GENERATION / TRANSMISSION POWER SYSTEM RELIABILITY EVALUATION BY MONTE-CARLO SIMULATION ASSUMING A FUZZY LOAD DESCRIPTION

J. Tomé Saraiva, Vladimiro Miranda
INESC - Inst. de Engenharia de Sist. e Computadores, and
FEUP/DEEC - Faculdade de Engenharia da Univ. do Porto
Largo de Mompilher, 22, 4007 Porto Codex, Portugal
Fax: +351.2.2084172 Tel. +351.2.2094230

L.M.V.G. Pinto
PUC/RJ - Pontifícia Universidade Católica
Rio de Janeiro
C.P. 38063 Gávea - CEP 22452 RJ
BRASIL

Abstract - This paper presents a Monte-Carlo algorithm considering loads defined by fuzzy numbers. In this methodology states are sampled according to the probabilistic models governing the life cycle of system components while fuzzy concepts are used to model uncertainty related to future load behavior. This model can be used to evaluate generation/transmission power system reliability for long term planning studies as one uses the more adequate uncertainty models for each type of data. For each sampled state a Fuzzy Optimal Power Flow is run so that one builds its power not supplied membership function. The paper proposes new indices reflecting the integration of probabilistic models and fuzzy concepts and discusses the application of variance reduction techniques if loads are defined by fuzzy numbers. A case-study based on the IEEE 30 bus system illustrates this methodology.

I. INTRODUCTION

Reliability evaluation is an important issue in the scope of planning studies. The Monte-Carlo simulation is certainly one of the most powerful available methods to evaluate system reliability. This method can be used to evaluate generation/ transmission power system reliability by analyzing a large number of system states identified by sampling component outages. Acceleration techniques can be adopted to reduce the number of the analyzed system states [1,2,3].

Traditionally, the Monte-Carlo approach is based on a probabilistic modelization of the behavior of the components. This is conceptually correct as the rules governing their life cycle are not likely to change in the future. As for loads, one usually uses deterministic values or probabilistic models. The first assumption is surely questionable specially for long term planning studies. Load values can be affected by many factors some of them being hardly forecasted. The probabilistic modelization of loads usually tries to incorporate information about the uncertainty affecting daily, monthly or yearly consumptions by sampling values according to a classified load diagram.

For long term planning studies, the focusing should be somewhat different. One easily recognizes that, in this case, the main concern should not be related to the variation of consumptions in a period due to the daily life cycle or to weather conditions, for instance. Rather, the planner should try to capture the uncertainties affecting the evolution of load values along a planning horizon. In this situation, the use of probabilistic models may not be adequate as load growth pattern can change due, for instance, to economic or legal factors or even to exogenous hardly predictable and, in many cases, unique events. In this case, the uncertainty affecting load values has not random nature and, therefore, a different framework is needed.

In this paper this uncertainty is modeled using concepts from Fuzzy Set Theory [4,5]. Fuzzy sets can be interpreted as sets of lumped and nested intervals each one having a membership value. Therefore they allow the user to interpret in an holistic way how uncertainties are reflected in the results. In the scope of fuzzy models, it should be referred that no values are sampled since the information inherent to a fuzzy set is treated as a whole. This is a definite contribution to increase the computational efficiency of these algorithms. References [6,7] present an overview of some experiences and results obtained with the introduction of Fuzzy Sets in power systems. References [8-11] address some specific topics as fuzzy power flow and fuzzy optimal power flow models.

This paper presents a Monte-Carlo simulation approach to generation/transmission reliability evaluation assuming loads defined by fuzzy numbers. Data uncertainties will be modeled more adequately: system component outages are represented by probabilistic models and load uncertainties are modeled by fuzzy numbers. For each sampled state, one can obtain the power not supplied membership function by running a fuzzy optimal power flow [11]. To reduce the computational burden of the simulation we will also discuss the use of convergence acceleration techniques. This methodology can, in a certain way, be considered an extension of the models presented in [1,2,3] which use linearized formulations to represent the operational features of the networks.

This methodology will allow the user to reflect in a very efficient way data uncertainties to the results. Traditional crisp reliability indices are no longer adequate since now they will have to reflect load uncertainty. Thus, new indices as the Expected Exposure and Robustness and the membership function of the Expected Power Not Supplied will be defined to characterize system behavior and to reflect the integration of probabilistic and fuzzy concepts which is, by itself, an important feature.

