FUZZY RELIABILITY ANALYSIS OF POWER SYSTEMS

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Abstract - This paper presents the fundaments of a power system fuzzy reliability theory. It includes modeling the consequences of having, as data, fuzzy loads (type I) or fuzzy reliability indices (type II). It also gives an approach to decision models combining fuzzy data including fuzzy reliability values (type III).

1. INTRODUCTION

This paper is a first systematic description of fuzzy reliability models (or fuzzyfied models) applied to Power Systems, seen as a natural extension of present probabilistic models.

In general, power systems are assumed to be repairable and their stationary analysis is based on mean values of probability distributions. We will accept the assumption that these distributions are exponential and concentrate on reliability assessment for mid or long term purposes.

Fuzzy reliability (FR) is the necessary framework to develop reliability studies whenever at least one item of data is described by a fuzzy model.

If we have only a fuzzy description of loads (not a deterministic and not a probabilistic description), then we are in **type I** fuzzy reliability assessments. A fuzzy load is the consequence of incomplete knowledge about consumption behavior; it is typical of planning studies: (what the load will be, 15 years from now?) or of distribution networks (describing consumption patterns:: what is exactly an industrial load type?).

A fuzzy load may denote both a degree of uncertainty in its exact value and a linguistic description of its nature or possible range of values. It may be translated by simple expressions such as "more or less 10 MW" or more elaborate declarations such as "load will not be under 8 MW nor above 12 MW, but the best estimate is between 10 MW and 11 MW, with little uncertainty".

Type II FR assessments have component reliability indices as fuzzy, instead of crisp. Many reliability data are obtained by analogy from data bases associated to similar equipment, not exactly the one under analysis (not installed under the same conditions, subject to technologic evolution); and repair times depend not only on the components themselves but also on other systematic factors that include company efficiency. In some cases, there is just not enough data and estimates must be adopted, and this uncertainty is not just of the probabilistic type. We have then a hybrid model: stochastic (we are still dealing with a failure-repair cycle), and fuzzy (we cannot accurately describe all the conditions of the "experiments" that would represent a pure probabilistic model).

We can also speak of **type III** calculations, for models that deal with fuzzy reliability in a decision making environment, i.e., where fuzzy values must be compared in order to reach conclusions or decisions.

In the following paragraphs we will address several innovative concepts and formulations for FR models, and will admit that the reader has a basic knowledge in fuzzy set theory [1] and fuzzy numbers (FN).

2. TYPE I FR MODELS

Type I fuzzy reliability relates to the uncertainty in defining the power consumption - and therefore, in measuring the impacts of failures of supply. The (fuzzy) uncertainty in a load affects the LOLP - Loss of Load Probability index. Furthermore a fuzzy load implies a fuzzy value of the power disconnected (PNS). The average annual energy not supplied ENS becomes also fuzzy; given the Unavailability U of supply (crisp), in hours per year, the following fuzzy equation applies:

$$\mathrm{ENS} = \mathrm{U\ PNS} \cong \lambda r\ \mathrm{PNS} \qquad (1)$$

with λ being the rate of interruption and r the average repair time, here represented by crisp numbers (the operations follow the rules of fuzzy arithmetics).

2.1. A fuzzy LOLP at HL 1

At HL1 - Hierarquic Level 1 (the Generating System), the calculation of the LOLP index requires a capacity outage probability table and a fuzzy load duration curve. The fuzzy LOLP calculation derives from

$$\text{fuzzy LOLP} = \sum_i p(C - L_i)\tilde{q}(L_i) \qquad (2)$$

where C - system installed capacity
$(C - L_i)$ - capacity outage
$p(C - L_i)$ - probability of outage of $(C - L_i)$
$\tilde{q}$ - fuzzy description of the probability of load exceeding available capacity
L_i - load value from a load duration curve

In expression (2) we have the addition of fuzzy numbers supported on [0;1], multiplied by positive constants with domain [0;1]. As all values are positive, a practical computation of the fuzzy LOLP, based on the regular partition of the interval [0,1] in several α steps, may be done by proceeding to classical LOLP calculations for the lower and upper extremes at every α.

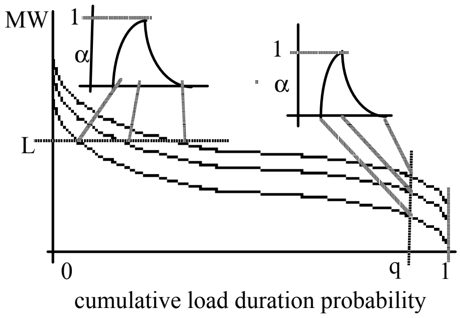

Fig. 1 - Fuzzy description of a load duration curve

If a crisp load curve is given by $L = f(q)$, then a fuzzy load curve may be defined, at a level α, by

$$L_\alpha = \left[(1 - \Delta_\alpha^-)f(q)\ ;\ (1 + \Delta_\alpha^+)f(q)\right] \tag{3}$$

with Δ_α^- and Δ_α^+ being two non-strictly monotone decreasing functions with α; we may also have a fuzzy description of the probability q through f^{-1} (Fig.1). This representation of an FN is called "interval of confidence" (based on $\alpha \in [0,1]$).

