A Control Allocation Approach to Manage Multiple Energy Sources in EVs

Ricardo de Castro, Pedro Melo, Pedro Pacheco, Rui Esteves Araújo, Diamantino Freitas
Faculdade de Engenharia da Universidade do Porto
R. Dr. Roberto Frias, 4200-465 Porto, Portugal
{de.castro, dee07007, ee05172, raraujo, dfreitas}@fe.up.pt

***Abstract*—This article is concerned with the design of an energy management system (EMS) for the hybridization of multiple energy sources (ES's) in electric vehicles, focusing in a particular configuration composed by batteries and supercapacitors (SCs). As a first design step, we investigated an (non-causal) optimal power allocation, targeting the minimization of the energy losses over a complete driving cycle. Albeit the solution obtained with this formulation demands the advance knowledge of the vehicle driving cycle, it also provides a useful benchmark solution to assess the performance of causal EMS's. A more practical EMS is then derived, based on the control allocation (CA) concept. This approach, typically employed in redundant control systems, enable us to address the various objectives and constraints that appear in EMS design problem, such as the DC bus voltage regulation, SC state of charge tracking, minimization of power losses, current and state of charge limits, etc. Simulation results show the effectiveness of the proposed CA based EMS, yielding performances very close to the optimal non-causal power allocation.**

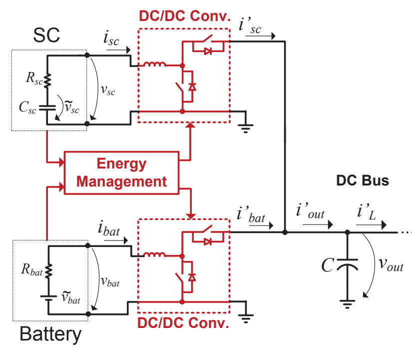

Figure 1. The Energy Management System (EMS), composed by two power sources (SC and Bat.) and two DC/DC bidirectional boost converters (b).

I. Introduction

The Energy Storage (ES) plays an important role in defining the most relevant performance metrics of Electric Vehicles (EVs), such as the vehicle range per charge and costs. The widespread use of this new transportation system will only be achieved if the EV ES feature low cost, long cycle life and high power and energy densities [1]. However, given the current state of technology, these features are difficult to combine into a single class of storage [2]. Consequently, in the absence of this ideal power source, several ES hybridization strategies have been proposed in the literature [3]–[7], driven by the idea of combining different storage technologies to take advantage of the best properties that each ES has to offer. In this context, the current work aims to develop an Energy Management System (EMS) for an EV based on the hybridization of supercapacitors (SCs) and batteries. From the various possibilities to interconnect these ES's (direct connection, one power converter or two power converters), we focus our attention in the configuration where each ES has its own power converter, since it allows a greater flexibility in the energy management [2] (see Figure 1).

As a starting point to the EMS design, we formulated an optimal non-causal problem to extract the ES's power allocation that minimizes the energy losses during a given driving cycle. By employing numerical methods, a solution to this optimal problem can be obtained, enabling us to gain some insight about the best power allocation and define the theoretical maximum efficiency that the EMS can reach. A more practical controller is then derived, based on the Control Allocation (CA) framework [8], often used in aeronautics [9], maritime [10] and automotive [11] applications, but adapted in this work to the EMS problem. The CA main responsibility is to allocate the current (or power) requested by the DC bus controller among the two (redundant) ES's (see Figure 1). This redundancy in the ES can then be explored to pursue secondary objectives, like minimizing power and energy performance indexes, related with local/"short term" and global/"long-term" perspectives. The short-term perspective is mainly concerned with the minimization of the instantaneous (power) losses and has a close connection with the perception that SC should provide the current peaks during transients. However, this allocation policy does not offer the best option on the long-run, otherwise the most efficient energy source (the SC) would be quickly drained, and does not take into account the road profile where the vehicle will travel. As a consequence, it is necessary to introduce a strategic/"long-term" objective in the CA performance metric to take in consideration the most suitable power sharing over the long run of the vehicle. To this aim, we elected the SC SOC tracking as the strategic objective.

