A Measure for Mutual Refinements of Image Segmentations

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***Abstract*—In this paper, we recover a graph interpretation of the mutual partition distance, proposed recently by Cardoso and Corte-Real. We deduce some properties of this measure, and establish a correspondence with the partition distance introduced by Almudevar and Field and Gusfield, and independently by Guigues. We also present some different formulations for the computation of the mutual partition distance. Finally, a comparison is made with similar measures.**

***Index Terms*—Image segmentation, mutual refinement, partition distance, segmentation quality evaluation.**

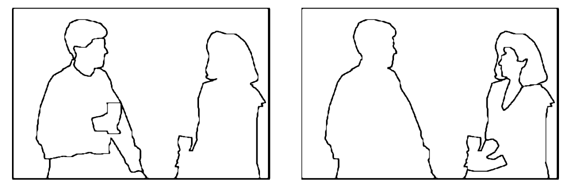

Fig. 1. Partitions A and B are a mutual refinement of each other. (a) Partition A. (b) Partition B.

I. Introduction

The concept of partition distance, introduced in [2] and calculated efficiently, in polynomial time, in [3], has found applications in many areas, ranging from image segmentation [1] to error-correcting codes [5] and natural sciences [2], [6], [7]. The same measure was independently introduced in [4] as part of a generic family of measures applied to the fusion of multidate images segmentations.

In order to evaluate the goodness of a partition produced by a segmentation algorithm or a human, in cases where the true partition is known, the partition distance between two partitions has shown to be effective, particularly for benchmarking [1]. However, in some applications, it is important to have measures insensitive to mutual refinements [8]. It is known that humans may segment an image differently: the same scene may be distinctively perceived; different subjects may attend to different parts of the scene; subjects may segment an image at different granularities. Nevertheless, segmentations of the same image tend to be consistent in the sense that they are mutual refinements of each other. [8]

Given a set S of N elements, a cluster is a nonempty subset of S. A partition of S is a set of mutually exclusive clusters whose union is S. A partition P is said to be a mutual refinement of a partition Q if and only if every cluster in P contains or is contained in a cluster in Q (Fig. 1). In [1], the *mutual partition distance* was defined as the minimum number of elements that must be deleted from S so that the two induced partitions (P and Q restricted to the remaining elements of S) are mutual refinements of each other. As easily reckoned, this is a symmetric measure.

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We first exploit the notion of mutual partition distance, giving a graph interpretation and establishing some useful properties of this measure. Next, we formulate the computation of the mutual partition distance as an integer programming problem. Finally, we apply the defined measure to the Berkeley Segmentation Dataset [9] and draw some conclusions.

II. Graph Interpretation of the Mutual Partition Distance

First, some definitions and notation are in order.

A bipartite graph $\mathcal{BG}$ is a graph whose set of vertices V can be split into two subsets, V_R and V_C, in such a way that each edge of the graph joins a vertex in V_R and a vertex in V_C. A bipartite graph with r vertices in V_R and c vertices in V_C is denoted by $\mathcal{BG}_{r,c}$.

A complete bipartite graph is a bipartite graph in which each vertex in V_R is joined to each vertex in V_C by an edge. The complete bipartite graph with r vertices in V_R and c vertices in V_C is denoted by $K_{r,c}$.

A tree graph is a simple, undirected, connected, acyclic graph. A tree with n nodes has $n-1$ edges. Conversely, a connected graph with n nodes and $n-1$ edges is a tree. The n-star graph, S_n, is a tree with $n+1$ nodes, with one node having vertex degree n and the others having vertex degree 1. The complete bipartite graph $K_{1,n}$ is the star graph S_n.

A. Graph Interpretation

The problem of computing the mutual partition distance can be casted naturally as a graph problem, on a graph derived from the partitions.

