Optimal Power Flow Analysis using Fuzzy Supply and Fuzzy Demand Functions

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ABSTRACT. The Optimal Power Flow (OPF) problem has been stated traditionally to optimize the electrical power system operation regarding minimal cost criteria. In the scope of the implementation of market mechanisms in supply and demand side, OPF analysis becomes in a useful tool to determine location marginal prices in order to send efficient economic signals to all market agents. Marginal pricing allows the allocation of power losses and congestion in the transmission network. In this scenario, production costs and elastic response of demand are uncertain. The model proposed in this paper uses Fuzzy Sets to represent demand and supply cost functions to deal with OPF problem. Membership functions are obtained related to optimum production strategies and consumption profiles. Fuzzy marginal prices are derived from economic interpretation of dual variables of the problem. The model has been tested for illustration proposes in a simply test case.

Keywords: Fuzzy optimal power flow, power system economics, price elasticity, transmission access pricing.

LIST OF SYMBOLS AND ACRONYMS

$\tilde{A}$	Fuzzy set
α	Cut level of hard fuzzy set $\tilde{A}_\alpha$
$\tilde{A}_\alpha$	Hard fuzzy set
a', b'	Demand curve coefficients
β	Price elasticity of demand (MWh^2/€)
b	Relative slope of demand curve (€/MWh^2)
B	Benefit function (€/h)
c', d', e'	Supply cost curve coefficients.
Δ	Fuzzy deviations
i	Subindex: node i
jk	Subindex: line jk
F	Production cost function (€/h)
G	Cost of quantity not supplied (€/MWh)
λ	System lambda (MWh)
ρ	Electricity prices (€/MWh)
Q_D	Power quantity demanded (MWh)
Q_G	Power quantity produced (MWh)
Q_{ij}	Power quantity flow associated to line ik (MWh)
Q^{max}_{jk}	Maximum power flow allowed in line ik (MWh)
Q^{max}_{G}	Maximum power production allowed (MWh)
QNS	Quantity of power not supplied (MWh)
$\mu_{\tilde{A}}$	Membership function of a fuzzy set Ã
γ_{jk}	Sensitivity coefficients associated to line jk (MWh)
x	Element of a fuzzy set Ã
OPF	Optimal Power Flow
FOPF	Fuzzy Optimal Power Flow
~	Fuzzy operator

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1. INTRODUCCION

The genesis of Optimal Power Flow problem began in 1962 when Carpentier [1] introduced a non-linear programming formulation of the economic dispatch problem regarding minimal production cost criteria, including operating constraints. Later, several approaches of OPF problem have been proposed to deal with different cost functions to be optimized [2][3]. Some of these techniques have derived in flexible and reliable on-line OPF applications. However, the crisp treatment of the problem imposes falls sometimes in unrealistic suppositions. Many authors worldwide have addressed the uncertainty integration in power production and demand in the OPF problem using fuzzy set approaches [4][5].

In fact, in the scope of the implementation of market mechanisms in supply and demand of electric industry, OPF studies become today in a useful tool to determine location marginal prices, in order to allocate transmission capital costs, power losses and congestion charges among all market agents. Hence, uncertainties in marginal production costs and demand response behavior should be taken into account [6].

This work proposes a Fuzzy Set representation of demand and supply functions to deal with OPF problem. Membership functions are obtained related to optimum production strategies and consumption profiles. Linear demand and marginal production curve are achieved from quadratic customer benefit and production cost function respectively. The model has been tested for illustration proposes in a simply test case.