With this approach we hope to find a suitable balance between all objectives present in the EMS, namely the DC bus regulation, the minimization of instantaneous power losses and (hopefully) the global driving cycle energy losses.

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Table I
COMPARISON OF THE BAT. + SC (OPTIMUM NON-CAUSAL SHARING STRATEGY) AGAINST THE CONFIGURATION WITH "PURE BATTERY"

Driving Cycle	Energy Sources	Energy E_{in} [Wh]	Energy Cycle E_{cycle}[Wh]	Energy Losses in Bat. + SC [Wh]	Energy Losses in Bat. + SC % of E_{in}	Efficiency $\frac{E_{cycle}}{E_{in}}$[%]	P_{bat} [kW] Mean	P_{bat} [kW] Max	P_{sc}[kW] Max
ECE	Bat.	58.97	55.66	3.32	5.63	94.37	1.09	11.44	-
	Bat. + SC	58.58	57.36	1.22	**2.09**	**97.91**	0.89	**4.00**	7.37
NEDC	Bat.	1073.68	960.84	112.84	10.51	89.49	3.17	42.18	-
	Bat. + SC	1051.35	978.05	73.30	**6.97**	**93.03**	3.17	**23.37**	12.65
FTP75	Bat.	1502.40	1360.04	142.36	9.48	90.52	2.88	29.48	-
	Bat. + SC	1457.40	1390.25	67.14	**4.61**	**95.39**	2.77	**14.21**	15.56
FTP-Highway	Bat.	1894.18	1752.07	142.11	7.50	92.50	8.91	22.83	-
	Bat. + SC	1894.10	1776.55	117.55	**6.21**	**93.79**	8.91	**13.96**	8.84
Artemis Urban	Bat.	381.37	328.00	53.37	13.99	86.01	1.38	24.36	-
	Bat. + SC	354.02	338.56	15.46	**4.37**	**95.63**	1.29	**7.12**	16.12
NYCC	Bat.	139.57	122.29	17.28	12.38	87.62	0.84	25.02	-
	Bat. + SC	131.31	126.60	4.71	**3.58**	**96.42**	0.87	**6.29**	16.64
SFTP SC03	Bat.	485.96	429.06	56.90	11.71	88.29	2.94	33.73	-
	Bat. + SC	461.63	439.10	22.54	**4.88**	**95.12**	2.64	**12.00**	20.01

II. NON-CAUSAL OPTIMAL POWER SHARING

In this section a non-causal optimal controller for allocating the power between the ES's will be studied.

A. Simplified ES models

The state of charge (q) and the charge (Q) in the SC and battery can be calculated as [1]:

$$\frac{dQ_j(t)}{dt} = -i_j(t), \quad q_j(t) = \frac{Q_j(t)}{Q_{0,j}}, \quad j \in \{bat, sc\} \tag{1}$$

where $Q_{0,j}$ is the ES nominal capacity and $i_j(t)$ the ES output current. The equivalent model of SC can be represented as an ideal capacitor (C_{sc}) in series with a resistor (R_{sc}), while in the battery model the capacitor is replaced by a variable DC source (see Figure 1). In both cases, the ES output voltage (v) is defined as:

$$v_j(t) = \tilde{v}_j(t) - R_j i_j(t), \qquad j \in \{bat, sc\} \tag{2}$$

where $\tilde{v}_j$ is the ES internal voltage. If the open-circuit voltage in the battery has a linear dependence on $q(t)$ [1], then the open-circuit voltage in both ES's is given by:

$$\tilde{v}_j(t) = \kappa_j + m_j q_j(t), \qquad j \in \{bat, sc\} \tag{3}$$

where κ_j is the offset of the open circuit voltage and m_j the SOC gain. Although more precise mathematical models for the SC [12] and batteries [13] are available, the simplified ES models considered in this work are a sufficient mean to analyze the main energy phenomena in the EMS, enabling us to gain some insight about the optimum power sharing, without introducing unnecessary complexity in the mathematical structure of the problem. With these definitions in mind, the power transmitted by the ES's is given by:

$$P_{in}(t) = P_{bat}(t) + P_{sc}(t) = \sum_{j\in\{bat,sc\}} \tilde{v}_j(t) i_j(t) \tag{4}$$

which must cover the losses in the energy sources and DC/DC converters, as well as the powertrain power requests.

