FUZZY LOAD FLOW - NEW ALGORITHMS INCORPORATING UNCERTAIN GENERATION AND LOAD REPRESENTATION

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ABSTRACT

Fuzzy modelling of power systems can take in account the qualitative aspects and vagueness or uncertainty that have not a random nature and therefore cannot be modelld by a probabilistic approach. This paper presents new fuzzy load flow analysis tools which enable one to incorporate in power system modelling information which is not deterministic nor stochastic about loads and generated powers, often expressed in a qualitative or linguistic way, and obtain results under the form of fuzzy information about voltages, active and reactive power flows and losses. To achieve this, a conceptual framework appropriate to assess the possibility of events is adopted. New fuzzy DC and AC formulations are presented, and the results under the form of possibility distributions obtained for an application study are examined. Adequate linearizing techniques have been adopted to overcome the difficulties related to some special cases where the resulting possibility distributions for currents and losses are described by membership functions that greatly differ from the ones assumed as data.

1. INTRODUCTION

There are situations, in power system planning, where deterministic or even probabilistic models and methods presently adopted may not be adequate because they lack the property of capturing qualitative knowledge emerging from the experience of the engineer, or fail to describe the types of uncertainty involved in proposing and assessing future scenarios or forecasts. Fuzzy set theory provides a theoretic basis to include in physical models such vague situations that often characterize human activities.

It is easily accepted that, in several situations, the available data are neither deterministic nor probabilistic. For instance, judgements such as: "Load in bus 10 will be *approximately* 12 MW", or "Load in bus 5 is *mainly* of industrial type" or even "Power flow in line 2 will be *almost* 10 MW", are clearly not deterministic nor probabilistic. This is particularly true in forecasting problems, where projecting data from the past into the future is more than a consistent procedure over stationary phenomena, because the human nature is always involved, samples are not enough significative and the environment is strongly and quickly varying. Many models so called "probabilistic" are, in fact, trying to capture this vagueness: but while the concept of probability is related to the repetition of events or experiments, vagueness derives from incomplete human knowledge and from the imprecision of natural language statements.

Fuzzy set theory (1,2,3,4) gives an adequate basis to model such kind of imprecision and has been applied, with success, to several real systems and domains. The methodologies based on fuzzy set theory are increasingly playing an important role in decision making processes as they can give a wide and global vision on the behavior of systems depending on large data sets.

This paper presents the new developments introduced by the authors in their fuzzy load flow approach (5,6). We follow a very new perspective on the problem of the description of the power system by adopting a conceptual framework appropriate to assess the **possibility** of events. The results obtained for the new AC load flow model show a significative improvement over the methodology presented in the previously published papers. It is now possible to derive possibility distributions not only for injected powers and line power flows, but also for line currents and power losses in the system. However, computational procedures and constraints remained in the level of simplicity as before, far from the heavy burden associated to convolution procedures well known in probabilistic load flow studies.

2. A FUZZY APPROACH TO LOAD FLOW STUDIES

2.1. <u>Fuzzy loads and generations</u>

In a fuzzy load flow environment, it is essential that linguistic declarations about loads may be translated into fuzzy numbers or, from a related point of view, that possibility distributions may be associated to the injected powers presumed at the system busbars.

Building fuzzy load diagrams is surely one subject open to research; one technique proposed may be found in (5). Techniques to aggregate different types of loads under linguistic declarations may also be inspired in the methodology proposed in (7). However, when one is concerned with one fuzzy value for some busbar, then the adoption and representation of a suitable membership function which can be understood as a possibility distribution is a problem for which some techniques are presented in (4).

In the following paragraphs, we will consider that some loads and generations can be precisely determined and fixed; on the other hand, at some buses, loads or generations may only be described in terms of "*more or less* this and that value". To represent such a fuzzy value, we will adopt in this paper trapezoidal or triangular shape membership functions, although the methodology presented may be used with, for example, any L-R form such as proposed in (4), within the limits of approximation of this representation. Of course, at the cost of a greater computer effort, α-cut procedures can always be used to evaluate results departing from any form of representation of the possibility distributions (4).

