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Optimal design of work-in-process buffers

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Abstract

When customers demand for very short lead-times and for a strict respect of the delivery schedules, it becomes difficult to avoid the existence of relatively large work-in-process buffers. As the buffers will also increase dramatically the production costs, their design (location and capacity) should be carefully analyzed and optimized, a task that becomes rather complex for large production systems. This paper firstly introduces a series of fundamental concepts related to the reliability of production systems. Then, the rationale of an analysis method oriented to this class of systems will be presented, together with its two main components, the modelling framework and the evaluation algorithm. The method allows analysing several issues of the design and operation of the production systems, namely the redundancy of equipment, the layout of the cells and the maintenance policies. To illustrate its practical usefulness and enlighten the kind of results that the method can provide, a numerical example concerning the design of work-in-process buffers of a just-in-time manufacturing system will be presented in the final part of the paper.

Keywords: Production systems; Reliability; Modelling; Evaluation; Economical analysis

1. Introduction

Nowadays, in order to remain competitive, manufacturing companies have to offer their clients high-quality products at low cost and short lead times (very short, in fact). These demanding requirements are driving manufacturing companies to increase their cooperation within value-added networks, where participants are tied by just-in-time (jit) deliveries and where the failure of a single manufacturing unit may have a dramatic consequence on the overall performance of the network. In spite of the worldwide intensive efforts made during the last two decades in order to reduce working inventory levels through the

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adoption of jit techniques, there are situations such that the existence of relatively large work-in-process (wip) buffers can not be avoided, as they are the only way to guarantee the strict deliver schedules required by the networked operation. Buffers filter the unbalance of manufacturing cells having different production rates and prevent the propagation of the disturbances from the faulty manufacturing cells to the downstream cells. However, they have a major drawback—the dramatic increase of the operational cost—so that their design should come from an economical analysis (Mahadevan and Narendran, 1993) that balances the implementation cost (stored materials and occupied area) with the level of service provided to the consumers (Berg et al., 1994) and the productivity improvement achieved by the fact that the overall production flow becomes less sensitive to the failures of the individual equipments.

This paper presents a reliability analysis method that will help managers in the design of the work-in-process buffers. Existing methods and tools for the analysis and performance evaluation of production systems often impose severe restrictions on the structure and behaviour of the systems that limit its application to relatively simple systems. For example Giordano (2002) considers the optimization of the safety stock for a single-part type, single unreliable machine production system, Van Ryzin et al. (1993) investigates optimal production control for a tandem of two machines, and Moinzadeh (1997) analyses an unreliable bottleneck assuming constant production and demand rate, constant restoration time and exponential failure processes. In comparison, the method that will be presented here considerably relax the basic assumptions and extends the application scope. Some of its main features are the ability to deal with processes having arbitrary distributions (and not only exponential distributions as it often happens in reliability tools), the use of a hierarchical modelling framework, that allows representing the overall structure of the system and the internal behaviour of each manufacturing unit at different modelling levels; the use of a standard canonical model to represent the behaviour of each production subsystem; and the evaluation of the overall reliability indices from a global system model obtained from the aggregation of the subsystems' canonical models.

After this introduction, a structural and behavioural analysis of the production systems will be presented in Section 2, in order to introduce a set of concepts related to the reliability of these systems that will clarify the rationale and the requirements of the method. Sections 3 and 4 will discuss the two components of the method: The modelling framework and the evaluation algorithm. The practical application of the method will be illustrated in Section 5, through a numerical example concerning the design of the buffers of a just-in-time manufacturing system. Finally, Section 6 discusses current limitations of the method and some extensions that are being developed.

2. Production systems analysis

A production system is seen as a network arrangement of cells that interact according to a producer/consumer scheme. As shown in Fig. 1, the output of a manufacturing cell may be linked directly to the input of one or more downstream cells but, also, it may exist an intermediate buffer between the cells, containing wip material. A *production failure* will occur when the normal flow of materials within the production system is disturbed and a materials shortage occurs. A shortage at the output of a cell may occur because that cell has halted its operation due to an *endogenous failure*, i.e., a failure of one of its internal equipments, or to an *exogenous failure*, i.e., a failure of a external equipment that has caused a material shortage at the input of the cell. When a production failure occurs at a cell, it will not be propagated immediately to the downstream cells if there is a buffer at the output of the failed cell. In such case, there will be a *propagation delay* before the downstream cells "see" the materials shortage. Typically, this will be a non-exponential random delay that depends on the quantity of material existing in the buffer when the shortage has occurred (as it will be discussed within the context of the numerical example of

