OPTIMIZATION STUDIES IN DISTRIBUTION NETWORKS INCLUDING RELIABILITY CALCULATIONS

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ABSTRACT

There are a few papers presenting mathematical models for the optimal (or near optimal) automatic design of radial networks. In this paper, we present a methodology that allows taking in account the operation costs (calculated from reliability indices) in the definition of the objective function to be minimized. A small example illustrates the technique and justifies the mathematical model presented.

1. INTRODUCTION

Low voltage networks represent an important fraction of the investment dealt with by electric power utilities. However, few works have been published about the optimization or reliability studies in low voltage systems, especially when compared with the number of publications on the generation and transmission systems.

The classical approach to network optimization is to minimize investments through the definition of the organization and structure of the network (choice of locations for LV substations, branch layout, line compositions, etc.).

The assessment of the quality of supply provided by such design is left to the experience and feeling of the engineers, which leads to the introduction *a posteriori* of changes such as the inclusion of open loops to allow eventual alternative supplies.

The non inclusion of operation costs resulting from the chosen system configuration may lead to unacceptable results. These costs depend on the network reliability and may be evaluated by quantifying the unavailability of supply and the average annual energy not supplied, reflecting both social costs incurred and additional repair and maintenance costs.

In this paper, we aim at establishing an integrated mathematical model for the computational calculation of radial distribution networks, with application in new urban areas.

The conditions of operation and reliability indices have deserved a special attention; therefore, we present an improved definition of the supply restoration time and new reliability indices [1], aiming at obtaining the closest possible representation of reality.

2. CRITICAL REVIEW OF RELIABILITY INDICES

The reliability assessment approach that has more success in the representation of real systems, although recent, has been the object of a number of publications [2, 3, 4]. It quantifies the reliability of a power system departing from each of its elements. The reliability indices considered are:

λ_I - failure rate of component i

r_i - mean repair time of component i

$U_i \approx \lambda_I r_i$ - unavailability of component i

An analysis of published works leads us to the conclusion that an identification is usually made between mean repair time and mean service restoration time. This identification, although valid in concentrated systems, doesn't seem to be in agreement with a true picture of power systems with a wide geographical dispersion such as distribution networks.

There are, in fact, two important contributions to the service restoration time following a failure. The first is the mean repair time of the component; this may be estimated from a statistical analysis of a set of similar components, and may be admitted as equal for all components installed or operated in similar conditions. The second depends mainly on peripheral circumstances, such as the particular location of a component, the type and geographical dispersion of the system including it, the efficiency of the repair crews of each utility, etc. In fact, it represents the time for failure detection, decision making and access to the location of the failed component.

It is in general accepted that the behavior of a component may be evaluated from the study of the behavior of a large set of similar components; this means that one year of useful life of 100 similar components are taken as equivalent to 100 years of useful life of a single component.

This reasoning seams adequate for the repair of a component, but it is not for the mean service restoration time - this latter time depends in a great extent on the second contribution referred to above. One should therefore contribute to create data bases on distribution network component behavior. This same reason cautions us to adopt in one utility data from another one, or even the blind adoption of indices from one region of an utility into another region.

Proceedings of CIRED'79, Rap.59, session 6, ed. A.I.M., Liège, Belgium, 1979

2.1. Mean power disconnected and energy not supplied [1]

The knowledge of failure rates, mean repair times and unavailabilities of a system id not enough to asses the quality of supply. The example in makes it clear.

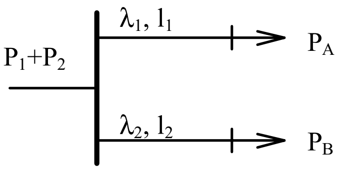


Figure 1

We take the relations

$l_1 = l_2$; $\lambda_1 = 2\ \lambda_2$; $P_A = 4\ P_B$

(λ_I and L_i represent the failure rate and the mean restoration time of line i).

Which one is the most reliable node, A or B?

The reliability calculation gives

$$U_A = \lambda_1 l_1 \qquad \frac{U_A}{U_B} = 2$$
$$U_B = \lambda_2 l_2$$

(we take the origin as 100% reliable)

The average power disconnected in A will be P_a^m and in B, P_B^m. Admitting that the loads in A and B are of the same type, we can calculate the energy not supplied in each node due to failures

$$E_A = U_A P_A^m \qquad \frac{E_A}{E_B} = 0.5$$
$$E_B = U_B P_B^m$$

Therefore:

- from the client point of view, bus B is the best one (its unavailability is half of A)
- from the utility and social cost points of view, the priority of investments may go to bus B.