Given a set S of N elements and two partitions of S, P and Q, define the bipartite graph $\mathcal{BG}(P,Q)$ with one node in V_R for each cluster in P and one node in V_C for each cluster in Q (see Fig. 2). Connect two nodes x and y by an undirected, weighted edge if and only if x and y intersect each other, assigning to the weight the number of elements in the intersection.

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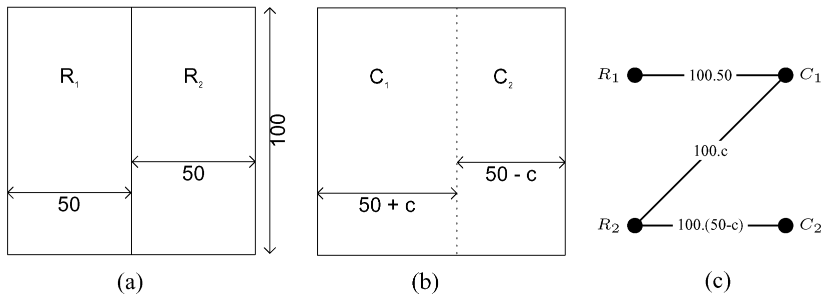


Fig. 2. Graph interpretation of mutual partition distance. (a) Segmentation 1. (b) Segmentation 2. (c) Associated bipartite graph.

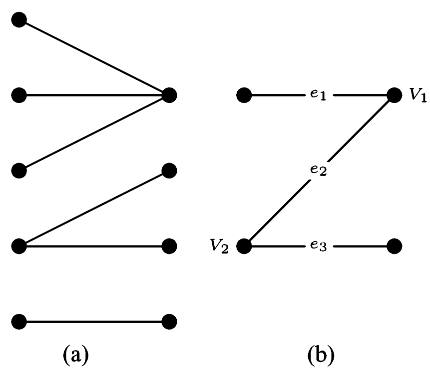


Fig. 3. Graph corresponding to a mutual refinement has paths of length at most two. (a) Bigraph of a mutual refinement—no paths' lengths greater than two. (b) Path length greater than 2.

We claim that partitions P and Q are a mutual refinement of each other if and only if the associated bigraph has only paths of length no greater than two.

Proof: Recognizing that a node (cluster) of degree one is contained in the node (cluster) to which it is connected, if we have only paths of length one or two, every node of degree greater than one is connected only to nodes of degree one—star configuration [see Fig. 3(a)]. Now, let $\{e_1, e_2, e_3\}$ be a path of length 3. Let v_1 be the vertex incident both to e_1 and e_2 and v_2 the vertex incident both to e_2 and e_3. Then, v_1 is not contained in v_2 (e_1 is not incident to v_2) and v_2 is not contained in v_1 (e_3 is not incident to v_1) [Fig. 3(b)]. ■

The mutual partition distance can now be formulated in the associated bigraph as the minimum sum of weights of pruned edges that such the induced (pruned) bigraph has paths of length at most two.

The idea of modelling the relation between two segmentations of the same image by the associated bigraph has been suggested earlier by Guigues [4], where a family of nesting relations between two partitions of the same set are introduced and applied to the fusion of multidate image segmentations. The mutual partition distance can be seen as one element of such family of similarity measures.

B. Properties of the Mutual Partition Distance, $d_{\rm mut}$

From this definition, a useful set of properties can be deduced. Let P, Q, R be partitions defined in a set S of N elements.

1) $d_{\rm mut}(Q,P) \geq 0$ and $d_{\rm mut}(Q,P) = d_{\rm mut}(P,Q)$, following directly from the definition.

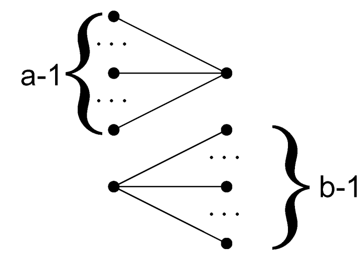


Fig. 4. $d_{\rm mut} \leq N - (a+b-2)$ in $K_{a,b}$.