In the ideal scenario, the vehicle driving cycle is known beforehand, i.e. the vehicle speed profile $v_{cycle}(t), t \in [0, T]$ and the road angle profile $\alpha_{cycle}(t)$ are available at the beginning of the journey ($t = 0$). Based on this information and applying simple Newton laws, it is straightforward to find the power $P_{cycle}(t)$ requested to the ES's [14].

B. Optimal Power Allocation

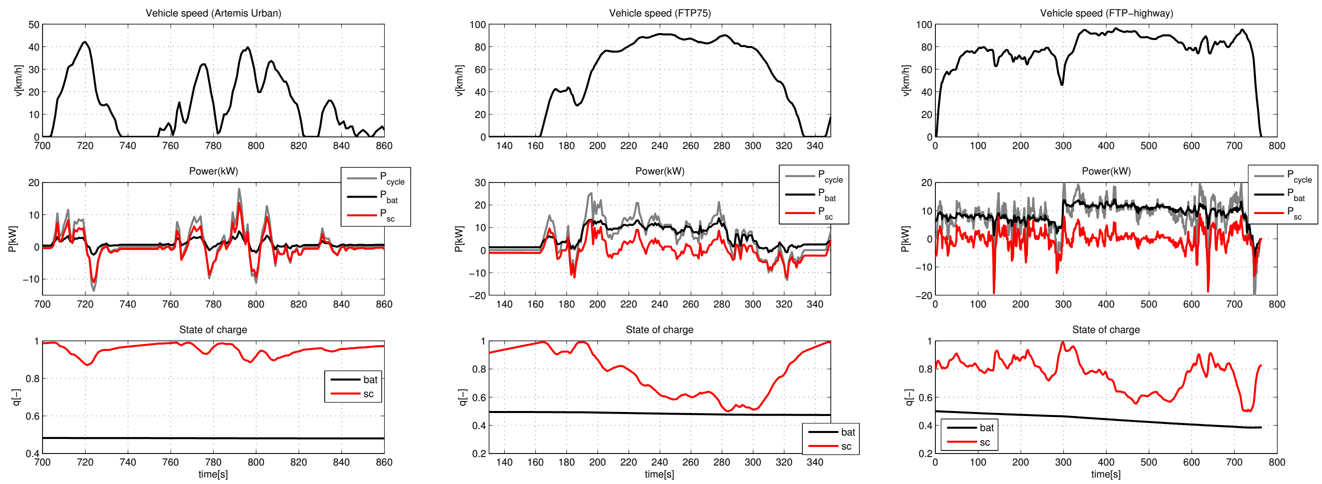
The main objective of the EMS is to find the power allocation that minimizes the energy delivered by the ES's or, instead, the energy losses in the process. Following the simplified reasoning initiated in the previous section, we neglect the DC/DC converter losses and assume that the Joule losses represent the dominant loss factor in the EMS:

$$P_{loss}\left(i_{bat}(t), i_{sc}(t)\right) = \sum_{j\in\{bat,sc\}} R_j \big(i_j(t)\big)^2 \tag{5}$$

As a result, the optimal power allocation can be mathematically formulated as follows:

$$\begin{aligned}
&\min_{\mathbf{I}_{bat},\mathbf{I}_{sc}\in\mathbb{R}^{N+1}} \sum_{j\in\{bat,sc\}} R_j \|\mathbf{I}_j\|^2 T_s \\
&s.t. \quad Q_j[k+1] = Q_j[k] - I_j[k]T_s, \quad Q_j[0] = Q_j^0 \\
&\qquad \tilde{v}_j[k] = \kappa_j + \frac{m_j}{Q_{0,j}} Q_j[k] \\
&\qquad \sum_{j\in\{bat,sc\}} \Big(\tilde{v}_j[k] I_j[k] - R_j \left(I_j[k]\right)^2\Big) = P_{cycle}[k] \\
&\qquad \underline{I_j} \le I_j[k] \le \overline{I_j}, \quad \underline{q}_j Q_{0,j} \le Q_j[k] \le \overline{q}_j Q_{0,j} \\
&\qquad k \in \{0, \ldots N\}, \quad j \in \{bat, sc\}
\end{aligned} \tag{6}$$