This example with its contradictory conclusions is enough to demonstrate the importance of the adoption of indices E_i and P_i in reliability assessments.

2.2. Definition of reliability indices

We present now the definition of the reliability indices adopted in this paper:

λ_I - failure rate of component i

λ_i^a - active failure rate of component i

r_I - mean repair time of component i

S_j - mean switching time of component j (to isolate component i following a failure)

A_I - mean decision and access time to component i

l_I - mean restoration time following a failure in component i

P_I^m - mean power disconnected due to failures in component i

U_I - unavailability of component i

E_I - mean annual energy not supplied due to failures in component i

The following relations are accepted among these indices

$$U_i = \lambda_i l_i$$
$$E_i = \lambda_i l_i P_i^m = U_i P_i^m$$
$$l_i = r_i + A_i + S_j$$

(in certain cases, A_i and S_j may be very small or zero so that $l_i \approx r_i$).

3. OPTIMIZATION WITH RELIABILITY CALCULATIONS

3.1. General comments

From the analysis of the optimization criteria adopted by utilities, for the design of new urban networks, we verify that often the choice of the final configuration depends on the initial investment necessary. However, results produced by computer programs do not agree with traditional practice; it is well know the reluctance of planners to change the cross section of lines, believing preferable to pay the extra cost of prolonging a larger cable to introducing a "weak" point in the network. The optimal option cannot be found unless with the inclusion and quantification of the quality of supply assured to customers by the final design, with global minimization of initial investment and operation costs during the useful life of the system. Valuing the energy not supplied is the path adopted in this paper to attain such objective.

3.2. Kirchhoff Law

Hindi and Brameller developed an algorithm for the optimal design of urban LV distribution networks[5]; their basic model to represent a network is adopted in this paper. Customers are admitted as evenly distributed along supplying lines (fixed paths). The load is then shared between the two nodes delimiting each branch, taking in account diversity factors.

Therefore, for each node j one has a load L_j.

There are also branches that do not supply ant clients (optional paths). As in each path the power may flow in any direction, we represent a line by two opposed directed branches, with flows F_k and F_k'.

The Kirchhoff law is given by

$$\sum_{i=1}^{n_{PT}} w_{ij} P_i + \sum_{k=1}^{N} T_{kj}\left(F_k - F_k^{'}\right) = L_j \quad , \quad j = 1,\ldots,M \tag{1}$$

M - total number of nodes (reference excluded)

N - total number of branches

n_{PT} - number of possible locations for LV substations

P_i - power at LV substation i

F_k - power flow in directed branch k

$$W_{ij} = \begin{cases} 0 & \text{if } P_i \text{ does not touch node j} \\ 1 & \text{if } P_i \text{ enters node j} \end{cases}$$

$$T_{kj} = \begin{cases} 0 & \text{if } F_k \text{ does not touch node j} \\ +1 & \text{if } F_k \text{ enters node j} \\ -1 & \text{if } F_k \text{ leaves node j} \end{cases}$$

In matrix form

$$\left[W \vdots T\right]\begin{bmatrix} P \\ \cdots \\ F \end{bmatrix} = \left[L\right] \tag{2}$$

where [W T] is a very sparse matrix.

3.3. Costs

The cost of each LV substation is represented by an expression such as $C_i = a_i + b_i P_i$ and the cable laying costs are given by $c_k = d_k + e_k F_k$, while keeping all information about the real costs. It is easy to include the cost of power losses in conductors as well.

3.4. Initial investment

We define two Boolean variables σ and π associated with each possible location for a LV substation and for each optional path. Each variable will have value 1 if the location or path is chosen; if not, its value will be 0.

The limits for power flow are given by

$$\begin{aligned} &0 \le P_i \le \sigma_i P_i^{max} && i = 1,\ldots,n_{PT} \\ &0 \le F_k, F_k^{'} \le \pi_k F_k^{max} && k = 1,\ldots,n_{op} \\ &0 \le F_k, F_k^{'} \le F_k^{max} && k = n_{op}+1,\ldots,N \end{aligned} \tag{3}$$

n_{op} - number of optional paths

The initial investment will be

$$\begin{aligned} D = &\sum_{i=1}^{n_{PT}}\left(\sigma_i a_i + b_i P_i\right) + \sum_{k=1}^{n_{op}}\left[\pi_k d_k + e_k I_k\left(F_k - F_k^{'}\right)\right] + \\ &+ \sum_{k=n_{op}+1}^{N}\left[d_k + e_k I_k\left(F_k - F_k^{'}\right)\right] \end{aligned} \tag{4}$$

with $I_k = \begin{Bmatrix} 1 & , & \text{if } F_k > F_k^{'} \\ -1 & , & \text{if } F_k \le F_k^{'} \end{Bmatrix}$

