

# Optimal Reactive Power Provision of Wind Farms in Liberalized Markets – A Generation Viewpoint

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**Abstract--** This paper proposes a methodology for the optimal management of the reactive power of non-dispatchable distributed generators. The optimization problem is stated from the generation utility viewpoint, aiming to improve the revenue in the active power sale and considering a power loss allocation policy based on incremental approach. The proposed methodology permits to evaluate the impact of reactive power injection in the active locational marginal prices of each generation bus. Both pricing and power quality policies imposed by the regulatory board are considered in the formulation. This model is suitable to be applied in the real time operation of the distributed generators. The proposed strategy maximizes the individual revenue, without changes in the active power output. The model has been tested and results are discussed from an illustrative distribution test system.

**Index Terms--** Distributed Generation, Embedded Generation, Reactive Pricing, Reactive Power, Optimization.

## I. NOMENCLATURE

$\rho$	locational marginal price;
$\lambda$	system market price;
$\eta$	tariff of use of network;
$L$	total active power losses;
$L_{Gi}, L_{Di}$	losses allocated to generators and loads of bus $i$ ;
$P_{Gi}$	active power output of generators of bus $i$ ;
$P_{Di}$	active power input of demands of bus $i$ ;
$Q_{Gi}$	reactive power output of generators of bus $i$ ;
$Q_{Di}$	reactive power input of demands of bus $i$ ;
$V_i, \theta_i$	voltage and angle of bus $i$ ;
$\theta_{ij}$	difference between angles $i$ and $j$ ;
$G_{ij}, B_{ij}$	real and imaginary part of admittance matrix $Y$ ;
$n$	number of buses.
$sl$	slack bus
$ILF^P$	incremental loss factors for active power;
$ILF^Q$	incremental loss factors for reactive power;
$R, X$	line resistance and reactance;
$B_{CAP}$	half of total line charging susceptance;
$S_{BASE}$	Power base;

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## II. INTRODUCTION

**T**HE INCORPORATION of dispersed power producers embedded into the medium voltage distribution networks is transforming the conception of how distribution utilities should be accessed, operated and regulated [1]. Distributed resources has been facing a heavy growth in the recent years and an important debate has been raised about whether distributed energy injections must be charged or rewarded by their contribution to the increase or reduction of system losses. Traditionally, distribution losses are considered as an additional load and allocated among all consumers, using average values [2]. However, in liberalized environment, regulator aspires to know the contribution of each utility in the system losses, aiming to distribute their cost among the energy transactions in a fair way. Different algorithms to allocate the total losses among generators and consumers [3-7], have been proposed. Comparisons between some of them can be found in [8-9].

The implementation of a loss allocation policy requires an access-pricing framework based on locational or uniform prices, affecting the income of distributed producers. In most of the European systems, the active power production of wind farms (like others renewable generators) is not dispatched and it is strongly related to the wind availability. Generally, active power controllability is rather difficult or not economically interesting for this kind of generators. Several system operators (as in Spain) require a communication of the forecasted active power of wind farms in a short time horizon [11].

As the active power generation, the reactive power of distributed power producers is not expressly dispatched. However, quality standards related to reactive generation are often required (for example, maximum range of voltage in the generation bus, power factor average, and others) in European systems.

This paper proposes a new initiative for the determination of how much reactive power must be supplied by a wind farm to the distribution system, in order to improve the revenue in the active power sale. In the formulation, a loss allocation policy based on hourly locational marginal prices of active power [3] and the power quality constraints imposed by the regulatory board are considered.

The active power generation of the wind farm depends of both wind availability and efficiencies of the plant. In contrast, wind farm operator can to control the reactive power production, inducing a modification of the active power locational price at the generation bus.

The proposed framework is as a tool for the individual-revenue maximization, encouraging non-dispatched distributed resources to be economically efficient under incremental loss-allocation policies.

In this first formulation, the strategic interactions between non-dispatchable producers are not considered. In the solution of the optimization problem for each distributed generator, it is assumed that the reactive supplies of the other agents or competitors are specified, measured or estimated. Different possible scenarios are represented and discussed, in a test case network.

Following, the regulatory framework considered in this work is presented.

