

Reactive Power Response of Wind Generators Under an Incremental Network-Loss Allocation Approach

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Abstract—The reactive power capability of wind generation (WG) producers can be used to provide volt/voltampere reactive (var) support in electrical distribution systems. In practice, the distribution network operator (DNO) is not able to dispatch reactive power of all generators, and such units are referred to as nondispatchable generators. Then, the reactive power of these nondispatchable generators is not a defined value, and it should accomplish the limits settled by national grid codes. This paper addresses the optimal reactive power provision of nondispatchable WG units under a loss allocation strategy based on incremental generation use of the system (GUoS) tariffs. The optimization problem is set from generators' viewpoint aiming to find the reactive power provision that minimizes the loss charges subject to operational constraints. Optimal solutions are compared with the DNOs mandatory dispatches using a reactive optimal power flow tool. Two test systems are analyzed: a 10-kV 28 bus and 60-kV 55-bus system throughout 672 and 168 h, respectively. The proposal permits to identify which units can be dispatched by the DNO due to its effect on the network.

Index Terms—Loss allocation, losses, mathematical programming, reactive power, wind power generation.

NOMENCLATURE

$GUoS_k$	Generation use of the system tariff at bus k in dollars per megawatthour.
LIP_k	Locational incremental price at bus k in dollars per megawatthour.
λ	System market price in dollars per megawatthour.
L	Total real power losses in kilowatt.
L_{Gk}, L_{Dk}	Losses allocated to generators and loads at bus k in kilowatt.
P_k, Q_k	Real and reactive power injected at bus k in megawatt.
P_{Gk}, P_{Dk}	Real power of generators and demands at bus k in megawatt.
Q_{Gk}, Q_{Dk}	Reactive power of generators and demands at bus k in megavoltampere reactive.

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V_k, θ_k	Voltage and angle of bus k per units and radians.
θ_{ik}	Difference between angles i and k in radians.
G_{ik}, B_{ik}	Real and imaginary part of admittance matrix Y in per units.
S_{ik}	Power flow between bus i and bus k in megavoltampere.
n	Number of buses.
n_g	Number of generating buses.
sl	Slack bus
ILF^P	Incremental loss factors for real power.
ILF^Q	Incremental loss factors for reactive power.
R, X	Line resistance and reactance in per units.
B_{CAP}	Half of total line charging susceptance in per units.
S_{BASE}	Power base in megavoltampere.
LR_{Gk}	Loss revenue of generators at bus k in dollars per hour.

I. INTRODUCTION

MOST of the installed renewable distributed generation (DG) facilities are based on wind generation (WG) units connected at a distribution level. In general, the distribution network operator (DNO) does not explicitly control or dispatch the reactive power provision of WG units. This task is difficult and costly for the DNO because the distribution system is not entirely observable. As a result, the reactive power of WG units is *nondispatchable* and it must accomplish some operational rules [1]. To do this, regulatory boards design grid codes either by setting voltage limits or by establishing power factor ranges at the generating buses, in order to reduce power losses and improve voltage profiles [2]. Many authors mention that the current national grid codes are not enough to provide more efficient ancillary services [3]–[5], claiming for better economic incentives to invest in reactive power supply. For instance, regulators such as the U.S. Federal Energy Regulatory Commission (FERC) have required better procedures for the remuneration of distributed generation, related to a volt/voltampere reactive (var) control based on switchable capacitors/inductors or on static synchronous compensator (STATCOM) or static var compensators video coding (SVC) [6]. Recently, some countries with an increased penetration level of distributed resources (like Spain) are sending economic incentives to producers, aiming to achieve benefits by reducing voltage deviations and power losses in the system [7].

In this context, the European Directive EU54/2003 [8] expressly indicates that regulatory authorities should revise the use of the system tariff model applied to the distribution networks, in order to include the costs and benefits introduced by

DG units on the network. In this sense, the volt/var support provided by the DG units is a valuable benefit introduced by the WG units.

Incremental pricing is broadly recognized as the core approach to an efficient economic valuation identifying the impact of each market agent on the benefit/cost structure of the network [9]. Locational incremental or marginal prices (denoted LIPs or LMPs) have been extensively applied to allocate losses and congestion charges between generators and demands. The loss component of LIPs varies across location and time. It must be computed for real and reactive powers by using incremental loss factors (ILFs) obtained from standard power flow solutions [10]. At the distribution level, losses are typically allocated by using average values, but some efforts are being made in order to introduce LIPs as a way to send economic signals to DG units [11], [12]. Consequently, a broad debate is being carried out toward the implementation of use of the system charging models for distributed generator through GUoS tariffs [16], [17].

The reactive power optimization and incremental power pricing have been extensively studied in literature [18]–[23]. It is observed that the application of real power incremental factors prevails in many proposals for power loss allocation [11]–[15].

However, as indicated by Hogan [22] and Baldick [23], reactive power injections drastically affect the pattern of ILFs across the network. Hence, if a WG unit is considered nondispatchable, then the reactive power provision is not a defined value, and generators can modify their own locational price by changing the reactive injection in the generating bus [24]. Therefore, a WG operator is able to modify the reactive power operation point (within the grid code's specified limits) in order to diminish its loss charges for getting additional revenues. The DNO should appraise this ability in order to estimate the impact of each WG unit on the system.

This paper discusses a new methodology to assess the optimal behavior of nondispatchable WG units under an incremental loss allocation policy. The optimization problem is stated from the point of view of the generation utility as an *individual welfare optimization problem*, thereby aiming to find the best operating point that minimizes the loss charges (or maximizes the loss revenues) of WG promoters under location-specific and time-varying GUoS tariffs.

The methodology allows evaluating the impact on network performance of the reactive power injection in the ILFs at each generating bus of the system. The real power demand and wind power generation injections are estimated from forecasting studies or obtained on a real-time basis from supervisory control and data acquisition (SCADA) systems.

The operating point obtained by the proposed methodology was compared with mandatory reactive power dispatches established by the DNO by means of a reactive optimal power flow (R-OPF) or by fixing a power factor at the generating bus. Depending on the impact of each generator in the system, the proposed methodology is a valuable tool to identify which generators must be dispatched in the reactive power. The model has been evaluated and results are discussed from two real sys-

tems: a 15-kV 28-bus distribution network and a 60-kV 55-bus distribution network.

The remaining sections of this paper are outlined as follows. Section III describes the pricing framework based upon GUoS tariffs. Section IV illustrates how reactive power injections affect the GUoS tariff structure. The proposed model is presented in Section V. The mandatory dispatches performed by the DNO are described in Section VI. Finally, results and conclusions are drawn in Sections VII and VIII, respectively.