Admit a generating system with 4 similar 10 MW units, each with a f.o.r. of 0.9, and a crisp cumulative load duration curve $L = 28.846 - 19.25\,q$. Let a triangular fuzzy load curve be defined within L±8%, such as

$$L_\alpha = \left[(1-(0.08-0.08\alpha))(28.846-19.25q)\ ;\ (1+(0.08-0.08\alpha))(28.846-19.25q)\right] \tag{4}$$

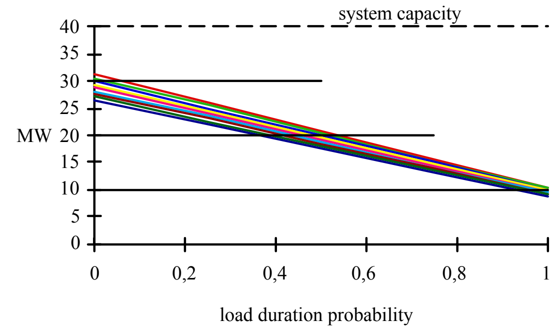

Fig. 2 - Fuzzy load curve and how it is intersected by the capacity outages

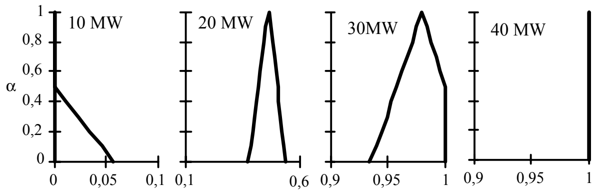

Fig. 3 - Fuzzy contribution of each outage state

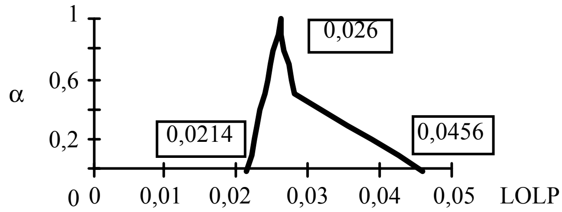

Fig. 4 - Fuzzy LOLP for the example in the text.

This fuzzy load curve is in Fig.2, together with the possible capacity outages. The contribution of each outage state is in Fig. 3; these, multiplied by the outage state probabilities and added up, give the result depicted in Fig.4. If the uncertainty range in loads could be kept above α of aprox. 0.5 (equivalent to say in the ±4% bandwidth), the LOLP would have a relatively small uncertainty; for larger uncertainties in load, the LOLP becomes quite disturbed and its uncertainty grows remarkably.

If the load curve is represented by a step diagram, then the resulting fuzzy LOLP is defined in a discrete domain, due to the discontinuities of q(L). Admit, for the same 4 generator system, that the fuzzy load duration curve is defined by triangular numbers, such as in Fig. 5. Fig. 6 displays the four cases that may occur, when an outage state interacts with one step or two consecutive steps.

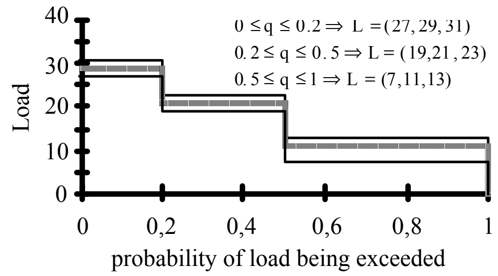

Fig.5 - Step fuzzy load duration curve

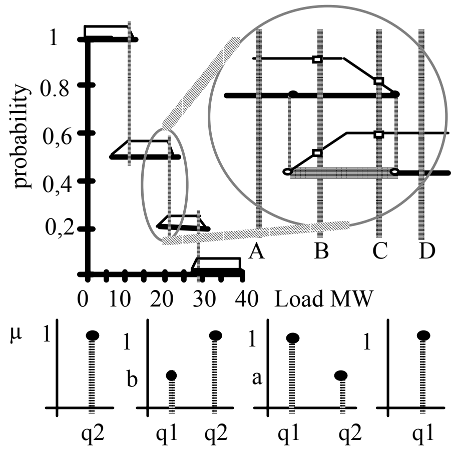

Fig. 6 - Four cases of how an outage state may intersect the load duration curve, and the fuzzy quantities representing the probability of the load exceeding a given value, from A (left) to D (right)

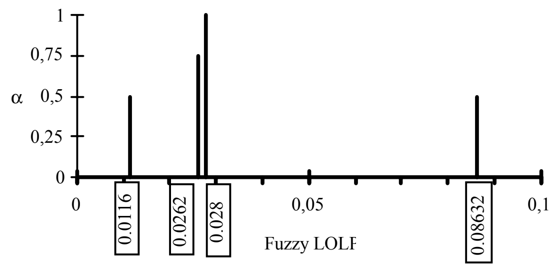

Fig. 7 - Discrete Fuzzy LOLP

In cases A and D, the result is crisp, but in cases B and C the result is a fuzzy quantity, with two probability values having two different possibility degrees.

Each outage state will therefore have one of A to D representation, with an associated probability. These cannot however be freely combined (added, weighted by their probabilities) to give the result: we cannot combine sub-domains to the left of the mode α=1 with sub-domains to the right of the mode (combining these cases would mean that we were accepting in a load step a load lower than the mode and in another step a load greater than the mode, which contradicts the concept of fuzzy load duration curve defined). With this pre-caution, the resulting fuzzy LOLP will be as in Fig. 7.

2.2. Fuzzy reliability assessments at HL2

The fuzzy reliability evaluation of a system design at HL2 may be achieved with a Monte Carlo approach, in a process that will be called Fuzzy Monte Carlo (FMC).

