# Experimenting in the Optimal Capacitor Placement and Control Problem with Hybrid Mathematical-Genetic Algorithms


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**Abstract - Genetic algorithms (GA) have shown how natural evolution and genetic based procedures can be used as a powerful optimization tool. Although a GA alone is able to handle optimization problems, its performance can be improved with the combination of genetic search with conventional mathematical gradient search. In the work reported, we describe how we have made experiments in finding the best optimization approach based on GA for a discretized optimal capacitor placement and control problem. The final results achieved clearly indicate that a hybrid approach, where the gradient search is used to repair or improve chromosomes without disrupting the evolutionary characteristics of the method, is the best approach.**



**Keywords:** Genetic algorithms, optimization, gradient search, voltage/VAR control, VAR planning.


## I. INTRODUCTION

Adequate capacitor placement and control scheme plays an important role in distribution or transmission power systems since it leads to loss minimization and voltage improvement which, in general, provide more stable operation conditions. The issues of this problem are to determine the best location, optimal size and type of capacitors, and the optimal control setting of installed capacitors at different load levels, which provide the maximized loss reduction and voltage profile improvement.

Among other methods, Genetic Algorithms (GA) have been applied to solve the problem mentioned above since they have attractive features such as capability of finding global optimum solutions, ability in quite easily handling different kinds of constraints and, most importantly, effectiveness in handling the discrete nature of problems[1]. Genetic Algorithms are based on the Darwin's survival-of-the-fittest strategy, from where the principles of natural genetics and natural selection are imitated. The reactive power planning problem may be modeled as a multi-objective constrained optimization problem and it can easily be formulated with genetic based algorithms [2].

Several models have been proposed for the problem of capacitor bank sizing and location, e.g. [1][3][4][5][6]. Approaches using heuristics or classical mathematical models cannot be said to have had a great success, either because of over-simplifications or because of the lack of confidence in the optimality of results. A good comparison of these methods may be found in [7] and [8]. This explains why one has witnessed the publication of new models based on Genetic Algorithms [9].


Paper ISAP2001– – accepted for presentation at the IEEE ISAP2001 Conference, Budapest, Hungary, June 18-21, 2001)


Recently, some authors have explored the possibilities offered by hybrid models, matching the might of GA with the power of the information content of mathematical description of systems [10]. In particular, we illustrate in this paper the case often argued by other authors that the information content of mathematical models might help in speeding up the convergence of GA. To achieve this result, we make use of a gradient search technique based on sensitivities.

Furthermore, it is known that in problems with discrete variables, GA are efficient in zooming into the optimum region, but have difficulties in reaching the exact optimum.

Miu et al. [11] also proposed the use of sensitivities; however, they organized their approach in a two-phase model, where the GA acts in a first stage and a sensitivity-based heuristic performs a sort of post-optimization.

In our model, we use information about the gradient of the objective function (e.g., minimizing losses) to repair chromosomes and improve solutions, giving a push in the right direction to the evolution procedure. The case dealt with in this paper shows that convergence is greatly improved with the adoption of a hybrid mathematical-genetic model.

The illustration is based on a comparison among three models: 1) a Simple Genetic Algorithm; 2) a Hybrid Genetic Algorithm/Evolutionary Programming model; and 3) a Hybrid Mathematical/Genetic model.

## II. PROBLEM FORMULATION

The problem of efficient capacitor placement and control belongs to a class of combinatorial optimization problems. These problems were attempted to be solved by heuristic approaches as well as mathematically based algorithms. In this study, GA have been applied to find the global optimum in terms of losses and voltage profile improvement.

The general objective (fitness) function was created based on three different functions - capacitor investment cost, energy cost of losses, voltage limit penalty - and subject to a number of constraints related to the problem. The formulation can be described in mathematical terms as follows:

Minimize $$OBJ = [\, CC + EC + VL \,] \quad (1)$$

Subject to: $$f(X,u) = 0 \quad (2)$$
$$S_{\min} \le S_i \le S_{\max} \quad (3)$$
$$F_{\min} \le F_i \le F_{\max} \quad (4)$$

where

$CC$ = capacitor investment function
$EC$ = energy cost of losses function
$VL$ = voltage limit penalty function
$f(X,u)$ = the equality constraints of the power flow problem;
$F_i$ = quantity of fixed capacitors at bus i
$S_i$ = quantity of switched capacitors at bus i