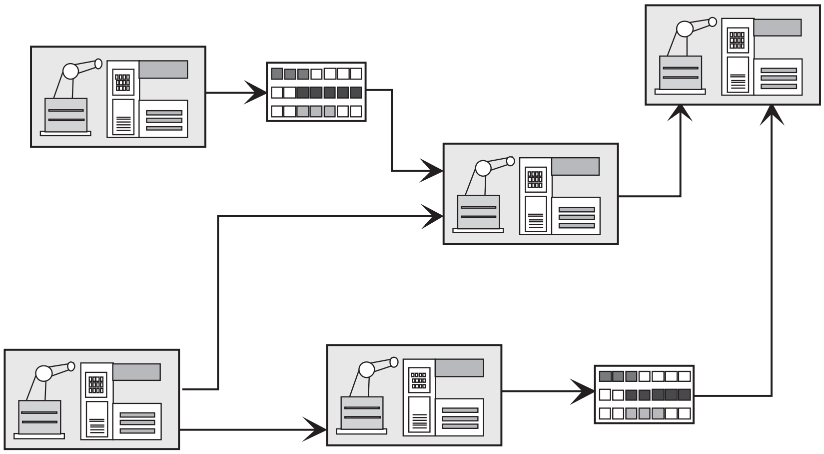

Fig. 1. Production system.

Section 5, an important feature of the method is the ability to deal with such non-exponential processes).

Each time a production failure occurs, there will be a corresponding economical loss. Hereafter, such a loss will be designated by a *production loss*, which may be driven by the duration of the shortages, or by the rate of occurrence of such shortages. These two components of the loss are designated by α *loss* and β *loss*, respectively. The first loss component corresponds to the situations where the halting of a manufacturing unit has a cost equivalent to a decrease on productivity. In such a case, the cost will be proportional to the duration of the failure. The other loss component is particularly significant in the situations where a transient disturbance of the manufacturing process can cause the deterioration of a large amount of in-process material (typical case: Continuous process industries). In this case, the number of failures will drive the loss. According to these concepts, the rationale for the method that will be presented in the next sections can be stated, for short, as follows:

(i) The optimal design of the buffers should come from an economical analysis that balances the implementation cost and the production losses.

(ii) A production loss consists of an economical damage caused by a shortage of material at the output of a manufacturing cell.

(iii) The production losses are caused by the manufacturing equipment failures, they propagate accordingly to the flow of materials within the production system, and their values may be driven by the duration or by the frequency of the shortages. Fig. 2 summarizes these ideas, namely, the physical flow of the consequences of the equipment failures (Fig. 2a), and the logical flow of the reliability analysis process (Fig. 2b).

3. The modelling framework

In order to perform the reliability analysis of a production system, both the internal behaviour of each cell and the global structure of the system have to be known. To capture this data, a two-level modeling framework was defined. At the *global level*, the model represents the overall structure of the production system in terms of the units that compose it, and of the physical flow of materials between them. At the *local level*, a set of models represents the internal behaviour of the individual cells of the system. As an example, Fig. 3 sketches the global level model of a system made by three cells and two buffers, along with the local level model of each cell. It is important to note that local level models just describe the internal behaviour of the cells, i.e., the part of the behaviour that depends on the internal

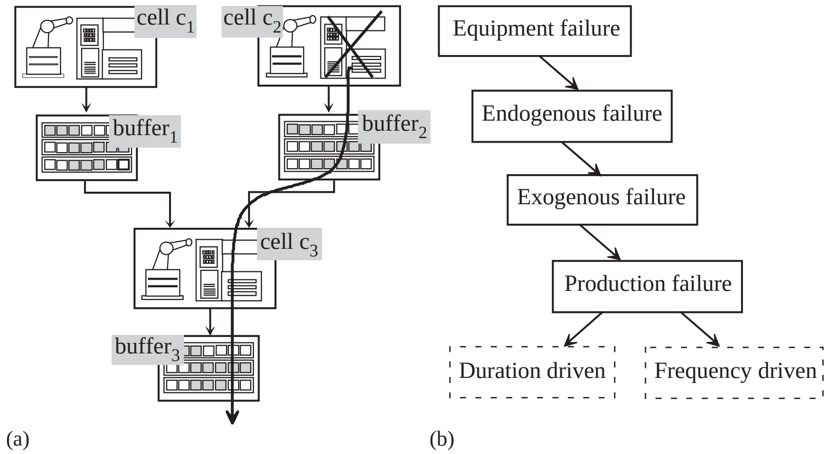

Fig. 2. Production failures.