This methodology will be illustrated with results from a case study using the IEEE 30 bus system.

II. DEFINITION OF FUZZY LOADS

The concept of a fuzzy load is related to a qualitative assessment, for instance through a linguistic declaration as "*load may occur between L_1 and L_4 MW but it is likely to be between L_2 and L_3*". This can be translated into a trapezoidal fuzzy number as sketched in fig. 1 and can be interpreted as expressing the uncertainty around an interval. If $L_2=L_3$ the trapezoidal number turns into a triangular one. This can express the uncertainty around a central value having maximum membership degree.

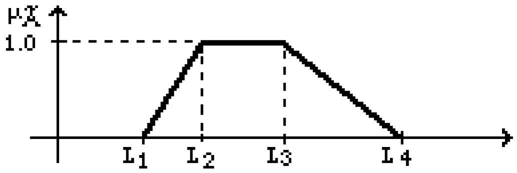

Fig. 1 - Trapezoidal fuzzy load.

Several questions related to the acquisition of a membership function deserve discussion. Reference [5] addresses the general problem of membership function measurement and [6] describes an interface to help the acquisition of load membership functions.

III. FUZZY MONTE-CARLO - **FMC** - SIMULATION

A. *Monte-Carlo sampling*

The proposed algorithm deals with uncertainties of load values and to the non ideal nature of components. As for load data no values are sampled as the fuzzy optimal power flow model used to analyze each state deals with this uncertainty as a whole. As for components a sequence of system states is sampled according to the forced outage rate of generators and branches. This is accomplished by generating a sequence of pseudo random numbers. Depending on generator and branch forced outage rates, component outages are identified under a classic non chronological strategy.

B. *Analysis of the sampled states*

The optimal power flow problem can be viewed as a process aiming at identifying the best generation values (driven by an economic criterion) subject to operational and security constraints. When, at least, one specified load is modeled by a fuzzy set the problem turns into a Fuzzy Optimal Power Flow - **FOPF**. Reference [11] presents a DC-FOPF algorithm (to be used in operation planning or for planning purposes) assuming loads represented by trapezoidal fuzzy numbers. The operational features of the network are modeled using the DC power flow model. This means that one can evaluate membership functions of phase angles or branch flows as soon as a generation strategy is identified.

The DC-FOPF algorithm starts with a deterministic DC-OPF to identify a feasible and optimal solution, according to the power limits of generators and branches and the active power balance equation. Using the central values (see Appendix) of load membership functions to define power injections, the algorithm uses a Linear Programming formulation to get a solution that will be called the central point. To take in account load uncertainties the method integrates a set of steps [11] summarized as follows:

a) a number of vertices of the hypervolume enclosing all possible load values are identified. This is accomplished by checking, for each branch and generator, the set of load values that maximize and minimize branch flow and generations;
b) optimal dispatch studies are run for all load scenarios lying on the lines between each vertex and the central point. The algorithm uses a parameter related to the membership value of each scenario;
c) the results of the parametric studies performed for all vertices are aggregated, forming membership functions for generations, branch flows and power not supplied (PNS).

The FOPF algorithm is very efficient. On average, its computing time has been evaluated as only 140% of a DC OPF performed for the central values of load membership functions.

For each analyzed state a FOPF exercise also provides the values of the Robustness and Exposure indices. As an example, let us consider the total system active load and PNS membership functions depicted in fig. 2.

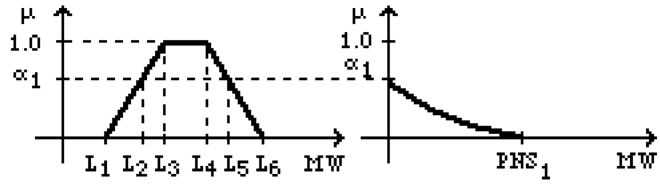

Fig. 2 - System total active load and PNS membership functions.

In this case, for α values greater than α_1 the system is robust. But when the uncertainty in the actual value of the loads becomes larger (for $\alpha<\alpha_1$), we can no longer guarantee that no load disconnection will ever be needed. Therefore, we have a possible value of PNS>0 for uncertainty ranges larger than the one at α_1. In other words, for load scenarios in $[L_2 , L_5]$ one can guarantee that no PNS values will occur and operational and security constraints will not be violated.