2. FUZZY SETS BASIC CONCEPTS

A fuzzy set Ã is described by a membership function $\mu_{\tilde{A}}(x)$ [7] relating each element x to its compatibility or association degree in Ã:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\} \quad (1)$$

These degree ranges of membership function $\mu_{\tilde{A}}(x)$, in normalized fuzzy sets, goes from 0 to 1 leading to a gradual transition between a complete belonging of x to $\tilde{A}$ and no belonging of x to $\tilde{A}$. We can describe an α-*cut* of a fuzzy set $\tilde{A}$ like a hard set A_α defined in X for each $\alpha \in [0,1]$ such that:

$$\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\} \quad (2)$$

A fuzzy number – as the presented in Fig. 1 – is a convex fuzzy set defined on the real semi-plane such that $\mu_{\tilde{A}}(x)$ is normalized between 1-cut and 0-cut and piece wise continuous. A trapezoidal fuzzy number as shown in Fig. 1 is specified declaring an interval $[x^a, x^b, x^c, x^d]$ and can be used to represent the uncertainty in the interval $[x^a, x^d]$ with different degree as function of α-cut level

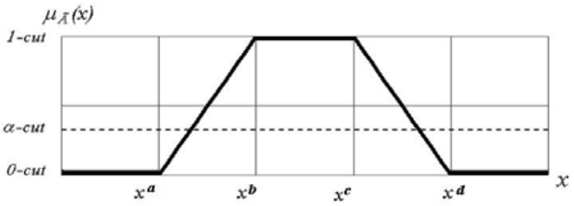

Figure 1. Trapezoidal fuzzy number.

The central value of trapezoidal fuzzy number is the average of its 1-cut level, corresponding to the mean of x^b and x^c.

2. DEMAND-SIDE UNCERTAINTY

Presently, demand is almost completely insensitive to price in most electric power markets because price fluctuations are not usually passed on to retail consumers. Nevertheless, competitive retailers shall be forced to implement progressively real-time pricing. In this scenario, demand becomes responsive to price in concordance with Marshallian consumer theory [8].

When electricity markets are fully liberalized consumers become exposed to volatile prices ρ and may decide to adjust their demand profile Q_D to reduce electricity costs. This demand-price relationship may be illustrated by the curve showed in Fig. 2.

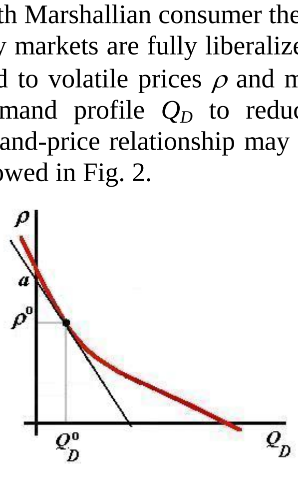

Figure 2. Demand curve.

This curve is difficult to quantify and often linearized around a given point (ρ^0, Q^0_D). Then, we can define the price elasticity of demand as follow [8]:

$$\beta = \frac{\Delta Q_D / Q^0_D}{\Delta\rho / \rho^0} \quad (3)$$

We will assume that all prices and demands have been normalized with respect to a given equilibrium point in order to obtain linear demand curves [9]. Therefore, the inverse of elasticity can then be written as the relative slope of this curve as:

$$b = \frac{\Delta\rho}{\Delta Q} = \frac{a - \rho^0}{Q^0_D} \quad (4)$$

and, the linear demand function can be expressed as:

$$\tilde{Q}_D(\rho) = \frac{a - \rho}{b} = a' + b'\rho \quad (5)$$

Uncertainty in linear demand function is incorporated using a' and b' factors as fuzzy numbers [6]. Then, a fuzzy demand function can be expressed as follows.

$$\tilde{Q}_D = \tilde{a}' - \tilde{b}'\tilde{\rho} \quad (6)$$

where

$$\tilde{a}' = a'^0 + (\Delta^a_a : \Delta^b_a : \Delta^c_a : \Delta^d_a) \quad (7)$$

$$\tilde{b}' = b'^0 + (\Delta^a_b : \Delta^b_b : \Delta^c_b : \Delta^d_b) \quad (8)$$

Right-hand parameters Δ represent these uncertainties as deviations from a central value a^0 and b^0. Fig. 3 shows graphically four different approaches of the fuzzy demand functions:

a) Inelastic response. Load does not change respect to price [10].