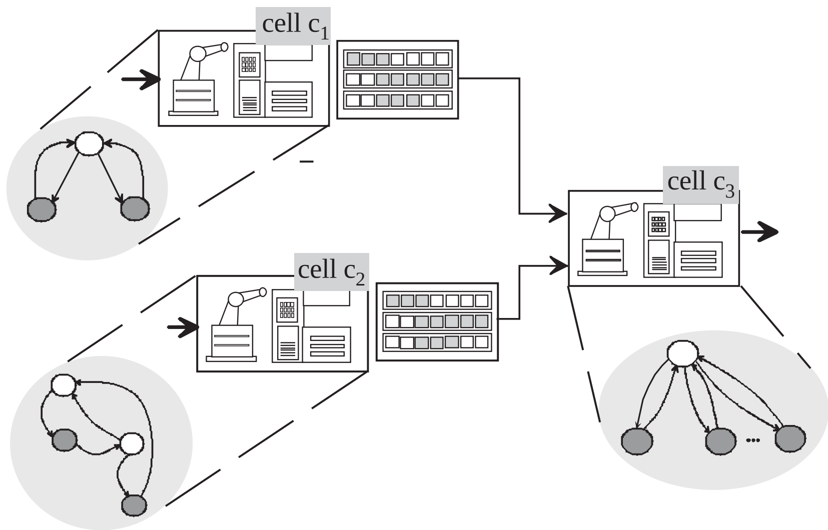

Fig. 3. Two-level modelling framework.

processes. The behaviour dependencies induced by the flow of materials between the units of the production system are implicitly represented by the structure of the global model.

3.1. The canonical model

From the point of view of the downstream system, the behaviour of a manufacturing cell can be described in terms of a two-state model (Fig. 4a) in which the *up* state corresponds to the

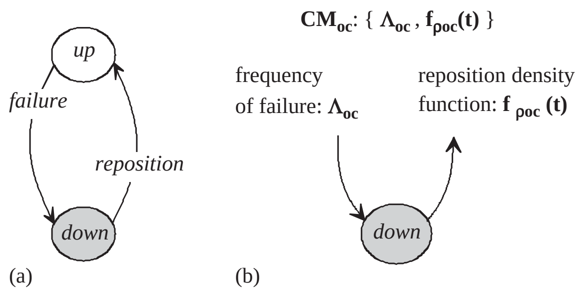

Fig. 4. Canonical model.

situations where the cell produces its output accordingly to the schedule, and the *down* state, represents the situations where the cell is halted and the normal flow of materials is interrupted. For the failure processes, normally, it is reasonable to assume that the time between failures is exponentially distributed, but the same can not be said for the reposition processes. Often, these processes are deterministic or quasi-deterministic, so that their probability density functions are close to the Dirac or to the step function, as it will be seen in Section 5. Therefore, the behaviour of an upstream cell c will be fully characterized by the couplet $\{\Lambda_{oc}, f_{\rho oc}(t)\}$ where Λ_{oc} is the rate of the failure and $f_{\rho o}(t)$ is the probability density function of the reposition process (Fig. 4b). Hereafter, this couplet will be designated as the *output canonical model* of the cell c, and it will be denoted by M_{oc}.

As pointed out before, the output unavailability of a cell has two components, one endogenous to the cell, and another induced by the upstream cells. The canonical model concept can be used to model these two components. In fact, they can be used in the following three situations: (i) modelling the internal behaviour of a cell, (ii) modelling the behaviour at the output of a cell and (iii) modelling the behaviour at the output of a buffer. In the first situation—internal behaviour of a cell—the failure state of the canonical model will represent the situations where the cell is unable to produce the desired output due to an internal failure. In the second situation—behaviour at the output of a cell—the failure state will represent the situations where the cell halts its operation due to an internal failure or to a failure of an upstream unit. In the third case—behaviour at the output of a buffer—the failure state will correspond

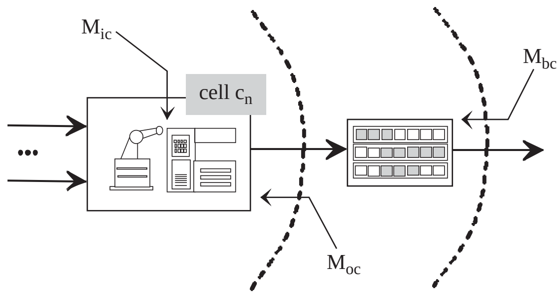

Fig. 5. Internal and external canonical models.

to the situations where the buffer is unable to feed the downstream cells. As shown in Fig. 5, the three canonical models of a cell c will be designated, respectively, as M_{ic}, M_{oc} and M_{bc}.

An important point concerning the canonical models is the fact that the model at the output of a cell c_n, M_{ocn}, can be obtained by the combination of the internal model of that cell, M_{icn}, and the model of the upstream buffer, M_{bcn-1}. In the next Section, it will be shown that this allows obtaining the canonical model equivalent to a set S of manufacturing cells by successively combining the canonical models of the cells belonging to S.

4. Evaluation algorithm

The reliability evaluation algorithm allows obtaining reliability indices such as the *unavailability* of the materials and the *rate of occurrence* of the shortages, at any point of the production system. It is closely related to the canonical model concept and it involves the following three steps that will be discussed in the subsequent paragraphs:

1. For each cell of the global model → obtain its internal canonical model.
2. For each node of the global model → obtain the upstream canonical model.
3. For each node of the global model → evaluate the α and β production loss.