In a classical MC, given a crisp load scenario, the process is roughly the following: a scenario of outages is sampled; a special form of an Optimal Power Flow (OPF) is run, to determine the minimum load disconnected; and the results are combined to give the average answer sought.

In a Fuzzy Monte Carlo [2,3], we need a special Fuzzy OPF [4] to deal with the fuzzy powers and we must aggregate the results with the rules of fuzzy arithmetics.

We will recall here from [3] just an illustrative result obtained for the IEEE Reliability Test System with triangular fuzzyfied loads, defined as a maximum uncertainty range of 7.5% around the central value. The resulting fuzzy description of the expected power not supplied PNS index is depicted in figure 8 (notice that this result is not triangular nor symmetric).

The FMC and the fuzzy indices highlight risks derived from system design. For instance, although the maximum range of uncertainty (at α level 0.0) is ±7,5% of the load central value, the corresponding range of uncertainty for the fuzzy PNS is of (-77%, +278%).

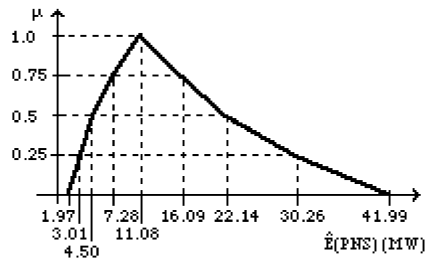

Fig. 8 - E(PNS) possibility distribution (example)

3. TYPE II FR MODELS

Classical reliability studies are based on two assumptions: *the probabilistic assumption*, which states that the system behavior can be fully described through probabilistic models (the laws and axioms of probability apply and the behavior is characterized in the context of probability measures); and *the binary state assumption*, which requires that the state of an equipment, of failure or functioning, shall be completely defined.

But the consideration of some fuzzy concepts leads to several reliability models [5]; we have therefore:

- the PROBIST model: assuming the probabilistic assumption and the binary state assumption;
- the PROFUST model: keeping the probabilistic assumption, but introducing a fuzzy state assumption;
- the POSBIST model: introducing a possibilistic assumption as to the description of the events and the laws governing their repetition, together with the binary state assumption; and
- the POSFUST model: with the possibilistic assumption and the fuzzy state assumption.

This chapter will be devoted to the PROFUST model; but it is necessary to clarify the meaning of the fuzzy state assumption: one cannot precisely define the state of a component, having therefore a fuzzy success and a fuzzy failure state; or one can define exactly the success and the failed states, but one is unable to define precisely how the transition occurs, namely how often. Therefore, the probability of finding a particular component at one state is given by a fuzzy number.

However, because the probability assumption is retained, particular probabilities of instantiated outcomes must add up to 1 in the whole state space. This introduces a dependency between fuzzy probability values that is not found in other fuzzy models.

The raw data are usually the failure rate λ and the mean repair time r. We may have, for a λ value, instead of a crisp number such as 0.01 fl./year, a fuzzy description such as a best estimate of (0.01) and an interval of confidence of [0.008, 0.012] fl./year. Therefore, λ_α= [0.01- (0.002 (1-α)), 0.01+(0.002(1-α))] will represent a triangular fuzzy failure rate, with $\lambda_\alpha = [\lambda_\alpha^-, \lambda_\alpha^+]$ being the interval of confidence at level $\alpha \in [0; 1]$.

3.1. Fuzzy exponential distribution

We can now consider the extension of the Reliability function R(t) to the fuzzy case [6]. For every t, the boundaries of the λ interval define

$$R_\alpha(t) = \left[R_\alpha^- = e^{-\lambda_\alpha^+ t} ; R_\alpha^+ = e^{-\lambda_\alpha^- t} \right] \qquad (5)$$

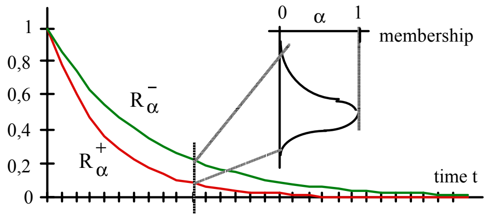

Fig. 9 - Fuzzy reliability function, and an illustration of its membership values at a certain t

We have, at each α, an interval of confidence delimited by a lower survival law $R_\alpha{}^+$ and an upper law $R_\alpha{}^-$. This defines a fuzzy reliability function $R_\alpha(t)$, such as suggested in fig. 6. The mean time to failure of a classical Reliability function is given by MTTF = $1/\lambda$. Taking in account eq. (5), one may define a fuzzy MTTF for the fuzzy reliability function $R_\alpha(t)$, based on the interval of confidence representation

$$MTTF_\alpha = \left[\frac{1}{\lambda_\alpha^+} ; \frac{1}{\lambda_\alpha^-} \right] \qquad (6)$$

This definition is straightforward. We define an interval mean time to repair r_α in relation with repair rates $\mu_\alpha{}^-$ and $\mu_\alpha{}^+$ such as $r_\alpha = \left[\frac{1}{\mu_\alpha^+} ; \frac{1}{\mu_\alpha^-} \right]$ (7)

This means that the fuzzy numbers r and μ are calculated as the inverse of each other: $r = 1/\mu$. This operation is valid because the fuzzy numbers r and μ are defined on the positive axis of the real line.

