in the distribution system. Besides, other economic factors must be considered, such as maintenance, etc.

One can hardly tell which is the optimal decision, because only the very simple cases can easily be evaluated. Large networks involve very complex calculations, as the benefits of one switching device depend not only of its position in the system but also of the existence and location of other switching devices.

The general problem can be stated as follows: given a radial network, define the optimal system configuration (location and type of switching devices) by minimizing an objective function which includes the cost associated with the switching devices and the economic consequences of the power disconnected and energy not supplied due to failures in the distribution system.

A switching device is considered in this paper to be either a switch breaker or an isolator (with local or remote control). All this equipment shares the property of limiting the supply interruptions to a sub-tree of the whole radial system, allowing consumers in not directly affected areas to have their energy supply recovered.

In the following sections a new algorithm and an efficient computer program are presented. Reliability concepts are used along with dynamic programming techniques in order to establish an objective function and obtain the desired optimized network configuration.

2. RELIABILITY COST

Admit that the economic value associated to a supply interruption is given by a polynomial expression

$$Y = P\sum_{j=0}^{n} a_j T^j \tag{1}$$

where

P - mean power disconnected during the interruption

T - time elapsed during the interruption

a_j - real constant

Y - interruption cost

If one considers failures to happen at random times and with random durations, we get

$$E\{Y\} = E\{P\}.E\left\{\sum_{j=0}^{n} a_j T^j\right\} \tag{2}$$

where E{} stands for the mathematical expectation. Equation (2) becomes

$$E\{Y\} = E\{P\}.\sum_{j=0}^{n} a_j E\{T^j\} \tag{3}$$

from where we can conclude that the mean value of the cost associated with failures depends on the moments of the interruption duration distribution function.

The expectation E{P} is readily identified with the reliability index L (mean load disconnected) mentioned in recent literature [1]. When a consumer follows a given load curve, L equals the mean value of this curve.

Considering only the first two terms of (1) we get

$$E\{Y\} = \left(a_0 + a_1\mu^{-1}\right)L \tag{4}$$

having μ^{-1} as the mean value of the interruption time distribution function, either this be normal or exponential. In the latter case, μ is just the repair rate.

Taking "f" as the failure frequency, we can define the "mean annual cost" D_0 as

$$D_0 = f.E\{Y\} \tag{5}$$

Remembering that $f\mu^{-1}$ = U (unavailability of supply, h/year) we get

$$D_0 = a_0 fL + a_1 UL \tag{6}$$

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The capitalized value of D_0 may be useful when one considers the possibility of decreasing interruption costs by investing in the supply system. Taking D, g and h as the capitalized values of D_0, a_0 and a_1, we have

$$D = gfL + hUL \tag{7}$$

The second term of (7) is related with the annual energy not supplied $E = UL$.

This simple and convenient model seems to fit into results of research works and field studies in several countries, allowing us to adopt the linear formulation above.

The published values strongly suggest, in accordance to engineering feeling, that several consumer classes should be considered, with their own values for the kW disconnected and the kWh not supplied [2].

Due to the characteristics of the electric energy system components, one may take in most cases failure rate "λ" instead of failure frequency "f", as they are numerically close.

3. FAILURES IN A RADIAL NETWORK

A radial network is composed of a set of branches under the form of a tree. For each branch j there is a value L_j corresponding to its mean power flow in its upper section.

In a simple configuration such as in Figure 1, and ignoring the existence of switching devices, a failure in any branch will trip the protection devices at the substation (disconnecting L_1); this is equivalent to consider all failures of the "active" type.

Second order events (such as failures occurring during interruptions of supply caused by other failures) will not be considered in the next sections. The switching devices will be supposed to be always placed in any branch next to the node that is closer to the substation.

Every branch that has no other branches depending on it will be called "terminal branch".