3.5. Objective function

The objective function to be minimized is

$$\Phi_1 = D + V_c \sum_{m=1}^{n_{PT}} \sigma_m \left[P_m^m U_m^{'} + \sum_{j=1}^{Sd_m} P_{mj}^m \left(U_j^{''} + U_j^{'''}\right)\right] + QV_Q \tag{5}$$

where

I - D is the initial investment (4)

II - V_c - capitalized cost of the kWh not supplied

III - $U_m^{'} = \sum_{i=1}^{nc_m} \lambda_i l_i$ - unavailability due to failures in the LV substation m (breakers excepted)

λ_I - failure rate of component i of the LV substation (MV equipment, transformer, busbars, etc.)

$l_i = r_i + A_{PT}$ - mean time of restoration of supply

nc_m - number of series components in LV substation m

IV - $U_j^{''} = \sum_{i=1}^{t_{mj}} \lambda_{mji} l_{mji}$ - Unavailability resulting from failures in cables or other series elements in the network and from the breaker at line j from LV substation m

λ_{mij} - failure rate

$l_{mij} = r_{mij} + A_{mij}$ - mean restoration time

t_{mij} - number of components in line j from LV substation m

V - $U_j^{'''} = \sum_{\substack{s=1 \\ s \ne j}}^{Sd_m} \lambda_{ms}^{a} S_m$ - unavailability of line i from LV substation m, as a consequence of active failures in the line protections of other lines

VI - P_{mj}^m - average power disconnected in line j of LV substation m

P_m^m - Average power disconnected for LV substation m

Q^m - Number of T-derivations or unions with change of cross section

The constraints of this problem are (2) and (3). Moreover, all flow values must be non-negative.

3.6. Partial solutions

We can define the following sets

I - set of integer variables

I_1 - set of integer variables with value 1

I_u - set of integer variables with undecided value

If during the procedure of solving the optimization problem we fix at some stage some integer variables to 0 or 1, we have another optimization problem which has a minimum equal or higher than problem (5), because there are less degrees of freedom:

$$\operatorname{Min} \Phi_2 = D_2 + V_c \sum_{m \in I_1} \left[P_m^m U_m' + \sum_{j=1}^{Sd_m} P_{mj}^m \left(U_j'' + U_j''' \right) \right] + V_c \sum_{m \in I_u} \sigma_m \left[P_m^m U_m' + \sum_{j=1}^{Sd_m} P_{mj}^m \left(U_j'' + U_j''' \right) \right] \quad (6)$$

where

$$D_2 = \sum_{m \in I_1} \left(a_m + b_m P_m \right) + \sum_{m \in I_u} \left(\sigma_m a_m + b_m P_m \right) + \sum_{k \in I_1} \left[d_k + e_k I_k \left(F_k - F_k' \right) \right] + \sum_{k \in I_u} \left[\pi_k d_k + e_k I_k \left(F_k - F_k' \right) \right] + \sum_{k=n_{op}+1}^{N} \left[d_k + e_k I_k \left(F_k - F_k' \right) \right] + QV_Q$$

with constraints

$$\begin{array}{ll} 0 \le P_{mi} \le P_m^{max} & m \in I_1 \\ 0 \le P_m \le \sigma_{mi} P_m^{max} & m \in I_u \\ 0 \le F_k, F_k' \le \pi_k F_k^{max} & k \in I_u \\ 0 \le F_k, F_k' \le F_k^{max} & k \in I_1 \cup k = n_{op}+1, \dots, N \end{array} \quad (7)$$

$\pi_k, \sigma_m = 0$ or 1 , but unknown for the moment

3.7. Lower bounds

By replacing the cost in each path by a new cost function ($\gamma_m P_m$, $\delta_k F_k$) we allow a continuous variation of π and σ between 0 and 1. If w also ignore the existence of active failures in line protections and the existence of unions and t-joints, and if we also replace in each path or LV substation its average power disconnected by a value proportional to the power flow, we face a new problem resulting from the relaxation of the constraints of problem (6) - δ and γ are the slopes of the straight lines between the origin and the maximum cost in the graphs of path costs or LV substations.