3.2. Series and parallel systems

The FR of a serial system of n components is given by

$$R^{ser} = R_1.R_2.....R_n = \prod_{i=1}^{n} R_i \qquad (8)$$

We have here the product of fuzzy numbers, all defined in the positive axis. In terms of intervals of confidence, if each $R_\alpha = \left[R_\alpha^- ; R_\alpha^+ \right]$, we have

$$R_\alpha^{ser} = \left[R_{\alpha 1}^- . R_{\alpha 2}^- ..... R_{\alpha n}^- ; R_{\alpha 1}^+ . R_{\alpha 2}^+ ..... R_{\alpha n}^+ \right] \qquad (9)$$

The result of such operation over triangular R_i does not give a triangular result; however, a triangular approximation is usually acceptable.

The FR of a n-parallel system is given by

$$R^{par} = 1 - \prod_{i=1}^{n} (1 - R_i) \qquad (10)$$

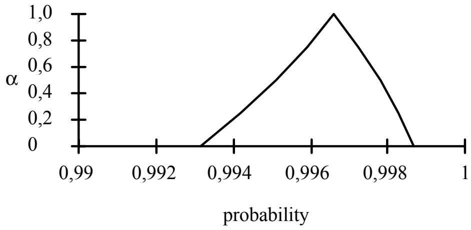

Fig. 10 - FR of a 3-component parallel system

Admit a parallel of 3 similar components, each of them with a reliability described as a triangular fuzzy number [0.81;0.85;0.89]. Applying eq. (10), one reaches a result depicted in Fig. 10: it is again NOT a triangular fuzzy number but still here this is a reasonable approximation.

3.3. Fuzzy Markov models

In the sense of PROFUST, it is possible to define Markovian models with the following characteristics:

a) the space state is completely defined and crisp;
b) the transitions between states are assumed as obeying to the general probabilistic laws;

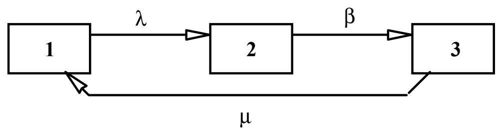

Fig. 11 - A 3-state Markov model

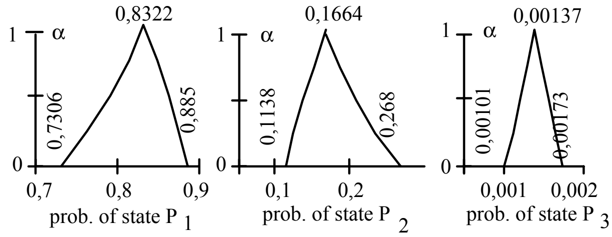

Fig. 12 - Results for the 3-state numerical example

c) the actual values of the transition rates between states are described as fuzzy numbers.

Consider a 2-state Markov model of a component, with fuzzy failure rate λ and fuzzy repair rate μ, and fuzzy probabilities P_s and P_f of finding the component in either state. The well known result for classical crisp models is:

$$P_s = \frac{\mu}{\lambda + \mu} \quad ; \quad P_s = \frac{\lambda}{\lambda + \mu} \qquad (11)$$

However, it would be wrong to just replace in (11) the crisp rates λ and μ by fuzzy definitions: the result would display a much larger uncertainty than necessary, because one would be using <u>more than once</u> the same fuzzy variable in the calculations. The correct form of the expressions for the fuzzy values of P_s and P_f is

$$P_s = \frac{1}{1 + \frac{1}{\mu}\lambda} \quad ; \quad P_s = \frac{1}{1 + \frac{1}{\lambda}\mu} \qquad (12)$$

For a 3-state model such as in Fig. 11 the solutions, arranged so that a fuzzy result may be found, are

$$P_s = \frac{1}{1+\lambda(\frac{1}{\beta}+\frac{1}{\mu})}\ ;\ P_s = \frac{1}{1+\beta(\frac{1}{\lambda}+\frac{1}{\mu})}\ ;\ P_s = \frac{1}{1+\mu(\frac{1}{\lambda}+\frac{1}{\beta})} \tag{13}$$

Fig. 12 displays the results when:

λ - "more or less 0.2" fl./year -> [0.18; 0.2; 0.22]
β - "around" 1 transition/year -> [0.6;1;1.4]
r =1/μ - "approximately" 72 hours -> [64; 72; 80]

Notice that he extreme values of the fuzzy results do not add up to unity - the non linearities of the solution may force the extreme values of the state probabilities to be obtained with non extreme values of some of the transition rates, at every α interval of confidence.

In a last example, take a 2 component and 4 state model, such as in Fig. 13. The fuzzy state probabilities in this case are given by the following fuzzy equations:

$$P_1 = \frac{1}{1+\frac{\lambda_1}{\mu_1}}\,\frac{1}{1+\frac{\lambda_2}{\mu_2}}\ ;\ P_2 = \frac{1}{1+\frac{\mu_1}{\lambda_1}}\,\frac{1}{1+\frac{\lambda_2}{\mu_2}} \tag{14a}$$

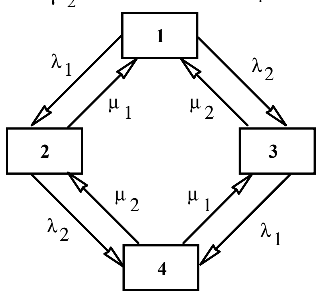

Fig. 13 - A 2 component / 4 state Markov model

$$P_3 = \frac{1}{1+\frac{\lambda_1}{\mu_1}}\,\frac{1}{1+\frac{\mu_2}{\lambda_2}}\ ;\ P_4 = \frac{1}{1+\frac{\mu_1}{\lambda_1}}\,\frac{1}{1+\frac{\mu_2}{\lambda_2}} \tag{14b}$$

These examples show one way of solving some fuzzy Markov processes: the crisp solutions for the limit stationary state probabilities may be fuzzyfied, but only after conveniently arranged, such that each variable never gets, directly or indirectly, divided by itself or subtracted from itself. An easy way to guarantee this is to have each variable only represented once in an expression.

