WHY *RISK ANALYSIS* OUTPERFORMS *PROBABILISTIC CHOICE* AS THE EFFECTIVE DECISION SUPPORT PARADIGM FOR POWER SYSTEM PLANNING

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Abstract - This paper demonstrates that a classical stochastic optimization is, in many cases, not convenient for power system planning. Instead, a risk analysis approach is proposed. In a comparison of both planning paradigms, the probabilistic approach is in occasions not adequate, is half blind to compromise solutions and leads, in numerous, cases to riskier decisions. The technical discussion is illustrated with a distribution planning example.

Keywords: Power System Planning, Risk Analysis, Probabilistic Choice, Decision Making.

1. INTRODUCTION

The general expansion planning problem in Power Systems has been developed traditionally under the Probabilistic Choice (PC) paradigm, which may be described as following: admit that one has defined a "cost function" that measures the goodness of a solution (*latu sensu*: we refer as "cost" any criterion that may be considered and adequately transformed into an objective of minimization); given a set of futures, each one with a probability assigned, the optimal solution should be chosen among those that minimize the expected cost over the set of futures considered.

In recent years, however, the generalized adoption of this planning paradigm has been growingly challenged. As an alternative, a Risk Analysis approach has been suggested. The case for this option has been very clearly defended in [1], and it has been chosen, in greater or lesser degree, in several papers published since then, as for example [2,3]. Also in some recent books [4] this approach has been defended.

The Risk Analysis (RA) paradigm indicates a preferred solution as one that minimizes the regret felt by a Decision Maker (DM) *after* verifying that the decisions he had made were not optimal, given the future that in fact has occurred.

Why has been the RA approach so attractive (although not many models have included it explicitly yet)? One of the reasons may well be that it reflects with much more accuracy the way people think.

In fact, the traditional models in planning, namely those working under the PC paradigm, concentrate their analysis on the *solutions* of the problem, while the RA paradigm is mainly about *decisions*. Factors as risk aversion or risk attraction associated with DMs, the concept of *hedging* (paying an extra to avoid adverse futures) and the measurement of regrets felt, all are reflected in the RA way of dealing with a problem, and are very well understood by those planners that have a daily contact with real and practical problems.

However, models developed by academics and scientists have insisted one way or the other in a kind of stochastic optimization, where none of these concerns are correctly or completely addressed.

Recognizing that Power System planning is a matter of Decision Making, and not of Optimization, has been one major step into a new perspective on this activity, with deep practical consequences.

However, recognizing that the optimum on the average of futures may not be the best decision, from the point of view of Decision Making, has been surely a bit harder.

So far, this discussion, in the Power System area, has been limited to some philosophical arguments, from which sometimes one may suspect that an unbiased appreciation may be absent in some degree. People used to "stochastic thinking" will need some time to adapt to the meaning and use of other paradigms, while people that do not fully understand probabilistic models will tend to over-evaluate risk analysis as a new universal tool. Neither position seems to us as deserving to be sustained.

This paper is devoted to giving a clear and rigorous demonstration of why a PC paradigm may be the wrong choice, in Power System planning, and why the RA approach may be superior, from a decision making point of view, in a number of cases.

2. PC vs. RA

2.1. Applicability of the PC paradigm

To compare the PC and RA paradigms, we admit facing a planning problem for which several futures have been defined as possible. Furthermore, without discussing the concept, we admit that for each future k one may assign a "subjective" probability value w_k.

A future (or a scenario, these designations are often used as equivalent) may be a dynamic sequence of events, and not only a static image. The dynamic nature of future uncertainty may be illustrated by the concept of "tree of futures", such as depicted in Figure 1; a scenario will in this case be a path in the tree of futures, and not a "frozen" state at a certain time T. Even so, a path k can have assigned a probability w_k.

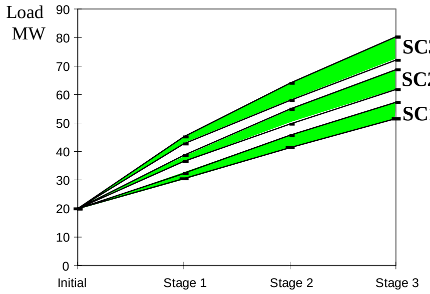

Figure 1 - Scenario tree with fuzzy loads.