This information can be expressed through robustness and exposure indices assuming values $1.0\text{-}\alpha_1$ and α_1 in this case. Therefore, a fully robust system would have a robustness index of 1.0, meaning that it is adequate no matter what future occurs.

C. *Fuzzy indices*

Let PNS_i, $Irob_i$ and $Iexp_i$ be the membership function of the Power Not Supplied, the Robustness and Exposure Indices of state i. If a sample of N system states is analyzed, (1) to (3) give the estimates of E(PNS), E(Irob) and E(Iexp). In expression (1) $\sum$ represents the addition of fuzzy numbers.

E(PNS) = 1/N . **Error!** (1)
E(Irob) = 1/N . **Error!** (2)
E(Iexp) = 1/N . **Error!** (3)

D. *Convergence criterion*

According to [1,3] the convergence criterion of a Monte-Carlo algorithm can be based on the relative uncertainty (4) monitored along the simulation. In this expression, E(PNS) is the current estimate of the PNS expected value and V(PNS) is the variance evaluated by (5). The calculated β value, $\beta calc$, should be compared with a target one, $\beta spec$, so that the simulation process should finish as soon as $\beta calc \leq \beta spec$.

$$\beta^2 = V(PNS) / (N.E(PNS)^2) \quad (4)$$

V(PNS) = 1/(N-1). **Error!** (5)

This criterion can still be applied if one considers fuzzy loads. In fact, for each sampled state, one runs a FOPF integrating a deterministic DC-OPF study. Thus, convergence can be monitored using the PNS values obtained from these deterministic DC-OPF studies.

E. *Convergence acceleration techniques*

The computational burden of the Monte-Carlo simulation is often reported as a drawback of this method. Expression (6) - obtained by solving (4) in terms of N - shows that the size of the sample to get a target β value can be reduced by adopting variance reduction techniques [1,3]. Subsections E.1 and E.2 describe the application of two of these techniques to the Fuzzy Monte-Carlo - FMC - problem.

$$N = V(PNS) / (\beta.E(PNS))^2 \quad (6)$$

E.1. *Antithetic sampling*

This technique is based on the fact that the variance of the sum of two random variables depends on their covariance. If these random variables are negatively correlated the variance of their sum will be smaller than the sum of their variances. The antithetic sampling strategy, described in [3], is based on this conclusion to reduce the computational burden of the simulation.

This can be applied to the FMC simulation provided that for each sampled state a so-called antithetic state is identified. This can be done by using the sequence of pseudo-random numbers (7) used to the identify the sampled state. The antithetic sequence of (7), given by (8), can be used to identify a new system state. A new estimator is defined by (9) where $PNS_1(x)$ and $PNS_2(x)$ are the PNS fuzzy numbers related to each sampled state and its antithetic one. The new sequence of PNS(x) numbers is then used to monitor the convergence of the simulation.

$$u_1, u_2, u_3, ..., u_n \quad (7)$$
$$1.0\text{-}u_1, 1.0\text{-}u_2, 1.0\text{-}u_3, ..., 1.0\text{-}u_n \quad (8)$$
$$PNS(x) = (PNS_1(x) + PNS_2(x))/2 \quad (9)$$

E.2. *Regression function technique*

This technique uses a regression function, Z, to obtain an approximation of the expected value of the function, F, to be analyzed. For each sampled state a residual is evaluated by subtracting F(x) and Z(x). According to [1], a new estimator F* for F is obtained by (10). In this expression E(Z) is the expected value of the regression function usually evaluated by an analytical method external to the simulation process. If Z and F are strongly correlated the value of the residual function will be small and the variance of F* will be smaller than the variance of F. The expected values of F and F* coincide and are given by (11).

$$F^*(x) = F(x) - Z(x) + E(Z) \quad (10)$$
E(F) = E(F*) = E(Z) + 1/N . **Error!** (11)

Two issues must be dealt with in order to apply this technique to the FMC simulation: the choice of a regression function and the fuzzification of (11) so that one can incorporate information about fuzzy loads. References [1,3] consider as very efficient the regression function corresponding to the Power Not Supplied due only to deficiencies of the generation subsystem. The expected value of this function, E(Z), is readily calculated using the capacity outage probability table. If loads are modeled by fuzzy numbers, expression (12) is used to estimate E(PNS).