Among the average powers disconnected and the power flows we can establish the relations

$$P_k^m, P_k^{'m} = \alpha_1 F_k, \alpha_1 F_k' \quad ; \quad P_m^m = \alpha_2 P_m$$

Here is the formulation of the lower bound problem:

$$\operatorname{Min} \Phi_3 = \sum_{m \in I_1} \left(a_m + b_m P_m \right) + \sum_{m \in I_u} \left(\gamma_m P_m \right) + \sum_{k \in I_1} \left[d_k + e_k I_k \left(F_k - F_k' \right) \right] + \sum_{k \in I_u} \left[\delta_k I_k \left(F_k - F_k' \right) \right] + \sum_{k=n_{op}+1}^{N} \left[d_k + e_k I_k \left(F_k - F_k' \right) \right] + \sum_{k \in I_u \cup I_1} \left[V_c \lambda_k l_k I_k \left(P_k^m - P_k^{'m} \right) \right] + \sum_{m \in I_1 \cup I_u} U_m' P_m^m \quad (9)$$

or then

$$\operatorname{Min} \Phi_3 = \sum_{m \in I_1} \left(a_m + \left(b_m + V_c U_m' \alpha_2 \right) P_m \right) + \sum_{m \in I_u} \left(\gamma_m + V_c U_m' \alpha_2 \right) P_m + \sum_{k \in I_1} \left[d_k + \left(e_k + V_c \lambda_k l_k \alpha_1 \right) I_k \left(F_k - F_k' \right) \right] + \sum_{k \in I_u} \left[\left(\delta_k + V_c \lambda_k l_k \alpha \right) I_k \left(F_k - F_k' \right) \right] + \sum_{k=n_{op}+1}^{N} \left[d_k + \left(e_k + V_c \lambda_k l_k \alpha_1 \right) I_k \left(F_k - F_k' \right) \right] \quad (10)$$

with constraints

$$\left[W \,\vdots\, T \right] \begin{bmatrix} P \\ \cdots \\ F \end{bmatrix} = [L]$$

$$0 \le P_{mi} \le P_m^{max} \qquad m \in I_1$$

$$0 \le F_k, F_k^{'} \le F_k^{max} \qquad k \in I_1 \cup k = n_{op} + 1, \ldots, N$$

The domain of the solutions of this problem is convex. Its minimum is smaller or equal to the minimum of problem (6). In fact, this is a capacitated transhipment problem, for which there are well known efficient solving procedures from the point of view of speed and computer memory.

3.8. Solving procedure

The search for an optimal solution follows a branch and bound procedure [6] in a binary decision tree where each node represents a transhipment problem and each branch represents an integer variable fixed to 0 or 1.

The nodes and branches are created during the process and the solution at each node is checked for feasibility. The root of the tree (S_0) is problem (10) with full constraint relaxation (no integer variables fixed). Fixating some integer variables leads to more constrained problems. At each node, the feasibility of problem (10) is checked and its cost calculated. The cost of the first feasible solution found (having, besides mathematical feasibility, a radial structure and including all fixed paths) is taken as reference. When we find a solution of type (10) with cost higher than the reference, we abandon the search from that tree node. The search is also abandoned from any node with unfeasible solution.

The real costs control the evolution of the process. These costs are used for each fixed path or optional path selected; furthermore, the cost of any integer feasible solution is calculated with all real costs.

To take in account voltage drop limits, one must memorize the best n solutions and submit them to a post-optimization procedure for optimal cross section correction.

The value n is based on experience, aiming at improving the chance of obtaining the global optimum. In any case, before and after this post-optimization procedure, we may verify the interest in having or not distinct cross sections in each line, considering the costs of unions and the possible reduction in reliability resulting.

3.9. Inclusion of open loops

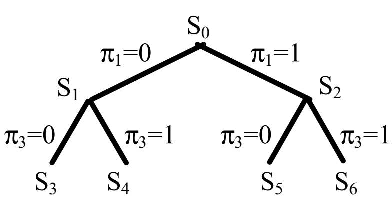


Figure 2 - Decision tree

The inclusion of an open loop in the network has a large impact in the reduction of the unavailability of the supply, especially for failures in LV substations; in many cases, these failures constitute a major fraction of the motives that lead to interruption of supply.