$$\tilde{Q}_D = Q^0_D + (\Delta^a_Q : \Delta^b_Q : \Delta^c_Q : \Delta^d_Q) \quad (9)$$

b) Uncertainty in consumer income shifts the linear demand curve preserving the same incremental behavior [11].

$$\tilde{Q}_D(\rho) = \frac{\tilde{a} - \tilde{\rho}}{b} = \tilde{a}' + b'\tilde{\rho} = (a' + \Delta_a) + b'\tilde{\rho} \quad (10)$$

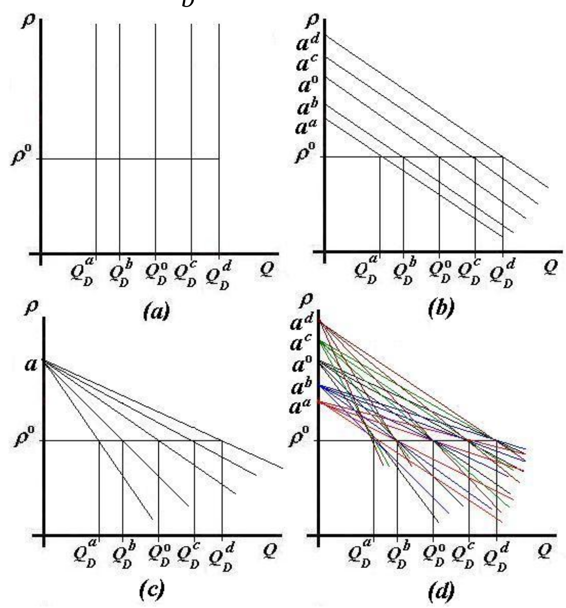

Figure 3. Fuzzy demand curves.

c) Consumers income is fixed and they are not in agreement with paying more than a maximum price. Then, consumer responds elastically changing their consumption pattern [11].

$$\tilde{Q}_D(\rho) = \frac{a - \tilde{\rho}}{\tilde{b}} = a' + \tilde{b}'\tilde{\rho} = a' + (b' + \Delta_b)\tilde{\rho} \qquad (11)$$

d) Consumers income and price sensitivity are uncertain.

$$\tilde{Q}_D(\rho) = \frac{\tilde{a} - \tilde{\rho}}{\tilde{b}} = \tilde{a}' + \tilde{b}'\tilde{\rho} = (a' + \Delta_a) + (b' + \Delta_b)\tilde{\rho} \qquad (12)$$

Finally, for each fuzzy demand function we may apply some algebra to obtain the fuzzy customer benefit function using the classic definition [8]:

$$\tilde{B}(\tilde{Q}_D) = \int \tilde{\rho}\, d\tilde{Q}_D \qquad (13).$$

3. PRODUCTION COST UNCERTAINTY

Traditional power load dispatch with thermal generators utilizes quadratic functions to represent the total production costs as function of the power quantity produced Q_G [12].

$$F(Q_G) = \frac{c}{2}Q_G^2 + dQ_G + e \qquad (14)$$

Hence, marginal production costs can be described as linear functions. Then, uncertainty can be incorporated considering c and d factors as fuzzy numbers:

$$\tilde{\rho}(\tilde{Q}_G) = \frac{\partial F}{\partial \tilde{Q}_G} = \tilde{c}\tilde{Q}_G + \tilde{d} \qquad (15)$$

where

$$\tilde{c} = c^0 + (\Delta_c^a : \Delta_c^b : \Delta_c^c : \Delta_c^d) \qquad (16)$$

$$\tilde{d} = d^0 + (\Delta_d^a : \Delta_d^b : \Delta_d^c : \Delta_d^d) \qquad (17)$$

Parameters Δ represents these uncertainties as deviations from a central value c^0 and d^0. Fig. 4 shows graphically four different approaches of the fuzzy marginal production cost functions