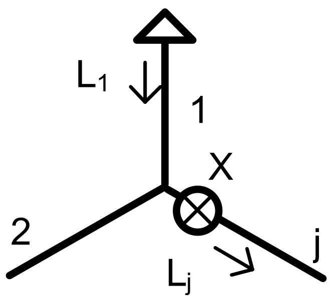

Figure 1 - Simple radial structure

3.1. Switching devices in terminal branches

Consider the network in Figure 1 and the possibility of including a device at X.

Without this equipment, failures in any branch will cause power L_1 to be disconnected. The associated capitalized cost is given by

$$D = (gf_1 + hU_1)L_1 + (gf_2 + hU_2)L_1 + (gf_j + hU_j)L_1 \tag{8}$$

When there is a breaker at X, this cost becomes

$$\begin{aligned} D = {} & (gf_1 + hU_1)L_1 + (gf_2 + hU_2)L_1 + (gf_X + hU_X)L_1 + \\ & + (gf_j + hU_j)L_j + V \end{aligned} \tag{9}$$

where f_X and U_X stand for the reliability indices associated with device X, and V is its cost.

If instead of a breaker, we have an isolator, the time S needed to operate the equipment must be considered:

$$\begin{aligned} D = {} & (gf_1 + hU_1)L_1 + (gf_2 + hU_2)L_1 + (gf_X + hU_X)L_1 + \\ & + (gf_j + hf_jS)L_1 + hU_jL_j + V \end{aligned} \tag{10}$$

It is possible to consider an equivalent branch "*", that would lead to the same cost value. Taking, for instance, equation (9), we get

$$D = (gf_1 + hU_1)L_1 + (gf_2 + hU_2)L_1 + (gf^* + hU^*)L_1 \tag{11}$$

where

$$f^* = f_X + f_j \frac{L_j}{L_1} \tag{12}$$

$$U^* = U_X + U_j \frac{L_j}{L_1} + \frac{V}{hL_1} \tag{13}$$

The branch with the switching device can therefore be replaced by a single terminal branch, with reliability indices f* and U*. This enables us to identify, in equations (8), (9) and (10) , the contribution of terminal branch j to the total cost, regardless of the existence of other branches on the system.

It is assumed to be clear that any sub-system with no switching devices is easily made equivalent to a single terminal branch having its indices calculated by just adding up the frequency rates and unavailabilities of its components.

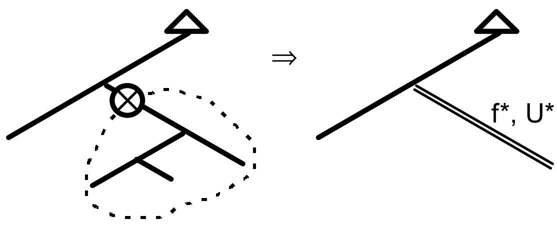

Figure 2 - Sub-system made equivalent to a single terminal branch

The described equivalencies (suggested in Figure 2) make it possible to condense a complex problem into a simple configuration which can be evaluated by an equation such as (8).

3.2. Inclusion of switching devices subject to the prior existence of other switching devices

Consider now the general case of having to decide whether to include or not a device in terminal branch j, knowing that in branch m, placed between j and the substation, there is another device. Of course, the existence of a switch-breaker at the substation is assumed.

We will define

$D_j(m)$ - contribution of terminal branch j to the total cost, assuming that there is a switching device in branch m upwards, and that the option is not to include a device in branch j.

$D'_j(m)$ - the same contribution if switching device is included in branch j.

Furthermore, the branches of the network will be numbered in such a way that a branch cannot be labeled with a number which is lower than the label of any other branch that lies in the path between the considered branch and the root of the tree (the substation).