The capacitor investment function was calculated based on costs of capacitor and installed capacity. Different costs for fixed and switched capacitor types were used, with higher value for switched capacitor banks, reflecting the incorporation of costs of switching devices into the nominal cost of capacitor alone. Linearized costs were used for installation cost of capacitors, assuming that the installation cost is directly proportional to installed capacity. The capacitor investment function, given as the multiplication of total installation cost of capacitors by a scale factor, is (5):

$$CC = sf_C \left[ \left( \sum_{i=1}^{n} NFC_i \times CF \right) \times \left( \sum_{i=1}^{n} NSC_i \times CS \right) \right] \qquad (5)$$

where
$sf_c$ = scale factor of capacitor investment function
$NFC_i$ = quantity of fixed capacitor installed in bus i
$NSC_i$ = quantity of switched capacitor in bus i
$CF$ = per unit cost of fixed capacitors
$CS$ = per unit cost of switched capacitors

The total power losses of the system were used as the main factor to create the energy cost of losses function. The total energy loss is calculated from system power losses and time duration of the planning period. The energy cost function formulation was as follow:

$$EC = sf_E \left[ \sum_{i=1}^{m} PL_i \times T_i \times MC \right] \qquad (6)$$

where
$sf_E$ = scale factor of energy cost function
$PL_i$ = power loss of load level i
$T_i$ = planning time duration of load level i
$MC$ = energy marginal cost of the system

In the case of the voltage limit penalty function, a combination of two different approaches was used to separate the priority of two different zones. A square penalty function is used for the case where voltage profiles were within a specified limit and a more sloped penalty function (a linear penalty function multiplied by a larger coefficient) is used when voltage profiles are outside of such limit. The voltage limit penalty function can be mathematically described by (7):

$$VL = sf_V \left[ \sum_{i=1}^{n} \left( \sum_{k=1}^{m} SF_k^i + sf_M \sum_{k=1}^{m} LF_k^i \right) \right] \qquad (7)$$

where
$sf_V$ = scale factor of voltage limit penalty function
$sf_M$ = scale factor of linear penalty function

$$SF_k^i = \begin{bmatrix} \left(V_k^i - VN_k^i\right)^2 & \text{if } V_k^i \rangle \text{ limit} \\ 0 & \text{if } V_k^i \langle \text{ limit} \end{bmatrix} \qquad (8)$$

$$LF_k^i = \begin{bmatrix} \left(V_k^i - VN_k^i\right) & \text{if } V_k^i \langle \text{ limit} \\ 0 & \text{if } V_k^i \rangle \text{ limit} \end{bmatrix} \qquad (9)$$

$V_k^i$ = voltage at bus k in load level i
$VN_k^i$ = nominal voltage of bus k in load level i

Using the same objective function mentioned above, three different algorithms, Simple Genetic Algorithm (SGA), Genetic Based Algorithm (GBA) and Genetic Based Algorithm with Gradient Search (GBAGS), were created based on different strategies and different genetic control operators.

As a general rule, the number of genes in each chromosome is taken as equal to the number of PQ buses times the number of time periods to represent the load levels.

In the SGA, as described by Goldberg [12], we defined a non-overlapping population, where every individual of each generation is entirely replaced with newly generated individuals. We built each chromosome as a binary string and adopted a flip mutator and single point crossover.

In the GBA we defined each gene as an integer, representing the number of capacitors of a given reactive power to be used in series at each location and time period. We adopted also an overlapping population, where only some individuals were replaced by new ones coming out from genetic operations. We used uniform crossover and modified gaussian mutation.

The uniform crossover is achieved by generating a random string of bits with length equal to a chromosome. Then, a child individual is built by adding the first parent gene if the random string shows a 1 and the second parent gene if the string shows a 0. The gaussian mutation is just the adoption of the mutation technique of Evolutionary Programming. The gaussian mutation was modified to be compliant with a discrete type problem: randomly generated values according to gaussian distribution are rounded to the closest integer value so that they can be used in problems with discrete domain.

The same strategy and genetic operators mentioned above were used in the third algorithm, Genetic Based Algorithm with Gradient Search. But selected individuals in current generation are repaired by a Gradient-based method described below. To get the global optimum, this gradient method is used to maintain diversity and provide guidance towards the global optimum. Since diversity has been recognized as playing the potential role which allows the exploration of the search space[13], it is certainly a wise decision to use gradient method to increase the diversity for the search of global optimal but not to be determinant in optimum solution finding (otherwise, the solution procedure might be trapped in local optima).