In Figure 1, a fuzzy description of the statement "active power generated is approximately 200 Mw and reactive power generated is 150 MVAr" is represented. There are other representations of fuzzy loads; however one must keep in mind that a fuzzy power value under the form of (P,Q) cannot be directly converted into the form (S,θ).

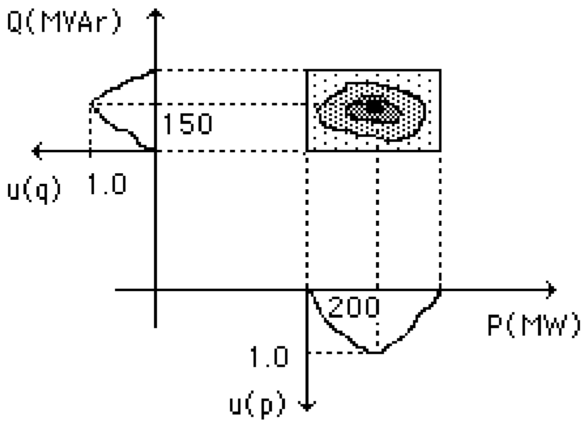

Fig. 1 Fuzzy description of a (P,Q) load.

2.2. Fuzzy load flow

2.2.1. DC model

The first approach to obtain a fuzzy description of the bus angles and active power flows consists of using an incremental DC model. In order to obtain their possibility distributions the following steps should be considered:

a) a deterministic DC load flow is previously run using the specified injected active powers associated with the medium point of their possibility distributions. Thus, deterministic values for the angles (θ_d) and active power flows (P_{dik}) are obtained.

b) the possibility distributions (ΔP_i) of the deviations from the specified active injected powers (P_i) regarding the deterministic values (P_{di}) are evaluated.

c) the possibility distributions of both bus angle deviations and active power flow deviations can be evaluated considering the DC model matrix [B] and the sensitivity coefficient matrix [A]. The possibility distributions of bus angles and active power flows are obtained by superimposing their respective increments to the deterministic values.

$$[\Delta\theta] = [B]^{-1}.[\Delta P] \quad (1)$$

$$[\Delta P_{ik}] = [A].[\Delta P] \quad (2)$$

2.2.3. Operational Methodology for AC Fuzzy Load Flow

The AC fuzzy load flow allows the imprecisions of bus data to be reflected on the voltages (V), angles (θ), active and reactive flows (P_{ik} or Q_{ik}) and losses (P_{loss} or Q_{loss}), generated active and reactive powers (Pg and Qg) and currents (I). In order to build their possibility distributions the following steps should be considered:

a) The possibility distributions are built using an incremental technique, departing from a previous deterministic load flow using load values corresponding to the medium point of the associated possibility distributions where deterministic values of voltages (V_d), angles (θ_d), active and reactive flows (P_{dik} or Q_{dik}), active and reactive generated powers (Pg and Qg) and losses (P_{lossd} or Q_{lossd}) have been evaluated. The final possibility distribution values will be obtained by superimposing these deterministic values to their respective fuzzy increments.

b) once an operating point is obtained, it is possible to evaluate the fuzzy increments of the injected active (on PV and PQ buses) and reactive powers (on PQ buses), both referring to their deterministic values.

$$[\Delta Z] = [Z] - [Z_d] \quad (3)$$

In this expression the elements of [Z] are the possibility distributions of active and reactive injected powers while [ΔZ] are the distributions of their increments referring to the associated deterministic values, $[Z_d]$.

c) the increments of the angles (PV and PQ buses) and voltages (PQ buses), $[\Delta X]$, are evaluated using the jacobian matrix [J] built in the last iteration of the deterministic study as defined in the Newton-Raphson method.