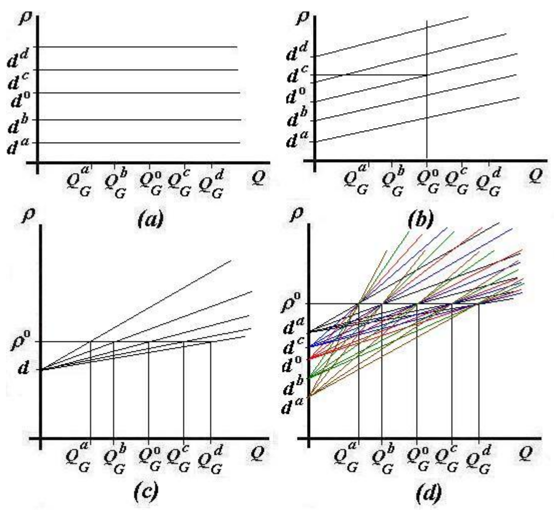

Figure 4. Fuzzy marginal production cost curves.

a) Marginal production costs are fixed and do not be changed as function of power output [12].

$$\tilde{\rho}(\tilde{Q}_G) = \tilde{d} = d^0 + (\Delta_d^a : \Delta_d^b : \Delta_d^c : \Delta_d^d) \qquad (18)$$

b) Uncertainty in fixed marginal costs shifts the linear production curve preserving the same incremental behavior [8].

$$\tilde{\rho}(\tilde{Q}_G) = \tilde{c}\tilde{Q}_G + d = (c + \Delta_c)\tilde{Q}_G + d \qquad (19)$$

c) Marginal production costs are fixed. It does not be affected by the output of plant [12].

$$\tilde{\rho}(\tilde{Q}_G) = c\tilde{Q}_G + \tilde{d} = c\tilde{Q}_G + (d + \Delta_d) \qquad (20)$$

d) Consumer’s income and price sensitivity are uncertain.

$$\tilde{\rho}(\tilde{Q}_G) = \tilde{c}\tilde{Q}_G + \tilde{d} = (c + \Delta_c)\tilde{Q}_G + (d + \Delta_d) \qquad (21)$$

4. THE FUZZY OPTIMAL POWER FLOW PROBLEM

A Fuzzy Optimal Power Flow methodology (FOPF) is an optimization procedure to build membership functions of dependent variables of problem identified by an economic criterion under uncertainty. Aggregating the objective and constraints, the problem can be mathematically formulated as a linear constrained optimization problem [10]:

$$min\ f(\tilde{Q}_G, \tilde{Q}_D) \qquad (22)$$

$$subject\ to\ \ g(\tilde{Q}_G, \tilde{Q}_D) = 0 \qquad (23)$$

$$h(\tilde{Q}_G, \tilde{Q}_D) \le 0 \qquad (24)$$

where g and h are the problem constraints

4.1 Fuzzy OPF with uncertainty in demand side.

The proposed methodology in based on a fuzzy linear formulation explained in detail by [10][13]. The basic idea consist in minimize global social cost; i.e. total production cost plus quantity not supplied cost due network constraints minus total consumer benefit, integrating a global balance and operation limits constraints.

$$min \sum_i F(Q_{G_i}) + G\sum_i \tilde{Q}NS_i - B(Q_{D_i}) \qquad (25)$$

subject to:

$$\sum_i \tilde{Q}_{G_i} = \sum_i \tilde{Q}_{D_i} - \sum_i \tilde{Q}NS_i \quad i = 1,...n \qquad (26)$$

$$-Q_{jk}^{max} \le \sum_i \gamma_{jk}^i (\tilde{Q}_{G_i} - \tilde{Q}_{D_i} + \tilde{Q}NS_i) \le Q_{jk}^{max} \qquad (27)$$

$$i, j, k = 1,...,n$$

$$\tilde{Q}_{G_i} \le Q_{G_i}^{max} \quad i = 1,...,n \qquad (28)$$

The resolution of FOPF with fuzzy demand functions is addressed using multiparametric linear programming. This process is explained in detail in [14] and flowchart process is shown in Fig. 3. It starts with a crisp OPF, using (25-28) equations Uncertainties are integrated as deviations regarding the correspondent central value. To obtain generation membership functions a multi-parametric optimization problem is formulated and vertex identification process must be made.