4.1. Determination of the internal canonical model

The first step of the evaluation algorithm consists of the determination of the internal canonical model, M_i, for each one of the cells of the production system. The procedure that provides the analytical expressions of the probability density functions of M_i will be illustrated for two typical situations: a cell made by n non-redundant equipments, and a cell containing equipment in passive redundancy.

4.1.1. Non-redundant components

Consider an upstream cell c that is composed by two non-redundant machines and whose internal

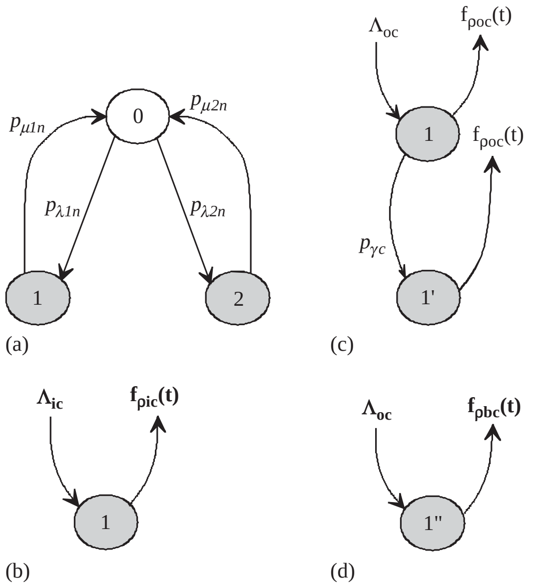

Fig. 6. Canonical model for a cell containing two non-redundant machines.

model is represented in Fig. 6a. Its internal canonical model M_{ic} (Fig. 6b) can be easily derived from:

$$P_0 = \frac{1}{1+\lambda_1 m_{\mu 1} + \lambda_2 m_{\mu 2}}, \tag{1}$$

$$\Lambda_{ic} = (\lambda_1 + \lambda_2)P_0, \tag{2}$$

$$f_{\rho ic}(t) = \frac{\lambda_1}{\Lambda_{ic}} f_{\mu 1}(t) + \frac{\lambda_2}{\Lambda_{ic}} f_{\mu 2}(t), \tag{3}$$

where P_0 and m_p denote, respectively, the probability of state s_0 and the mean of process p. For the general case of a cell made by k non-redundant machines, the parameters of the equivalent internal canonical model are given by the following expressions, where λ_j is the failure rate of machine j, and $f_{\mu j}(t)$ is the density function of its repair process, $p_{\mu j}$:

$$P_0 = \frac{1}{1+\sum_{j=1}^{k} \lambda_j m_{\mu j}}, \tag{4}$$

$$\Lambda_{ic} = \sum_{j=1}^{k} \lambda_j, \tag{5}$$

$$f_{\rho ic}(t) = \sum_{j=1}^{k} \frac{\lambda_j}{\Lambda_{ic}} f_{\mu j}(t). \tag{6}$$

As it was assumed that cell c is an upstream cell, its output canonical model will be identical to M_{ic}. The buffer canonical model will also be identical to M_{ic} if the cell feeds directly its downstream cells. When there is an intermediate buffer, the canonical model at its output, $\{\Lambda_{bc}, f_{\rho bc}(t)\}$, will be determined as follows (Figs. 6c and d). The failure rate Λ_{bc} comes from the product of the frequency of arrival to state s_1, and the probability of transition $s_1 \to s_{1'}$. If $f_{\gamma c}(t)$ is the density function of the buffer process,

$$\Lambda_{bc} = \Lambda_{ic} \int_0^\infty f_{\gamma c}(t_1) \int_{t_1}^\infty f_{\rho ic}(t_2)\, dt_2\, dt_1. \tag{7}$$

The density function of the reposition process comes from the ratio between the density function of the time of residence in state $s_{1'}$, given that the system has arrived to s_1,

$$\int_0^\infty f_{\gamma c}(t_1) f_{\rho ic}(t+t_1)\, dt_1, \tag{8}$$

and the probability of transition $s_1 \to s_{1'}$. Thus:

$$f_{\rho bc}(t) = \frac{\int_0^\infty f_{\gamma c}(t_1) f_{\rho ic}(t+t_1)\, dt_1}{\int_0^\infty f_{\gamma c}(t_1) \int_{t_1}^\infty f_{\rho ic}(t_2)\, dt_2\, dt_1} \quad \text{or}$$

$$f_{\rho bc}(t) = \frac{\int_0^\infty f_{\gamma c}(t_1) f_{\rho ic}(t+t_1)\, dt_1}{\Lambda_{bc}/\Lambda_{ic}}. \tag{9}$$

For a detailed presentation of a methodology that allows obtaining analytical expressions for the reliability indices of systems having non-exponential distributions see Faria (2001).