$$[\Delta X] = [J]^{-1}.\ [\Delta Z] \qquad (4)$$

The possibility distributions of voltages and angles, [X], are the superimposing of the deterministic values, $[X_d]$, and the distributions of their increments, $[\Delta X]$.

$$[X] = [X_d] + [\Delta X] \qquad (5)$$

d) the increments of active and reactive line flows are evaluated considering they are non-linear functions of both V and θ in the extreme buses. For instance, for a line between buses i and k:

$$P_{ik}=f_1(V_i,V_k,\theta_i,\theta_k) \qquad (6)$$

$$Q_{ik}=f_2(V_i,V_k,\theta_i,\theta_k) \qquad (7)$$

It is, therefore, possible to linearize these functions by considering the first terms of their expansions in Taylor series around the associated deterministic values. Considering, for example, P_{ik}:

$$f_1(V_i,V_k,\theta_i,\theta_k) = -G_{ik}.V_i^2 + V_i.V_k.(G_{ik}.\cos\theta_{ik}+B_{ik}.\sin\theta_{ik}) \qquad (8)$$

$$\delta f_1/\delta V_i = -2.G_{ik}.V_i+V_k.(G_{ik}.\cos\theta_{ik}+B_{ik}.\sin\theta_{ik}) \qquad (9)$$

$$\delta f_1/\delta V_k = V_i.(G_{ik}.\cos\theta_{ik}+B_{ik}.\sin\theta_{ik}) \qquad (10)$$

$$\delta f_1/\delta\theta_i = V_i.V_k.(-G_{ik}.\sin\theta_{ik}+B_{ik}.\cos\theta_{ik}) \qquad (11)$$

$$\delta f_1/\delta\theta_k = -\delta f_1/\delta\theta_l \qquad (12)$$

Gik and Bik are the real and imaginary parts of the ik element of the network Admittance Matrix and θ_{ik} is the difference between the voltage angles of buses i and k. Any deviation ΔP_{ik} can be, approximately, evaluated by:

$$\Delta Pik \approx \delta f_1/\delta V_{i(Vi=Vdi)}.\Delta V_i+\delta f_1/\delta V_{k(Vk=Vdk)}.\Delta V_k + \delta f_1/\delta\theta_{i(\theta i=\theta di)}.\Delta\theta_i+\delta f_1/\delta\theta_{k(\theta k=\theta dk)}.\Delta v\theta_k \qquad (13)$$

Considering in this expression the possibility distributions ΔV_i, ΔV_k, $\Delta\theta_i$ and $\Delta\theta_k$ referred to in the previous point, it is possible to obtain ΔP_{ik}. The possibility distribution P_{ik} is thus:

$$P_{ik} = P_{dik} + \Delta P_{ik} \qquad (14)$$

Using functions f_2 and a similar technique it is possible to obtain possibility distributions for Q_{ik}.

e) the possibility distributions of both the generated active power in the slack bus and generated reactive power in the slack and PV buses can be evaluated considering that they are non-linear functions of V and θ in all buses.

$$Pg = f_3(V_i,..., V_n,\theta_1,..., \theta_n) \qquad (15)$$

$$Qg = f_4(V_i,..., V_n,\theta_1,..., \theta_n) \qquad (16)$$

Using a similar technique, these functions are linearized by considering the first terms of their expansions in Taylor series around the associated deterministic values. Therefore, it is possible to obtain their possibility distributions by superimposing the deterministic values and the respective deviations.