Unfortunately, in general it will not be possible to arrange expressions in the desired way; one may then define a non linear programming (NLP) solution. If one has a problem with n states and m transition rates λ_j among the states, one may calculate, at each level α, the interval of confidence relative to the fuzzy probability P_k of a given state k at that level, defined as

$$P_k(\alpha) = \left[P_k^-(\alpha); P_k^+(\alpha)\right]\ , \quad \text{such that}$$

$$P_k^-(\alpha) = \min\left\{P_k(\lambda) \mid \forall\lambda_j : \lambda_j^-(\alpha) \le \lambda_j \le \lambda_j^+(\alpha)\right\} \tag{15}$$

$$P_k^+(\alpha) = \mathrm{Max}\left\{P_k(\lambda) \mid \forall\lambda_j : \lambda_j^-(\alpha) \le \lambda_j \le \lambda_j^+(\alpha)\right\} \tag{16}$$

For a given Markov process, one will have therefore, for the computation of maximum and minimum values of the probability P_k of state k, at level α:

$$P_k^-(\alpha) = \min P_k \quad ; \quad P_k^+(\alpha) = \mathrm{Max}\, P_k \tag{17}$$

subject to $(\mathbf{M\text{-}I})\,\mathbf{P} = \mathbf{0}$

$$[1\ \ 1\ \dots\ 1]\ \mathbf{P} = 1 \tag{18}$$

$$\boldsymbol{\lambda}^-(\alpha) \le \boldsymbol{\lambda} \le \boldsymbol{\lambda}^+(\alpha)$$

where **P** is the vector of instantiated state probabilities, **M** is the transposed of the classical stochastic transition matrix, **I** is the identity matrix, $\boldsymbol{\lambda}$ is the vector of instantiated transition rates and $\boldsymbol{\lambda}^-(\alpha)$, $\boldsymbol{\lambda}^+(\alpha)$ are the extremes of the intervals of confidence of the transition rates, at level α.

In constraints (18) we find **[**1 1 ... 1**]** **P** = 1. It means that the instantiated probability values must always add up to 1, and is the direct consequence of the probabilistic assumption of the PROFUST approach. In particular, this formulation allows the calculation of the fuzzy probability of a set of states considered together in an aggregate state G. In this case, in the objective functions (17), one only needs to replace P_k by $P_k = \sum_{j\in G} P_j$. This is not the same as calculating first each fuzzy P_j and then adding them to give (wrong) P_k.

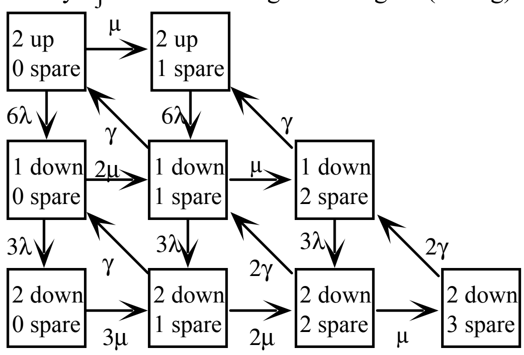

Fig. 14 - Two transformer banks with one spare; states numbered 1-9 from left to right and top to bottom

As an illustration, one may see in Fig.14 the Markov space state diagram, from an example in [7], where the problem of the reliability of a generating transformer substation with two identical three-phase transformer banks and a spare is discussed. Fuzzy triangular transition rates were defined as:

failure: [0.05;0.1;0.2] /yr
repair: [4;12;20] /yr
replacement: [120;183;183] /yr

Fig. 15 shows the fuzzy probabilities of states 1, 2 and the joint consideration of 1+2. Fig 16 shows the results for the aggregate states 1+2 (both transformers up), 3+4+5 (one up, one down) and 6+7+8+9 (two down).

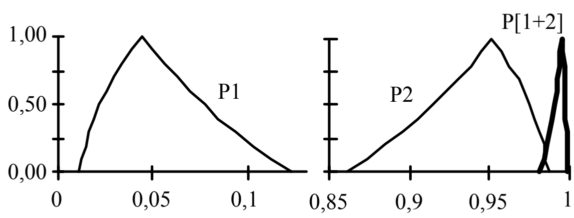

Fig. 15 - Fuzzy probabilities: P1 + P2 - P[1+2]

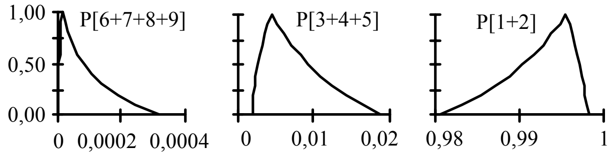

Fig. 16 - Fuzzy probabilities of aggregate states

3.4. Frequency and duration models

We now present some approximate formulas for fuzzy calculations in parallel and serial systems, arranged so that the fuzzy calculations may remain valid.