### III. REGULATORY FRAMEWORK

In the present work, the regulatory framework applied to the market agents includes two sets of characteristics: economic and quality ones. Firstly, the economic framework observes that active energy injected by distributed generators and consumed by loads must be remunerated under a wholesale energy market, at the clearing system price.

Incremental loss factors (ILF's) are computed and applied to all market agents in form of hourly or half hour locational marginal prices, in order to allocate the network power losses [3]. Locational marginal prices are applied after the operation, as an *ex-post* economic mechanism for losses recovery. In the formulation, it is not required the use of loss factors in all the buses; equivalent or uniform values of these factors can be used in the most of demand buses, using the concept of uniform marginal pricing at demand-side [10].

Regarding quality rules, it is considered that all distributed generators connected to the grid must supply reactive power to maintain the voltage in the generation buses between standard limits. Others quality system requirements can be also represented in the model.

#### A. Economic Framework

Under incremental analysis, power losses are allocated to producers and consumers through incremental loss factors. By definition, these coefficients measure the change in the power losses due to the incremental change of the power injections in each bus [12-13]:

$$ILF_i^P = \frac{\partial L}{\partial (P_{Gi} - P_{Di})} \quad (1.a)$$

$$ILF_i^Q = \frac{\partial L}{\partial (Q_{Gi} - Q_{Di})} \quad (1.b)$$

At a given operation point, the ILF's are computed by means of the Newton-Raphson Jacobean of an AC power flow solution, using the chain rule [3]:

$$\begin{bmatrix} ILF^P \\ ILF^Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_1} & \dots & \frac{\partial P_n}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_2}{\partial \theta_1} & \dots & \frac{\partial Q_n}{\partial \theta_1} \\ \frac{\partial P_1}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_2} & \dots & \frac{\partial P_n}{\partial \theta_2} & \frac{\partial Q_1}{\partial \theta_2} & \frac{\partial Q_2}{\partial \theta_2} & \dots & \frac{\partial Q_n}{\partial \theta_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_1}{\partial \theta_n} & \frac{\partial P_2}{\partial \theta_n} & \dots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial Q_1}{\partial \theta_n} & \frac{\partial Q_2}{\partial \theta_n} & \dots & \frac{\partial Q_n}{\partial \theta_n} \\ \hline \frac{\partial \theta_1}{\partial P_1} & \frac{\partial \theta_2}{\partial P_1} & \dots & \frac{\partial \theta_n}{\partial P_1} & \frac{\partial \theta_1}{\partial Q_1} & \frac{\partial \theta_2}{\partial Q_1} & \dots & \frac{\partial \theta_n}{\partial Q_1} \\ \frac{\partial \theta_1}{\partial P_2} & \frac{\partial \theta_2}{\partial P_2} & \dots & \frac{\partial \theta_n}{\partial P_2} & \frac{\partial \theta_1}{\partial Q_2} & \frac{\partial \theta_2}{\partial Q_2} & \dots & \frac{\partial \theta_n}{\partial Q_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \theta_1}{\partial P_n} & \frac{\partial \theta_2}{\partial P_n} & \dots & \frac{\partial \theta_n}{\partial P_n} & \frac{\partial \theta_1}{\partial Q_n} & \frac{\partial \theta_2}{\partial Q_n} & \dots & \frac{\partial \theta_n}{\partial Q_n} \\ \hline \frac{\partial V_1}{\partial P_1} & \frac{\partial V_1}{\partial P_2} & \dots & \frac{\partial V_1}{\partial P_n} & \frac{\partial V_1}{\partial Q_1} & \frac{\partial V_1}{\partial Q_2} & \dots & \frac{\partial V_1}{\partial Q_n} \\ \frac{\partial V_2}{\partial P_1} & \frac{\partial V_2}{\partial P_2} & \dots & \frac{\partial V_2}{\partial P_n} & \frac{\partial V_2}{\partial Q_1} & \frac{\partial V_2}{\partial Q_2} & \dots & \frac{\partial V_2}{\partial Q_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial V_n}{\partial P_1} & \frac{\partial V_n}{\partial P_2} & \dots & \frac{\partial V_n}{\partial P_n} & \frac{\partial V_n}{\partial Q_1} & \frac{\partial V_n}{\partial Q_2} & \dots & \frac{\partial V_n}{\partial Q_n} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \theta_1} \\ \frac{\partial L}{\partial \theta_2} \\ \vdots \\ \frac{\partial L}{\partial \theta_n} \\ \frac{\partial L}{\partial V_1} \\ \frac{\partial L}{\partial V_2} \\ \vdots \\ \frac{\partial L}{\partial V_n} \end{bmatrix} \quad (2)$$