Since the reactive power was treated as the main control variable for minimizing losses, it was necessary to select a relationship between changing reactive power and system losses. Changing of system losses can also be assumed as active power changing at reference bus; load is a constant value for the system and the incremental change in loss can be expressed as follow:

$$\Delta P_{LOSS} = \Delta P_G \tag{10}$$

where $\Delta P_{LOSS}$ = incremental change in system power losses

$\Delta P_G$ = incremental change in active power at ref. bus

Then, using the sensitivity coefficients of the reference bus derived from the nodal power injection equations and the inverse of the Jacobian of the Newton-Raphson method, the gradient vector of losses with respect to reactive power changing at bus i is given as:

$$\frac{\Delta P_{loss}}{\Delta Q_i} = \sum_{i=1}^{n} \frac{\partial P_G}{\partial \delta_i}\frac{\partial \delta_i}{\partial Q_i} + \sum_{i=1}^{n} \frac{\partial P_G}{\partial V_i}\frac{\partial V_i}{\partial Q_i} \tag{11}$$

The gradient vector of losses with respect to reactive power changing is used to find out a possible better reactive power setting in the current system status; the following equation is used:

$$Q_i^{new} = Q_i^{old} - \alpha \frac{\Delta P_{loss}}{\Delta Q_i} \tag{12}$$

where

$Q_i^{old}$ = reactive power at bus i

$Q_i^{new}$ = new reactive power setting at bus i after a gradient vector move

$\alpha$ = positive scale factor

During the procedure, the gradient search is included when necessary to increase the diversity, to overcome a possible local optimum and to move solutions in the right direction towards the global optimum solution without disturbing the evolutionary nature of the algorithm. The gradient search is only activated when the fitness of the current individuals is worse than a specified value (see in Figure 1).

The best individual's fitness value of a previous generation is selected as the judging value to decide which individuals of the current generation are to be repaired. The selection of such threshold value depends on the tradeoff between how-much-time-we-can-spend and how-good-results-we-want-to-achieve.

If the best fitness value of such a previous generation, close to the current generation, is used as the threshold value for repairing individuals, the number of individuals needing to be repaired are likely to be much higher than using the best fitness value of a previous generation, more distant from the current generation. The amount of individuals needing to be repaired in the current generation is dependent on the threshold value selection since the algorithm is evolving and the average fitness values are improving generation by generation.

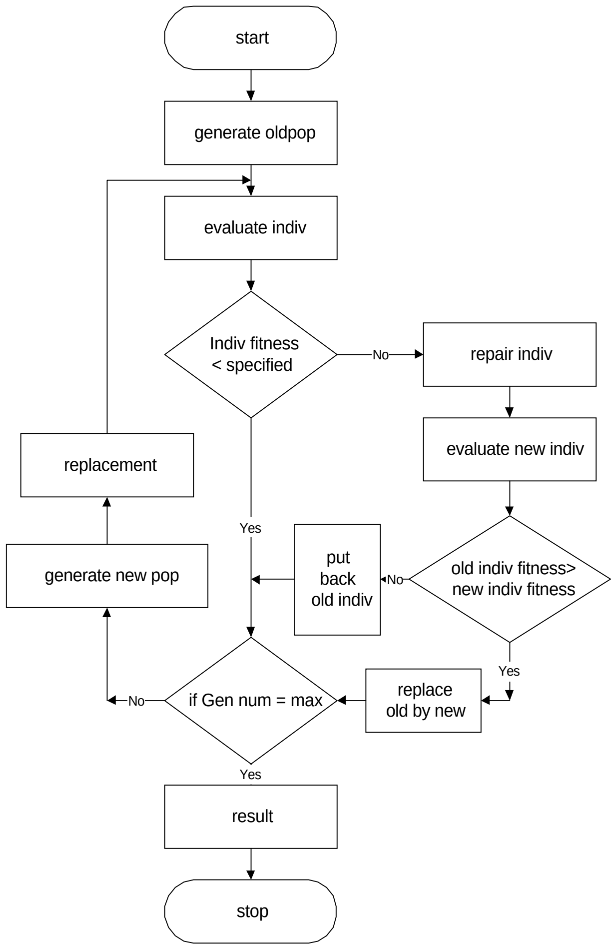


Figure 1. Flowchart of GBAGS

Although the gradient method was included in the algorithm, a full utilization of gradient method to find the new reactive power setting is not advisable: a) rounding off the gradient vector is necessary because of the discrete nature of the reactive power setting; and b) such a procedure risks to get trapped in local optima.