4.1.2. Passive redundancy

This second example concerns a cell composed by two machines, one of which in passive redundancy. The corresponding behaviour is represented in Fig. 7a. According to this model, when the first machine fails (state s_1), the reconfiguration process p_ξ puts the redundant machine into operation (state s_2). Then, either a new failure may occur (state s_3) or the repair of the first machine may be accomplished (state s_0). To

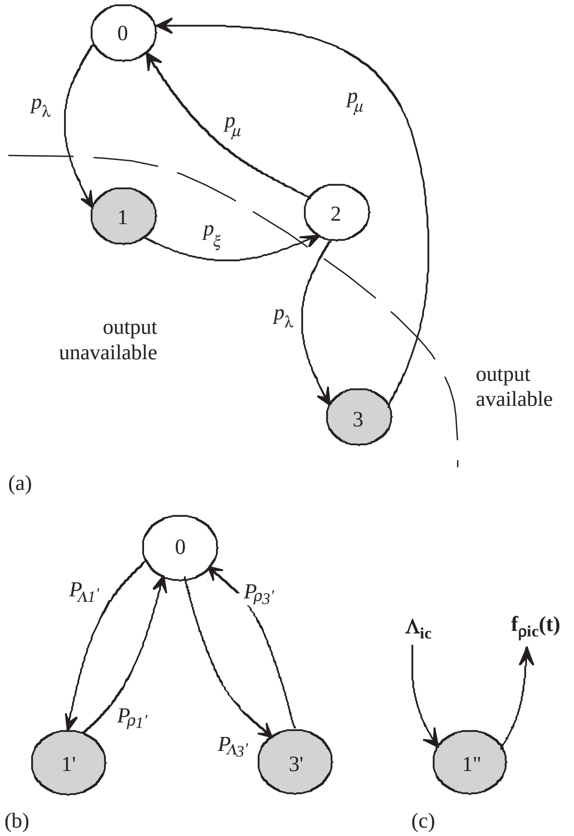

Fig. 7. Two components passive redundancy system.

obtain the equivalent canonical model in this case, a slightly more complex procedure must be employed. First, for each down state in the original graph, the expressions for the frequency of arrival and for the distribution of the reposition process are determined. Then, Λ_{ic} and $f_{\rho ic}(t)$ may be determined using expressions similar to (5) and (6). Therefore, the relevant expressions for the first failure state of Fig. 7b are

$$P_0 = \frac{1}{1 + \lambda(m_\xi + m_\mu)}, \tag{10}$$

$$\Lambda_{i1'} = \lambda P_0, \tag{11}$$

$$f_{\rho i1'}(t) = f_\xi(t). \tag{12}$$

For the second failure state, the equivalent failure rate and reposition process are

$$\Lambda_{i3'} = \Lambda_{i1'} \int_0^\infty f_\lambda(t_1) \int_{t_1}^\infty f_\mu(t_2)\,\mathrm{d}t_2\,\mathrm{d}t_1, \tag{13}$$

$$f_{\rho i3'}(t) = \frac{\int_0^\infty f_\lambda(t_1) f_\mu(t + t_1)\,\mathrm{d}t_1}{\int_0^\infty f_\lambda(t_1) \int_{t_1}^\infty f_\mu(t_2)\,\mathrm{d}t_2\,\mathrm{d}t_1}. \tag{14}$$

The equivalent internal model of the cell (Fig. 7c) can be obtained as before from

$$\Lambda_{ic} = \Lambda_{i1'} + \Lambda_{i3'}, \tag{15}$$

$$f_{\rho ic}(t) = \frac{\Lambda_{i1'}}{\Lambda_{ic}} f_{\rho i1}(t) + \frac{\Lambda_{i2'}}{\Lambda_{ic}} f_{\rho i2}(t). \tag{16}$$

If the cell has an output buffer, the corresponding canonical model can be obtained, as before from expressions (7) and (9).

4.2. Determination of the upstream canonical models

The second step of the evaluation procedure consists of the determination of the upstream canonical model, for each node of the production system. Those models can be obtained from the successive aggregation of the canonical models corresponding to the individual cells.

Suppose that, in the system of Fig. 3, the canonical model at the output of $cell_3$, M_{o3}, was to be determined, and suppose that models M_{i3}, M_{b1} and M_{b2} were already determined through a procedure similar to those presented in Section 4.1. The unavailability of materials at the output of c_3 has an endogenous component due to the failures of the internal equipment of the cell (M_{i3}), and an exogenous component due to the material shortages at its inputs (M_{b1} and M_{b2}).

