Algorithms for DSP implementation in coherent optical systems

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Abstract

The electrical compensation of chromatic dispersion and polarization mode dispersion in a coherent optical system exploiting polarization multiplexing is discussed in this paper. The benefits of combining a phase estimation algorithm with a decision directed least-mean-square equalizer in a feedback configuration is reported.

I - Introduction

Coherent optical communications have gained renewed interest due to the availability of high speed digital signal processing, low priced components as well as partly relaxed receiver requirements at high data rates. Coherent detection allows the optical field parameters (amplitude, phase and polarization) to be available in the electrical domain enabling new potential of multi-level signaling (M-ary PSK and M-ary QAM modulation), as well as the possibility of exploring polarization multiplexing [1]. Additionally it enables quasi-exact compensation of linear transmission impairments (CD and PMD) by a linear filter (equalizer) [2], which can operate adaptively to overcome time-varying impairments.

Equalizers can achieve adaptation of their coefficients either by transmitting a training sequence, known symbol statistics or decision-directed (DD) adaptation. When no training sequence is transmitted, the operation is referred to as blind equalization and the constant modulus algorithm (CMA) introduced by Godard and Treichler is the most used, essentially because of its robustness and ability to converge prior to phase recovery. However, for non-constant modulus constellations as QAM, the multimodulus algorithm (MMA) introduced by Yang et al improves upon the performance of CMA by obtaining low steady-state mean-squared error (MSE) [3], but its cost function is not carrier phase independent. An elegant solution consists in using the CMA for initial adaptation, avoiding bandwidth consuming training sequences, and subsequent independent carrier phase estimation. Once equalizer convergence has been achieved, there is a great benefit if the equalizer switches to DD mode, whereby the error signal is derived from the error between the baseband signal and the nearest, ideal point of the constellation, improving the demodulator SNR performance [4]. According to the author's knowledge, although this concept has already been suggested in [2,4], a detailed study of the phase estimation algorithm when working in feedback with the equalizer was never done.

This paper presents our developments relating to the MATLAB simulation of a coherent optical system employing 16-QAM using a receiver based on a phase and polarization diversity configuration [5]. The paper is organized as follows: In section II we review the theory, section III details the simulation model, section IV presents our proposed algorithm, in section V we present and discuss the simulation results followed by a conclusion.

II - Theory

A. Introduction

The compensation of fiber impairments in the digital domain, in coherent optical systems, means that, in principle, any linear distortion can be compensated in the DSP at 1 sample/bit, as long as an analog matched filter precedes the sampler. However, the matched filter has several drawbacks: It may be more difficult to design than its digital equivalent and the exact phase of sampling is required to be known to sample the signal at its maximum energy. Moreover, if the channel response is unknown or time-varying, an adaptive equalizer is necessary whereas an adaptive analog matched filter may be difficult to design. Even if the sample time is optimum, spectral overlap always occurs unless the system transfer function is a sinc function, which is not realizable in practice. A fractionally spaced equalizer (FSE) implements the matched filter and equalizer as a single unit [6].

B. Phase and polarization diversity receiver

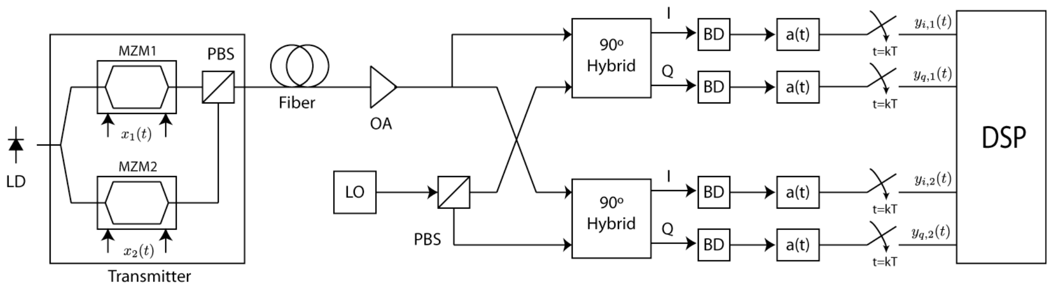

Fig. 1 – Coherent transmission system employing a phase and polarization diversity homodyne receiver. LD – Laser Diode, PBS – Polarization Beam Splitter, OA – Optical Amplifier, BD – Balanced Detector.