The four following cases must be considered:

A - Breaker at m - breaker at j ($1 \le m < j$)

$$D_j(m) = (gf_j + hU_j)L_m \tag{14.a}$$

$$D'_j(m) = (gf_X + hU_X)L_m + (gf_j + hU_j)L_j + V \tag{14.b}$$

B - Breaker at m - isolator at j ($1 \le m < j$)

$$D_j(m) = (gf_j + hU_j)L_m \tag{15.a}$$

$$D'_j(m) = (gf_X + hU_X)L_m + (gf_j + hf_jS)L_m + hU_jL_j + V \tag{15.b}$$

C - Isolator at m - breaker at j ($1 < m < j$)

$$D_j(m) = (gf_j + hf_jS)L_1 + hU_jL_m \tag{16.a}$$

$$D'_j(m) = (gf_X + hf_XS)L_1 + hU_XL_m + (gf_j + hU_j)L_j + V \tag{16.b}$$

In this case we have 1<m because branch 1 (the root of the tree) is supposed to be equipped with a breaker.

D - Isolator at m - isolator at j ($1 < m < j$)

$$D_j(m) = (gf_j + hf_jS)L_1 + hU_jL_m \tag{16.a}$$

$$D'_j(m) = (gf_X + hf_XS)L_1 + hU_XL_m + (gf_j + hf_jS)L_j + hU_jL_j + V \tag{16.b}$$

(here is also 1<m for the reasons stated above)

In these four cases it is assumed that no other switching device exists between m and j. Furthermore, in cases C and D no automatic breaking device is considered between m and 1 (root). If such device exists, say in branch k, two new situations arise:

E - Breaker at k - isolator at m - breaker at j ($1 \le k < m < j$)

F - Breaker at k - isolator at m - isolator at j ($1 \le k < m < j$)

The equations for these latter cases are similar to (16) and (17), requiring only the replacement of index 1 by index k.

4. OPTIMIZATION ALGORITHM [3]

The analysis of all possible configurations becomes unpractical for real sized networks. If one considers the inclusion of only one type of device, one gets 2^n cases for each n-branch radial network. Thus a new algorithm based on the concepts of dynamic programming and on the tree structure of the electrical network was developed; it allows the selections of the optimal switching device location under the criteria stated above.

We will adopt the following notation:

d_j - son-branch of branch j (depending on branch j)

$\phi_j(m)$ - optimal cost, associated with the inclusion or not of a device in branch j, assuming that there is another device in branch m, upwards in the tree.

It is assumed that no other device exists between branches m and j.

4.1. Recursive equations for the inclusion of one type of device

In this case, we have the following recursive expression:

$$\phi_T(m) = \min\{D_T(m); D'_T(m)\} \quad \text{(T - Terminal branch)} \tag{18.a}$$

$$\phi_j(m) = \min\left\{ D_T(m) + \sum_{d_j} \phi_{d_j}(m); D'_T(m) + \sum_{d_j} \phi_{d_j}(j) \right\} \tag{18.b}$$

$1 \le m < j < T$

The recursive process ends when the root of the tree representing the radial network is reached. During the process, terminal branches will be agglutinated in order to form equivalent branches and simpler network configurations, until a network such as in Figure 1 is obtained. A final decision for the two equivalent terminal branches is then possible, based on equations (8), (9) and (10). Once the status of these branches having been fixed, it is easy to obtain the optimal solution for the whole network by sheer decomposition of the equivalent branches previously formed.

This recursive algorithm is quite straightforward when only one type of device is considered. Its characteristics allow the specification of the existence of other devices (automatic or otherwise) in any branch of the network, the calculations being performed according to cases A to F.

The efficiency of this algorithm may be evaluated by the number of sub-problems that need to be solved. In fact, the number of decisions to be taken is equal to the internal path length of the information tree associated to the radial network (sum of the lengths of all the paths connecting the root to all the nodes of the tree).

It is possible to set the upper and lower bounds to the internal path length of any tree with given number of nodes [4].

For instance, an 11 node tree will have path length between 55 and 17 (if it is a ternary tree) while the comprehensive analysis of all the possible cases would lead to the consideration of 1024 possible network configurations.