E(PNS)=E(PNSg)⊕1/N . **Error!** (12)

In this expression, ⊕ and ∑ denote the addition of fuzzy numbers and Θ represents the deconvolution process (Appendix). $PNS(x_i)$ is the fuzzy number resulting from the FOPF exercise. $PNSg(x_i)$, calculated by (13), is the fuzzy number representing the PNS related to state x_i considering only generator outages.

$$PNSg(x_i) = f(P_l \,\Theta\, P_g^{max} \oplus P_g^{out}(x_i)) \quad (13)$$

In expression (13):
- P_l is the fuzzy number of the system total active load;
- P_g^{max} is the system total installed generation capacity;
- $P_g^{out}(x_i)$ is the addition of generator capacities out of service in state x_i;
- f is a fuzzy variable function that nulls the membership degree of negative values;

E.3. *Evaluation of E(PNSg)*

In expression (12), the membership function of the expected Power Not Supplied only due to generator deficiencies, E(PNSg), can be obtained by (14) using the capacity outage probability table. In this expression $PNSg(x_i)$ is given by (13), $p(x_i)$ is the probability of state x_i only considering generator outages and ∑ represents the addition of fuzzy numbers.

E(PNSg) = **Error!** (14)

IV. CASE STUDY

A. *System data*

The 30-bus / 41 branch IEEE system sketched in fig. 3 is used to exemplify the application of the FMC algorithm. Tables 1 and 2 include generator and branch data. Regarding generator data, values for the incremental costs and FOR are specified. The original number of generators was also altered as the number of installed generators in buses 1, 2, 8, 11 and 13 was increased, to add complexity to the simulation.

Table 3 presents the central value of each load membership function. The original load data does not stress enough the transmission system as the bulk system studies would show little impact from line outages. In the present days, this does not seem to be the more usual situation found in utilities. Therefore, we have increased loads by a factor of 1.5. Using these central values two load sets were used to describe load uncertainties:
- triangular fuzzy numbers - the extreme values of their 0.0-cut correspond to 0.925 and 1.075 of the central value;
- trapezoidal fuzzy numbers - the extreme values of their 0.0 and 1.0-cut are (0.9, 0.95, 1.05, 1.1) of the central value;

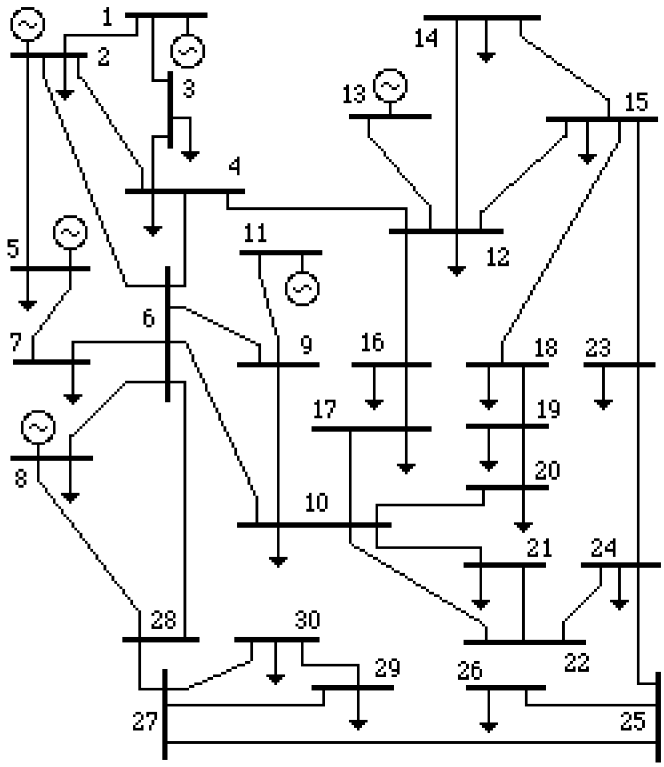

Fig. 3. IEEE 30 bus network.