where,

$$\frac{\partial P_i}{\partial \theta_j} = V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (3)$$

$$\frac{\partial P_i}{\partial \theta_i} = -B_{ii} V_i^2 - \sum_{j=1}^n V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (4)$$

$$\frac{\partial P_i}{\partial V_j} = V_i [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (5)$$

$$\frac{\partial P_i}{\partial V_i} = G_{ii} V_i + \sum_{j=1}^n V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (6)$$

$$\frac{\partial Q_i}{\partial \theta_j} = -V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (7)$$

$$\frac{\partial Q_i}{\partial \theta_i} = -G_{ii} V_i^2 + \sum_{j=1}^n V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (8)$$

$$\frac{\partial Q_i}{\partial V_j} = V_i [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (9)$$

$$\frac{\partial Q_i}{\partial V_i} = -B_{ii} V_i + \sum_{j=1}^n V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (10)$$

$$\frac{\partial L}{\partial \theta_i} = 2 \sum_{j=1}^n V_i V_j G_{ij} \sin(\theta_i - \theta_j) \quad (11)$$

$$\frac{\partial L}{\partial V_i} = 2 \sum_{j=1}^n G_{ij} [V_i - V_j \cos(\theta_i - \theta_j)] \quad (12)$$

In the linear system (2), the columns and rows associated to the slack bus are eliminated. The implementation of the ILF's requires an access price framework for active and reactive power. The economic framework established in this paper only considers locational marginal prices of active power (in \$/MWh). The application of locational marginal prices of reactive power will be considered in further analyses.

The active power provided by the distributed generator is remunerated at the wholesale market price. As this price does not directly depend on the loss allocation policy, its behavior is specified in the following analyses. The remuneration obtained by consumers and producers at the market system or spot price is modified through a specific locational price, in order to recover the cost of the losses in the operation.

Hence, locational marginal prices  $\rho$  are computed for each bus, as a function of the active ILF (equation 1.a) and the system market price  $\lambda$ :

$$\rho_i = \lambda(1 + ILF_i^P) \quad i=1, \dots, n \quad (13)$$

The additional term of the energy price ( $\eta_i = \lambda \cdot ILF_i^P$ ) aims to remunerate either produced or avoided power losses, related to the agents connected at bus  $i$ .

The polarity of ILF's must be interpreted in accordance with the following criteria

$$\text{When } ILF_i^P > 0; \rho_i > \lambda \quad (14.a)$$

$$\text{When } ILF_i^P < 0; \rho_i < \lambda \quad (14.b)$$

As shown in (14.a), when the  $ILF_i^P$  is positive, each generator connected to bus  $i$  is rewarded for the contribution to the decrease of the total losses of the system, obtaining for the active power production a locational price greater than the market price. On the other hand, when the  $ILF_i^P$  is negative (equation (14.b)), the generators connected to this bus are charged for increasing the total losses in the system operation, being remunerated by a locational price lower than the market price.

As observed in equations (2) and (13), the incremental loss factors depend on the reactive provision of all generators and loads. In consequence, a deviation in the reactive injection of either generator or load leads to a change in the active locational marginal prices, applied to the system agents. This circumstance affects the incomes and revenues expected by all market agents, particularly to the distributed generators.

### B. Power Quality Framework

As previously said, it is assumed that the regulatory board allows wind farms to modify their reactive power injection, while the voltage in the bus connection is enforced between the specified limits.

## IV. PROPOSED METHODOLOGY

In the real-time operation, distributed producer can access the network parameters and to estimate the demands and generations in the surrounding area of his plant. Generally, this information can be acquired from the system operator or (when available) through the monitoring process (SCADA) of the system. Besides this, the non-dispatchable producer knows the methodology that will be used (*ex-post*) by the system operator to allocate the losses. In this paper, it is supposed that this loss allocation is performed using active incremental loss factors, computed under a real-time after the operation [3]. In this scenario, the producer is able to calculate in real-time (for instance, in each half hour) the optimum value of reactive power generation, in order to modify its active locational price improving the wind farm profit.