where $\|.\|^2$ is the Euclidean norm and T_s the discretization step size. The first and second constraints are just stating that the ES model must be satisfied; the third constraint is a simple power balance, while the last constraint specifies the admissible current and SOC limits. We notice that, with the exception of the ES charge dynamics, the remaining constraints are algebraic and, since the charge dynamics are relatively slow, we selected a large step size, equal to 1s. For longer driving cycle, the number of variables can be very significant (e.g. superior to 10000 in the FTP75 cycle), but the numerical solver employed in the current work, the Interior

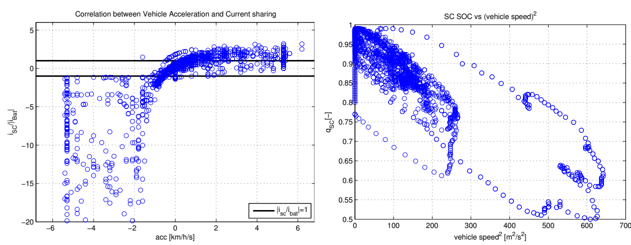


(a) Artemis Urban (zoom)

(b) FTP75 (zoom)

(c) FTP Highway (zoom)

Figure 2. Examples of optimum power sharing (non-causal) for some sections of typical driving cycles.



(a) FTP75

(b) FTP75

Figure 3. Correlation between acceleration and the optimum current sharing i_{sc}/i_{bat} (left figures) and SC SOC and square of vehicle speed (right figures).

Point OPTimizer (Ipopt) [15], was able to cope well with the problem.

C. *Evaluation of the (non-causal) Optimum Power Sharing*

To assess the performance of the optimal power distribution defined in (6), several numerical simulations were carried out for some typical driving cycles: ECE, NEDC, Artemis Urban, FTP75, FTP Highway, SFTP S03 and NYCC [16]. The characteristic of the vehicle used in these tests are briefly described in Appendix A, and initial SOC were taken as $q_{sc} = 0.8, q_{bat} = 0.50$. For each driving cycle, two scenarios were considered: $i)$ in the first case, only batteries were used as ES (the SC mass was also removed from the vehicle total mass); $ii)$ in the second case, the SC were added to the energy storage system, and the optimal power sharing (6) applied. The comparison between these two scenarios was performed taking in consideration the following metrics:

1) Energy delivered by the ES's: $E_{in} = \int_0^T P_{in}(t)dt$;
2) Energy requested by the powertrain to satisfy the driving cycle: $E_{cycle} = \int_0^T P_{cycle}(t)dt$;
3) The ES's losses: $E_{loss} = \int_0^T P_{loss}(t)dt$;
4) The ES's discharge efficiency: E_{cycle}/E_{in};
5) The maximum power requested to the battery: $\max_{t\in[0,T]} P_{bat}(t)$

Table I shows the overall results of the various configurations under study. It can be seen that, in all the driving cycles, the SC inclusion significantly reduces the energy losses in the ES's, contributing to an increase in the discharge efficiency and, eventually, the vehicle range per charge. Another factor to be highlighted is the reduction in the maximum power requested to the battery: when the SC aid the battery, the peak power in the battery is reduced in $50-80\%$, which will decrease the battery stress and extend its cycle life. In Figure 2 the optimal power sharing between the ES's, achieved during some typical driving cycles, can be observed. Based on these results, we can state the following remarks:

Remark 1: it is advantageous to charge the SC's when the vehicle is stationary (see the initial part of Figure 2(b).

Remark 2: when the vehicle is travelling at "cruising" speed, the battery should provide the average power, while the SC's should respond to the power oscillations (see Figure 2(c)).

Remark 3: during the acceleration and braking transients, the SC usage is normally higher that the batteries. (see Figure 2(a) and 3).