The proposed methodology was implemented in a μVAX 3300 using Pascal language.

bus	Gen. no.	Pg^{max} MW	Inc cost $/MWh	FOR	bus	Gen. no.	Pg^{max} MW	Inc cost $/MWh	FOR
1	1	40.0	2.0	0.08	8	2	20.0	3.5	0.1
1	2	40.0	2.0	0.08	8	3	30.0	3.5	0.1

1	3	40.0	2.0	0.08	8	4	30.0	3.5	0.1
1	4	40.0	2.0	0.08	11	1	50.0	4.5	0.05
2	1	40.0	3.0	0.02	11	2	50.0	7.0	0.05
2	2	40.0	5.0	0.02	13	1	30.0	1.5	0.12
5	1	50.0	2.5	0.01	13	2	30.0	5.5	0.12
8	1	20.0	3.5	0.1					

Table 1 - Generator characteristics.

extreme buses	x (pu)	p^{max} MW FOR	extreme buses	x (pu)	p^{max} MW FOR
1	2.0575	130.00545	12	13.1400	65.00049
1	3.1852	130.00815	12	14.2559	32.00040
2	4.1737	65.00046	12	15.1304	32.00029
2	5.1983	130.00849	12	16.1987	32.00029
2	6.1763	65.00051	14	15.1997	16.00029
3	4.0379	130.00905	15	18.2185	16.00029
4	6.0414	90.00285	15	23.2020	16.00032
4	12.2560	65.00171	16	17.1923	16.00034
5	7.1160	70.00055	18	19.1292	16.00041
6	7.0820	130.00737	19	20.0680	32.00043
6	8.0420	32.00040	21	22.0236	32.00037
6	9.2080	65.00171	22	24.1790	16.00034
6	10.5560	32.00171	23	24.2700	16.00032
6	28.0599	32.00041	24	25.3292	16.00046
8	28.2000	32.00039	25	26.3800	16.00043
9	10.1100	65.00046	25	27.2087	16.00040
9	11.2080	65.00046	27	28.3960	65.00171
10	17.0845	32.00040	27	29.4153	16.00030
10	20.2090	32.00041	27	30.6027	16.00045
10	21.0749	32.00040	29	30.4533	16.00050
10	22.1499	32.00039			

Table 2 - Branch characteristics.

bus	P (MW)	bus	P (MW)	bus	P (MW)
1	0.0	11	0.0	21	26.25
2	32.55	12	16.8	22	0.0
3	3.6	13	0.0	23	4.8
4	11.4	14	9.3	24	13.05
5	141.3	15	12.3	25	0.0
6	0.0	16	5.25	26	5.25
7	34.2	17	13.5	27	0.0
8	45.0	18	4.8	28	0.0
9	0.0	19	14.25	29	3.6
10	8.7	20	3.3	30	15.9

Table 3 - Central values of load membership functions.

B. *Sampled state: overlapping outage of generators 1/1 and 13/2*

Figures 4 and 5 present the PNS membership functions obtained for one sampled state - overlapping outage of generators 1/1 and 13/2. The fuzzy number of fig. 4 was obtained considering the triangular load set while fig. 5 corresponds to the trapezoidal one. According to fig. 4, if this outage occurs the system shows no capacity to meet load for cuts lower than 0.71. Therefore, the corresponding exposure index is 0.71 and the robustness one is 0.29. If the trapezoidal load set is considered, the system is less robust than in the previous situation. In fact, the system is not

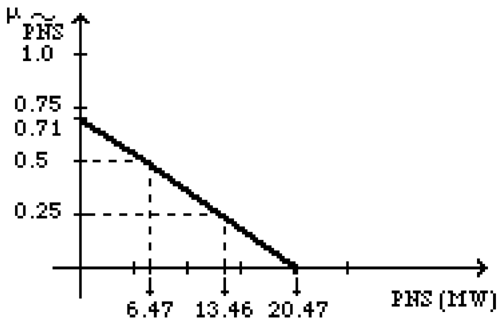

Fig. 4. PNS if generators 1/1 and 13/2 are out of service (triangular load set).

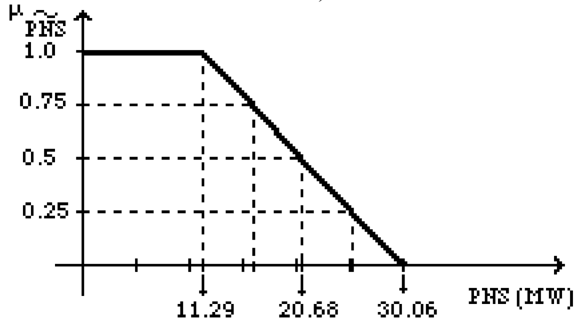

Fig. 5. PNS if generators 1/1 and 13/2 are out of service (trapezoidal load set).

robust for any α-cut and, therefore, the exposure and robustness indices are 1.0 and 0.0.