If the cost incurred in scenario k by an alternative I is f_{ik}, then the PC paradigm is characterized by

$$\min_i \sum_k w_k f_{ik} \tag{1}$$

We can immediately see that the PC paradigm depends on a linear composition of the future costs. As it will be shown, this may constitute a serious drawback, and the reason for challenging this method in its use as a decision making tool.

This appreciation is made *a priori*, i.e., it does not depend on the appreciation of the consequences of the decisions, compared to what could have been decided if one knew the future in advance. This is why we say that the PC paradigm deals with solutions and not with decisions. However, under some assumptions, we can understand how this process may be related with decisions.

If the future k were known in advance with no uncertainty, one would be able, in theory, to calculate an optimal solution for the expansion of a Power System, having cost f_k^{opt} - this will be called the conditional optimum. A *Regret* felt for a Decision Maker, for having chosen a given alternative i and then seeing future k happening, can be defined as a function $\mathrm{Regret}_{ik} = R\left(f_{ik} - f_k^{opt}\right)$. Although nothing can legitimate taking R as a linear function, let's assume for a moment that $\mathrm{Regret}_{ik} = f_{ik} - f_k^{opt}$. If we define a stochastic optimization of the possible regrets felt, we have

$$\min_i \sum_k w_k\left(f_{ik} - f_k^{opt}\right) \Leftrightarrow$$
$$\min_i \sum_k (w_k f_{ik} - w_k f_k^{opt}) \Leftrightarrow$$
$$\min_i \sum_k w_k f_{ik} \tag{2}$$

because $w_k f_k^{opt}$ is a constant for every k. In this case, therefore, the stochastic optimization of the solution costs is equivalent to the stochastic minimization of the decision regrets.

This, however, is not true in the general case of having R as a non linear function of the deviation from the conditional optima. The non-linearity of the regrets may perhaps be neglected, if the real costs are relatively close to the conditional optima; but the perspective of unwanted or even catastrophic events surely gives to the function R a highly non-linear characteristic - which means that scenarios with a very low probability of occurrence may in any case have a decisive influence in the final decision, namely when they lead to the acceptance of higher costs to avoid their consequences (this attitude is called *hedging*).

The concept of *unwanted* events is very important, because it contradicts the basic assumption behind the Probabilistic Choice paradigm: that bad situations will be compensated by good situations along time, so that one can evaluate a solution by its average behavior. However, if a catastrophic event or scenario occurs, no reasonable recovery will ever be possible and the PC assumption cannot be verified - the PC paradigm is not an useful context for decision making in planning, in this case.

And when can we use linear Regrets? Basically, in any problem context where the cost of any solution f_{ik} is not very different from an ideal cost f_k^{opt}, and where no unwanted events are likely to occur. Figure 2 displays an example of a Regret function with linear characteristic near the zone of no regret and increased risk aversion for extreme scenarios

$$\mathrm{Regret} = \begin{cases} f - f^{opt} \Leftarrow (f - f^{opt}) < 1 \\ \left(f - f^{opt}\right)^2 \Leftarrow (f - f^{opt}) \geq 1 \end{cases} \tag{3}$$

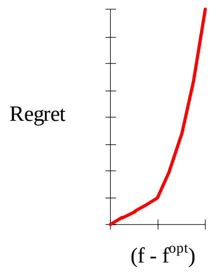

Figure 2 - Regret function with aversion to extreme cases

2.2. Applicability of the RA paradigm

The RA paradigm may be characterized by

$$\min_i \left\{ \max_k \left(w_k \mathrm{Regret}_{ik}\right) \right\} \tag{4}$$

By minimizing the maximum regret, one really tries to avoid selecting a solution that may have a bad performance in any future considered. This is not equivalent to the PC paradigm, not even in its "updated version" where the solution costs are replaced by the associated regrets, such as in

$$\min_i \sum_k \left(w_k \mathrm{Regret}_{ik}\right) \tag{5}$$

The application of the RA paradigm is associated with the concept of *robust* solution, meaning a solution that will be acceptable in all futures considered. *Exposure* relates a solution with adverse scenarios, and the concept of *hedging* results therefore naturally as a mean of reducing exposure or increasing robustness.