The optimization problem is established as the estimation of an optimum level of reactive injection  $Q_{Gk}$ , in order to maximize the individual revenue of producer  $k$  and considering the operational constraints: active and reactive power balance equations and the voltage restrictions in all buses. It is required to include the calculation of the ILFs, through equality constraints.

The model is presented as follows in equations (15-20):

$$\begin{aligned} & \max \\ & \min \end{aligned} \rho_k P_{Gk} = \lambda(1 + ILF_k^P) P_{Gk} \quad (15)$$

s.t.

$$P_{Gi} - P_{Di} = V_i \sum_{j=1}^n V_j [G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}] \quad i=1, \dots, n \quad (16)$$

$$Q_{Gi} - Q_{Di} = V_i \sum_{j=1}^n V_j [G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}] \quad i=1, \dots, n \quad (17)$$

$$\sum_{j \neq sl}^n \frac{\partial P_j}{\partial \theta_i} ILF_j^P + \sum_{j \neq sl}^n \frac{\partial Q_j}{\partial \theta_i} ILF_j^Q = \frac{\partial L}{\partial \theta_i} \quad i=1, \dots, n \quad i \neq sl \quad (18)$$

$$\sum_{j \neq sl}^n \frac{\partial P_j}{\partial V_i} ILF_j^P + \sum_{j \neq sl}^n \frac{\partial Q_j}{\partial V_i} ILF_j^Q = \frac{\partial L}{\partial V_i} \quad i=1, \dots, n \quad i \neq sl \quad (19)$$

$$V_i^{min} \leq V_i \leq V_i^{max} \quad i=1, \dots, n \quad i \neq sl \quad (20)$$

Specified Data:

$$P_{Gi}; i=1, \dots, n \quad i \neq sl$$

$$Q_{Gi}; i=1, \dots, n \quad i \neq sl \quad i \neq k$$

$$V_k^{max}; V_k^{min}; P_{Di}; Q_{Di}; i=1, \dots, n; V_{sl}; \theta_{sl}$$

Variables:

$$Q_{Gk}; P_{Gsl}; Q_{Gsl};$$

$$V_i; \theta_i; ILF_i^P; ILF_i^Q; i=1, \dots, n \quad i \neq sl$$

The optimization problem (15)-(20) is specified through  $(4n-2)$  equality constraints and  $(4n-1)$  variables. The specified data are: the active power of all distributed generators (including the active power of producer  $k$ ); the reactive power supply injected by the others generators; the active and reactive load demands; the voltage limits in all the buses; and both voltage module and angle of the slack node. As result of the optimization problem, the reactive provision of generator  $k$ , both active and reactive injections at slack bus, the incremental factors, and the voltages and angles in all the buses (excepting the slack one) are calculated.

The objective function (15) corresponds to the sum of the energy sold to the market at system market price ( $\lambda P_{Gk}$ ) and the loss revenue ( $\lambda P_{Gk} ILF_k^P$ ). The first term of the objective function is constant. The system market price ( $\lambda$ ) is established for the wholesale market. The wind farm supply ( $P_{Gk}$ ) are related to the wind power available at node  $k$ . In the second term of (19), the  $ILF$  of the bus depends of the reactive power injected by the wind farm in the operation. To identify the possible range of the revenue by losses, due to the reactive power injection strategy, the optimum reactive injection  $Q_{Gk}$  is calculated for both maximum and minimal revenue objectives. Equations (16) and (17) represent the power balance in each bus of the network. Expressions (18) and (19) were derived from equations (1.a), (1.b) and (2), to calculate the incremental loss factors for active and reactive power. The expression (20) is related to regulatory constraints. Each distributed producer is able to freely modify its own reactive injection  $Q_{Gk}$ , when accomplishing the bus voltage limits  $V^{max}$  and  $V^{min}$  specified by the regulatory board. However, to execute the dispatch, the system operator must guarantee that the voltage in all the buses is constrained between the standard limits. For this, equation (20) considers voltage limits in all the buses of the system. The reactive generation limits of the wind farm are not included in the model. Reactive power injection at the generation bus could be included using the generator reactive capability or, when economically attractive, adding local VAr compensation.