II. INCREMENTAL LOSS ALLOCATION FRAMEWORK

Under the incremental cost allocation approach, economical signals are sent to the agents by means of location-specific and time-varying GUoS tariffs. GUoS tariffs require the computation of the well-known ILFs [10]. The ILFs are defined as the variation in the real power losses due to the incremental change of the real power injections in each bus k , given by

$$\text{ILF}_k^P(P_{Gk}, Q_{Gk}) = \frac{\partial L(P_{Gk}, Q_{Gk})}{\partial P_k} = \frac{\partial L(P_{Gk}, Q_{Gk})}{\partial (P_{Gk} - P_{Gk})}, \quad (1)$$

The real ILF^P and the reactive ILF^Q can be obtained from the current state of the system by evaluating the Jacobean of a converged Newton–Raphson ac power flow. According to [11], the standard chain rule is applied to calculate the ILFs by means of intermediate variables, voltages, and angles obtained as

$$\begin{bmatrix} \text{ILF}_k^P \\ \text{ILF}_k^Q \end{bmatrix} = \begin{bmatrix} \partial L / \partial P \\ \partial L / \partial Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial Q}{\partial \theta} \\ \frac{\partial P}{\partial V} & \frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial L}{\partial \theta} \\ \frac{\partial L}{\partial V} \end{bmatrix}. \quad (2)$$

In order to recover the cost of the losses, the remuneration obtained by a producer connected at generating bus k is modified through a specific locational incremental price, as a function of the ILF^P at bus k and the system market price λ

$$\text{LIP}_k = \lambda + \text{GUoS}_k = \lambda [1 - \text{ILF}_k^P(P_{Gk}, Q_{Gk})] \quad (3)$$

The GUoS tariff ($-\lambda \cdot \text{ILF}_k^P$) intends to remunerate either the produced or the avoided power losses related to the agents connected at bus k . This economic framework considers only the real power-based locational incremental prices. The application of reactive power-based locational incremental prices is out of the scope of this paper.

The polarity of the ILFs is interpreted by using the following rule:

$$\begin{aligned} \text{ILF}_k^P < 0; \quad \text{LIP}_k > \lambda; \quad \text{GUoS}_k > 0 & \text{ [AVOIDED]} \\ \text{ILF}_k^P > 0; \quad \text{LIP}_k < \lambda; \quad \text{GUoS}_k < 0 & \\ & \text{ [PRODUCED]}. \end{aligned} \quad (4)$$

If an ILF^P is negative, then the locational incremental price LIP_k is higher than the market price. This means that a generator connected to bus k is rewarded with an additional fee GUoS_k due to the contribution so as to decrease the total operational losses. On the other hand, if ILF^P is positive, the locational incremental price LIP_k is lower than the market price λ . In this case, the generators connected to bus k are charged with an additional payment GUoS_k due to increase of losses.

Loss revenues or payments LR_k at a given generating bus depend on the incremental loss factors calculated at the operating point, the real power injected, and the system market price. As a result, a deviation in the power reactive injection leads to a change in the real power-based ILFs applied to all the generators. This condition affects the incomes and revenues expected by the distributed generators. The loss revenue (LR_k) of generators connected at bus k is expressed as follows:

$$LR_k = P_{Gk} \text{GUoS}_k^p = -\lambda P_{Gk} \text{ILF}_k^p. \quad (5)$$

Note that negative ILFs lead to earnings due to power losses reduction. Positive ILFs lead to charges due to power losses increase. Finally, at a given operating point ($V_i, V_k, \theta_{ik} = \theta_i - \theta_k$), the power losses can be computed by means of

$$L = \frac{1}{2} \sum_{i=1}^n \sum_{k=1}^n G_{ik} [V_i^2 + V_k^2 - 2V_i V_k \cos \theta_{ik}]. \quad (6)$$

III. DEPENDENCE BETWEEN THE ILFS AND THE REACTIVE POWER PROVISION OF WG UNITS

The ILFs can be computed using (2) at any operating point [11]. However, incremental values are strongly influenced by the reactive provision of generators. Hence, different ILF patterns can be obtained depending on the power factor at the generating bus. In this section, we show the direct relationship between incremental loss factors and the reactive provision of a WG. This dependence is exemplified using a simple two-bus system, where line parameters R_{ik}, X_{ik} , voltages V_i , angles θ_i , and real power injected at generating bus P_k are known variables.

At the generating bus k , the real net power balance is given by

$$G_{kk} (V_k)^2 + V_i V_k [G_{ik} \cos \theta_{ki} + B_{ki} \sin \theta_{ki}] = P_k. \quad (7)$$

On the other hand, the reactive net power balance at generating bus k is given by

$$-B_{kk} (V_k)^2 + V_i V_k [G_{ik} \sin \theta_{ki} - B_{ki} \cos \theta_{ki}] = Q_k. \quad (8)$$

Then, as the voltage at slack bus V_i , real power P_k , and line parameters R_{ik}, X_{ik} are known variables, if a known value of Q_k is injected at bus k , voltage V_k , and angle θ_{ki} are directly calculated by solving (7) and (8). Finally, the incremental loss factors ($\text{ILF}_k^p, \text{ILF}_k^q$) are acquired by applying (2). Each derivative is a function of the calculated V_k and θ_{ki} , given by

$$\begin{aligned} \text{ILF}_k^p &= \frac{\partial L}{\partial P_k} (V_k, \theta_{ki}) \\ &= \frac{\partial Q_k / \partial \theta_k \partial L / \partial V_k - \partial Q_k / \partial V_k \partial L / \partial \theta_k}{\partial Q_k / \partial \theta_k \partial P_k / \partial V_k - \partial Q_k / \partial V_k \partial P_k / \partial \theta_k} \end{aligned} \quad (9)$$

$$\begin{aligned} \text{ILF}_k^q &= \frac{\partial L}{\partial Q_k} (V_k, \theta_{ki}) \\ &= \frac{\partial P_k / \partial \theta_k \partial L / \partial V_k - \partial P_k / \partial V_k \partial L / \partial \theta_k}{\partial Q_k / \partial V_k \partial P_k / \partial \theta_k - \partial Q_k / \partial \theta_k \partial P_k / \partial V_k}. \end{aligned} \quad (10)$$

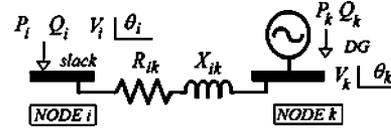


Fig. 1. Two-bus network.