## III. IMPLEMENTATION AND RESULTS

The system shown in Figure 2 was used as the test case for this study. It is a simplified diagram from a real system in the Azores islands.

Only PQ buses were selected as candidate buses to install capacitors, reflecting the idea that capacitor adding at the reference bus or at voltage controlled buses would not affect power losses (one would have to include in the study the influence of voltage control, then). Capacitor banks were used as a reactive power supply to the system. Since a continuous formulation for capacitor sizes would not reflect the practical situation where capacity of capacitor banks is not changing continuously, a discretized capacitor size was used instead.

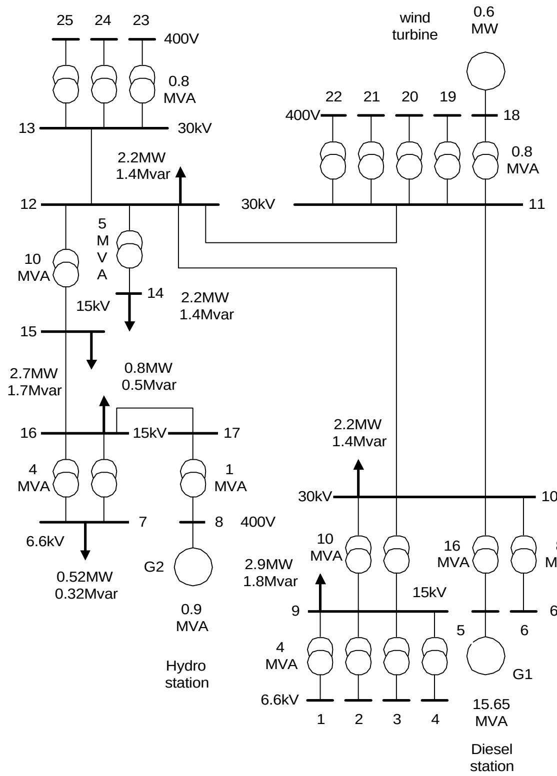


Figure 2. Single line diagram of test system

Two different types of capacitor bank were adopted: fixed capacitor bank, which provides the base amount of reactive power to the system, and switched capacitor bank.

The installation costs of 90 US$ and 150 US$ were used as the cost of each 100 kVAr installation of fixed and switched type respectively. Typical average marginal costs of 0.06 US$/kWh were taken as the energy cost for evaluating the costs due to energy losses.

Three different load levels, off-peak, peak and normal, were used to reflect a realistic situation of a load curve. In the cases of peak and off-peak load levels, a shorter time duration was selected comparing with time duration for the normal load level.

Both different and the same load for three load levels were used to test the ability of the algorithms. Satisfactory results were achieved from both tests. But the results from three load levels with the same load were selected to be presented in this paper, because they provide a quick check on the fact that fixed type capacitor installation is preferable over switched type for the flat load situation. Furthermore, they also provided a quick check on the quality of the solutions, because the Genetic algorithm should provide the same solution for all periods, if the load were the same for all of them. As we shall see, that is not always happening.

The size of population used for all three algorithms was 30 and typical values for crossover and mutation, 0.9 and 0.005, were used in creating SGA. But a higher mutation value (0.05) was found to work better for GBA and GBAGS. Each algorithm was run until a termination criterion was satisfied and 300 generations was chosen as a limit generation number. In this study and for GBAGS, the best individual of 10 generations before the current generation were taken as candidate to define the specified value for selecting individuals need to be repaired.

Each algorithm was run ten times and the fitness values were recorded for the 100th, 150th, 200th, 250th and 300th generation. And other important results such as losses, installation costs and voltage profile were also collected at the end of each run. Then, the average values of the results were calculated, providing the possible approximate results of the programs, and they were used to compare the performance of the algorithms.

The comparisons among the three algorithms were made based on fitness values, installation costs, costs of losses and voltage profile at buses. Average values, calculated from the results, are mainly taken as the parameters to compare the performance of algorithms in this study except for control setting of installed capacitors.

The comparison of fitness values has been made to highlight the overall performance of the algorithms. Figure 3 represents the average fitness value comparison of the three algorithms and it clearly indicates that gradual improvements have been achieved when moving from SGA to GBA to GBAGS. The comparison of the fitness value of the best run is shown in Fig 4 and having the same trend as in Figure 3 it indicates that average results are enough to reflect the overall performance of the algorithms.