The RA paradigm is applicable whenever one cannot assume a probabilistic compensation of bad and good results, because there will not be enough frequent repetition of events or situations (validating a PC paradigm through the law of large numbers); this may result either from a too low frequency of the event cycle (compared with the planning time horizon) or from the occurrence of unwanted or catastrophic situations that constitute a disruption in the stochastic process of repetition of "experiences" and compensation of consequences.

3. THE PC PARADIGM IS HALF BLIND

We have so far justified why the PC and the RA paradigms may lead to different planning decisions, and why *the PC paradigm may result unjustified due to the non-linearity of the Regret concept and the possibility of occurrence of unwanted or even catastrophic scenarios*.

We will now explain why the PC paradigm may also be inconvenient for indicating a good compromise solution in a planning environment including uncertainty.

Admit that one has a set of m planning alternatives. Admit also that the planning uncertainty is represented by a set of n futures or scenarios, each one having assigned a subjective probability. Admit that we have evaluated, for each alternative i, its regret in each future k. We can therefore imagine these alternatives represented by their regrets in an n-dimensional space of scenarios.

Applying some multi-objective decision making concepts, we can define the planning problem as the simultaneous minimization of m objective functions, each one being the minimization (weighted by the probabilities) of the regret in each future k:

$$\begin{aligned} &\min \; w_1 \, \text{Regret}_1 \\ &\ldots \\ &\min \; w_m \, \text{Regret}_m \end{aligned} \tag{6}$$

We can further admit that the set of solutions has already been screened and that we have retained only those that are non dominated. We recall that an alternative is said to be non dominated or Pareto optimal if any other alternative that is better than the former in one criterion is also worse in at least one other criterion - this means that we cannot improve in one criterion without losing somewhere else. These non dominated solutions are universally accepted as being the natural candidates for a final decision.

The blindness of the PC paradigm results from equation (5), or equation (1): it represents a linear combination of the criteria. Therefore, varying the weights w_k, eq. (5) will only discover those solutions that lie on the convex hull of the non dominated set. However, the surface joining all non dominated solutions is not necessarily convex and, in fact, in problems with integer variables, this non-convexity most often happens. Therefore, many possibly interesting compromise solutions will be missed, if the search is conducted by a PC paradigm. This substantiates the statement that *the PC paradigm is half blind*.

Certainly, an approach which allows "invading" the concave parts of the non dominated border will be necessary to uncover new alternatives that may constitute good compromise solutions. The RA paradigm, translated by equation (4), has precisely this ability.

4. THE PC PARADIGM IS RISKIER

The last appreciation of the relative merits of the PC and RA paradigms refers to the consequences of the decisions produced by both approaches.

It is easy to demonstrate the following: *the PC paradigm consistently tends to propose riskier decisions*.

Although the PC solution is the best on the average of the futures, if the future does not show up to be close to that average or to the most probable forecasted scenario, which is most usual, the regrets incurred are in general higher than those that would be generated by decisions proposed by the RA paradigm.

If we compare the PC and the RA paradigms in the space of scenarios, equations (4) and (5) may receive the interpretation that one is trying to find the solution that is closest to the *ideal* one. The concept of *ideal* has been introduced by Zeleny [5] as a virtual solution having the best value in all criteria, chosen from the values found in the non dominated set - this would be the solution chosen by the DM, if it were feasible.

In fact, equations (4) and (5) represent the minimization of the distance to the ideal (in this case, represented by the origin, which would mean a solution with no regrets in all futures, a fully robust solution).

The PC paradigm measures this distance in a L_1 metric; the RA paradigm measures this distance in a L_∞ metric. Zeleny defined *compromise set* as the set of solutions close to the ideal, that would be obtained by varying the metric used to evaluate this distance, namely from L_1 to L_∞.