2) The transitive property does not hold, i.e., $d_{\rm mut}(P,Q) = 0, d_{\rm mut}(Q,R) = 0 \not\Rightarrow d_{\rm mut}(P,R) = 0$.
Consequently, the triangular inequality does not hold, either.

3) Let the complete bipartite graph $K_{a,b}, b \geq a > 1, N = ab$ be the graph associated with partitions P and Q. Note that every edge has weight one. Then, $d_{\rm mut} = N - (a+b-2)$.
Proof: That $d_{\rm mut} \leq N - (a+b-2)$ can be easily seen in Fig. 4, showing a possible pruning of $K_{a,b}$ leading to a graph with paths of length at most two by removing $N - (a+b-2)$ edges. On the other hand, a tree in $K_{a,b}$ has $a+b-1$ edges. That implies $d_{\rm mut} \geq N-(a+b-1)$—otherwise, a cycle would exist in the remaining graph. Simultaneously, any subgraph $K_{2,2}$ of $K_{a,b}$ cannot be connected after pruning, as is trivial to verify. So, $d_{\rm mut} \geq N - (a+b-2)$. ■

4) Let $\mathcal{BG}_{a,b}, a = \lceil (N/\lceil\sqrt{N}\rceil) \rceil, b = \lceil\sqrt{N}\rceil$, be the bipartite graph associated with partition P and Q, with every edge with weight one. Then, $d_{\rm mut} = N - (a+b-2)$.
Proof:
- $d_{\rm mut} \geq N - (a+b-2)$—as in last item;
- $d_{\rm mut} \leq N - (a+b-2)$
  —$N > a(b-1)$ so at least 1 node from P is connected to all nodes from Q;
  —$N > (a-1)b$ so at least 1 node from Q is connected to all nodes from P.

Then, as in last item, we can keep $(a-1)+(b-1)$ edges. ■

5) From last item, given a set of N elements, we can always find two partitions $N - (\lceil (N/\lceil\sqrt{N}\rceil) \rceil + \lceil\sqrt{N}\rceil - 2)$ elements apart. So, $\max(d_{\rm mut}) \geq N - (\lceil (N/\lceil\sqrt{N}\rceil) \rceil + \lceil\sqrt{N}\rceil - 2)$. Normalizing the mutual partition distance by the same factor as the partition distance was normalized in [1], $N-1$, gives a normalized distance, ranging from 0 to approximately 1, for typical values of N in image segmentation (N equals the number of pixels in the image): $(N - (\lceil (N/\lceil\sqrt{N}\rceil) \rceil + \lceil\sqrt{N}\rceil - 2))/(N-1) \approx 1$.

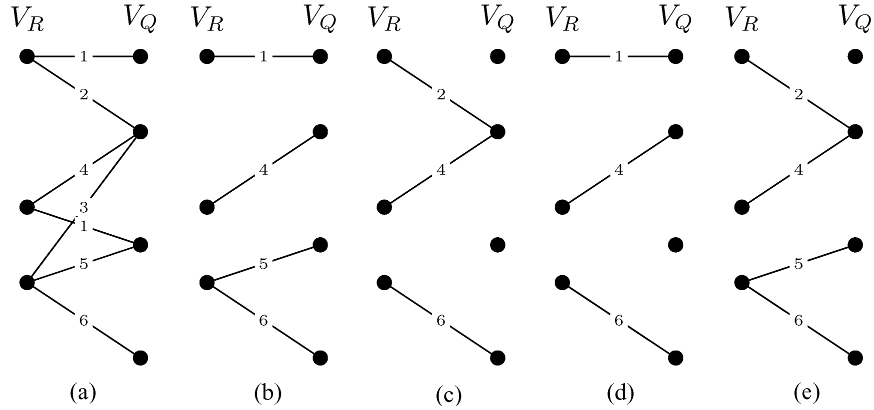


Fig. 5. Original and resulting graphs for all measures. (a) Original graph. (b) $d_{\text{asy}}(R,Q)$. (c) $d_{\text{asy}}(Q,R)$. (d) $d_{\text{sym}}(R,Q)$. (e) $d_{\text{mut}}(R,Q)$.