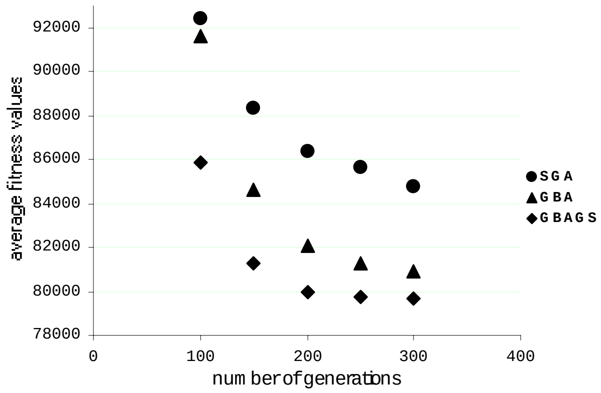


Figure 3. Average fitness value comparison

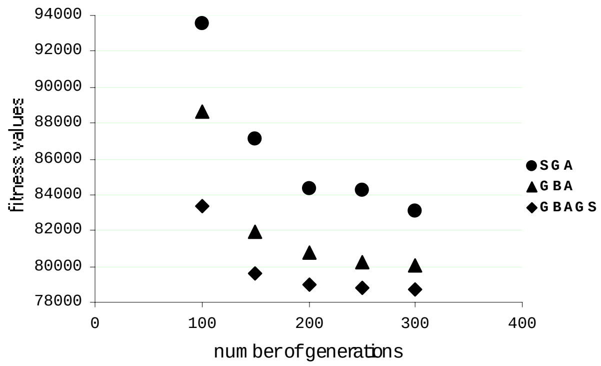


Figure 4. Fitness value comparison of best run

The performance of algorithms based on the average installation costs and costs of losses can be seen in Fig. 5. Although the costs of losses have not significantly changed for the three algorithms, a great achievement is obtained with capacitor installation with a monthly reduction of 48%, from 6283.3 US$ to 3254.8 US$, between the cases of system without and with capacitors. And another significant improvement is seen on the installation cost of capacitors: it is down to 6531 US$ from 10272 US$, reducing installation costs of 3741 US$ a 37 % reduction of the total installation cost between SGA and GBAGS.

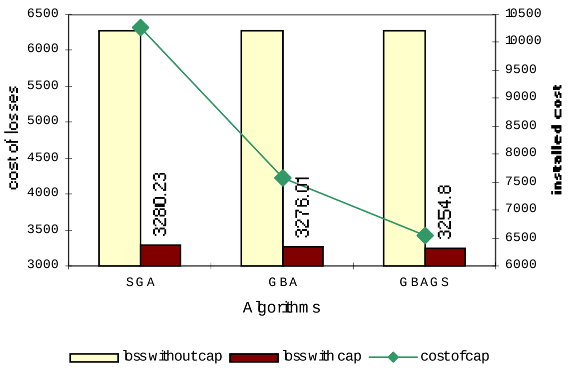


Figure 5. Economical performance of algorithms: monthly costs of losses and installation costs

As for voltage profile comparisons, the buses with voltage level outside limits in the initial situation (buses 7, 14, 15, 16, 17) were taken as the candidate buses for comparison of algorithms. The worst voltage profile, 0.8859 p.u., occurred at bus 14 in the initial system configuration. In Figure 6 we observe that even the worst voltage profile at bus 14 was improved up to within the specified limit of ±5% from nominal value; besides, all three algorithms showed the ability to improve the voltage profiles into limits with good voltage profile results achieved.

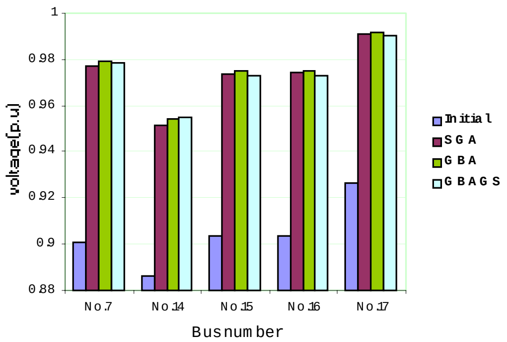


Figure 6. Voltage profile performance

Results from the best run were also used to investigate the performance of the algorithms. We see in Table 1 that costs of losses and installation costs were in the same trend as average results but switched type capacitors were installed with SGA even with flat load situation. It reminds us that the results we achieved with SGA were not good. In case of GBA and GBAGS, only fixed type capacitors were installed, indicating that the solutions achieved were logically correct. The control settings of the capacitors are shown in Table 2.