## V. RESULTS

The proposed methodology has been tested in an illustrative 4-bus test network with two wind farms. Initially, an optimal power flow is performed, in order to obtain the active and reactive dispatch for minimal power losses conditions. This operational point is used in the following analysis as a substitute of the dispatch performed by the system operator to the next interval.

However, other different operation point could be used instead. From this operational point, the proposed formulation (15-20) is applied, calculating both maximal and minimal revenues of Wind Farm 2 and Wind Farm 3. Finally, a sensitivity analysis was performed to calculate the behavior of the optimal revenue of each wind farm, when reactive power injected by the other distributed generator is modified.

In Fig. 1, the test case is presented. Each wind farm has 3MW of installed capacity. The load is connected to bus 4 (5.0 MW) and remains unchanged for all simulations. The load represented in bus 4 can be the unique demand in the system or an equivalent demand bus.

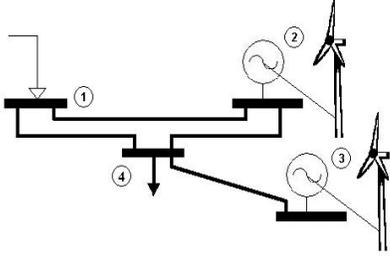


Fig. 1. The 4-bus test network

All lines have  $R = 0.4$  pu,  $X = 0.15$  pu and  $B_{CAP} = 0.02$  pu, considering  $S_{BASE} = 10$ MVA and  $V_{BASE} = 10$ kV. Bus 1 corresponds to the interconnection bus with the transmission system and it is used as slack bus. Voltage operation level has been established between 90% and 110%, in all buses.

### A.- Minimal active power losses point

In order to obtain an initial operation point, an optimal power flow is performed, aiming to minimize the total power losses, as shown in equation (21). The constraints of this optimization problem are the balance equations for active and reactive power, equations (16) and (17), and the allowable voltage limits (equation 20).

$$\min L = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)] \quad (21)$$

For a minimum of losses in the system, the operational point is shown in Table I.

TABLE I  
OPTIMAL POWER FLOW RESULTS FOR MINIMIZATION OF POWER LOSSES

Node	$P_G$		$Q_G$		$P_D$		$Q_D$		V	$\theta$	ILF <sup>P</sup>	Revenue		
	MW	MVAr	MW	MVAr	pu	rad	-	Energy				Losses	Total	
								\$/h				\$/h	\$/h	
1	1.61	-0.42	-	-	1.000	0.000	0	16.11	0.00	16.11				
2	1.75	-0.06	-	-	1.006	0.000	0	17.54	0.00	17.54				
3	1.77	0.04	-	-	1.011	0.000	0	17.67	0.00	17.67				
4	-	-	5.00	0.00	0.982	-0.069	0.0536	-50.0	-2.68	-52.68				

In Table I,  $P_G$  (node 1) represents the energy imported from the transmission system and bought at the system market price ( $\lambda = 10$ \$/MWh). In the operational point showed in this

table, the power injected by wind farms 2 and 3 are remunerated at the same system market price, because both wind farms do not pay or receive incomes due to power losses ( $ILF = 0$ ). It is observed that all the active power losses (130.9 kW) are charged to the load, in bus 4.

### B.- Minimum and maximum revenue of the producers

In Table II, the results of the proposed strategy to maximize and minimize the revenue of the Wind Farm 2 are shown. The active and reactive power of Wind Farm 3 and the active power production of Wind Farm 2 are settled in the initial operational point (Table I), 1.77 MW, 0.04 MVAr and 1.75 MW, respectively.