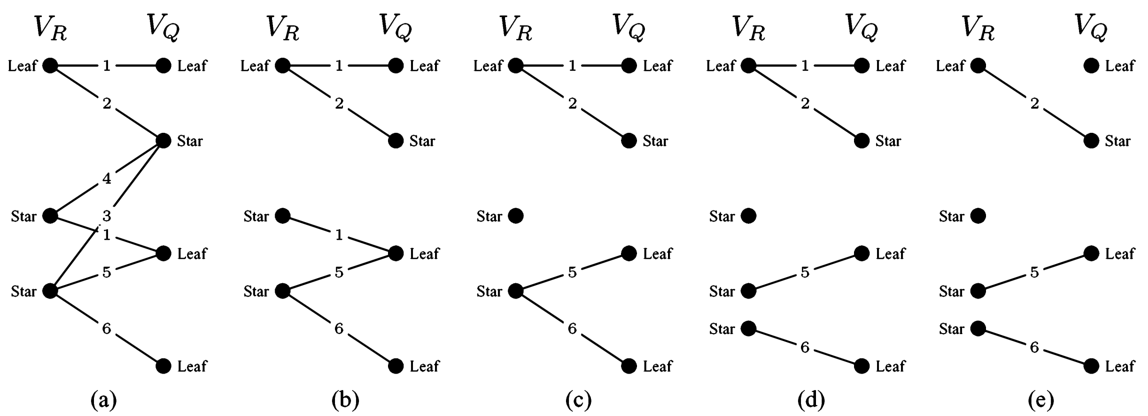


Fig. 6. Mutual partition distance with constrained star centers. (a) Original graph, with a chosen labelling. (b) Step 1. (c) Step 2. (d) Step 3. (e) Step 4.

C. Connection to the Partition Distance

In [1], besides the mutual partition distance, a set of different measures to evaluate the quality of an image segmentation Q was proposed, when comparing it to a reference segmentation R:

- a symmetric measure $d_{\text{sym}}(R,Q)$, the partition distance between the two partitions, given by the minimum number of elements that must be deleted from the original set S, so that the two induced partitions in the remaining elements are identical;
- an asymmetric measure $d_{\text{asy}}(R,Q) = d_{\text{sym}}(R \cap Q, Q)$, tolerant to over segmentations, given by the minimum number of elements that must be deleted from S, so that the induced partition Q is finer than the induced partition R;
- an asymmetric measure $d_{\text{asy}}(Q,R) = d_{\text{sym}}(R \cap Q, R)$, tolerant to under-segmentations, defined similarly to $d_{\text{asy}}(R,Q)$.

As established in the referenced work, the computation of d_{sym} is mapped in the traditional matching problem, in the corresponding bigraph; $d_{\text{asy}}(R,Q)$, a special and rather straightforward instance of the matching problem, is simply translated as "for each vertex in V_Q remove all but the edge of biggest weight;" $d_{\text{asy}}(Q,R)$ is computed in a similar way (Fig. 5).

The resulting graph for $d_{\text{asy}}(R,Q)$ is partitioned in disconnected $K_{1,m}$ subgraphs with the star center always in V_R. The resulting graph for $d_{\text{asy}}(Q,R)$ has the star centers in V_Q. The original partition distance does not allow stars (or only degenerate stars $K_{1,1}$). The mutual partition distance weakens the constraints on the number and position of the star centers.

It is not hard to prove that, by keeping constant the number and position of the star centers (possibly some in V_R and others in V_Q), the problem of computing the mutual partition distance simplifies to a matching problem.

Proof: Start by labelling arbitrarily each vertex either as a star center or as a leaf. Impose the following additional constraints:

- two vertices labelled as star centers can not be connected by an edge;
- a vertex labelled as a leaf can not have degree greater than one.