$$Pg = Pdg + \Delta Pg \qquad (17)$$

$$Qg = Qdg + \Delta Qg \qquad (18)$$

f) for line currents and losses one could also think of using similar techniques. However, an extensive study has led us to the conclusion that it is not a satisfactory method for lightly loaded lines or for lines where reversing of power flows may occur. In those situations, it is possible to build sensitivity matrices either for active or reactive losses or currents given by:

$$[S] = [D].[J]^{-1} \qquad (19)$$

The elements of [D] are the partial derivatives of the active or reactive losses, or currents in order to V and θ in the extreme buses of the line considered, so that [S] relates the injected specified powers to the active or reactive losses or currents. Therefore, it is possible to obtain an approximation to the injected power deviations regarding the deterministic values which lead to the extreme values of the variable under consideration. For instance, the extreme values of the active losses in line ik will be approximately obtained by:

$$Ploss_{ik} = Ploss_{d\ ik} + \sum_j S_j.\ \Delta Z_j \qquad (20)$$

The deviations ΔZ_j must be chosen according to the algebraic sign of the sensitivity coefficients:

- if the maximum value is desired the maximum possible value of ΔZ_j if $S_j>0$ should be used, while the minimum possible value of ΔZ_j should be used if $S_j<0$.
- if the minimum value is desired the maximum possible value of ΔZ_j if $S_j<0$ should be used, while the minimum possible value of ΔZ_j should be used if $S_j>0$.

Once these deviations identified, a new set of injected powers can be obtained and used as data in a deterministic load flow study; the value searched for is then available.

This technique can be adopted considering successive α-cuts in the possibility distributions of the injected power deviations. In this way, possibility distributions whose "shape" substantially diverges from the original data membership functions may still be derived. As stated before, this is the case for losses or line currents in lightly loaded lines or in lines where a reversion of power flow has a possibility to occur, because in these cases the first degree approximation such as in equation 13 is no longer acceptable.

3. AN APPLICATION STUDY

The methodology described above was used to study some scenarios for the expansion of the 60 kV electrical distribution system of Oporto, Portugal. The simplified network is shown in Fig. 2.

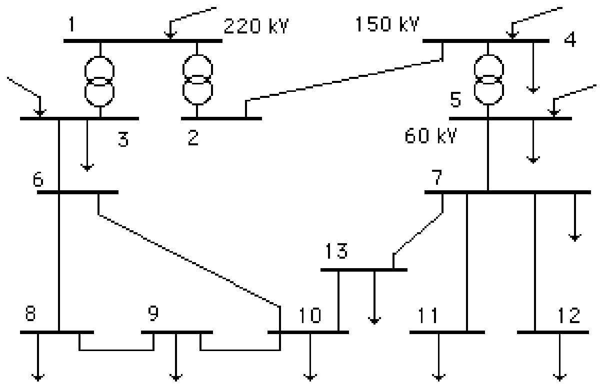

Figure 2 -Oporto simplified 60 kV distribution network

The network data such as line and transformer characteristics, specified voltages in the Slack and PV buses, load possibility distributions in all buses, possibility distributions of generated active power in PV and PQ buses and generated reactive power in PQ buses (a 500 MVA power base was considered) are presented in Tables 1 to 3 in appendix 1. These distributions were assumed as trapezoidal fuzzy numbers represented by values P_1, P_2, P_3 and P_4 according to figure 3.

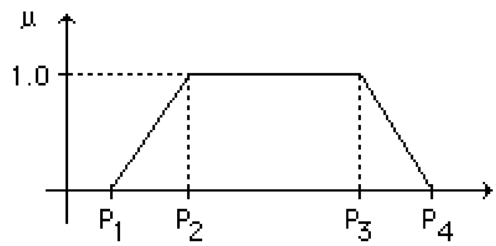

Figure 3 Representation of a trapezoidal fuzzy number

The results for the possibility distributions of voltages and angles in some busbars as well as the values obtained through two deterministic load flows (using the Newton-Raphson method), considering the maxima and minima load values and generated powers of each possibility distribution, are presented in Tables 4 and 5 in appendix 1.

The calculated possibility distributions of the active generated power at the slack bus and the reactive generated power at the slack and PV buses as well as the values obtained with deterministic load flow studies already mentioned are presented in Tables 6 and 7 in appendix 1.

The results for the possibility distributions of active and reactive power flows in some lines as well as the values obtained through deterministic load flows already referred are presented in Tables 8 and 9 in appendix 1. These distributions are assumed as keeping a trapezoidal shape which stands as a fairly good approximation in this case.