TABLE II  
RESULTS FOR MAXIMUM AND MINIMUM REVENUES OF WIND FARM 2

		min Revenue WP2	max Revenue WP2
Reactive Power $Q_{G2}$	MVAr	-2.13	3.80
Active Power $P_{G2}$	MW	1.75	1.75
Voltage $V_2$	pu	0.946	1.100
ILF <sup>P</sup> <sub>2</sub>	-	-0.00086	0.00427
Loss Revenue: $\lambda \cdot P_{G2} \cdot ILF^P_2$	cts\$/h	-1.5081	7.4933
Global Power Losses	kW	182.36	261.34

In this case, an increment in the reactive power generation of Wind Farm 2 (from -0.06 MVAr in the initial operational point to 3.8 MVAr) modifies the  $ILF^P_2$ , obtaining a revenue of 7.49 cts\$/h. On the contrary, a decrease in the reactive power generation (from -0.06 MVAr in the initial operational point to -2.13 MVAr) modifies the  $ILF^P_2$ , obtaining a negative revenue of -1.5 cts\$/h.

The loss-revenue of Wind Farm 2 ( $\lambda \cdot ILF^P_2 \cdot P_{G2}$ ) may derive in positive or negative values reflecting the use of the network according to the avoided or produced system losses. It must be stressed that the access-price policy does not modify the revenues obtained from wind energy sold to the market at the system market price ( $\lambda \cdot P_{G2}$ ).

Table III details the results of the proposed strategy, when applied to Wind Farm 3. The active and reactive powers of Wind Farm 2 are preset in 1.75 MW and -0.06 MVAr and the active power generation of Wind Farm 3 is fixed in 1.77 MW (operational point of Table I). As shown in Table III, the Wind Farm 3 improves his revenue in the active power trade in 0.83 %, changing the reactive power production from 0.04 to 1.48 MVAr.

TABLE III  
RESULTS FOR MAXIMUM AND MINIMUM REVENUE OF WIND FARM 3

		min Revenue WP2	max Revenue WP2
Reactive Power $Q_{G3}$	MVAr	-1.41	1.48
Active Power $P_{G3}$	MW	1.77	1.77
Voltage $V_3$	pu	0.900	1.100
ILF <sup>P</sup> <sub>3</sub>	-	-0.00965	0.00833
Loss Revenue: $\lambda \cdot P_{G3} \cdot ILF^P_3$	cts\$/h	-17.0504	14.7223
Global Power Losses	kW	204.29	178.86

The difference between an optimal strategy and the less favorable one is (17.05+14.72 = 31.77 cts\$/h), 1.80 % of the total revenue of the wind farm. This value is not negligible when compared with the total of active power losses in the system, value inferior to 2.0% of the active power generation in all simulations.

The Wind Farm 3 obtains maximum and minimum revenues (shown in Table III) if the generator in bus 2

supplies the reactive power required by the operational point (Table I). This situation can happen if the generator in bus 2 is dispatched. If this latter producer is a distributed (and non-dispatched) generator, the revenue in the optimal reactive power generation of Wind Farm 3 is variable, as function of the reactive power production of the generator allocated in bus 2. Similar analysis can be performed to Wind Farm 2. In the next section, the maximum and minimum revenues for each generator are calculated, as a function of the reactive power injection of the other generator.

*C. - Maximum and minimum revenue of the Wind Farm, as function of the reactive injection of the other generator*

In Fig. 2, is shown the maximum and minimum revenues of Wind Farm 2 whether the reactive power production of the Wind Farm 3 varies between  $-2.3$  and  $2.5$  MVar.

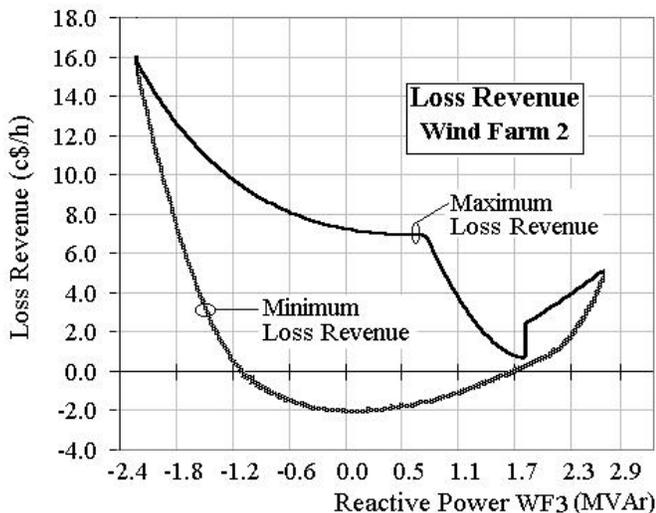


Fig. 2. Minimum and maximum loss revenue of Wind Farm 2, as a function of the reactive power injection in Wind Farm 3