The model that describes the unavailability of materials at the output of c_3 can be obtained from the combination of M_{i3}, M_{b1} and M_{b2}, as sketched in Fig. 8. As the structure of this model follows Fig. 6a, a similar approach can be used to obtain M_{o3}. The failure rate at the output of the downstream c_3 comes from the sum of the endogenous and exogenous failures rates, that is

$$\Lambda_{o3} = \Lambda_{b1} + \Lambda_{b2} + \Lambda_{i3}. \tag{17}$$

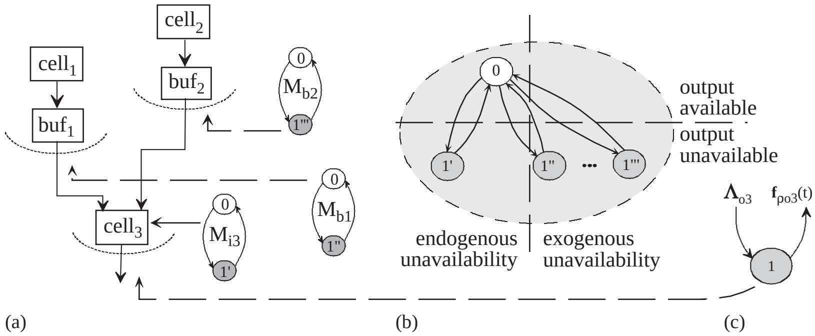

Fig. 8. Endogenous and exogenous unavailability.

As far as the service reposition process at the output of cell c_3 is concerned, its distribution comes from the weighted average of the three reposition processes involved:

$$f_{\rho o3}(t) = \frac{\Lambda_{i3}}{\Lambda_{o3}} f_{\rho i3}(t) + \frac{\Lambda_{b1}}{\Lambda_{o3}} f_{\rho b1}(t) + \frac{\Lambda_{b2}}{\Lambda_{o3}} f_{\rho b2}(t). \quad (18)$$

If there is a buffer at the output of cell c_3, buffer, M_{b3} may be obtained, as before, from (7) and (9).

If this same procedure is invoked repeatedly, starting from the upstream cells, it will allow obtaining the canonical model equivalent to any specified subset of the production system. For example, the full procedure to obtain M_{b3} for the system of Fig. 3 will involve the following steps:

1. determination of M_{i1}, M_{i2} and M_{i3}.
2. determination of M_{o1} and M_{o2} (in this case, they are identical to M_{i1} and M_{i2}).
3. determination of M_{b1} and M_{b2} (aggregation of processes b_1 and b_2 to M_{o1} and M_{o2}).
4. determination of M_{o3} (aggregation of M_{b1}, M_{b2} and M_{i3}).
5. determination of M_{b3} (aggregation of process b_3 and M_{o3}).

4.3. Evaluation of the production losses

The third step of the evaluation algorithm consists of the assessment of the production loss. According to the discussion of Section 2, at the output of each cell, two loss components are to be considered, one proportional to the unavailability of materials and another to the frequency of failure. Therefore, the total production loss L of the system will be given by:

$$L = H_y \sum_{j=1}^{k} [\alpha_j \bar{A}_j + \beta_j \Phi_j], \quad (19)$$

where H_y is the number of working hours per year (typical value is 5.000 hours), k the number of nodes of the production system, $\bar{A}_j$ the materials unavailability at the output of node$_j$; Φ_j is the shortages rate at the output of node$_j$ and α_j and β_j are the loss drivers for the α and β losses, respectively.

If M is the canonical model equivalent to the upstream system of node$_j$, whose parameters are Λ and $f_\rho(t)$ then the reliability indices $\bar{A}_j$ and Φ_j required for the calculation of the loss at that node can be readily calculated from

$$\bar{A}_j = \frac{\int_0^\infty t f_\rho(t)\,dt}{1 + \Lambda \int_0^\infty t f_\rho(t)\,dt}, \quad (20)$$

$$\Phi_j = \frac{\Lambda}{1 + \int_0^\infty t f_\rho(t)\,dt}. \quad (21)$$

5. Numerical application example

In this Section, a practical application example concerning the design of the wip buffers of a jit manufacturing system will be presented. The system has a structure identical to that of Fig. 3.

It contains two manufacturing cells (cell c_1 and cell c_2) that feed an assembly cell (cell c_3) operating jit, which means that the finished parts produced at its output are delivered directly to the clients without any temporary storage. Cell c_1 is made by two identical non-redundant machines, whereas cell c_2 is made by two machines in passive redundancy, so that their internal canonical models are identical to those analysed in Section 3.1. Cell c_3 is made by a large assembly line that is modelled as a single equipment, i.e., overall failure and repair processes are assigned to this cell.