4.2. Simultaneous inclusion of isolators and switch-breakers

The attempt to consider simultaneously the optimal location of non-automatic and automatic switching devices leads to an increase in the number of sub-problems to be solved and decisions to be taken. In fact, one must evaluate for each branch a higher number of hypothesis taking in account not only the existence of other isolators but also the position of the nearest automatic device up in the tree. The number of sub-problems becomes equal to the sum of internal lengths of all the paths in the tree between the root and any node. Given a tree with 11 nodes, the number of decisions to be taken lies between 222 (for a degenerate tree) and 24 (for a ternary tree with levels full) while the number of possible configurations reaches 2673.

The recursive relationships that may be derived are similar to equations (18).

4.3. Computer program DINAMINA

A program named DINAMINA was developed in BASIC for a WANG PCS-II mini-computer, in which the radial network is represented as an ordered information tree. The visit to the nodes is carried out level by level from the leaves to the root, until the optimal solution at this node is obtained; the optimal network configuration is then derived by visiting the nodes again according to a pre-order traversal algorithm.

The natural sparsity of radial structures is fully explored by its tree representation and other sparsity techniques adopted. The flexibility of network representation is high and the main limitation is the maximum tree internal length or internal sum of path lengths allowed.

The first version of the two programs developed runs in a very small 16 k-byte CPU system; nevertheless, it supports the analysis of real sized networks such as those represented by trees up to 80 nodes and 360 branches. Other versions were developed and run presently in more powerful computers.

The use of mini-computers operating with BASIC must be emphasized, for it constitutes one of the main options in the research work developed. In fact, the profit of the development of new mathematical and algorithmic tools will be pushed to a maximum only f the access to them by the engineers can be made easy, ready and cheap. In many cases, this objective can be achieved in a very simple way by the use of mini-computers, discarding the need of large and heavy computer systems.

5. EXAMPLE

DINAMINA was used in a study of a 15 kV overhead radial distribution network of Electricidade de Portugal - EDP (northern area board, DODN). It spans itself over the districts of Albergaria-a-Velha and Sever do Vouga, as shown in Figure 3.

This network is connected to low-voltage substations located in agricultural and industrial complexes, as well as in small towns and villages for public service (domestic, commercial, traffic, lights, etc.). The values of annual peak demand and energy consumption for each substation are known. The network forms a tree with 77 branches, 40 of them being terminal.

Several analysis were performed, from the simple inclusion of one type of device to the complex mixed strategies, either ignoring the existent equipment or taking it into account and trying to improve the present configuration.

As base costs and reliability indices the values included in Table I were adopted.

Due to space restrictions it is not possible to present in detail the obtained solutions [3]; however, the following conclusions could be reached:

1. Without any switching devices, the reliability cost would rise up to 155 million escudos.

2. The optimal inclusion of 32 isolators would considerably reduce this value; taking into account the equipment cost, the optimal solution is associated with a value of 24.8 million escudos.

3. The existent network has precisely 32 isolators, nevertheless it can be improved by adding 6 more at the appropriate places; the solution cost would then come to 25.1 million escudos, which is not far from the above optimum.

4. The inclusion of breakers only leads to a solution which is not better than the optimum possible with the isolators, because the price of the 17 devices used has a considerable weight in the total cost.

5. The best solutions by far are obtained following mixed strategies, leading to a total cost of 16.9 million escudos with the help of 30 isolators and 3 breakers; this represents a reduction of at least 49% in comparison with solutions that use only one type of device.

TABLE I - Example data	λ (/year)	r (h)	Cost (escudos)
Overhead lines (/km)	0.025	212	
breakers 15 kV	0.005	75	300,000.
isolators	0.001	75	80,000.
kW disconnected (cap.)			24.74
kWh not supplied (cap.)			247.38

6. CONCLUSIONS

In this paper a new approach to the problem of the optimal location of switching devices in a radial distribution network is presented.