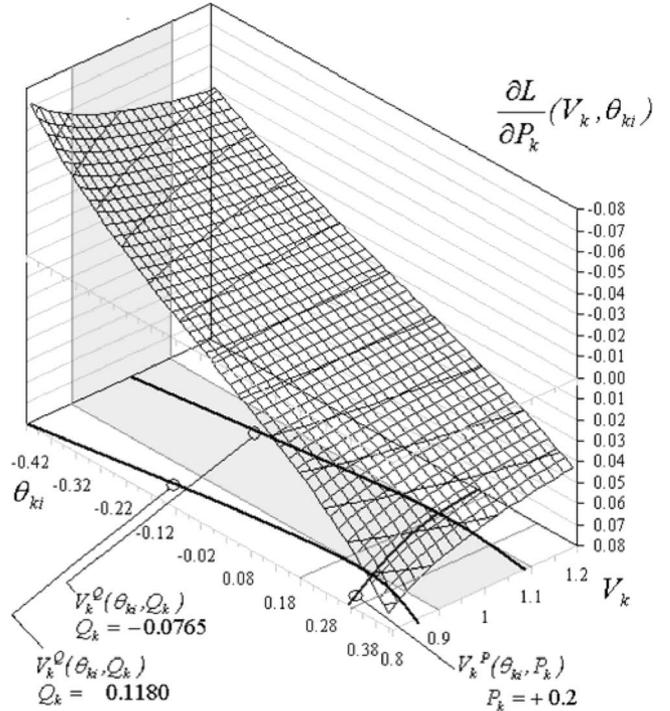


Fig. 2. ILFs surface for a two-bus system and power flow solutions.

As a result, when the reactive power is specified, Q_k, V_k , and θ_{ki} are directly obtained by solving (7) and (8), and therefore, incremental loss factors ($\text{ILF}_k^p, \text{ILF}_k^q$) are automatically specified by evaluating (9) and (10). This means that WG units connected at bus k can drive the incremental loss factors by using their own reactive power capability within the reactive power, power factor, or voltage limits imposed by the grid code.

In order to illustrate this relationship, a simple numerical example is provided. It is supposed that the two-bus network shown in Fig. 1 has $R_{ik} = 0.05$ p.u. and $X_{ik} = 1.0$ p.u. Bus i is the reference ($V_i = 1.0$ p.u. and $\theta_i = 0$ p.u.) and the real power injection at bus k is fixed $P_k = 0.2$ p.u.

The ILF_k^p distribution can be drawn as a 3-D graph in Fig. 2. The ILFs points are given by the power flow solution when the reactive provision Q_k is changed in a range (from -0.14 to 0.25 p.u.). Voltages at the generating bus k go from 0.775 to 1.225 p.u. Then, when Q_k is specified, there is θ_{ki} that simultaneously accomplishes (7) and (8). Fig. 3 shows the resultant curves of ILF_k^p and ILF_k^q as function of the reactive provision Q_k and the voltage V_k .

If voltage at bus k goes from 0.90 to 1.10 p.u., then the ILF_k^p goes from 0.0240 to 0.0182 . In all the cases, the ILFs are greater than zero. It means that, in all operating points, an increase of real power leads to an increase of losses.

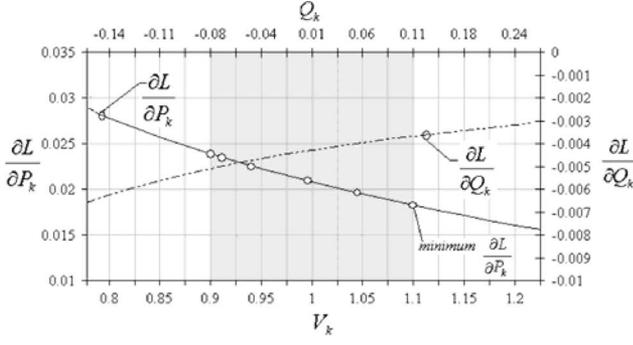

 Fig. 3. Relationship between reactive provision at bus k (Q_k) and ILFs.

Fig. 4 shows six operating points for ILF_k^P in the V_k – θ_k plane. Note that the minimum value of ILF_k^P (that leads to the minimum WG's loss payments) is reached when the voltage at bus k is 1.10 p.u. (boundary condition). However, this point may depend on other limits as the maximum line flow capacity. Therefore, a more general optimization model is required to find the best ILF_k^P for the WG unit. This issue is described in detail in the next section.

Although the ILF_k^P values are always positive—charging the WG units by the increased losses—the ILF_k^Q values always remain negative. If only reactive power signals are applied, the best operating point for the units connected at bus k correspond to a voltage of 0.90 p.u. (opposite boundary condition). Then, an increase in the reactive power injection leads to a decrease of system losses, and the WG units must be rewarded. It means that the ILF_k^Q signal is opposite to ILF_k^P . Consequently, a combination of both signals (ILF_k^P and ILF_k^Q) seems to be more appropriate to send economical signals to the WG units. However, there is no consensus about the application of reactive incremental signals, and a broad discussion in the regulatory forum is raised [20]. Despite some advances that have been made in order to introduce incremental signals in distribution systems [11], this scheme based on real power-based signals seems inadequate to provide efficient operational signals to DGs. The weakness of reactive price signals enforces a mandatory reactive power dispatch by the DNO in order to avoid undesired voltages and power losses.

IV. PROPOSED METHODOLOGY: OPTIMAL REACTIVE POWER RESPONSE FOR WG UNITS

This section describes an individual welfare methodology to find the optimum level of reactive power injection Q_{Gk} that maximizes the loss revenue $L_{Rk} = -\lambda P_{Gk} \text{ILF}_k^P$ of a WG unit. The real power injection P_{Gk} and the system price λ depend on external factors such as the wind availability and the energy market. As these values are known, the objective is to minimize the incremental loss factor ILF_k^P .