Figure 1 represents a diagram of the coherent transmission system under study. In the transmitter optical IQ-Modulators are used whereby the laser signal light is split into two orthogonal carriers and then modulated by Mach-Zehnder-Modulators (MZM), biased at minimum transmission, and driven by multi-level RF signals. For 16-QAM these RF signals have 4-levels. Then, the two data streams, polarized in orthogonal directions, are combined in a Polarization Beam Splitter (PBS) and launched into the transmission channel (optical fiber). The signal is amplified to overcome attenuation in the long haul transmission. Additive White Gaussian Noise (AWGN) comes from the amplified spontaneous emission (ASE) of optical amplifiers which dominates over LO shot noise [7].

The optical multi-level modulation signal can be detected by an homodyne IQ-receiver, whose general configuration is valid for any M-PSK and M-QAM modulation format. In the configuration depicted in fig. 1, the received signal is split into two orthogonal components, each going to a separate 90º hybrid, and then is coherently detected using four balanced photodiodes. If the LO power is low, single ended photodiodes can be used, as the LO relative intensity noise (RIN) is minimum. The four electrical output signals correspond to the I and Q components associated with the parallel and orthogonal LO polarizations. Finally, the signals are low-pass filtered by anti-aliasing filters a(t) and sampled at the Nyquist rate.

III – Simulation Model

A. Model description

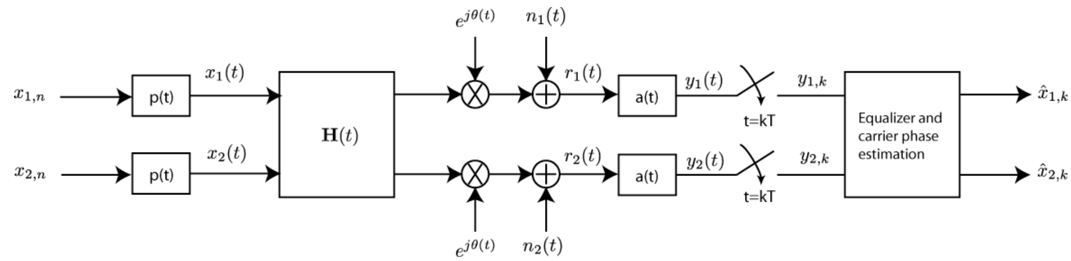

Fig. 2 – System canonical model

The figure above represents the canonical model of a coherent optical system employing polarization multiplexing. The transmitted signals, in each polarization are given by:

$$x_l(t) = \sum_n x_{l,n} p(t - nT_s)$$

Where $x_{l,n}$is the *n*th symbol transmitted in the *l*th polarization and *p*(t) is the transmitted pulse shape. The received signal is then of the form:

$$\begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} h_{11}(t) \; h_{12}(t) \\ h_{21}(t) \; h_{22}(t) \end{bmatrix} \otimes \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}$$

The matrix h(t) represents the fiber impulse response, completely describing a dually polarized channel, which is mathematically represented as a 2x2 Multiple Input Multiple Output (MIMO) channel[8]. The frequency response of the fiber can be described by:

$$H(w) = T(w) \times e^{-\frac{1}{2}\beta_2 L w^2}$$

Where $T(w)$ is the fiber Jones Matrix accounting for PMD [9], β_2 is the Group Velocity Dispersion (GVD) parameter and L is the fiber length. Chromatic dispersion is accounted up to second order. Then, the signal is noise loaded, with both AWGN noise and phase noise. Phase noise is usually characterized as a Wiener process, described in the discrete domain as:

$$\phi_k = \sum_{-\infty}^{k} \nu_m$$

where the ν_m's are independent identically distributed (i.i.d.) Gaussian random variables with zero mean and variance $\sigma_p^2 = 2\pi\Delta\nu T$. $\Delta\nu$ is generally assumed to be the 3-dB combined linewidths of the signal and LO lasers (also known as the beat linewidth), and T is the symbol period[10]. In our numerical model, the transmitter laser is assumed to be ideal, while the LO laser is assumed to have phase noise equal to the sum of the linewidths of the two lasers [2]. Letting $q(t) = a(t) \otimes h(t) \otimes p(t)$ and $n_l^{'} = a(t) \otimes n_l(t)$ we can write the signal after the anti-alias filters as:

$$y_l(t) = \sum_n \sum_{m=1}^{2} x_{m,n} q_{lm}(t - nT_s) e^{j\phi(t)} + n_l^{'}(t)$$