One's attention is drawn to line 10-13, where the possibility of power flow in either direction is clearly detected by the correspondent possibility distribution but not by the deterministic extreme case analysis. Such a situation is exemplified in Fig. 4 corresponding to active power flow in line 10-13.

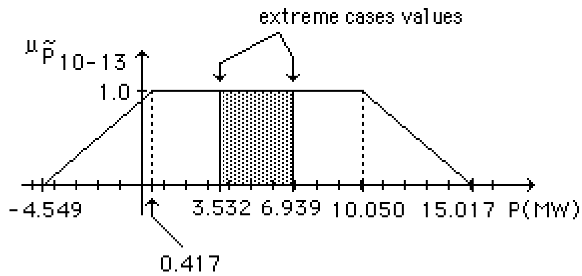

Figure 4 Possibility distribution of active power flow in line 10-13, and the extreme case analysis results.

Line current and power loss possibility distributions can also be approximately evaluated with trapezoidal shape functions, in the general case. However, line 10-13 shows one of the special cases where one must use the method associated to equations 19 and 20. In Fig. 5, a sketch of the possibility distributions of both power active losses and current for line 10-13 is shown. It should be stressed that the current scale is not linear in respect to the power loss scale, because its possibility distribution is related to the square root of the possibility distribution of the active power losses. The figure is distorted to evidenciate the breaking points of the membership function.

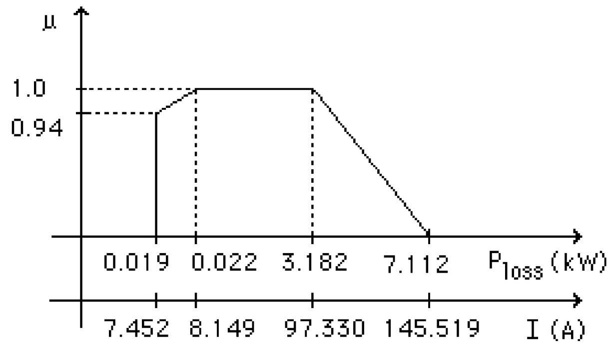

Figure 5 Current and active power losses in line 10-13

From the analysis of these results and the results obtained by the authors for other case studies, one may in general conclude that:

- These results are significantly better than those presented in (5) or (6) as the models have been strongly improved.

- The possibility distributions calculated cover in general a wider range of values than the interval delimited by the values obtained with the two deterministic load flow studies using maxima and minima loads and generated powers.

- This methodology reveals the possibility of power flow occurring in either direction at some lines (due to different combinations of loads and generated powers), which could remain undetected if only some deterministic scenarios were studied.

- In some cases, power losses and currents are not conservative in the sense that they do not even approximately preserve the trapezoidal shape of the specified injected powers. For such cases as in line 10-13 above, although a definite possibility of zero flow of both active and reactive power flows occurs for this line, the active losses or current assume the zero value with a null possibility (or, in other words, they will never reach zero value). This is explained by the fact that active losses or current would be nulled only if active and reactive power flows would simultaneously go through zero, which is by no means guaranteed.

4. CONCLUSIONS

In this paper, an improved version of a Fuzzy Load Flow algorithm is presented. It has been stressed through the paper that its aim is to model qualitative imprecision and to allow linguistic descriptions of loads or injected powers to be dealt with.

It is never argued that possibilistic modelling is to replace probabilistic modelling. Both approaches may be merged as the theoretic developments of fuzzy set and probabilistic set theory demonstrate. The actual and practical ways to achieve it in power system analysis, however, remain an open subject to research. In reliability analysis, for example, the authors believe that major steps are to be accomplished with such mixed approach.

The uncertainty in generated powers may be also interpreted in terms of distribution possibilities in dispatch decisions. Integrating the fuzzy load flow within a dispatch framework would be a logical step to be taken, specially in a planning environment.