The mutual partition distance, constrained as above, simplifies to a matching problem (Fig. 6):

1) remove every edge connecting two star centers;
2) for each leaf, remove every edge to a star center, except the biggest of them (the others are *dominated* by this)—this step is not strictly necessary;
3) split every star center S_i in deg(S_i) vertices, each with one and only one of the deg(S_i) edges;
4) perform the traditional matching in the resulting graph.

Because the result of such algorithm is in fact a mutual refinement in the original graph, then d_{mut} is less or equal than the result of any labelling.

It is also easily reckoned that the mutual partition distance equals one of such labelling: In the resulting graph for d_{mut}, label each vertex with degree greater than one as star center and the others as leaves. Then, d_{mut} is equal to the result for this particular labelling. ■

We see that the mutual partition distance can be interpreted as a generalization of the partition-distance problem.

It should also be apparent that the number of stars in the optimal solution will be no greater than $\min(\#V_R; \#V_Q)$: Every star makes use of, at least, one vertex of V_R and V_Q.

D. A General Framework for the Comparison of Image Segmentations

The bigraph associated with two image segmentations, as presented previously, can be used as a *factory* of indices of similarity between partitions. It has already been used to define the partition distance d_{sym}, the asymmetric measure d_{asy} and the mutual partition distance d_{mut}.

Guigues [4] has already defined a family of symmetric measures on this graph, where d_{sym} and d_{mut} are just special cases. Although the simplest way is to assign the area of intersection to the weight of each edge that can be replaced by any cost function expressing the importance of a region intersection.

More generally, rules can be defined on the vertices and edges of the bigraph to create suitable measures. In fact, many of the previously proposed measures in the literature can be accommodated under this framework. To illustrate, the LCE measure introduced in [8] can be effectively computed as

$$\frac{1}{N} \sum_{\forall \text{ edge } e_i} w_i \cdot \min\left\{ \frac{w_{r_i} - w_i}{w_{r_i}}, \frac{w_{c_i} - w_i}{w_{c_i}} \right\}$$

where w_i is the weight of the edge e_i, r_i and c_i are the nodes incident to edge e_i, w_{r_i} is sum of the weights of all the edges incident to node r_i, w_{c_i} is sum of the weights of all the edges incident to node c_i. Later on in this study, we will make use of this general framework to develop a tuned measure for a specific application.

III. Mutual Partition Distance as an Optimization Problem

The computation of the mutual partition distance can be performed directly on the corresponding bipartite graph by removing every possible combination of clusters' intersections (edges) and assessing the validity of the resulting graph. The search space can be traversed using a gray code counter [10]: In each iteration, only a single edge needs to be added or removed to the graph under evaluation. This leads to an exponential-time algorithm, as a function of the number of clusters' intersections (number of edges of the graph). Another possibility would be to use the results of Section II-C, and compute every possible matching problem. This would lead to an exponential-time algorithm, as a function of the number of clusters (number of vertices of the graph). However, it is easy to show that the computation of the mutual partition distance fits the definition of an integer optimization problem.

For the mathematical model, use the following decision variables:

$$\begin{aligned} &\mathbf{X} = \mathbf{1} - \mathbf{Y}, \quad \text{with} \\ &\mathbf{X} = [x_1, \ldots, x_n]^T, \quad \mathbf{Y} = [y_1, \ldots, y_n]^T, \quad \text{and} \\ &x_i = \begin{cases} 1, & \text{if edge } e_i \text{ is kept} \\ 0, & \text{if not.} \end{cases} \quad \text{for each edge } e_i \end{aligned}$$

Setting $\mathbf{W} = [w_1, \ldots, w_n]^T$, where w_i is the weight of edge e_i, we formulate the mutual partition distance as the following integer constrained minimization problem:

$$\begin{aligned} d_{\mathrm{mut}} = &\min \mathbf{W}^T\mathbf{Y} \\ &\text{s.t. } y_i + y_j + y_k \geq 1, \text{ for each trio of edges} \\ &\qquad\qquad e_i, e_j, e_k \text{ forming a path of length 3} \\ &y_i \in \{0, 1\}. \end{aligned} \tag{1}$$