The sampling occurs at a rate of 2 samples per symbol, which was previously shown to be sufficient [11]. In this way, the results become almost insensitive to the sampling time error, since the FSE synthesizes via its transfer characteristic the necessary phase adjustment [6]. Linear equalization follows, by performing convolution with a bank of four complex valued T/2 spaced FIR filters, arranged in a butterfly structure [11]. The linear equalizer takes $N = 2L + 1$

samples of the incoming signal and calculates the minimum-mean-squared-error (MMSE) estimate of the *k*-th symbol $\tilde{x}_k$. The optimum solution for the coefficients is obtainable by the Wiener-Hopf equation [8]. However, in practice, H is time-varying due to PMD, and an adaptive equalizer is necessary. Therefore, the least mean square (LMS) or the recursive least squares (RLS) algorithms can be used to update the coefficients, and track the time varying minimum of the cost function.

B. Carrier phase estimation

In our canonical model, the multiplication by the phase noise $e^{j\phi(t)}$ manifests as a rotation of the received constellation. Additionally, phase noise is a Wiener process with temporal correlation, which means the phase at any symbol period is likely to have a value similar to the phases at adjacent symbols, which allows the usage of signal processing techniques in order to mitigate it. Assuming perfect compensation, the general form of the signal at the output of the equalizer, is:

$$\tilde{x}_k = x_k e^{j\theta_k} + n_k$$

where x_k is a complex valued transmitted symbol at the *k*-th symbol period, θ_k is the carrier phase, and $n_k$is AWGN. The goal of the phase estimation process is to find θ_k, which will allow de-rotation of the signal by multiplying it with $e^{-j\theta_k}$, followed by a symbol-by-symbol detector to find x_k. Ip and Kahn proposed an algorithm[10], which uses a two stage iteration process for finding the carrier phase. The first stage is a soft decision phase estimator, which computes soft estimates of the carrier phase without taking into account temporal correlation. Then this soft estimate ψ_k is passed through the second stage, which is a linear filter $W(Z)$ whose output is the MMSE estimate of θ_k. The soft phase estimator can be either non-decision aided (NDA) or decision-directed (DD). The NDA algorithm is especially well suited to M-ary PSK transmission, since raising the received signal to the M-th power eliminates the phase modulation, due to the M-fold rotational symmetry of an M-PSK constellation, allowing θ_k to be estimated without any symbol decisions. However, this algorithm is asymptotically optimal for low SNR. On the other hand, the decision directed algorithm replaces the known symbols $x_k$with the output of a decision device, which is asymptotically optimal for high SNR. Additionally, if the system constellation is non-PSK (non-constant-envelope), the DD algorithm must be used. One disadvantage of this algorithm is that it requires an initial estimate ($\widehat{\theta_k}$) of the phase noise in order to find ψ_k.

IV - Proposed Algorithm

According to [10], taking into account the temporal correlation of phase noise, it can be shown that the optimum filter for phase estimation is a Wiener filter, consisting of two exponentially decaying sequences that are symmetric about n=0, causal and anti-causal. The causal coefficients are given by:

$$w_n = \frac{\alpha r}{1-\alpha^2}\alpha^n \ n \geq 0$$

With $\alpha = (1 + r/2) - \sqrt{(1 + r/2)^2 - 1}$ and $r = \sigma_p^2/\sigma_{n'}^2$. σ_p^2 is the variance of the phase noise and $\sigma_{n'}^2$ is the variance of the phase of Gaussian amplitude noise. Therefore, the ratio of phase noise to the phase of amplitude noise r, determines the rate of decay of the filter exponential. Additionally, it can be shown that the Wiener filter is a specialization of the Kalman filter, for the case of zero delay, and after convergence of the Kalman filter gain. The Kalman recursion is given by $\tilde{\theta}_{k+1} = \tilde{\theta}_k + G(\psi_k - \tilde{\theta}_k)$. Although the gain factor G is supposed to converge during operation, we have noticed that this stabilization happens very quickly in our type of scenario, and therefore we can pre-compute its steady state value, which can be shown to give $G = 1 - \alpha$, yielding:

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k + (1-\alpha)\cdot(\psi_k - \tilde{\theta}_k) = \theta_k + (1-\alpha)\cdot \text{angle}(\tilde{x}_k e^{-j\tilde{\theta}_k}\cdot \text{conj}(d_{ref}))$$

The diagram of the equalizer is shown in figure 3. Basically, the error signal for the equalizer is derived after carrier "de-spin", so that the equalizer output is still a constellation with ringed shape. Unlike [10], we have used a different approach to the determination of ψ_k, without needing to perform phase unwrapping, wherein we compute $\psi_k - \tilde{\theta}_k$ directly [12]. We have also introduced a new way of calculating the value of $\sigma^2_{n'}$. In [10] it is calculated from the SNR. However, as the SNR might be unknown, we do it iteratively, by periodically evaluating the variance of a statistical sufficient window size of the quantity: $\text{angle}(\tilde{x}_k e^{-j\tilde{\theta}_k}\cdot \text{conj}(d_{ref}))$. One extremely important aspect of this structure is the fact that the Kalman filter has zero delay. While this is not optimum in a phase noise estimation problem, when operating in a loop with the equalizer this is the best compromise. The recursive nature is one of the very appealing features of the Kalman filter since it makes practical implementations much more feasible than of a Wiener filter[13] which is designed to operate on all of the data directly for each estimate. The Kalman filter instead recursively conditions the current estimate on all of the past measurements [4].

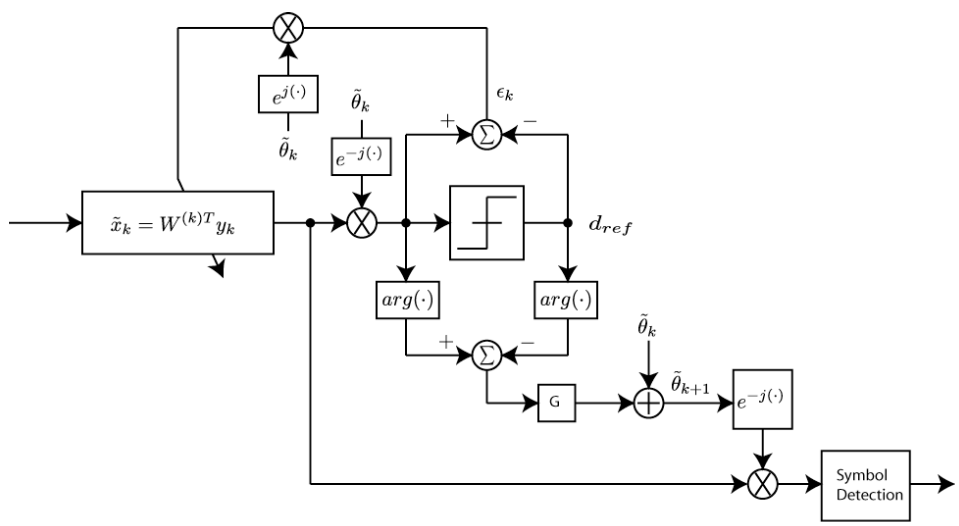

Fig. 3 –Diagram of the equalizer and subsequent carrier phase estimation

V - Results

In our numerical simulations, we have used a Non-Return to Zero (NRZ) pulse shape $p(t)$, obtained by passing an ideal rectangular pulse train by a 5th order low pass Bessel filter with a 3-dB bandwidth of 80% of the symbol-rate. For the anti-alias (AA) filter $a(t)$, 3rd order low pass Bessel filters were used.

In figure 4, we have plotted the symbol error rate versus input SNR per symbol.