These values can be easily understood considering that the uncertainty of the trapezoidal load set is larger than the one related to the triangular load set.

C. *Fuzzy Monte-Carlo - FMC - analysis*

The FMC simulation was performed for the two load sets considering a target value for β of 10%. Figures 6 and 7 present the membership functions of the expected PNS for these two situations. These fuzzy numbers are closely related to these load sets. In fact, as the amplitude of load uncertainty grows the α-cut of the E(PNS) membership functions also gets larger meaning that the system is more exposed to load uncertainties.

These two membership functions also reveal that the FMC simulation does not preserve the shape of the specified loads. In fact, the right branch of these functions are distorted in a convex way while a concave distortion occurs for the left branch. The distortion of the right branch, for instance, can be explained considering that the operational conditions of the system are rather stressed. In fact, central load values were obtained by multiplying the original ones by 1.5 and branch limits were not increased. Therefore, a small load growth is likely to originate non zero PNS values. The effect of aggregating a large number of PNS

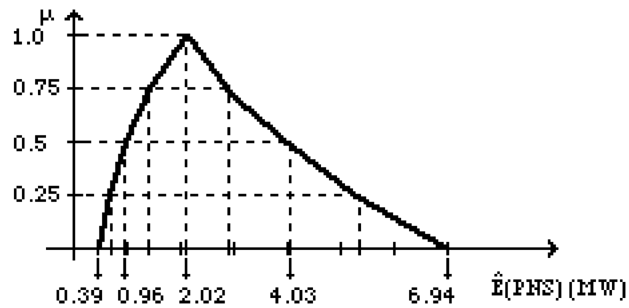

Fig. 6. E(PNS) membership function (triangular load set).

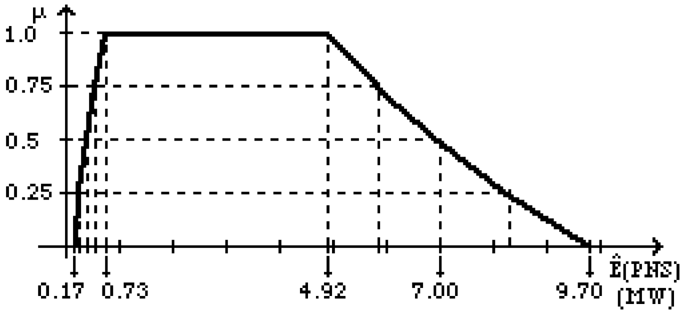

Fig. 7. E(PNS) membership function (trapezoidal load set).

membership functions, some of them similar to the one sketched in fig. 4, originates the distortion of the right branch of E(PNS).

Table 4 includes information about the expected values of the Exposure and Robustness Indices. As expected the exposition to future load uncertainties is larger when one considers the trapezoidal load set.

Type of Loads	Exposure Index	Robustness Index
Triangular load set	0.177	0.823
Trapezoidal load set	0.255	0.745

Table 4 - Expected Robustness and Exposure indices.

Regarding the variance reduction techniques, the FMC simulation was run for three situations - using the regression function, using the antithetic sampling and combining these two techniques. In a fourth situation none of these were used. Table 5 presents the number of analyzed states to get convergence in these four situations. In the first place, it is important to notice that N does not depend on the analyzed load set. This is explained considering that the FOPF methodology starts by running a deterministic DC-OPF study for the set of central load values. According to III.D, convergence is monitored using the sequence of these deterministic PNS values. Therefore, one easily concludes that N does not depend on the load sets provided that central load values in the two load sets are the same.

In the second place, results presented in table 5 indicate that the regression function is very efficient in reducing the computational burden of the simulation. In fact a reduction of about 30% is accomplished if one compares situations 1 and 4. It is also important to notice that the antithetic sampling is not effective in reducing the variance of the estimator. This conclusion can be derived if one compares situations 1 and 3, for one side, and 2 and 4, for the other. A similar result was also reported in [3].