The model considers the following operational constraints: real and reactive power balances in all buses, voltage limits in all buses, and power flow restrictions in all lines. In the formulation, the calculation of the ILFs is included through equality constraints. The model to assess the optimal response

of one generating bus k is presented as follows:

$$\max -\text{ILF}_k^P(Q_{Gk}) = \min \text{ILF}_k^P(Q_{Gk}) \quad (11)$$

subject to

$$P_{Gi} - P_{Di} = V_i \sum_{j=1}^n V_j [G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}], \quad (12)$$

$$Q_{Gi} - Q_{Di} = V_i \sum_{j=1}^n V_j [G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}], \quad (13)$$

$$\sum_{\substack{j=1 \\ j \neq sl}}^n \frac{\partial P_j}{\partial \theta_i} \text{ILF}_j^P + \sum_{\substack{j=1 \\ j \neq sl}}^n \frac{\partial Q_j}{\partial \theta_i} \text{ILF}_j^Q = \frac{\partial L}{\partial \theta_i}, \quad i \neq sl \quad (14)$$

$$\sum_{\substack{j=1 \\ j \neq sl}}^n \frac{\partial P_j}{\partial V_i} \text{ILF}_j^P + \sum_{\substack{j=1 \\ j \neq sl}}^n \frac{\partial Q_j}{\partial V_i} \text{ILF}_j^Q = \frac{\partial L}{\partial V_i}, \quad i \neq sl \quad (15)$$

$$S_{ij}^{\min} \leq S_{ij} \leq S_{ij}^{\max}, \quad i \neq j \quad (16)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \quad i \neq sl \quad (17)$$

where the specified data is: $P_{Gi}, V_i^{\max}, V_i^{\min}, i = 1, \dots, n, i \neq sl$; $Q_{Gi}, i = 1, \dots, n, i \neq sl$; $P_{Di}, Q_{Di}, i = 1, \dots, n, i \neq sl$; $S_{ij}^{\min}, S_{ij}^{\max}, i, j = 1, \dots, n, i \neq j$ and the variables are: $Q_{Gk}, P_{Gsl}, Q_{Gsl}, S_{ij}, i, j = 1, \dots, n, i \neq j; V_i, \theta_i, \text{ILF}_i^P, \text{ILF}_i^Q, i = 1, \dots, n, i \neq sl$.

The specified data comprises the real power generation of WG units (including producer k); the reactive power supply injected by all generators except the producer k , and the real and reactive power demands. The operational limits are the voltage limits in all the buses; the capacity of all lines. The voltage and angle of the slack bus are also specified.

The reactive provision of WG units connected at bus k , both real and reactive injections of the slack bus, the incremental factors ILFs, and the voltages and angles in all the buses (excepting the slack one) are calculated.

The objective function (11) corresponds to the minimum incremental loss factor ILF_k^P of the k -generating bus. In order to compute the loss revenue L_{Rk} , the energy price (λ) is established for the wholesale market and the real power supply (P_{Gk}) is related to the energy resource availability at bus k . Both variables are fixed and known. Equations (12) and (13) represent the real and reactive power balances at each bus of the network. Expressions (14) and (15) were derived from (1) and (2), to calculate the incremental loss factors for real and reactive power. Mathematical derivation of (14) and (15) and detailed expressions of partial derivatives can be found in the Appendix. Equation (16) expresses the capacity line limits. Expression (17) is related to voltage limits settled by a grid code. As discussed in Section IV, the voltage limits are strongly related to the reactive capacity allowed in the generating buses. Then, the reactive power or power factor limits can be used instead of the specified voltage limits.

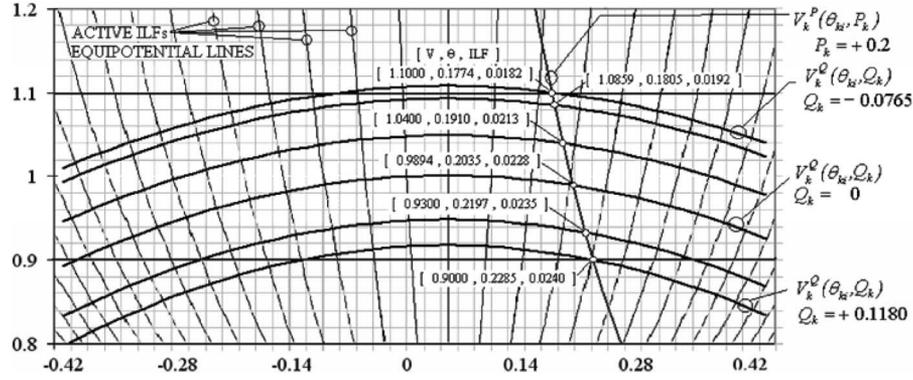


Fig. 4. ILFs operating points for a two-bus system in the $V_k - \theta_k$ plane.

Finally, as an additional analysis, we include the condition of maximum global revenue for all WG units (the sum of all individual revenues). In this case, the objective function (11) can be substituted by the global welfare objective as

$$\max - \sum_{k=1}^{n_g} \text{ILF}_k^P(Q_{Gk}) = \min \sum_{k=1}^{n_g} \text{ILF}_k^P(Q_{Gk}). \quad (18)$$

This point corresponds to the maximum benefit of the n_g producers connected to the grid.

V. REACTIVE POWER DISPATCH PERFORMED BY THE DNO

The reactive power provision of most of installed WG units is considered nondispatchable by the DNO. Thus, WG operators should ensure that reactive power injections are within the Grid Code limits. However, in some cases, the DNO might remove the nondispatchable condition by performing a mandatory reactive dispatch in order to reduce power losses and improve voltage profiles in the system.

In this paper, we defined for comparison purposes two types of mandatory dispatches: First, the settlement of a specific power factor value, and second, by running a R-OPF analysis. Results of mandatory dispatches will be compared with solutions provided by proposed individual welfare.

A. Specifying Power Factor Limits

The DNO specifies a given power factor at the then generating bus. Simulations of this paper consider that WG units are forbidden to provide reactive power to the system ($\cos \varphi = 1$).

B. Reactive Optimal Power Flow

The DNO performs an R-OPF in order to get the reactive provision of WG units in order to minimize the power losses subject to the system limits [3]. The R-OPF formulation employed here is written as

$$\min L = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}] \quad (19)$$

subject to (12), (13), (16), and (17) constraints.

where the specified data is P_{Gi} , V_i^{\max} , V_i^{\min} $i = 1, \dots, n$ $i \neq \text{sl}$; S_{ij}^{\min} , S_{ij}^{\max} , $i, j = 1, \dots, n$ $i \neq j$; P_{Di} , Q_{Di} , $i = 1, \dots, n$

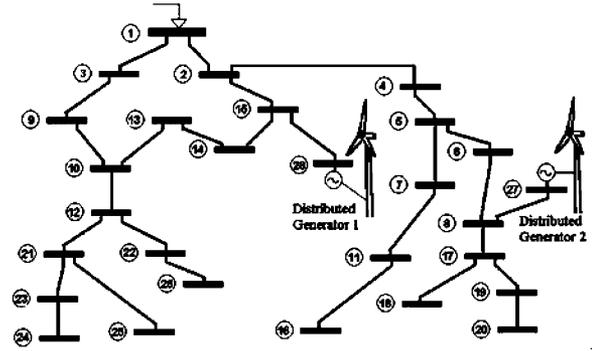


Fig. 5. T15-kV 28-bus test network.

and the variables are P_{Gsl} ; S_{ij} $i, j = 1, \dots, n$ $i \neq j$; Q_{Gi} , $i = 1, \dots, n$; V_i , θ_i , ILF_i^P , ILF_i^Q $i = 1, \dots, n$ $i \neq \text{sl}$.