<u>2-Parallel systems</u>

System failure rate -> $\lambda = \lambda_1\lambda_2\,(r_1 + r_2)$

System unavailability -> $U = \lambda_1 r_1\, \lambda_2 r_2$

System mean repair time -> r = \f(1,\f(1,r_1) + \f(1,r_2)) (19)

<u>2-Serial systems</u> (no overlapping failures, or neglected)

System failure rate -> $\lambda = \lambda_1 + \lambda_2$

System unavailability -> $U = \lambda_1 r_1 + \lambda_2 r_2$ (20)

System mean repair time -> Not straightforward. One can derive exact formulas, but it is more easy to follow a deconvolution approach. In fact, as the following fuzzy equation must hold: $U = \lambda\, r$ (21) then an easy way of obtaining r from (21) is by finding the fuzzy number r that multiplied by the fuzzy λ would give the fuzzy U; this is not the same as solving $r = \frac{U}{\lambda}$ by applying the rules of fuzzy arithmetic.

3.5. Min cut set method with fuzzy indices

The min cut set method is a well known approach to the assessment of the reliability of general systems. It is not an exact method, but within the relative values for component reliability indices usually found in power systems, it gives a satisfactory accuracy. It is appropriate to find nodal or local indices, and is specially adequate for continuity assessment, in the design of power stations or substations, and also in sub-transmission or distribution systems. A relevant paper referring to this technique is [8]. The fuzzy model has been first discussed in [9].

The min cut set method demands one to determine which component failure mode (a "cut") causes the supply to a node to be interrupted; then, reliability indices are calculated for each cut - applying, if necessary, formulas for components in parallel; and finally, the reliability calculations are performed as if the cuts were independent entities in series - namely, applying the fuzzy formulas for component series.

Although one will have, during calculations, repeated entries of the same fuzzy indices (for instance, two 2nd order cuts may share one component), we can apply directly the "fuzzyfied" formulas of the crisp method because, in series or parallel components, we only find additions and multiplications - therefore, a variable will never divide itself or subtract from itself.

We can get also a fuzzy index for the average annual energy not supplied ENS. Recall equation (1); now, he fuzzy ENS results from the multiplication of a crisp number PNS for a fuzzy number U

$$ENS = U.PNS \qquad (22)$$

3.6. Conditional probability - the fuzzy case

Within the PROFUST approach, for conditional probability calculations the operation rules are not straightforward, from the fact (already found) that the fuzzy probability of a set of states cannot be calculated by the simple fuzzy sum of state probabilities. In general, a conditional probability approach may derive from an expression such as

$$P(A) = P(A|B)\,P(B) + P(A|\overline{B})\,P(\overline{B})$$
$$= P(A|B)\,P(B) + P(A|\overline{B})\,(1 - P(B)) \qquad (23)$$

To calculate the fuzzy description of P(A), based on fuzzy descriptions of P(B), P(A|B) and $P(A|\overline{B})$, the direct fuzzyfication of expression (23) is incorrect: P(B) appears twice, adding and subtracting. Therefore, the following approach must be followed, at each level α, where intervals of confidence $[P^-_\alpha\,;\,P^+_\alpha]$ are defined for P(B), P(A|B) and $P(A|\overline{B})$:

- <u>a lower bound</u> $P^-_\alpha(A)$, defined by a NLP (non linear programming problem), is

$$\min \Phi = P(A|B)\,P(B) + P(A|\overline{B})\,(1 - P(B)) \qquad (24)$$

subj. to

$$P^-_\alpha(A|B) \le P(A|B) \le P^+_\alpha(A|B)$$
$$P^-_\alpha(B) \le P(B) \le P^+_\alpha(B)$$
$$P^-_\alpha(A|\overline{B}) \le P(A|\overline{B}) \le P^+_\alpha(A|\overline{B})$$

This formulation admits that P(A|B) and $P(A|\overline{B})$ are independent. From the objective function, we see that

$$\min\Phi \Rightarrow \left(P(A\mid B) = P^-_\alpha(A\mid B)\right) \wedge \left(P(A\mid \overline{B}) = P^-_\alpha(A\mid \overline{B})\right)$$

An equivalent NLP is then

$$\min\Phi' = \left(P^-_\alpha(A\mid B) - P^-_\alpha(A\mid \overline{B})\right)P(B) + P^-_\alpha(A\mid \overline{B})$$

subj. to

$$P^-_\alpha(B) \le P(B) \le P^+_\alpha(B)$$

Its solution depends on the relative values of $P^-_\alpha(A|B)$ and $P^-_\alpha(A|\overline{B})$. It may be summarized as follows:

- <u>a lower bound</u> $P^-_\alpha(A)$ depends on the relative values of $P^-_\alpha(A|B)$ and $P^-_\alpha(A|\overline{B})$:

$$P^-_\alpha(A) = \begin{cases} \left(P^-_\alpha(A|B) - P^-_\alpha(A|\bar{B})\right) P^-_\alpha(B) + P^-_\alpha(A|\bar{B}) \\ \quad \Leftarrow P^-_\alpha(A|B) \geq P^-_\alpha(A|\bar{B}) \\ \left(P^-_\alpha(A|B) - P^-_\alpha(A|\bar{B})\right) P^+_\alpha(B) + P^-_\alpha(A|\bar{B}) \\ \quad \Leftarrow P^-_\alpha(A|B) < P^-_\alpha(A|\bar{B}) \end{cases}$$

- an upper bound $P^+_\alpha(A)$ is defined in an analog way:

$$P^+_\alpha(A) = \begin{cases} \left(P^+_\alpha(A|B) - P^+_\alpha(A|\bar{B})\right) P^+_\alpha(B) + P^+_\alpha(A|\bar{B}) \\ \quad \Leftarrow P^+_\alpha(A|B) \geq P^+_\alpha(A|\bar{B}) \\ \left(P^+_\alpha(A|B) - P^+_\alpha(A|\bar{B})\right) P^-_\alpha(B) + P^+_\alpha(A|\bar{B}) \\ \quad \Leftarrow P^+_\alpha(A|B) < P^+_\alpha(A|\bar{B}) \end{cases}$$

4. TYPE III FR MODELS

Type III models deal with decision making problems. So far, types I and II only required arithmetic operations with fuzzy numbers; but type III calculations demand the ranking of fuzzy options. These models are typical of planning activities, combining technical and economical factors. In fact, many economic parameters may be uncertain, such as costs of equipments, interest rates or costs of power disconnected or energy not supplied.