As there is an inverse relation between the cost of establishing open loops and the value of energy not supplied, we cannot beforehand determine the direction of change of the objective function. This problem is dealt with in the following manner:

- in every node of the decision tree where an integer feasible solution is found, calculate its cost C_I
- apply to the solution a procedure to include open loops
- calculate the cost C_c of such alternative
- If $C_c > C_I$, abandon the new solution
- If $C_c \le C_I$, replace the old by the new solution, unless the former is the reference - then, keep both solutions
- the reference solution is always a design with no loops
- a list of the best n solutions is kept excluding the reference; this may be added to the list, in the end of the process, if its cost is smaller that most expensive of the n best ones.

4. PRACTICAL EXAMPLE

To illustrate the solution method proposed, we present a small example. Its configuration and data are in Figure 3 and Tables 1 to 4. We have not considered voltage drop limits to keep the problem simple and clear. We have just considered the expansion of the LV network of the substation.

Table 1 - Loads

Node	L1, L2	L3, L4	L5, L8	L6, L7
Load (kW)	0	80	40	60

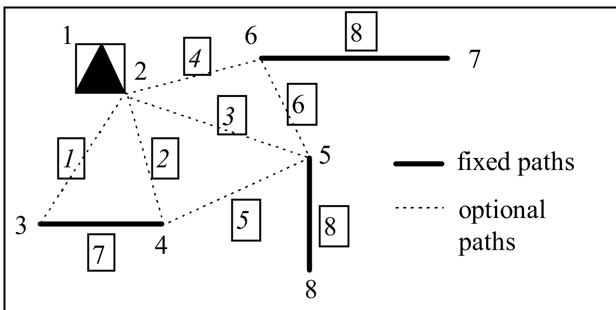


Figure 3

Table 2 - Paths

Type	n.	length (m)	Access time (h)
Fix	7	114	2
	8	150	2
	9	70	3.5
LV sub	-		2
Optional	1	69	1.5
	2	80	1.5
	3	40	2
	4	77	2
	5	80	2
	6	67	3.5

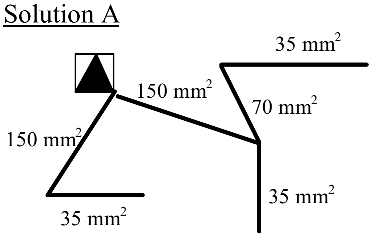
Table 3 - Costs

	PTE ($)
Capitalized value of the kWh not supplied, at 25% interest rate and 20 years	201.65
Isolators	7 500
Unions, t-joints	2 500
Breakers	3000
Cables NAYBY (/m)	840 + 2.77 P

Table 4 - Reliability Indices

	λ (/year)	r (h)	λ^a (/year)
Breakers	0.005	20	0.0025
Cables (/m)	5.3×10^{-5}	26	5.3×10^{-5}
Unions, t-joints	0.005	26	0.005
isolators (open)	0.005	30	0.005

Solution A



I - NO reliability calc.

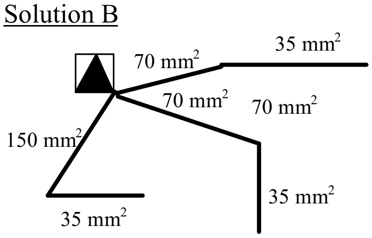
Total cost

C_A' - 545 304.00

C_B' - 545 814.00

ΔC_I - 510.00

Solution B



II - WITH reliability calc.

Mean annual energy not supplied

E^m_A - 193.75 kWh

E^m_B - 129.52 kWh

Average power disconnected

P^m_A - 129.52 kW $\quad$ P^m_B - 83.75 kW

Cost of interruptions of supply

C_A'' - 39 070.00 $\quad$ C_B'' - 30 573.00

ΔC_{II} - 8 497.00

III - Comparison between solutions

$C_A = C_A' + C_A'' = 584\ 374.00$

$C_B = C_B' + C_B'' = 576\ 387.00$

$\Delta C = 7\ 987.00$

5. CONCLUSIONS

In this paper we have presented a new formulation of the problem of the optimal design of distribution networks. The methodology takes in account the initial investment and the operation costs by quantifying the quality of supply.

The model has been developed as a computing procedure. The difference of PTE 8.000 between the optimal solutions without and with reliability calculations (in an example with only three optional paths used) justifies the progress of research in this area.

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Proceedings of CIRED'79, Rap.59, session 6, ed. A.I.M., Liège, Belgium, 1979