In what concerns the computational time, the Fuzzy Monte-Carlo simulation is very efficient as, in average, it only takes 40% more time than a classical simulation for deterministic loads. This figure can be easily understood if one remembers that the FOPF algorithm also takes, in average, 40% more computational time than a deterministic DC-OPF study (see III.B). Due to the convergence criterion adopted for the FMC simulation, the number of analyzed states in this simulation and in a classical one are the

	Antithetic Sampling	Regression Function	Number of States, N
1	no	yes	993
2	yes	no	1836
3	yes	yes	1060
4	no	no	1412

Table 5 - Number of analyzed states to get convergence.

same. Therefore, the relation of the computational times of these two simulations, for one side, and between the FOPF and a deterministic DC-OPF study, for the other, should be the same.

Several runs were also performed considering a target β value of 1%. These runs still indicate that the FOPF only takes, in average, 40% more computational time than a deterministic run and that the use of the regression technique is very effective in reducing the computational time.

D. *Expansion strategies to reduce system exposure*

Fuzzy loads reflect uncertainty in human knowledge about the future. As a consequence, decision makers usually adopt, at the end of the planning process, some hedging strategies aiming at reducing the risk implicit in the decisions they are about to make. In other words, they wish to reduce the regret they may experience for making a decision according to a possible development if a different and adverse future occurs. Concerning system reliability, investments in generation and transmission capacity could be foreseen as a means of reducing the risk of having load disconnections. This can be interpreted as an attempt to reduce the E(PNS) and the exposure index.

As an example, we present results for two expansion plans. These two plans correspond to increase the active power limit of branch 6-10 to 65 MW, for one side, and to increase branch 9-11 limit to 100 MW, for the other. Figures 8 and 9 present the two E(PNS) membership functions considering the trapezoidal load set. For these two simulations the expected exposure index is 0.255 and 0.219. These results reveal that the reinforcement of branch 6-10 has little impact both in the E(PNS) function and in the expected exposure index while the second strategy is far more efficient in reducing E(PNS) uncertainty.

Using this type of information the planner can obtain trade offs between investments and robustness values. This can provide valuable information in a decision aid environment in order to select the investment according to the degree of risk the user is ready to accept.

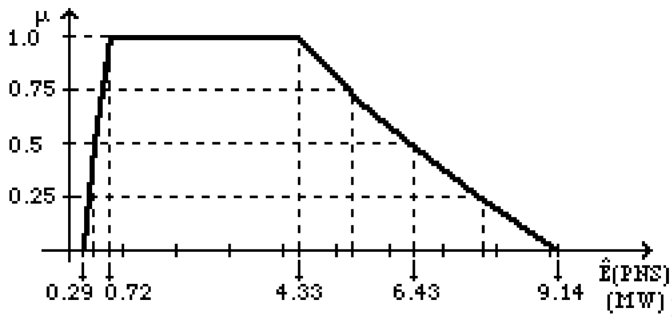

Fig. 8 - E(PNS) membership function for the reinforcement of branch 6-10 to 65 MW.

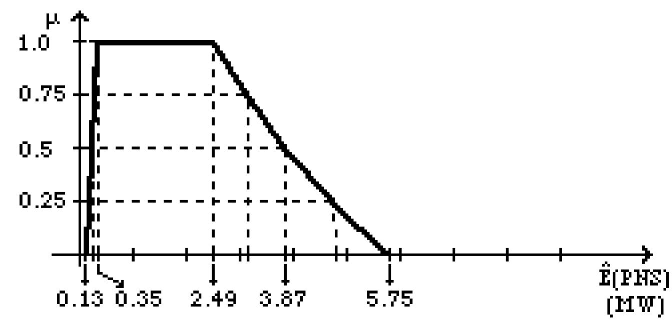

Fig. 9 - E(PNS) membership function for the reinforcement of branch 9-11 to 100 MW.

V. CONCLUSIONS

This paper presents a Monte-Carlo simulation approach to evaluate the reliability of generation/transmission power systems. In this algorithm, two kinds of uncertainties are dealt with: the uncertainties regarding future load values (modeled by fuzzy concepts) and the ones related to the non ideal nature of system components (represented by probabilistic concepts). This is an important feature by itself and it reflects the authors belief that fuzzy models will not replace probabilistic approaches. In fact, each uncertainty model should be used where adequate, and hybrid models will come as a result.