For $K_{a,b}$, this translates to $a \cdot b$ decision variables and $a(a-1)b(b-1)$ constraints. However, noting that a graph corresponds to a mutual refinement if and only if every $\mathcal{BG}_{2,2}$ subgraph has, at most, two edges, the mutual partition distance can be alternatively defined as

$$\begin{aligned} d_{\mathrm{mut}} = &\min N - \mathbf{W}^T\mathbf{X} \\ &\text{s.t. } \sum_{e_i \in \mathcal{BG}_{2,2}} x_i \leq 2, \text{ for each } \mathcal{BG}_{2,2} \text{ subgraph} \\ &x_i \in \{0, 1\}. \end{aligned} \tag{2}$$

This reduces the number of constraints in $K_{a,b}$ to $(a(a-1)b(b-1))/(4)$. This formulation has the additional benefit of reducing the set of continuous feasible solutions. In fact, although both formulations are equivalent for binary variables, they would differ if the decision variables were relaxed to [0, 1]. As easily reckoned, the feasible solution set for the second formulation would be a subset of the first. This may result in a faster convergence of the second formulation, as algorithms for solving the integer problem are usually based on the continuous counterpart formulation.

A. Reformulation With a Compact Convex Domain

Equations (1) and (2) are a brute-force NP-hard integer minimization problem. In general, there is no efficient way of (optimally) solving such type of problems. Nonetheless, it can be shown to be equivalent to the following concave minimization problem [11]:

$$\begin{aligned} d_{\mathrm{mut}} = &\min \mathbf{W}^T\mathbf{Y} + \mu\mathbf{Y}^T(\mathbf{1} - \mathbf{Y}) \\ &\text{s.t. } y_i + y_j + y_k \geq 1, \text{ for each trio of edges} \\ &\qquad\qquad e_i, e_j, e_k \text{ forming a path of length 3} \\ &y_i \in [0, 1] \end{aligned} \tag{3}$$

where μ is a sufficiently large positive number. Provided that μ is large enough, the global minimum is attained only when $\mathbf{Y}^T(\mathbf{1} - \mathbf{Y}) = 0$.

B. Reformulation as a Generalization of the Partition Distance

Yet another formulation can be devised, by adopting a different viewpoint, based on the results of Section II-C. Introduce the additional binary variables

$$r_i = \begin{cases} 1, & \text{if vertex } vr_i \in V_R \text{ is a star center} \\ 0, & \text{if not} \end{cases} \qquad \text{for each vertex } vr_i \in V_R$$

$$c_j = \begin{cases} 1 & \text{if vertex } vc_j \in V_C \text{ is a star center} \\ 0, & \text{if not} \end{cases} \qquad \text{for each vertex } vc_j \in V_C.$$

The mutual partition distance can now be computed as

$$\begin{aligned} d_{\mathrm{mut}} &= \min N - \mathbf{W}^T\mathbf{X} \\ &\text{s.t. } x_i + r_{e_i} + c_{e_i} \leq 2, \forall \text{ edge } e_i, \text{with} \\ &\qquad vr_{e_i} \text{ and } vc_{e_i} \text{ incident to } e_i \\ &\sum_{\forall e_i \text{ incident to } vr_k} x_i \leq 1 + r_k \cdot \deg(vr_k) \\ &\sum_{\forall e_i \text{ incident to } vc_k} x_i \leq 1 + c_k \cdot \deg(vc_k) \\ &x_i, r_i, c_j \in \{0, 1\}. \end{aligned} \tag{4}$$

The first condition expresses that two star centers cannot be connected; the second and the third express that if a vertex is a leaf, it can only have a edge incident with it—if the vertex is a star center, the inequality is always satisfied.