VI. APPLICATION EXAMPLES

The aforementioned key features of the proposed optimization model were illustrated using two networks: a 15-kV 28-bus system and a 60-kV 55-bus system considering real wind conditions throughout 672 and 168 h, respectively.

A. Simulation Platform

The proposed methodology described in Section V and the conventional R-OPF optimization problem stated in Section VI were solved using the MATLAB's built-in nonlinear programming optimization tool FMINCON [27].

B. 15-kV 28-Bus Test System

The topology of the 15-kV 28-bus test system is shown in Fig. 5. Two studies were performed: 1) a single-point analysis for a specific hour (h 92) and 2) during a period of 4 weeks (28 days, 672 h). There are two wind farms rated 15 MW each. The first one is connected in the bus 27 at the end of a radial branch. The second one is connected in the bus 28. In the remainder of this paper, both generators are denoted by WG_{27} and WG_{28} . Fig. 6 shows the wind power generation output and Fig. 7 shows energy prices and load demand profile. Economical and technical data were taken from the Spanish market and the SCADA. Bus 1

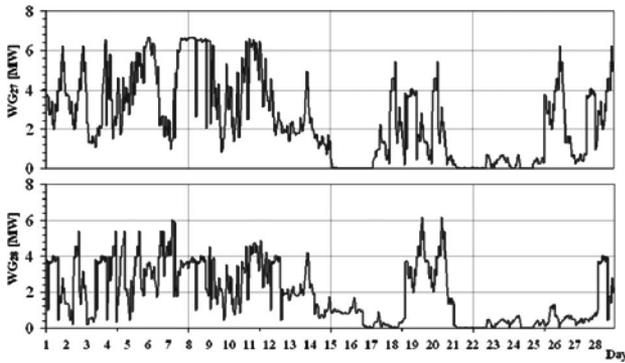


Fig. 6. Wind real power generation profile at bus 27 and bus 28.

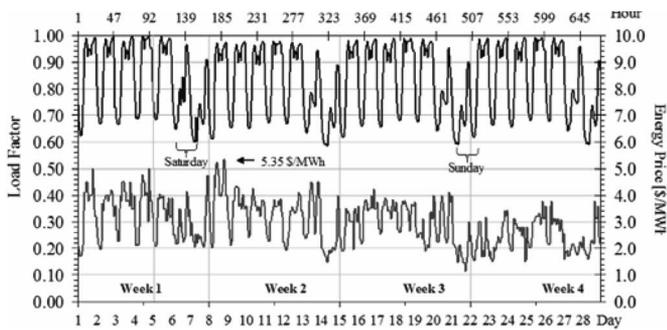


Fig. 7. Energy prices and load factor profile.

is the grid supply point (GSP), and it is used as a slack bus where no loss compensation is performed. According to grid code requirements, the voltage operating level should be set within 90% and 110%, in all buses. Line data can be requested from the authors. It is considered that WG units do not have reactive power limitations, being able to give volt/voltampere reactive support via installed capacitors or their own reactive capability.

1) *Single-Point Analysis*: In order to illustrate the response of WG units under incremental loss pricing (Section IV) and compare them with fixed DNOs reactive dispatches (Section V), calculations were performed for a specific hour (92). At this operating point, the system has a load factor equal to 1, power injection at both wind farms is 30% ($P_{G27} = 4.65$ MW and $P_{G28} = 4.65$ MW), and the energy price is 30.0 \$/MWh.

Five operational conditions are considered as follows:

- 1) No reactive injection allowed by DNO. $Q_{G27} = 0$ and $Q_{G28} = 0$. Variables are computed directly from the power flow solution using (1)–(6).
- 2) DNO R-OPF. Q_{G27} and Q_{G28} fixed by the DNO for minimal losses solving (19), (12), (13), (16), (17).
- 3) Individual Welfare model: Maximizing LR_{27} Maximizing- ILF_{27}^P when $Q_{G28} = 0$. The optimal reactive power provision of WG_{27} is obtained solving (11)–(17) for $k = 27$.
- 4) Individual welfare model: Maximizing LR_{28} , maximizing ILF_{28}^P when $Q_{G27} = 0$. The optimal reactive power

provision of WG_{28} is obtained by solving (11)–(17) for $k = 28$.

- 5) Global welfare model: Maximizing $LR_{27} + LR_{28}$, maximizing $ILF_{27}^P - ILF_{28}^P$. The optimal reactive power provision is obtained by solving (18), (12)–(17) for $k = 27$ and $k = 28$.

For each case, the following variables are computed: power losses (L), the ILFs (ILF_{27}^P and ILF_{28}^P) and the LR of each WG generating bus (LR_{27} and LR_{28}).

Table I shows the results associated to a single point (h 92). If reactive power generation is not allowed, the loss revenue level reached by WG_{27} and WG_{28} is similar to that obtained when a R-OPF dispatch is performed by the DNO. Note that LR_{27} is around 2.5 \$/h and LR_{28} is around -0.9 \$/h.

If the proposed individual welfare methodology is applied, each generator achieves considerable gains. Note that WG_{27} increases its loss revenue by 68% with respect to the R-OPF dispatch. On the other hand, when profits of WG_{28} are maximized the loss revenue goes from a negative value -0.92 \$/h (WG pays by losses) to 0.51 \$/h (WG does not pay by losses). As expected, minimal power losses (482 kW) are achieved under the R-OPF dispatch. However, when we maximize the loss revenue of WG_{27} and WG_{28} , power losses grew significantly until reaching 805 kW (67%) and 1291 kW (167%), respectively. These operating points are achieved when the voltage level reaches a limit.

Finally, the global welfare condition is reached when $Q_{G27} = 8.68$ Mvar and $Q_{G28} = -14.03$ Mvar, and power losses achieve 2 MW. At this point, the best revenue condition for both generators (LR_{27} is 7.73 \$/h and LR_{28} is 0.93 \$/h) is reached. Observe that the loss revenue of WG_{27} represents the 5.5% of the real power sale (7.73 \$/h/139.5 \$/h). This result is meaningful because both generators could make a deal in order to maximize their global revenues. However, the increase of losses is very important and it might be unbearable by the system. In this case, DNOs mandatory reactive dispatch of reactive power of WG_{27} is necessary.