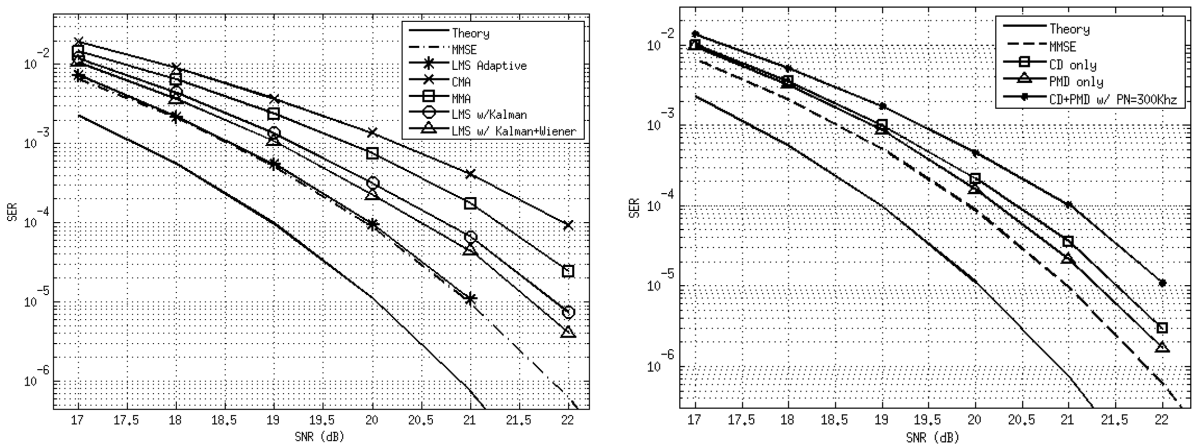

Fig 4 –Symbol error rate versus SNR. Left: Comparison between LMS with phase noise (circles and triangles) with CMA ("x") and MMA (squares). Right: Impact of CD and PMD with phase estimation.

The SNR is defined as: $SNR = \frac{P_x T_s}{N_0} = \frac{E_s}{N_0}$where P_x is the mean symbol power, E_s is the symbol energy and N_0 is the noise power spectral density. On the left, the bottom curve represents the theoretical performance of a 16-QAM system without impairments and ideal pulses (sinc) and AA filters. The MMSE curve was obtained by calculating the optimum 13-tap vector coefficients in the MMSE sense that maximizes the output SNR [6] in the presence of an ideal channel, whilst considering the Bessel filters in both transmitter and receiver. Keeping the same channel, all other results in the same plot were obtained by 13-tap adaptive filters, with Monte-Carlo simulations, which were evaluated after convergence. The result corresponding to the LMS adaptive filter closely matches the MMSE result (asterisk). The following two curves are still for the case of LMS filters, but now with 300 kHz of phase noise combined linewidth, where we have employed the previously referred feedback configuration for phase noise estimation and equalizer adaptation. The curve represented by the diamonds symbol is for the case where the Kalman filter is used within the loop. The curve corresponding to the triangles symbol was obtained with the same processing but additionally a Wiener filter was employed, that is without feedback. The latter enables one to assess the gain achieved by using also the anti-causal part of the filter. The upper two curves are the result of the CMA (squares) and MMA (circles), but without phase noise. It is very clear that decision-directed operation in feedback with carrier phase estimation clearly outperforms algorithms based on non-decision aided cost functions. On the figure of the right, we can observe the system performance when fiber impairments are present. We have considered a fiber length $L = 200\,\mathrm{km}$, CD of $20\,\mathrm{ps/(nm \cdot km)}$ and PMD coefficient of $3\,\mathrm{ps}/\sqrt{\mathrm{km}}$, which is about 30 times more than a common commercial fiber. The triangles are for the case of only considering PMD, the squares correspond to CD and finally the asterisk is for the case when both PMD and CD are present, as well as 300 kHz of phase noise. As we can see, the penalty originated from phase noise is within 0.5 dB.

VI - Conclusions

We have investigated in detail the implementation of digital signal processing algorithms in order to mitigate system impairments such as CD, PMD and laser phase noise, in a coherent system. We have shown the advantages of combining a Decision Directed equalizer with a carrier phase estimation stage in feedback.

Acknowledgments

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