A distance to the origin, in a system of n coordinates and in a L_i metric, is given by

$$\text{distance}_i = \sqrt[i]{x_1^i + \ldots + x_n^i} \tag{7}$$

The Euclidean metric is L_2, of course. The L_∞ metric converts into the expression

$$\text{distance}_\infty = \max(x_j, j = 1, \ldots n) \tag{8}$$

which is reflected in the RA paradigm. Because of this definition of the L_∞ metric, the distance$_\infty$ value will not be affected by any simultaneous transformation of the coordinates by a increasing monotonous function; when applied to equation (4), this means that the RA paradigm may be represented just by the difference $f - f^{opt}$, and that a Regret function such as the one in Figure 2 will not affect the decisions proposed under this paradigm. This is why the RA paradigm may be seen as the most conservative one, in terms of risk aversion.

Any intermediate metric, such as the Euclidean, will therefore tend to propose solutions in between the ones offered under the riskier PC paradigm and the conservative RA paradigm.

This will be illustrated in the example in the following section.

5. WORKED EXAMPLE

In order to illustrate the concepts defined in the previous sections, we have performed a planning exercise on a electrical distribution system composed by 54 nodes, 16 lines and 2 substations in the initial system and 45 lines and 2 substations in project (Figure 3). The example was developed for a planning horizon of 3 years divided in 3 time stages.

We have considered 3 possible load scenarios, referred as SC1, SC2 and SC3, with probabilities P1, P2 and P3 respectively; in each scenario path the load has a fuzzy definition, leading to the fuzzy scenario tree shown in Fig. 1.

Three criteria were considered:

- Investment (+losses) cost.
- Solution inadequacy. This criterion derives from the fuzzy definition of loads and may be interpreted as the consequence of adverse scenarios in which the system capacity remains below the forecasted load, leading to the need for corrective measures or to repressed demand. A way to measure this attribute is described in [6]
- Reliability - evaluated through the assessment of energy not supplied.

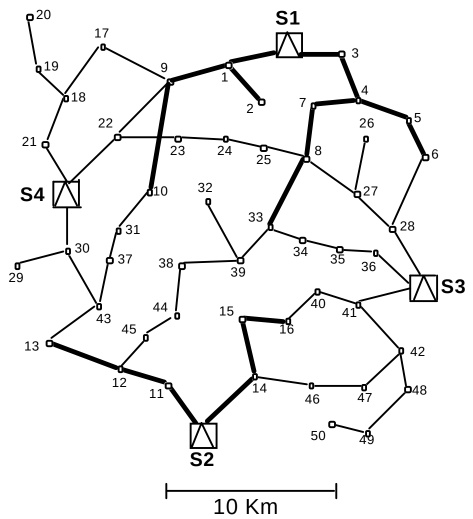

Fig. 3 - Initial system. Thick lines represent existing branches and thin lines represent possible sites for the expansion of the system. Substations S3 and S4 are considered to be in project.

In order to find a set of non-dominated solutions, we have applied a Genetic Algorithm tool [3, 7]. The 3 criteria above were combined to a single value for each scenario, by the use of appropriate fuzzy tradeoff weights, which were varied to present a picture of the non-dominated border of the problem.

The fuzzy tradeoffs reflected an uncertain cost of the kWh not supplied (reliability) and of the peak kW not suppliable (inadequacy).

As it can be seen, we have incorporated uncertainty in the model, through fuzzy modeling. This aspect is not described in detail, because it is not essential to the paper objective; extensive descriptions may be found in [3]. The results presented hereafter result from the defuzzification of fuzzy values obtained.

The Genetic Algorithm was able to find 49 non-dominated solutions (expansion plans). Note that the solutions are non-dominated in the space of criteria and not necessarily in the space of scenarios. The defuzzified weighted cost (including effects from investment and losses, power not supplied and system inadequacy) for each scenario is shown in the next table for some selected solutions (cost in PTE.10^6). The ideals for each scenario are shaded - Solutions 28, 30 and 32 are the best for scenarios 2, 3 and 1 respectively.