4.1. Robustness, exposure

A fuzzy number PNS_{ab} describing the power not supplied for a given contingency, such as in Fig. 16, as a result of a type I calculation departing from fuzzy loads, can be the object of further useful interpretations.

We define the **exposure** index $\alpha^{exp} \in [0,1]$) of a power system, given a set of fuzzy loads, as the level α above which PNS becomes 0, regardless of which load values are instantiated (it means that the system accommodates data uncertainties in the range [a,b], at α level); conversely, (1-α) is defined as a **robustness** index.

Exposure (and robustness) are, in this context, new reliability indices, emerging from FR analysis. They allow the comparison of two designs by telling how much uncertainty in load can a system cope with, in either case. A more robust system design will be more insensitive to the uncertainty in data, namely load forecasts, which is a characteristic most desirable. For the reliability evaluation of a power system, given a set of n contingencies, a Global Exposure G(α) index is

$$G(\alpha) = \sum_{k=1}^{n} \alpha_k^{exp} p_k \qquad (25)$$

where α_k^{exp} is the exposure index for contingency k and p_k is the probability of outage k. A Global Robustness index can be defined similarly.

4.2. Hedging

In risk analysis, hedging occurs when one invests some extra to avoid the consequences of some adverse futures. In power system planning, hedging means reducing the value of the Exposure index (generating a solution more insensitive to uncertainties). Within the framework of fuzzy set models, some hedging approaches have already been proposed [3,4], related to investments in the generation or transmission system, that lead to a ranking of investment decisions associated with increased robustness of design. This can also be seen in Fig. 17, if one imagines that moving from option 'ab' to 'cd', reducing exposure to adverse load scenarios, has been done with some extra expenditure on a given system.

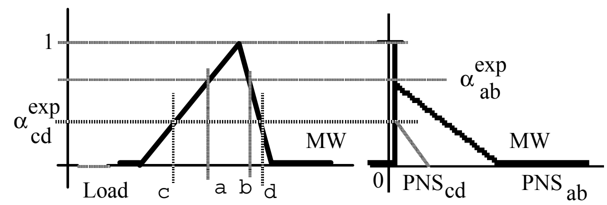

Fig. 17 - Load uncertainty and power disconnected for two system designs; the alternative leading to α\s(exp,ab) is less robust than the other one: for this option, only inside [a,b] the system will have no load disconnected.

4.3. Fuzzy regret

In planning, if costs are uncertain, wouldn't one wish to know how good a decision is, i.e., to know whether a ranking of alternatives would not change if other values for the costs were considered?

Fig. 18 displays the typical decision making dilemma, in comparing the cost of two alternative plans A and B. If crisp calculations (at α=1) were made, it would seem that plan B should be preferred, because it is less expensive; however, a fuzzy modeling of data uncertainties makes one hesitate. One realizes that, below α_1, B may exceed A, and this becomes obvious below α_2, i.e., if the decision maker has uncertainties in data larger than the ones defined at the α_1 level, then he must weight the risk of making a regrettable choice [9].

If uncertainties could be constrained to remain above α_1, then his decision in favor of plan B would be the best whichever instantiation of the data values would occur.

There are some methods to defuzzify and rank fuzzy numbers (for instance, the Centre of Mass G technique). But they should not be applied directly to rank costs. One important question that direct ranking of fuzzy costs may not answer is: "how much will one regret, for choosing one alternative instead of another?".

Let's assume a context of minimization. We now introduce the new concept of **Fuzzy Regret** Reg(A|B), the possible regret felt by a decision maker when he chooses A instead of B, and then B happens

$$\mathrm{Re\,g}_\alpha(A|B) = \left\{ \max\left\{0;(A_\alpha^- - B_\alpha^+)\right\}; \max\left\{0;(A_\alpha^+ - B_\alpha^-)\right\}\right\} \tag{26}$$

Of course, if one chooses A, and A results better than B, no regret will be felt - that's why expression (24) does not allow for negative regrets.

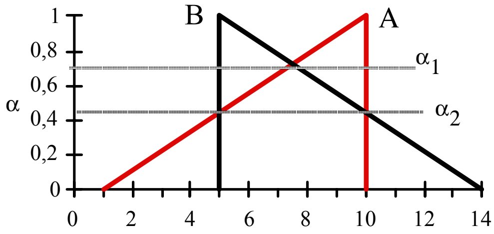

Fig. 18 - Triangular fuzzy costs of two planning alternatives. Plan B seems to cost less than plan A, but is perhaps riskier

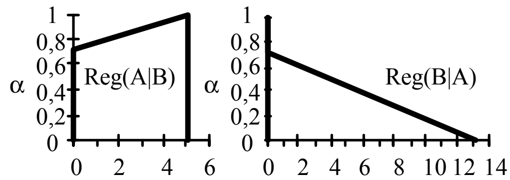

Fig. 19 - Fuzzy Regrets, referring to fig. 10. Left - regret for choosing A instead of B; right - regret for choosing B instead of A

Fig. 19 shows Reg(A|B) and Reg(B|A) in relation to Fig.18. The decision dilemma lies now between these two regrets. We can now apply the Center of Mass technique and rank the two fuzzy regrets: we get Reg(B|A)<Reg(A|B) - therefore, decision B would be preferable to decision A, in this pairwise comparison (however, applying the Center of Mass technique directly to the fuzzy costs A and B would have ranked A<B; this shows the importance of changing the point of vue from objective function values to decisions).