2) *4-Week Analysis (672 h)*: The models described before have been applied during a period of 4 weeks (672), taking into account high variability in wind power, market energy prices, and load consumption (see Figs. 6 and 7). The results of the proposed methodology and mandatory DNOs dispatches are condensed in Table II.

First, we note that the loss revenue of both generators was similar to the reactive provision derived from the ROPF analysis. If the individual welfare model is applied, WG_{27} and WG_{28} revenues are improved in 90% and 300%, respectively (2.5% and 1.2% of real power sale). However, the losses increase significantly passing from the minimal power losses (4.12% of global energy consumption) to 5.35% and 12.19%. The point-to-point results are drawn in Figs. 8 and 9.

Figs. 8 and 9 show the reactive provision at each generating bus and the reactive dispatch provided by the R-OPF. In addition, the real power-based ILFs and loss revenue patterns are shown. Note that when the real power generation is low (days 17, 18, and 21–27), the reactive provision of WG_{27} is less variable than when the wind production is high. Conversely, high values of

TABLE I
SINGLE-POINT ANALYSIS RESULTS

Variable	Units	No Reactive Injection	Reactive OPF	Maximum $-\text{ILF}_{27}^P$	Maximum $-\text{ILF}_{28}^P$	Maximum $-\text{ILF}_{27}^P - \text{ILF}_{28}^P$
Q_{G27}	Mvar	0.00	1.68	6.40	0.00	8.68
Q_{G28}	Mvar	0.00	-1.05	0.00	12.13	-14.03
V_{27}	pu	0.9610	0.9992	1.1000	0.9919	1.1000
V_{28}	pu	1.0109	1.0072	1.0270	1.1000	0.9000
P_{G27}	MW	4.65	4.65	4.65	4.65	4.65
P_{G28}	MW	4.65	4.65	4.65	4.65	4.65
Losses	kW	531.46	482.78	805.18	1291.61	2004.48
ILF_{27}^P	-	-0.0178	-0.0188	-0.0300	-0.0198	-0.0555
ILF_{28}^P	-	0.0060	0.0066	0.0042	-0.0037	-0.0067
Market Price Hour 92	\$/MWh	30.0	30.0	30.0	30.0	30.0
Energy Sold G_{27}	\$/h	139.5	139.5	139.5	139.5	139.5
Energy Sold G_{28}	\$/h	139.5	139.5	139.5	139.5	139.5
Loss Revenue G_{27}	\$/h, %	2.48 (1.8%)	2.62 (1.9%)	4.19 (3%)	2.76 (2%)	7.73 (5.5%)
Loss Revenue G_{28}	\$/h, %	-0.83 (0.6%)	-0.92 (0.6%)	-0.58 (0.4%)	0.51 (0.3%)	0.93 (0.6%)

TABLE II
28-BUS SYSTEM FOUR-WEEKS ANALYSIS RESULTS

Variable	Units	No Reactive Injection	Reactive OPF	Maximum ILF_{27}	Maximum ILF_{28}
P_{G27}	MWh	1687.14	1687.14	1687.14	1687.14
P_{G28}	MWh	1286.14	1286.14	1286.14	1286.14
Load Demand	MWh	8658.20	8658.20	8658.20	8658.20
Losses	MWh	382.32	356.53	463.17	1055.28
Losses	%	4.42%	4.12%	5.35%	12.19%
Energy Sold G_{27}	\$	54082.88	54082.88	54082.88	54082.88
Energy Sold G_{28}	\$	40681.77	40681.77	40681.77	40681.77
Loss Revenue G_{27}	\$	785.37	746.34	1398.43 (2.5%)	866.14
Loss Revenue G_{28}	\$	177.45	128.81	214.47	514.50 (1.2%)

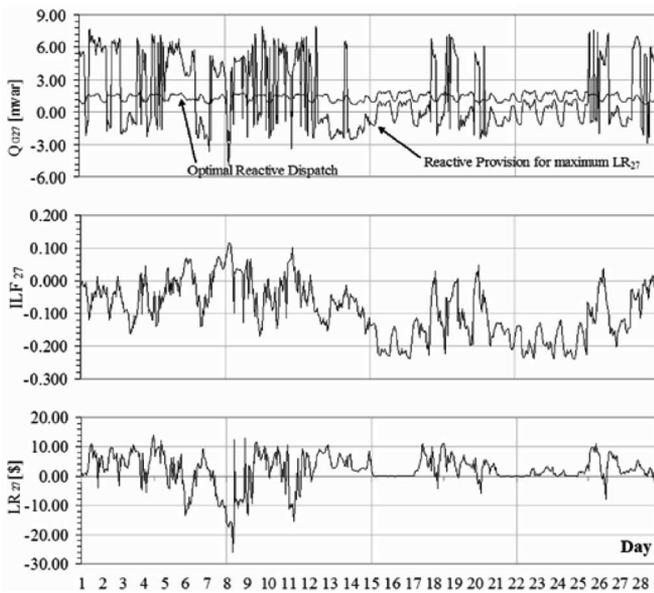


Fig. 8. Optimal reactive provision of WG_{27} .

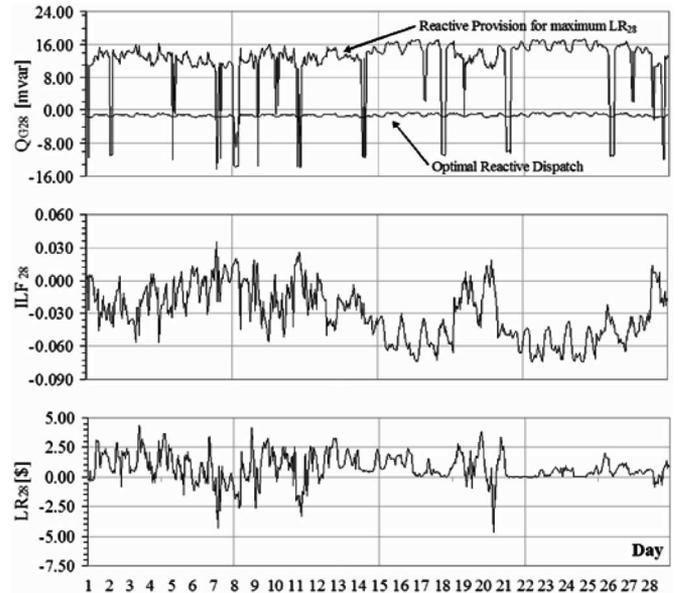


Fig. 9. Optimal reactive provision of WG_{28} .

reactive provision of WG_{28} are associated to high voltages. In general, results lead to restricted operating points.