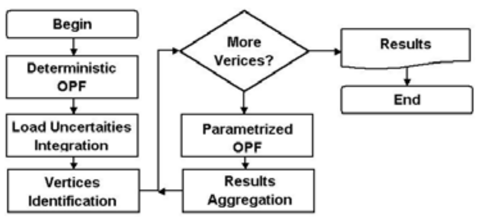

Fig. 5. Basic Fuzzy Optimal Power Flow Algorithm

The production injections and demand consumption membership functions are obtained aggregating the solutions of these parametric studies. Fuzzy local marginal prices are associated to system lambda obtained from dual variable of global balance constraint of primal problem [14].

4.2 Fuzzy OPF with uncertainty in production costs.

The resolution of OPF problem considering uncertainty in production costs should be obtained using previous algorithm applying a dual formulation of optimization problem stated in (29-31).

Primal FOPF

$$\min\ (cost + \Delta_{cost})^t\, x \qquad (29)$$

$$\text{subject to}\ \ Ax \le b \qquad (30)$$

$$x \ge 0 \qquad (31)$$

Dual FOPF

$$\max\ wb \qquad (32)$$

$$\text{subject to}\ \ wA \le (cost + \Delta_{cost}) \qquad (33)$$

$$w \ge 0 \qquad (34)$$

Therefore, deviations in production costs are integrated in dual problem constraints and primal solution are obtained from the weak duality property [15].

5. A SIMPLY TEST EXAMPLE

A simply two-bus transmission network without losses is shown in Fig. 6.

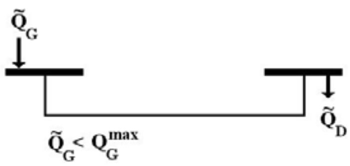

Fig. 6. Two-bus example

FOPF study consists in obtaining the membership functions of production and demand consumption under un-certainty in production costs and demand response. Additionally, a fuzzy system lambda is obtained from Lagrange multiplier associated with global balance con-straint, given us information about system congestion. The cost of the quantity not supplied G is equal to 2 and maximum capacity of line is 1.3.

5.1 Demand is a fuzzy number and marginal production cost is a deterministic value

Demand and supply values are given by:

$$\tilde{Q}_D = \tilde{a}' - 0\rho = (0.6;0.8;1.2;1.4) \qquad (35)$$

$$\tilde{\rho}(\tilde{Q}_G) = 0\tilde{Q}_G + d = 1 \qquad (36)$$

and the crisp OPF problem is stated as:

$$\begin{aligned} &\min \quad dQ_G + G.QNS \\ &st. \quad Q_G + QNS = a'^0 + \Delta_a \\ &Q_G \le Q_G^{max} \quad Q_G, QNS \ge 0 \end{aligned} \qquad (37)$$

Membership functions associated to power quantity produced Q_G, quantity not supplied QNS and system lambda λ are showed in Fig. 7.

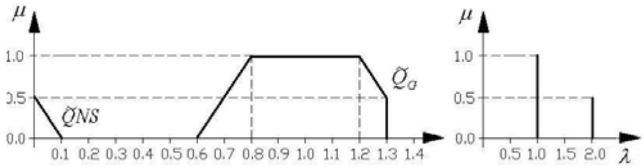

Fig. 7. Case N° 1 - Membership functions.

Note that limits in transmission network produce a cut in the production membership function and an increase of quantity not served. Obviously, this condition derive in shift in the system lambda from d=1.0 to G=2.0.