It is assumed that the relevant production losses are those corresponding to the shortage of materials at the output of c_3. There is a component proportional to the duration of the shortages (α driver $= 2000€\ h^{-1}$), and another component proportional to the rate of occurrence of the shortages (β driver $= 1000€$). In order to minimize the impact of the failures of cell c_1 and cell$_2$ upon the output of the production system, two intermediate buffers, b_1 and b_2, will be implemented. The annual implementation cost of these two buffers, c_{b1} and c_{b2} are, respectively, $10.000\Delta_{b1}€$ and $15.000\Delta_{b2}€$, where Δ_{b1} is the capacity of b_i, expressed in terms of the equivalent hours of production the buffer is able to store.

Now suppose that system managers want to determine the capacity of both buffers b_1 and b_2 that minimize the total operational cost of the system, i.e., the sum of the production loss and the implementation cost of the buffers. That capacity depends on the way the contents of the buffers are managed: a buffer may be managed in order to maintain a constant inventory level I (Fig. 9a); alternatively, its contents may vary randomly depending on the instantaneous production unbalance between its input and output cells (Fig. 9b). In the first case, the density function of the propagation delay introduced by the buffer will be close to the Dirac function: $f_{bi} = \delta(t - \Delta_{bi})$, whereas in the second situation the density function will be close to the step function:

$$f_{bi} = [h(t) - h(t - \Delta_{bi})]/\Delta_{bi}.$$

These are two typical, but extreme, situations. For other situations, the density function of the propagation delay will present an intermediate

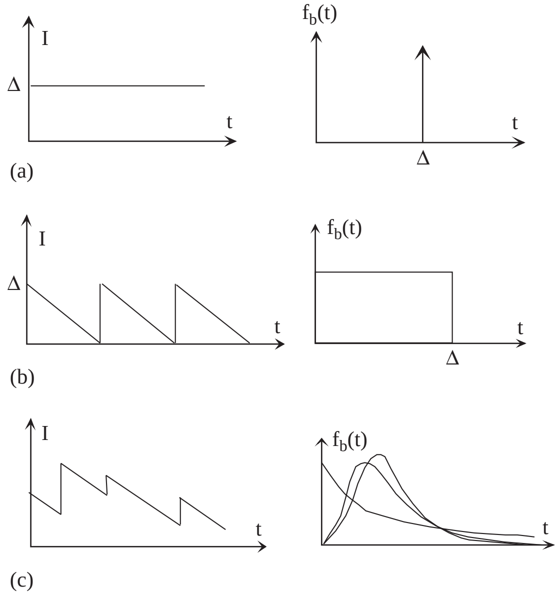

Fig. 9. Types of probability density functions.

shape that can be suitable described, for example, by n-order Erlang function (Fig. 9c)

$$f_{bi} = (n/\Delta_{bi})^n t^{(n-1)} \frac{e^{-nt/\Delta_{bi}}}{(n-1)!}.$$

The graph of Fig. 10a plots the evolution of the production loss and the total operational cost, with the capacity of b_1 and b_2, assuming that the content of the buffers remains constant. For this first scenario, the calculations were made using the expressions developed in Section 4 and the distributions of Table 1. Fig. 10b plots the evolution of the loss for a second scenario where the contents of the buffers vary randomly (step distribution), as shown in Table 2, and the other processes present the same distributions as before. All the calculations were performed using a general purpose mathematical tool such as those presented in Char et al. (1991) or Wolfram (1991).

The analysis of the numerical results for the total cost suggests that the optimal capacities of the two buffers are, respectively, 70 and 35 minutes, in the first scenario (constant buffer content), and 60 and 35 minutes in the second one (random buffer content). As far as the production loss and the total cost is concerned, their

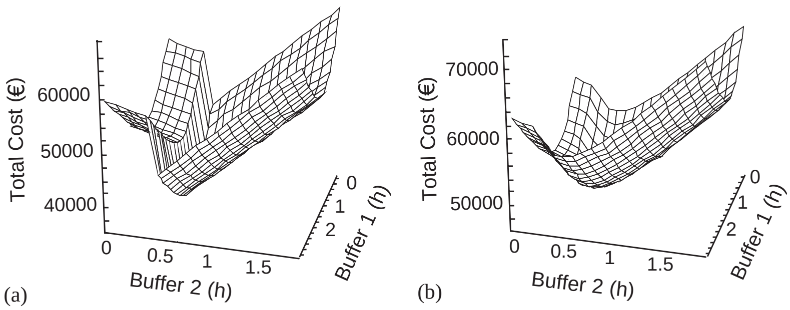

Fig. 10. Production loss evaluation results: (a) scenario 1; (b) scenario 2.