4.4. Fuzzy dominance

We come now to the last point discussed in this paper - the concept of dominance in multi-criteria problems. This discussion has its relevance due to the fact that in many power system planning problems, typically two criteria (at least) are weighted againt each other: costs (investment + operation) × reliability.

The concept of efficient, dominant or Pareto-optimal set of solutions, in a multi-criteria environment, is by now well understood. In short, a solution is said to dominate another if it is at least as good in all criteria and strictly better in at least one of them (this means that a rational decision maker would always prefer the dominant to the dominated solution). Those solutions for which one cannot find any others that dominate them form, precisely, the non-dominated set.

The situation is clear in fig. 20, which represents three solutions in a two-attribute space: C is dominated by A and B, and these are non-dominated, in a context of minimization.

The fuzzy case is somewhat more complicated. In fig. 21 we have three solutions D, E and F, with fuzzy values on each attribute (say, a fuzzy cost and a fuzzy power not supplied, as a consequence of uncertainties in data). To make it more evident, admit that this fuzzyness is just represented by intervals at α=0 (giving a rectangle) and a point at α=1, and suppose that these points would represent the Center of Mass of each fuzzy solution.

If a Decision Maker evaluates the solutions only based on the Centers of Mass, he will conclude that D dominates E and that both dominate F. However, the fuzzy representation shows that the uncertainties in data do not allow such a simplistic conclusion: for example, at level α=0, there is the possibility that E may dominate D (while still both dominate in every case F).

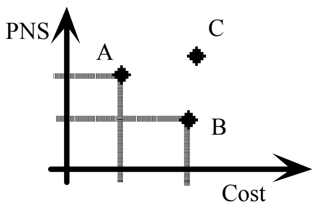

Fig. 20 - Three crisp multi-attribute solutions

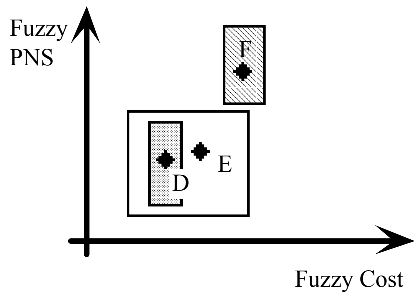

Fig. 21 - Three fuzzy multi-attribute solutions

This illustrates very well one of the important contributions of fuzzy modelling to the understanding of the problems dealt with, in Power Systems: contrary to crisp models, where the concepts of solutions and decisions are confused, in fuzzy models there is a clear separation between the (fuzzy) solution values and the decisions that must be made, and this distinction derives from the evaluation of risk, resulting from the simultaneous consideration of multiple possible scenarios.

In a crisp model, solutions may be ranked for decision according to their solution values, because the ordering of regrets, in pairwise comparisons, may be considered the same as the ordering of solution values.

But in a fuzzy model this is not necessarily true: the ordering of central points or of centers of mass does not reflect the possible regrets, as we have seen in 4.3. This happens also in a multi-criteria environment (one might in theory even reach situations of intransitivity of preferences).

The discussion of decision making in fuzzy context is a very interesting topic of research in itself. Related to the subjects discussed in this paper, the fuzzy modeling allows the identification of risks and the evaluation of regrets, when opting for one solution instead of another. For the multi-criteria case, an extension of the Fuzzy Regret concept, developed in section 4.3., must be considered.

CONCLUSIONS

This paper tries to serve two purposes: to present the fundamentals of Fuzzy Reliability analysis of power systems, introducing some new concepts and definitions and organizing ideas that were dispersed; and to serve as a basic reference to future research in the field.

The latest survey [10] on fuzzy set theory application in power systems clearly indicated that fuzzy reliability had not yet attracted the attention of research effort, contrary to other areas. Besides the author of the present paper and his team, only another team of researchers seems to have made public an organized effort in studying the possible advantages of a fuzzy dimension in reliability studies (see, for instance, ref [11]).

One question that must be considered is about the real need of fuzzy modeling. Some will argue that a probabilistic approach would suffice. A possible answer includes two perspectives. First, the past decade of success in developing and applying fuzzy models to many problems has demonstrated that this approach is technically valuable and engineering useful. Second, it is a fact that new perspectives about problems may be acquired.

For instance, the fuzzy results present a holistic sensitivity analysis to the uncertainty in data, different from local sensitivities translated by partial derivatives; and, because many scenarios of data are considered at a time, one can take a step into risk analysis concerning decisions, something so far not easily addressed by other techniques.

In any case, the material presented in this paper does not replace the well established probabilistic models - these are just extended, to serve in those occasions when the information or data available will be better modeled by a fuzzy approach.

The paper opens a few innovative paths into reliability analysis, and provides clues to solving the computation of fuzzy indices and other results. Although just a first step, one hopes that the material delivered will be stimulating enough.

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