5.2 Demand and marginal production cost are fuzzy numbers.

Demand and supply values are given by:

$$\tilde{Q}_D = \tilde{a} - 0\tilde{\rho} = (0.6;0.8;1.2;1.4) \qquad (38)$$

$$\tilde{\rho}(\tilde{Q}_G) = 0\tilde{Q}_G + \tilde{d} = (0.8;0.9;1.1;1.2) \qquad (39)$$

and the crisp OPF problem is stated as:

$$\begin{aligned} &\min \quad (d + \Delta_d)Q_G + G.QNS \\ &st. \quad Q_G + QNS = a'^0 + \Delta_a \\ &Q_G \le Q_G^{max} \quad Q_G, QNS \ge 0 \end{aligned} \qquad (40)$$

For each value of a', a dual of OPF problem is resolved:

$$\begin{aligned} &\max \quad aw + Q_G^{max}\gamma \\ &st. \quad w + \gamma = d + \Delta_D \quad and \quad w \le G \quad \gamma \le 0 \quad w\ free \end{aligned} \qquad (41)$$

Aggregating the results the membership functions, associated to power quantity produced Q_G, quantity not supplied QNS and system lambda λ are obtained and showed in Fig 8.

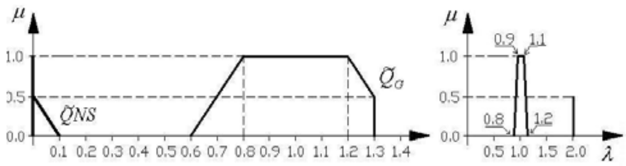

Fig. 8 Case N° 2 - Membership functions.

As seen before, the limits in transmission network produce a cut in a production membership function and an increase of quantity not served. However, this condition derives in a change in the system lambda from the membership function of production cost, d to the cost of quantity not supplied, G.

5.3 Demand is a fuzzy function and marginal production cost is a deterministic value

Demand function and supply value are given by:

$$\tilde{Q}_D = a' - \tilde{b}'\tilde{\rho}, \quad a' = 2, \quad \tilde{b}' = (0.6;0.8;1.2;1.4) \qquad (42)$$

$$\tilde{\rho}(\tilde{Q}_G) = 0\tilde{Q}_G + d = 1 \qquad (43)$$

For vertex analysis, the Crisp OPF problem is stated as follows:

$$\begin{aligned} &\min \quad dQ_G + G.QNS - \int \rho\, dQ_D \\ &st. \quad Q_G + QNS = Q_D = a' - (b'^o + \Delta_b)\rho \\ &Q_G \le Q_G^{max} \quad Q_G, Q_D, \rho, QNS \ge 0 \end{aligned} \qquad (44)$$

Aggregating the results, the membership functions are showed in Fig 9.

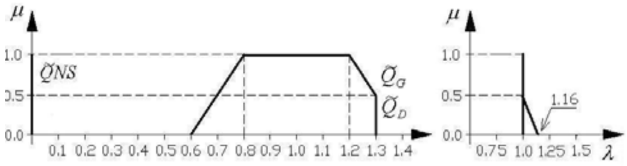

Fig. 9 Case N° 3 - Membership functions.

In this case, demand and production have the same membership function and no congestion condition is achieved. Quantity not supplied is equal to zero and system lambda goes from 1.0 to 1.16 as indicated in Q-λ diagram illustrated in Fig. 10.

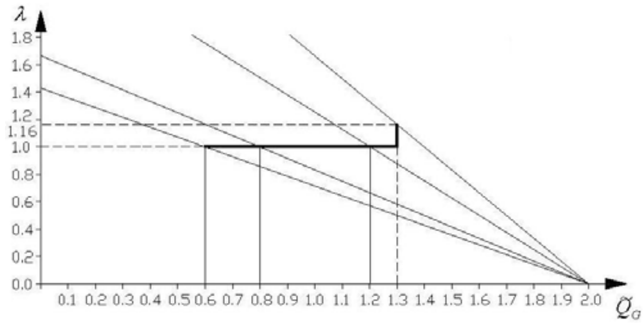

Fig. 10 Q-λ diagram for Case N° 3

6. CONCLUSIONS

In this paper, a fuzzy set model is proposed to represent uncertainty in demand and supply cost functions to deal with Optimal Power Flow problem. Membership functions are obtained related to optimum production strategies and consumption profiles. Fuzzy marginal prices are derived from economic interpretation of dual variables of the problem. The model has been tested for illustration proposes in a simply test case.

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