Fuzzy modelling, as it can be seen in the proposed fuzzy load flow, can give holistic insight to problems and system behaviour. Therefore, another opened path is the building of decision aid or decision support systems where symbolic and numeric computing would be integrated. Fuzzy modelling may certainly be used as a sort of interfacing between qualitative and quantitative assessments; it has the potential to be manipulated so as to condense and translate large amounts of data and results into meaningful perspectives manageable or understandable by decision makers or engineers.

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APPENDIX 1

In this appendix, the data and some of the results of a power system fuzzy load flow analysis are presented.

Table 1 Generated and load active powers

bus	type	Vsp(pu)	Generated Active Power (pu)				Load Active Power(pu)			
			P1	P2	P3	P4	P1	P2	P3	P4
1	Slack	1.05	-	-	-	-	0.0	0.0	0.0	0.0
2	PQ	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	PQ	-	0.07	0.076	0.084	0.09	0.494	0.498	0.504	0.508
4	PV	1.0505	0.392	0.396	0.404	0.408	0.132	0.134	0.138	0.140
5	PQ	-	0.054	0.056	0.060	0.062	0.082	0.084	0.088	0.090
6	PQ	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	PQ	-	0.0	0.0	0.0	0.0	0.088	0.09	0.094	0.096
8	PQ	-	0.0	0.0	0.0	0.0	0.098	0.1	0.104	0.106
9	PQ	-	0.0	0.0	0.0	0.0	0.068	0.07	0.074	0.076
10	PQ	-	0.0	0.0	0.0	0.0	0.049	0.051	0.055	0.057
11	PQ	-	0.0	0.0	0.0	0.0	0.108	0.11	0.114	0.116
12	PQ	-	0.0	0.0	0.0	0.0	0.07	0.072	0.076	0.078
13	PQ	-	0.0	0.0	0.0	0.0	0.053	0.054	0.056	0.057

Table 2 - Generated and load reactive powers

bus	Generated Reactive Power (pu)				Load Reactive Power (pu)			
	P1	P2	P3	P4	P1	P2	P3	P4
1 -	-	-	-	0.0	0.0	0.0	0.0	
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.21	0.228	0.252	0.27	0.247	0.249	0.252	0.254
4	-	-	-	-	0.0792	0.0804	0.0828	0.084
5	0.126	0.1307	0.1493 0.154	0.0475	0.0487	0.0510	0.0522	
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0352	0.036	0.0376	0.0384
8	0.0	0.0	0.0	0.0	0.0343	0.035	0.0364	0.0371
9	0.0	0.0	0.0	0.0	0.0306	0.0315	0.0333	0.0342
10	0.0	0.0	0.0	0.0	0.0245	.0255	0.0275	0.0285
11	0.0	0.0	0.0	0.0	0.0378	0.0385	0.0399	0.0406
12	0.0	0.0	0.0	0.0	0.028	0.0288	0.0304	0.0312
13	0.0	0.0	0.0	0.0	0.0238	0.0243	0.0252	0.0257

Table 3 - Line and transformer characteristics

line n.	send. end	rec. end	r (pu)	x (pu)	y/2 (pu)
1	2	4	0.007191	0.057528	0.0
2	3	6	0.034975	0.279802	0.0
3	5	7	0.031276	0.250212	0.0
4	6	10	0.008748	0.029322	0.002606
5	6	8	0.008172	0.024306	0.002172
6	8	9	0.020377	0.040027	0.000771
7	9	10	0.019181	0.037675	0.000726
8	10	13	0.015550	0.047124	0.002352
9	7	13	0.011274	0.034165	0.001706
10	7	11	0.014098	0.011944	0.004388
11	7	12	0.007764	0.014508	0.002064
12	1	3	0.0	0.125	0.0
13	1	2	0.0	0.142857	0.0
14	4	5	0.0	0.20	0.0