This formulation requires $ab + a + b$ decision variables and the same number of constraints in $K_{a,b}$. In this formulation, the variables x_i can be relaxed to the real domain [0, 1]. This follows directly from the established correspondence with the matching problem which, as well known, can be solved in the continuous domain [0, 1].

Because the efficiency of each formulation depends on the application, one should select the most suitable for the target task.

IV. Experimental Results

We implemented both our algorithm and Martin's LCE measure [8] in C++.[1] Equation (2) of the mutual partition distance was used in the software implementation. The linear programming problem was solved with the freely available *lp_solve v5.0* software. Tests were carried out on a regular PC (1-GHz AMD microprocessor, 256-MB RAM).

A. Results

To check the adequacy of the proposed measure to evaluate mutual refinements, the d_{mut} measure was applied to the Berkeley Segmentation Dataset [9], and compared with the proposed LCE measure in [8].

In Fig. 7(a), we plot the distribution of the d_{mut} and the LCE measures over the segmentation database, for pairs of segmentations of the same image; in Fig. 7(b), it is presented d_{mut} versus LCE for pairs of segmentations of the same image. In Fig. 8, the same information is depicted for pairs of segmentations of different images. As expected, both measures are portraying similar information.

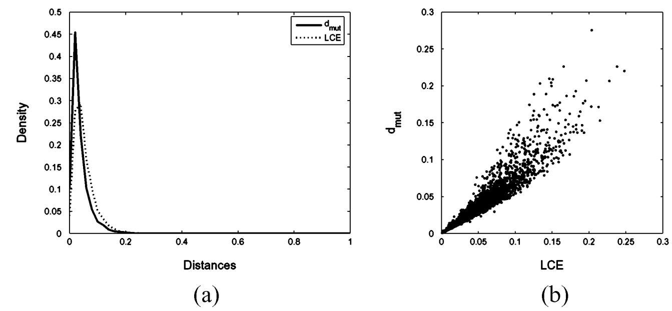


Fig. 7. Comparison over the Berkeley Segmentation Dataset for pairs of human segmentations of the same image. (a) Distribution of the d_{mut} and LCE [8] measures; (b) d_{mut} versus LCE [8].

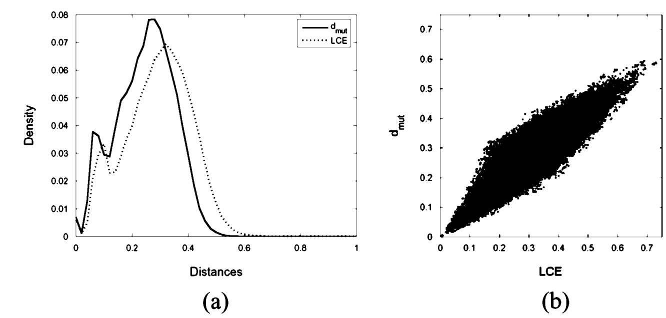


Fig. 8. Comparison over the Berkeley Segmentation Dataset for pairs of human segmentations of different images. (a) Distribution of the d_{mut} and LCE [8] measures; (b) d_{mut} versus LCE [8].

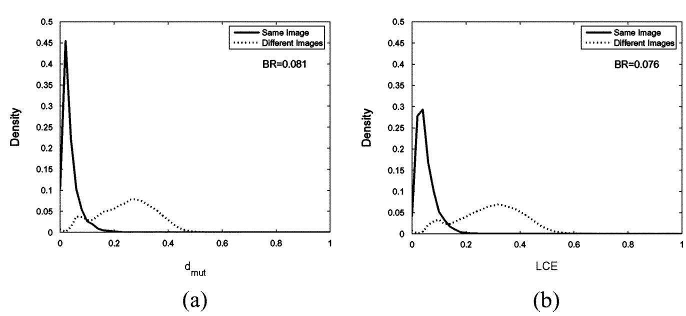


Fig. 9. Comparison of the Bayes risk. (a) Bayes risk for the d_{mut} measure. (b) Bayes risk for the LCE measure.