Solution	SC1	SC2	SC3
Sol28	1003,3	1059,2	1270,3
Sol29	1003,7	1066,4	1267,2
Sol30	1010,5	1072,7	1246,1
Sol31	1007,9	1063,3	1246,5
Sol32	968,8	1088,3	1342,7
Sol33	1010,3	1072,8	1246,3
Sol34	1008,7	1064,4	1248,5
Sol35	1004,5	1145,1	1557,9
Sol36	1001,2	1138,8	1536,8
Sol37	987,60	1125,3	1520,3
Sol38	986,5	1124,3	1519,4
Sol39	987,9	1117,4	1497,0
Sol40	994,2	1101,4	1356,2
Sol41	981,6	1089,0	1343,9
Sol42	980,1	1087,5	1342,4

Taking the planning problem as multicriteria over the three Scenarios, we solved it using: linear Probabilistic Choice as in (1); Euclidean distance as in (7); non-linear regrets with PC, as in (5) combined with (3); and Risk Analysis as in (4). Because we have calculated the criteria values of every solution in every scenario, and we had also the *ideal* in each scenario, we could easily derive the regret values in all cases.

The interesting results become apparent when we consider the w_k scenario weights as variable parameters, and try to find out the preferred solution in each case, for the whole range [0,1] of the weights. As we have

$$w_{SC1} + w_{SC2} + w_{SC3} = 1 \quad (9)$$

we are able to project the result of this exercise in a w_{SC1}/w_{SC2} plane, giving Figures 4 to 7.

This projection is a triangle, as a result of equation (9). The vertices of this triangle correspond to the three scenarios, because at each of these points one of the weights has value 1 (at the origin, we have $w_{SC3} = 1$); a point inside this domain corresponds to a specific mix of (subjective) weights.

In this particular problem, comparing the pure PC and RA paradigms, we can see that the PC model recommends solutions 30 and 32 (which are the ideals in SC3 and SC1) in a much larger range of weights than the RA model - the PC paradigm tends to recommend extreme alternatives, more than the RA paradigm, while the RA points out earlier to compromise solutions.

This conclusion is reinforced by the fact that the PC model suggests much less solutions than the RA model, as being the best decision for the whole ranges of the weights. The PC paradigm is blind to some of the possible compromises - it does not consider alternatives 29, 33 or 42.

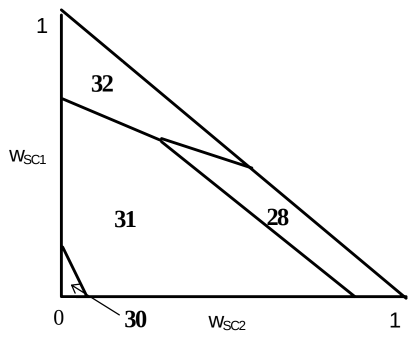

Fig 4. Best solution (numbers) for the linear PC model

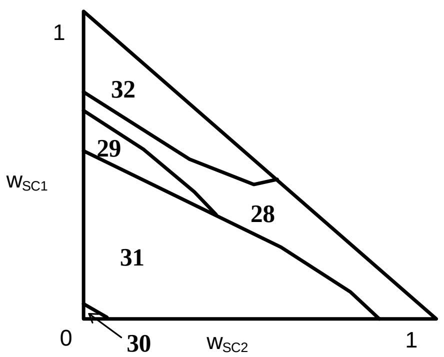

Fig. 5 - Best solution (numbers) for the Euclidean distance model

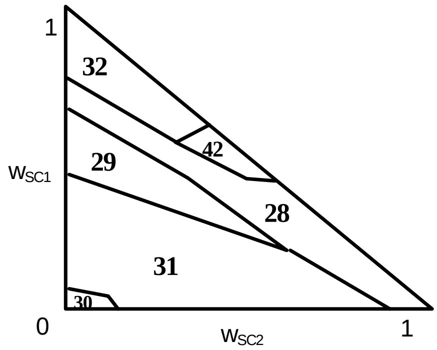

Fig. 6 - Best solution (numbers) for the non-linear regret PC model

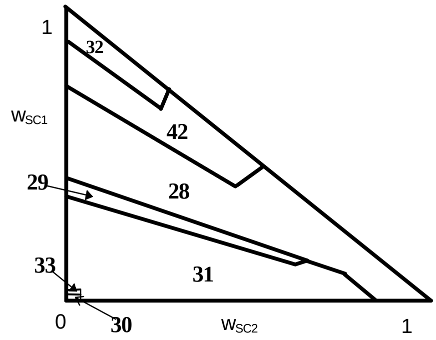

Fig. 7 - Best solution (numbers) with RA model

The PC paradigm would lead to the extensive adoption of solution 31, when the RA models point out that this decision will consistently lead to higher regrets in a very large set of scenarios, in the case that the future actually occurring is different from the average of futures calculated in the PC approach (which, if not by any other reason, from human experience, is most likely to happen).