In Fig. 10, we show the behavior of power losses during 672 hours for the proposed methodology and the R-OPF. Minimal

power losses are around 300–700 kW. Note that if LR_{28} is maximized, the power losses reach values around 2–2.5 MW. On the other hand, if LR_{27} is maximized, the optimal results are similar to minimal losses provided by the R-OPF analysis.

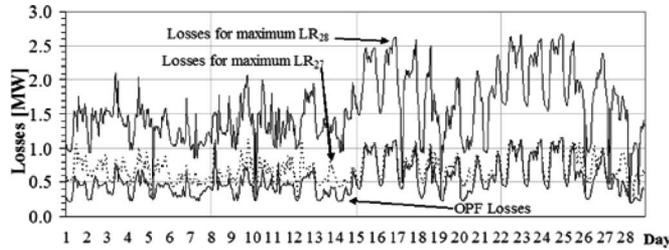


Fig. 10. Power losses in the 28-bus test network.

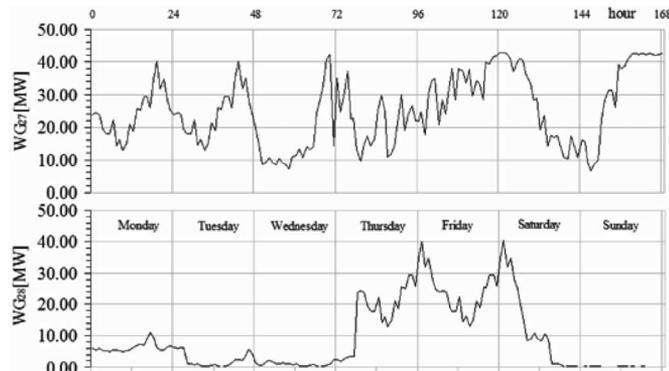


Fig. 11. Wind real power generation profile for the 55-bus test network.

In these circumstances, the proposed methodology permits to identify that WG_{27} does not affect the system losses. On the contrary, WG_{28} does drastically affect the system and it should be compulsorily dispatched by the DNO.

3) *Validation of Results:* The polarity and value of the ILFs calculated earlier can be verified using an incremental analysis by performing multiple power flow simulations. The PCFLO power flow program was used [28]. For a given bus k , it is necessary to run two power flow programs to reach a specific ILF_k^P . For instance, if the power injection at bus 28 is increased by a small amount (1 kW), the variation of power losses can be easily calculated using the loss equation (6). This validation process was successfully performed on all results reported in Figs. 8 and 9.

C. 60-kV 55-Bus Test System

The models described in this paper were also applied to a 60-kV 55-bus system during a period of 1 week (168 h). The system has two 100 MW wind farms connected to buses 27 and 28, respectively. Wind-based real power generation profile is presented in Fig. 11. System topology is shown in Fig. 14. The peak load is 320 MW, and the consumption is around 1920 GWh/year. Energy prices and load profile are drawn in Fig. 12. Bus 1 is the GSP, and it is used as a slack bus, where no loss compensation is performed. The voltage operating level has been established within 90% and 110%, in all buses. Line data can be requested from the authors.

Similar to a previous example, results are condensed in Table III. We observed that WG_{27} and WG_{28} slightly improved their loss revenue in 3.5% and 10.9%, respectively. The loss

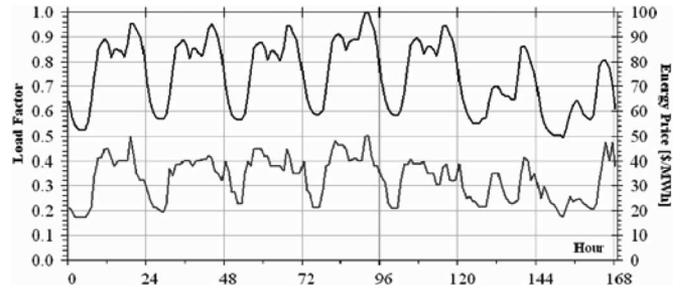


Fig. 12. Energy prices and load profile for the 55-bus test network.

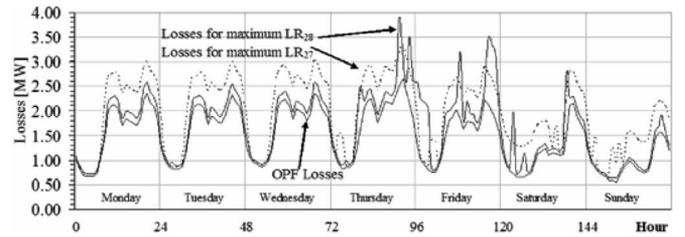


Fig. 13. Power losses for the 55-bus test system.

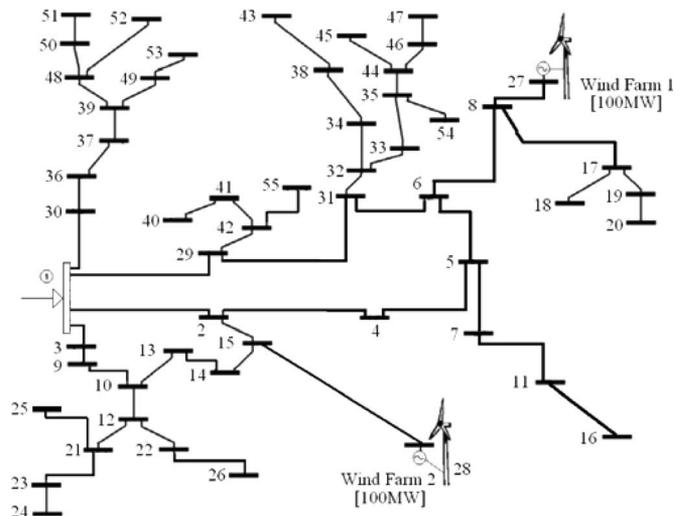


Fig. 14. 60-kV 55-bus test network.

revenues obtained only represent 0.7% and 0.2% of the incomes by energy sold to the market.

In this case, loss revenues are not relevant with respect to the 10-kV case. As the voltage level is higher and power losses are smaller, there is not much variability in the ILF pattern of the WG units. Numerical results show that LR at 60 kV is less than 0.7% of the power sale.