It may seem a bit disappointing, however, that the proposed measure has an inferior capability to discriminate segmentations of the same image from segmentations of different images, than the LCE measure, as evaluated by the Bayes risk (Fig. 9). That is probably a consequence of some human segmentation inconsistencies or errors (second pair in Fig. 10) and of degenerate pairs in the different-image pairs: segmentations that compare favorably with nearly any other because all the segments are small or fortuitous alignment of segmentations [8].

Although the proposed measure was not intended for the separation of same-image and different-image pairs of segmentations, it is not difficult to, using the general setting introduced in Section II-D, construct a measure with improved performance for this task.

[1]The code is available upon request to the authors.

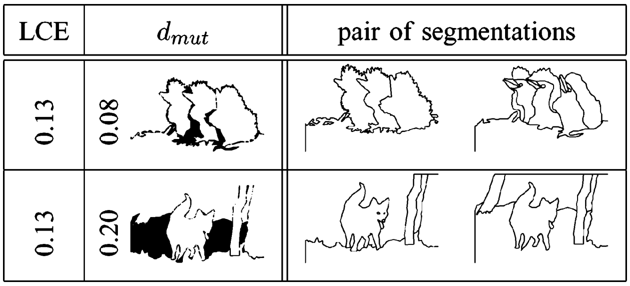
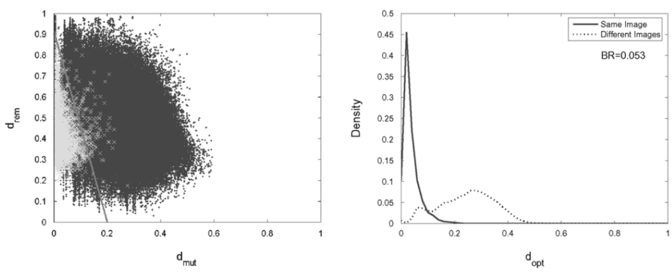


Fig. 10. Example pairs at various LCE and d_{mut} values.



(a) (b)

Fig. 11. Construction of a measure with improved discriminative capability. (a) Distribution of both populations over the $(d_{\mathrm{mut}}, d_{\mathrm{rem}})$ space. (b) Bayes risk for the d_{opt} measure.

To exemplify, and starting from the bigraph corresponding to a pair of segmentations, we considered as an additional feature the percentage of remaining edges in the calculation of $d_{\mathrm{mut}}, d_{\mathrm{rem}}$. Following a pattern classification approach [12], based on the SVM principle [13], several pairs of features were gauged: $(d_{\mathrm{mut}}, \mathrm{LCE}), (d_{\mathrm{mut}}, d_{\mathrm{rem}}), (\mathrm{LCE}, d_{\mathrm{rem}})$. An optimized measure for the separation of these two populations was obtained as $d_{\mathrm{opt}} = 0.82 d_{\mathrm{mut}} + 0.18 d_{\mathrm{rem}}$, with the boundary decision at $d_{\mathrm{opt}} = 0.165$, Fig. 11. Naturally, better measures for this dataset could be thought, either by adopting nonlinear boundaries in the selected two-feature space or by considering other features.

A side information of the proposed mutual partition distance is the indication of the erroneous pixels (the pixels to be removed), black pixels of the mask image in Fig. 10, information that can be useful for further processing.

V. Conclusion

In this paper, we have presented an elegant interpretation of the mutual partition distance. A link has been established between this measure and the partition distance, which can be seen as a special case of the mutual partition distance. Both measures have been integrated in a general framework, based on the bigraph corresponding to a pair of partitions. Binary integer linear programming formulations for the computation of the measure were also provided. The resulting algorithms have shown to be effective when applied to a real-world dataset.

Although the notion of mutual refinement may not capture all the unpredictability in human segmentations, it certainly models a wide category of variabilities. This may imply the need to complement it with other criteria, as exemplified by the two feature example.

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