Table 1. Performance based on best run of three algorithms

| type | cost of losses (US$) | cost of caps (US$) | capacitor(Mvar) (switched) | capacitor(Mvar) (fixed) |
|---|---|---|---|---|
| SGA | 3278.5 | 9690 | 2.5 | 6.6 |
| GBA | 3282.14 | 7380 | 0 | 8.2 |
| GBAGS | 3231.68 | 6570 | 0 | 7.3 |

Table 2. Control setting of capacitor banks based on best run of three algorithms

| Bus | SGA | | | GBA | | | GBAGS | | |
|---|---|---|---|---|---|---|---|---|---|
| No | off | nor | peak | off | nor | peak | off | nor | peak |
| 1 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 2 | 0 | 0 | 2 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 2 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 6 | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 0 | 0 |
| 7 | 3 | 3 | 2 | 2 | 2 | 2 | 4 | 4 | 4 |
| 9 | 8 | 7 | 8 | 5 | 5 | 5 | 6 | 6 | 6 |
| 10 | 2 | 0 | 4 | 6 | 6 | 6 | 0 | 0 | 0 |
| 11 | 3 | 4 | 3 | 8 | 8 | 8 | 0 | 0 | 0 |
| 12 | 3 | 3 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 13 | 6 | 5 | 5 | 3 | 3 | 3 | 4 | 4 | 4 |
| 14 | 2 | 1 | 0 | 0 | 0 | 0 | 9 | 9 | 9 |
| 15 | 3 | 1 | 3 | 4 | 4 | 4 | 0 | 0 | 0 |
| 16 | 15 | 14 | 14 | 9 | 9 | 9 | 13 | 13 | 13 |
| 17 | 15 | 15 | 14 | 15 | 15 | 15 | 15 | 15 | 15 |
| 18 | 11 | 13 | 12 | 15 | 15 | 15 | 9 | 9 | 9 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 2 | 1 | 2 | 3 | 3 | 3 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Each integer number represents the number of 100 kVAr size capacitors in a bus; the capacity of fixed capacitor type was counted as the lowest integer number in each bus. The difference between the lowest and the highest number in each bus represents the quantity of switched type capacitors. Taking in account the results from Table 1, where costs of losses were decreasing while installed capacity of capacitors was also reducing, loss reductions were mainly achieved from efficient control setting of capacitors. The control setting of installed capacitors is shown in Table 2 and the unique setting, which provides the lowest costs of losses and installation cost with fewer buses used for capacitor installation was achieved with GBAGS.

## IV. CONCLUSION

This study illustrates how optimal results were step by step achieved for the capacitor placement and control problems with genetic algorithms.

In the case of the simple genetic algorithm (SGA), the bus voltage profiles reach an acceptable level, losses have been reduced up to almost 48% but a large amount of capacitors was needed to achieve this result. The algorithm proposed switched capacitor banks even if a flat load was used to test the convergence of the algorithms. Furthermore, the SGA could not provide a coherent solution for all load periods, in the case of the flat load profile. A SGA approach is therefore, clearly not acceptable.

A significant improvement was achieved with the Genetic Based Algorithm (GBA) approach. With GBA, the required capacity of capacitors was dramatically reduced, no switched type capacitors were installed and greater loss minimization was obtained. The installation cost in capacitors was significantly reduced and the results were logically correct for the flat load case.

Further improvement was seen when the gradient search was included in the GBA to repair or improve chromosomes, forming GBAGS. Both losses and installation costs were reduced further, indicating that better control settings were found with this algorithm. The inclusion of the gradient search in the genetic algorithm was only as a tool for providing a higher quality within diversity - it did not disrupt the evolutionary characteristics of the method, and it was not used as a simple post-optimization heuristic.

Although there must be other concerns with the technical problem of capacitor installation (such as resonance with harmonic frequencies), it is obviously attractive to achieve loss minimization and voltage profile control with minimum investment cost if the installation is planned with an efficient algorithm.

This paper illustrates that hybrid mathematical-genetic algorithms, where the information content of mathematical models is used to reinforce the exploration of the solution space by the evolutionary procedure, have the greatest potential to address problems of this kind.

## ACKNOLEDGEMENTS

The authors would like to thank the financing obtained from the Portuguese PRAXIS research fund. The second author would also like to thank the Open Society Institute for the funding they have contributed for his study.

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