In the new algorithm developed, the decision criteria are based on the evaluation of an objective function corresponding to the sum of equipment cost and reliability cost, namely the capitalized values of the average power disconnected and energy not supplied. A computer program has been written in BASIC for a small mini-computer; nevertheless, it allows the detection of the optimal number, type and location of the devices that minimize the total economic cost mentioned above even for large networks.

In this paper, the attention was focused on isolators and switch breakers, but the developed algorithm shows a high degree of flexibility and may be adapted to perform other studies. As an example we shall mention the replacement of locally operated by remote controlled devices with switching times largely inferior to the previous ones; in this case, new equations must be taken in consideration but they are quite similar to the ones presented in Chapter 3 and may easily derived.

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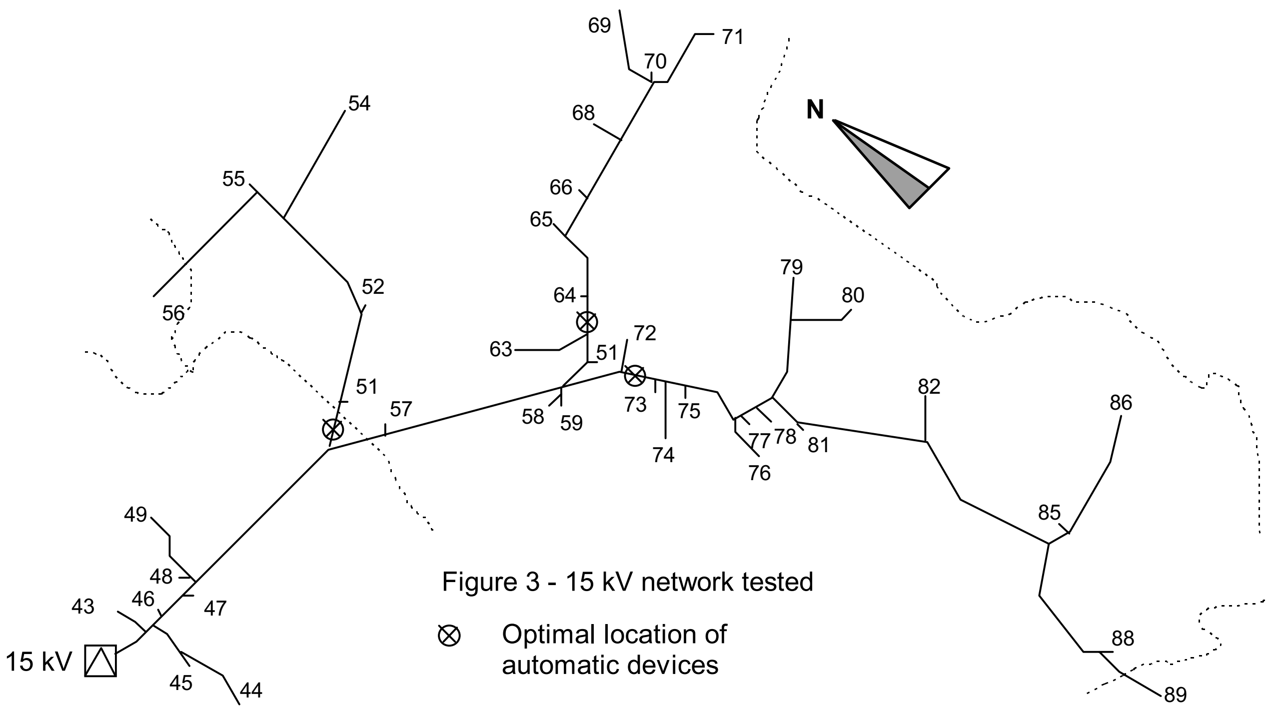

Figure 3 - 15 kV network tested

⊗ Optimal location of automatic devices