Furthermore, the fact that solution 28 is favored by the RA model is not in contradiction with the statement that the PC model is riskier. In fact, 28 is the ideal for SC2, *which is an intermediate scenario*, so in fact the RA model is favoring an intermediate solution.

The calculations with a pure Euclidean distance and with a PC non linear regret models give (not surprisingly) coherent results. These can be seen as intermediate between the PC and the RA models, uncovering more compromise solutions as one moves from the L_1 to the L_∞ metric.

The coefficients employed in the non-linear regret function used allowed us to uncover solution 42, because some of the regrets associated with solution 32 became strongly penalized for a set of scenario weights.

In any case, it is demonstrated that a non-linear regret PC model is NOT equivalent to a Risk Analysis approach: the results obtained are different, even if we disregard the distinct philosophical background of each model.

Picking up the pure PC and RA results, we can build Figure 7, where we identify what we call the “approximate stability areas” for the best decision: they correspond to the combinations of weights for which the recommended decision is constant, irrespective of the metric (or paradigm) chosen. For a planning or decision making point of view, these stability areas are quite interesting, because within them there is no place to discussion and the adepts or one or the other paradigm may, for practical purposes, sign a truce.

We believe that finding these areas is in itself an interesting decision aid exercise - one does not need, within them, to consider whether the losses will be compensated by profits in a sequence of events, or if the index of risk aversion must be taken in account more or less seriously.

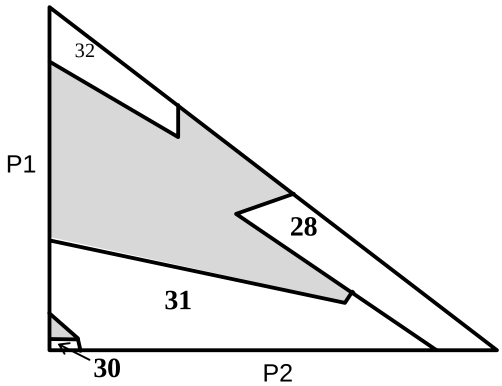

Fig. 8 - Approximate solution stability areas

6. CONCLUSIONS

Having discussed the fundamentals of the Probabilistic Choice and the Risk Analysis paradigms, in system planning, and having analyzed the practical results obtained with the application of both models to a planning problem, we have reached the following conclusions:

1. The assumptions behind the PC paradigm (that the repetition of events is achieved and that it has a large enough rate to give statistical assurance that bad outcomes will be compensated by positive outcomes) are not verified in many cases - either because unwanted or catastrophic events may be foreseeable or because the planning time horizon is too short compared to the event repetition cycle.

2. The PC paradigm is blind to many compromise solutions, because it has a linear nature, in a space of scenarios.

3. The PC paradigm tends to recommend riskier solutions, namely giving extended preference to extreme alternatives and allowing the possibility for low probability futures with high regret values.

These conclusions substantiate the statement that *Risk Analysis* outperforms *Probabilistic Choice* as the effective decision making paradigm for Power System planning (in a general or typical case).

The conclusion must however be taken with the precaution of analyzing if the probabilistic model assumptions are fully verified in each case.

We have also demonstrated that taking in account non-linear regrets, within a PC environment, is not equivalent to a Risk Analysis approach. However, non-linear regrets may be seen as an intermediate or hybrid model between the PC and the RA paradigms. This may have some virtues in particular cases, which remain to be explored.

Finally, we have shown that for a large set of probabilities allocated to scenarios, the two planning paradigms may in fact propose the same decisions as the best ones to be taken. The identification of these solution stability areas may be of great help in practical decision making, avoiding discussions about assumptions or paradigms and giving extra reassurance to the planners on how unquestionable their decisions may be.

ACKNOWLEDGMENT

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