On the other hand, power losses increase moderately going from the minimal power losses (0.61%) to 0.83% (when loss revenue of WG_{27} is maximized) and 0.71% (when loss revenue of WG_{28} is maximized). In some operating points, power losses achieve high values as seen in Fig. 13. However, in general, both WG_{27} and WG_{28} do not affect drastically the level of

TABLE III
55-BUS SYSTEM 1-WEEK ANALYSIS RESULTS

Variable	Units	No Reactive Injection	Reactive OPF	Maximum ILF ₂₇	Maximum ILF ₂₈
P _{G27}	GWh	4.25	4.25	4.25	4.25
P _{G28}	GWh	1.50	1.50	1.50	1.50
Load Demand	GWh	40.25	40.25	40.25	40.25
Losses	MWh	260.77	246.58	332.11	285.70
Losses	%	0.65%	0.61%	0.83%	0.71%
Energy Sold G ₂₇	\$	139123.31	139123.31	139123.31	139123.31
Energy Sold G ₂₈	\$	50776.97	50776.97	50776.97	50776.97
Loss Revenue G ₂₇	\$	1048.97	1046.69	1083.42 (0.7%)	1030.66
Loss Revenue G ₂₈	\$	105.99	107.36	107.24	119.10 (0.2%)

system losses and they could be considered as nondispatchable generators in reactive power by the DNO.

D. Final Considerations

The optimal operating points are within desired limits owing to rigid restrictions. Further analyses should consider charges due to infringement on operational limits. Fuzzy constraints might be used.

In this proposal, known data are taken from the monitoring system (SCADA). However, part of this data could not be available. Future research should focus on the integration of fuzzy optimal power flow techniques [25] and fuzzy state estimators [26], in order to cope with uncertainties and errors in data acquisition.

Finally, as the incremental loss allocation framework depends on the choice of the slack bus, the optimal reactive operating points are strongly affected by its localization. Moreover, in more complex distribution systems (with several connections with transmission networks), additional research might be made to assess the effect of several grid supply points (or slack buses) in the same grid.

VII. CONCLUSION

This paper addresses the optimal reactive power provision for WG units exposed to location specific and time-varying GUoS tariffs. The optimization problem is stated from the generation utility point of view, aiming to minimize charges (or maximize revenues) due to incremental-based loss allocation strategy. The proposed model has been evaluated, and results are discussed from two distribution systems under real wind, load, and energy market conditions.

The operating points obtained by the proposed methodology are compared with mandatory dispatches established by the DNO by means of a R-OPF.

The main conclusions addressed from the application examples are outlined as follows:

- 1) Wind power generation producers are able to drive the reactive power injection—within the grid code limits—in order to maximize loss revenues under location specific and time-varying incremental GUoS tariffs.
- 2) At medium-voltage distribution system (10 kV), simulations show the individual loss revenues can reach 1.2–2.5% of the real power sale. Global loss revenues can

reach until 5% of the real power sale. Results at higher voltage level system (60 kV) are less significant. In this case, loss revenues are less than 0.7% of the real power sale. In both cases, generators do not pay any charge.

- 3) The results of the proposed analysis prove that some WG units can produce an important increase in power losses when they are exposed to incremental prices. In this case, these generators should be obligatory dispatched in reactive power by the DNO.
- 4) The proposed analysis constitutes a valuable tool to identify which generators are suitable to be controlled in reactive power by the DNO under the incremental loss allocation policy.

APPENDIX

MATHEMATICAL DERIVATION OF (14) AND (15) OF THE MODEL

The system power losses are obtained through (6) as a function of all voltages V and angles θ . In order to get the real and reactive incremental loss factors, it is required to apply the chain rule

$$\frac{\partial L}{\partial \theta_k} = \sum_{i=1}^n \frac{\partial L}{\partial P_i} \frac{\partial P_i}{\partial \theta_k} + \sum_{i=1}^n \frac{\partial L}{\partial Q_i} \frac{\partial Q_i}{\partial \theta_k} \quad (20)$$

$$\frac{\partial L}{\partial V_k} = \sum_{i=1}^n \frac{\partial L}{\partial P_i} \frac{\partial P_i}{\partial V_k} + \sum_{i=1}^n \frac{\partial L}{\partial Q_i} \frac{\partial Q_i}{\partial V_k} \quad (21)$$

where the real and reactive power injections are given by

$$P_i = V_i \sum_{k=1}^n V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)] \quad (22)$$

$$Q_i = V_i \sum_{k=1}^n V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]. \quad (23)$$

Under incremental analysis, at a given operating point power losses can be allocated to producers and consumers simultaneously through the ILFs. By definition, these coefficients measure the change in real power losses due to the incremental change in power injections in each bus if the network

$$\text{ILF}_i^P = \frac{\partial L}{\partial (P_{Gi} - P_{Di})} \quad \text{ILF}_i^Q = \frac{\partial L}{\partial (Q_{Gi} - Q_{Di})}. \quad (24)$$

By definition, incremental loss factors at slack or reference bus are equal to zero: $\text{ILF}_{s1}^P = \text{ILF}_{s1}^Q = 0$. Substituting (24) in

(20) and (21), the following expressions are obtained:

$$\frac{\partial L}{\partial \theta_k} = \sum_{\substack{i=1 \\ i \neq sl}}^n \text{ILF}_i^P \frac{\partial P_i}{\partial \theta_k} + \sum_{\substack{i=1 \\ i \neq sl}}^n \text{ILF}_i^Q \frac{\partial Q_i}{\partial \theta_k}, \quad k \neq sl \quad (25)$$

$$\frac{\partial L}{\partial V_k} = \sum_{\substack{i=1 \\ i \neq sl}}^n \text{ILF}_i^P \frac{\partial P_i}{\partial V_k} + \sum_{\substack{i=1 \\ i \neq sl}}^n \text{ILF}_i^Q \frac{\partial Q_i}{\partial V_k}, \quad k \neq sl. \quad (26)$$

Derivatives $\frac{\partial P_i}{\partial \theta_k}, \frac{\partial P_i}{\partial \theta_j}, \frac{\partial P_i}{\partial V_k}, \frac{\partial P_i}{\partial V_j}, \frac{\partial Q_i}{\partial \theta_k}, \frac{\partial Q_i}{\partial \theta_j}, \frac{\partial Q_i}{\partial V_k}, \frac{\partial Q_i}{\partial V_j}, \frac{\partial L}{\partial \theta_i}, \frac{\partial L}{\partial V_i}$ are calculated directly from (22) and (23).

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