The maximal difference between the best and the worst strategies of Wind Farm 2 (maximization and minimization of equation 15, respectively) is achieved when the Wind Farm 3 supplies  $-500$  kVar, with  $9.33$  cts\$/h. As shown in Fig. 2, in Wind Farm 2 the variation between maximum and minimum revenues diminishes whether the generator in bus 3 either increases or decreases the reactive power production.

The variation of reactive power injections of Wind Farm 3, in the interval  $[-2.3$  MVar,  $2.5$  MVar], implies that the voltage in bus 3 reaches the limits imposed by the regulatory board (0.9 and 1.1 pu, respectively).

It is observed that solutions have similar operational points for both maximizing and minimizing revenue strategies. In Fig. 3 is shown the variation in the revenues of Wind Farm 3, when reactive power generation of Wind Farm 2 varies from  $-4.5$  to  $4.9$  MVAR. Unlike Wind Farm 2, Wind Farm 3 is connected to the load in a radial form. Then, the maximum and minimum revenues for this producer are related to the allowable voltage limits in the generation bus.

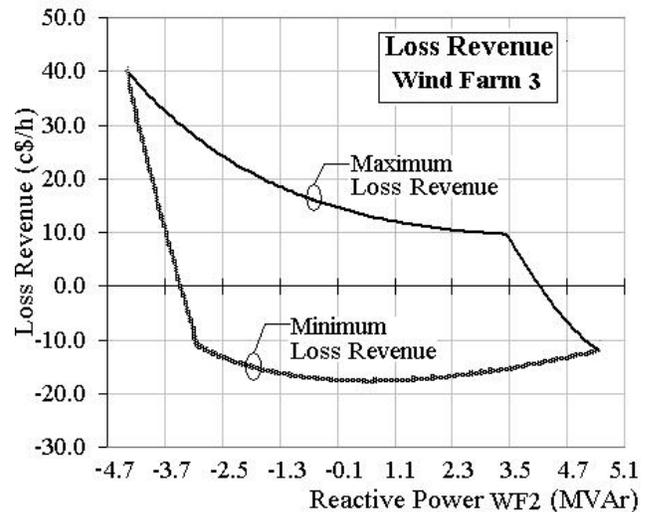


Fig. 3. Minimum and maximum loss revenue of Wind Farm 3, as a function of the reactive power injection in Wind Farm 2

When the maximum revenue of Wind Farm 3 is imposed, the voltage at bus 3 reaches the upper limit (1.1 pu) in all simulations. Correspondingly, if the minimal revenue is calculated, voltage at bus 3 is equal to 0.9 for all simulations. When the Wind Farm 2 increases the reactive power production, the system becomes more stressed and other voltage buses reach limits, decreasing the interval of possible action of the proposed strategy.

The difference between the best and worst operations varies from  $37.01$  to  $23.52$  cts\$/h, when reactive injection in Wind Farm 2 changes from  $-2.3$  to  $2.5$  MVar, respectively. This difference represents an average gain in the best operation of  $1.85\%$  of the total active power revenue of Wind Farm 3 (when compared with the worst operation), at market price.

## VI. CONCLUSION

This paper proposes a methodology based on a real-time calculation of the optimal reactive power provision to be supplied for non-dispatchable distributed generators. The producers are considered as market agents into a liberalized energy market and distribution losses are recovered through a loss allocation policy, based on hour or half-hour locational marginal prices computed under a real-time basis.

The optimization problem is established from the distributed power utility viewpoint, aiming the maximization of the individual revenue and subject to both pricing and power quality policies. The model has been applied and results were discussed from an illustrative distribution test system. The proposed strategy allows profits up to  $1.85\%$  of the revenue, without changes in the active power production.

In following works, the interaction between the distributed generators (by using simulations, gaming theory, and others methodologies) will be considered. In particular, the equilibrium problem in the market operation must be specially pondered.

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