Table 4 - Voltage possibility distributions

bus	voltage possibility distribution (kV)				results (kV) for	
	P1	P2	P3	P4	min and max powers	
2	63.036	63.042	63.048	63.054	63.042	63.048
3	61.668	61.836	62.106	62.274	61.848	62.100
6	59.448	59.676	60.108	60.336	59.910	59.868
10	59.304	59.532	59.976	60.210	59.784	59.724
12	59.310	59.532	59.994	60.216	59.802	59.718
13	59.292	59.520	59.976	60.198	59.784	59.706

Table 5 - Angle possibility distibutions

bus	angle possib. distributions (degrees)				results (degrees) for	
	P1	P2	P3	P4	min and max powers	
2	-0.889	-0.779	-0.563	-0.453	-0.620	-0.722
3	-4.892	-4.754	-4.514	-4.376	-4.558	-4.709
6	-9.374	-9.075	-8.506	-8.207	-8.492	-9.089
10	-9.601	-9.294	-8.704	-8.395	-8.687	-9.310
12	-9.636	-9.312	-8.694	-8.370	-8.678	-9.328
13	-9.648	-9.331	-8.724	-8.406	-8.708	-9.347

Table 6 - Generated active power possibility distribution

bus	Pg possibility distributions (MW)				results (MW) for	
	P1	P2	P3	P4	min and max powers	
1	363.948	380.580	410.851	427.483	385.885	405.563

Table 7 - Generated reactive power possibility distributions

bus	Qg possibility distributions (MVAr)				results (MVAr) for	
	P1	P2	P3	P4	min and max powers	
1	60.184	73.033	93.569	106.388	91.798	74.902
4	65.929	73.171	90.267	97.484	82.488	81.002

Table 8 - Possibility distributions of active power flows

line		possibility distribution P_{ik} (MW)				results (MW) with	
i	k	P1	P2	P3	P4	min and max powers	
5	7	154.162	158.844	167.880	172.561	157.918	168.808
6	10	59.683	63.500	70.893	74.712	62.954	71.444
6	8	66.921	69.098	73.391	75.567	67.764	74.726
8	9	16.794	18.489	21.816	23.510	18.680	21.624
10	9	12.244	14.075	17.678	19.509	15.345	16.407
10	13	-4.549	0.417	10.050	15.017	3.532	6.939

Table 9 - Possibility distributions of reactive power flows

line		possibility distribution Q_{ik} (MVAr)				results (MW) with	
i	k	P1	P2	P3	P4	min and max powers	
5	7	63.273	67.146	74.908	78.757	67.937	74.142
6	10	13.093	15.482	20.315	22.683	16.248	19.520
6	8	19.506	20.684	23.051	24.223	20.614	23.114
8	9	3.878	4.891	6.923	7.931	5.381	6.425
10	9	6.697	7.781	9.960	11.049	8.488	9.261
10	13	-8.044	-4.790	1.810	5.086	-2.145	-0.825

APPENDIX 2

Fuzzy set theory can be understood as an extension of n-valued logic when the number of the admissible logic values tends to infinity. For example, given an universe X and a subset X_1 of X, the membership value of an element x_1 to X_1 belongs to:

- {0, 1} in the Boolean logic.
- {0, 0.5, 1} in the trivalued Lukasiewicz´s logic.
- [0, 1] if normalized fuzzy sets are considered.

Therefore, a fuzzy set Ã is characterized by a membership function $u_{\tilde{A}}(x)$ associating an element x_1 to its compatibility degree to X_1. In this sense, the transition between full and no membership is gradual rather than abrupt.

$$\tilde{A} = \{(x_1, u_{\tilde{A}}(x_1)) , x_1 \in X_1\} \quad \text{(A1)}$$

An α-cut of a fuzzy set Ã can be defined as the crisp set A_α of elements whose membership value to Ã is not inferior to α.

$$A_\alpha = \{x_1 \ X_1 : u_{\tilde{A}}(x_1) \geq \alpha\} \quad \text{(A2)}$$