It should also be emphasized that the Fuzzy Monte-Carlo simulation is very efficient as, in average, it only takes 40% more computational time than a classical one using deterministic load values. This seems to be a very modest price to pay for an important increase in the information obtained. We have also shown that some well known variance reduction techniques can be adapted to the fuzzy case.

The interest of the fuzzy load model is further enhanced by the fact that it also allows the definition of a hedging policy. This policy aims at reducing the risk inherent to adverse futures, by a selective choice of branches and generators to reinforce.

Finally, this methodology using the linear DC model should be seen as a first attempt to integrate probabilistic and fuzzy models in the scope of power systems. New developments regarding the automatic identification of expansion strategies to reduce system exposure are also encouraged so that a better understanding of long term system behavior is accomplished.

VI. APPENDIX - BASIC FUZZY SET CONCEPTS

A fuzzy set $\tilde{A}$ is associated with a membership function $\mu\tilde{A}(x)$ relating each element x_1 to its compatibility degree with X_1 (A.1). Therefore, one obtains a gradual transition between full and complete lack of membership rather than an abrupt one. An α-level set or an α-cut of $\tilde{A}$ is the hard set A_α obtained from $\tilde{A}$ for each $\alpha \in [0,1]$ according to (A.2).

$$\tilde{A} = \{(x_1, \mu\tilde{A}(x_1)),\ x_1 \in X_1\} \qquad (A.1)$$

$$A_\alpha = \{x_1 \in X_1 : \mu\tilde{A}(x_1) \geq \alpha\} \qquad (A.2)$$

A fuzzy number is a normal convex fuzzy set of the real line such that its membership function is piecewise continuous. A trapezoidal fuzzy number, usually represented by $\tilde{A} = (a_1; a_2; a_3; a_4)$, is sketched in figure A.1.

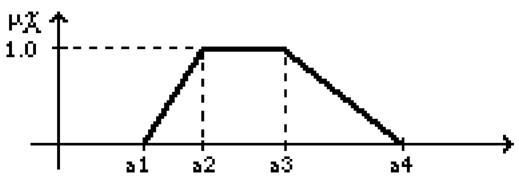

Fig. A.1. Trapezoidal fuzzy number.

If $\tilde{A}$ and $\tilde{O}$ are two trapezoidal fuzzy numbers then their addition is given by (A.3). X, obtained by submitting equation $X \oplus \tilde{O} = \tilde{A}$ to a deconvolution process, is given by (A.4) - notice that this does not correspond to the subtraction of fuzzy numbers.

$$\tilde{A} \oplus \tilde{O} = (a_1 + o_1; a_2 + o_2; a_3 + o_3; a_4 + o_4) \qquad (A.3)$$

$$X = \tilde{A} \ominus \tilde{O} = (a_1 - o_1; a_2 - o_2; a_3 - o_3; a_4 - o_4) \qquad (A.4)$$

The central value - CV - of a fuzzy number is defined as the mean value of its 1.0-cut. If the fuzzy number of figure A.1 is considered, the CV is given by $(a_2+a_3)/2$.

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João Tomé Saraiva was born in Oporto, Portugal, on August 18, 1962. He received his licenciate, an M.S. equivalent and Ph.D. degrees from the Faculty of Eng. of Oporto University (FEUP) in 1985, 1988 and 1993, respectively, all in electrical engineering. In 1985 he joined FEUP where he is Auxiliar Professor. In 1989 he joined also INESC - a Research Institute in Systems and Computer Engineering - as a researcher.

Vladimiro Miranda was born in Oporto, Portugal, on March 11, 1955. He received his licenciate and Ph.D. degrees from FEUP in 1977 and 1982 in electrical engineering. In 1981 he joined FEUP and currently holds the position of Associate Professor. In 1985 he joined also INESC where he currently is Project Manager - Head of Information and Decision in Energy Systems Area.

Leontina Maria V. Graziadio Pinto was born in Coimbra, Portugal in 1958. She received her Bsc in Electrical Engineering from PUC/RJ in 1979, and MSc and DCs degrees from COPPE/UFRJ and IM/UFRJ in 1980 and 1990 respectively. She is now with the Catholic University of Rio de Janeiro (PUC/RJ); her current interests are research and education in Planning and Operation of Electric Systems.