Table 1
Density functions for scenario 1

Cell_1	Cell_2	Cell_3
$f_{\lambda 1} = \lambda_1^{-\lambda_1 t}$ $\lambda_1 = 0.001$	$f_{\lambda 2} = \lambda_2^{-\lambda_2 t}$ $\lambda_2 = 0.002$	$f_{\lambda 3} = \lambda_3^{-\lambda_3 t}$ $\lambda_3 = 0.001$
$f_{\mu 1}(t) = (3_{\mu_1})^3 t^2 e^{-3\mu_1 t}/2$ $\mu_1 = 1$	$f_{\mu 2}(t) = (3_{\mu_2})^3 t^2 e^{-3\mu_2 t}/2$ $\mu_2 = 0.3$	$f_{\mu 3}(t) = (3_{\mu 3})^3 t^2 e^{-3\mu_3 t}/2$ $\mu_3 = 2$
$f_{b1}(t) = \delta(t - \Delta_{b1})$	$f_{\xi} = \delta(t - \Delta_{\xi})$ $\Delta_{\xi} = 1/3$	
	$f_{b2}(t) = \delta(t - \Delta_{b2})$	

Table 2
Density functions for scenario 2

Cell_1	Cell_2
$f_{b1}(t) = [h(t) - h(t - 2\Delta_{b1})]/2\Delta_{b1}$	$f_{b2}(t) = [h(t) - h(t - 2\Delta_{b2})]/2\Delta_{b2}$

corresponding values for the optimal design of the buffers are 16.213€ and 36.462€, respectively, in the first scenario, and 28.401€ and 46.650€, in the second one.

Very often, in reliability modelling, it is assumed that all the processes present exponential distributions because this highly simplifies the calculations. However, as it is shown in (Nunes et al., 2002), if a system contains non-exponential processes with similar time constants (as it will typically be the case with the repair, reconfiguration and buffer processes) such assumption may lead to very significant errors in the calculations. One of the main features of the method presented so far is the ability to deal with processes having arbitrary distributions. To stress this issue, let's consider the determination of the "optimal" design of the buffers if, for modelling and evaluation purposes, all the processes were assumed to have exponential distributions when, in fact, they have non-exponential distributions (as described in Table 1).

Fig. 11 shows the results obtained from such an "exponential" model. The total cost of the system would be minimal for a capacity of 48 minutes for b_1, and of 17 minutes for b_2. However, the correct value of the total operational cost corresponding to this design of the buffers (that is, the cost obtained when the correct, non-exponential distributions, are employed) are 48.837€ for the first scenario, and 49.626€ for the second one. The design obtained from the exponential model would then be clearly worse than the one obtained from the non-exponential model. This is particularly

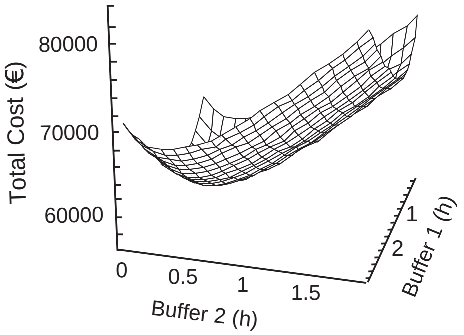

Fig. 11. Production loss for exponential processes.

true for the first scenario, where the difference reaches 34% (36.462€ versus 48.837€). These results show that, as far as the reliability analysis of production systems is concerned, the adoption of the exponential assumption may lead to rather ineffective management decisions.

6. Conclusions

The numerical example presented in the final part of the paper has shown that the accomplishment of a reliability analysis may be a valuable help in the design and in the management stages of production systems' life cycle. In practice, however, such a systematic analysis is often neglected, and a major reason for that is the inexistence of ground engineering methodologies and tools that may guide and support system designers and managers through the reliability analysis. The work presented in this paper is an attempt to modify this situation. It has been developed in the framework of a research project (Faria et al., 2002) aiming at the development of reliability evaluation tools for non-markovian systems. Within the project, a number of specific application domains have received a particular attention, one of them being the industrial production systems. Specialized tools were developed for each application domain, accordingly to its particular requirements. In fact, as it was shown in this paper for production systems, each domain presents its own specificities in terms of the structural and behavioural patterns and, if those specificities are fully understood, the reliability tools can be adapted in order to make the reliability modelling and evaluation more effective.

In the paper, a full analytical approach was used to derive the expressions for the relevant reliability indices. This approach proves to be very effective when the study requires sensitivity analysis, as it was the case with design of the wip buffers but, for large systems, it may lead to expressions too complex to be dealt analytically. In order to extend its application to large production systems, an alternative approach based on Monte Carlo simulation has been developed. This approach is also based on the modelling framework and on the canonical model concept presented in this paper but, now, the internal model of a cell is employed to obtain a histogram of the time of residence in the failure states which will, in turn, will be used as a random generator for the reposition process for simulation at the global level. Other topics currently under development concern the modelling and evaluation of multi-product/multi-component production systems with fuzzy parameters. The accomplishment of these developments is expected to extend the application domain of the conceptual framework presented in this paper to virtually any production system.

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