Remark 4: there is a small correlation between the SC SOC and the square of the vehicle speed (Figure 3).

Remark 5: Analyzing the power allocation in the FTP Highway, depicted in Figure 2(c), it is worth to notice the severe reduction in the SC SOC that appear before the vehicle final stop (around 720s); In practice, anticipate these situations without knowing the vehicle future course is extremely challenging.

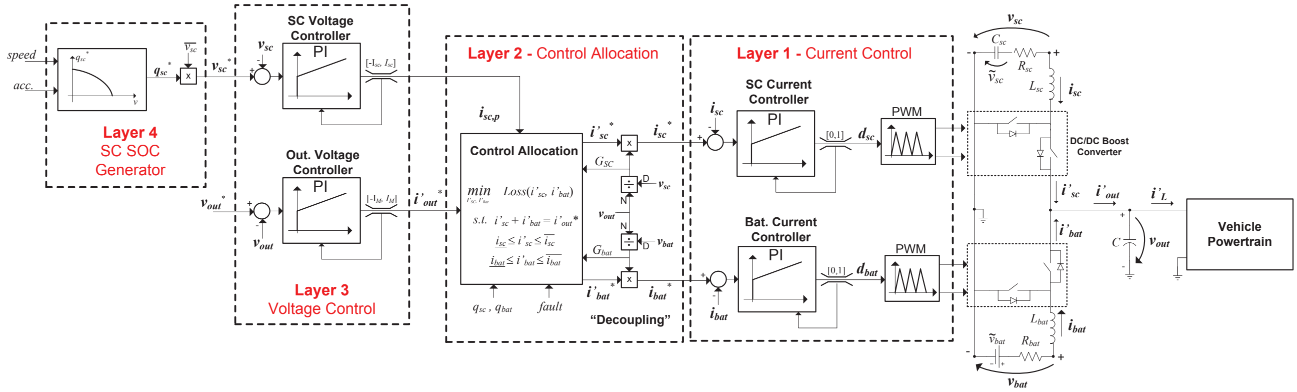
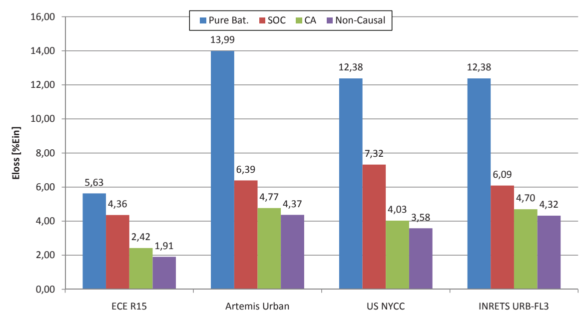
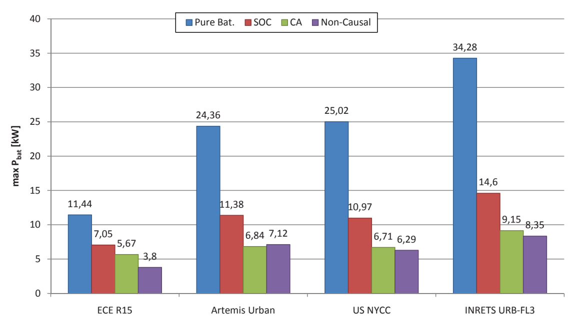


Figure 5. Final control diagram for the proposed EMS.



(a) Energy Losses



(b) Maximum Battery Power

Figure 4. Energy Losses in the ES's (a) and the maximum power requested to the battery (b), obtained for several urban driving cycles and allocation strategies.

III. Causal Power Allocation

In practice, implementing the controller (6) is not easy due to the undesirable non-causal property, i.e. the controller requires future information about the vehicle driving cycle. Therefore, in this section we describe a causal EMS based on the Control Allocation (CA) approach, depicted in Figure 5. For systematize the study of this EMS, we divided it in 4 main layers: current controllers (1), control allocation scheme to distribute the current among the ES's (2), voltage controllers (3) and the SC SOC generator (4).

On the lower level we have the current control loops: the DC/DC converters are current controlled, since the advantages of these inner loops are well known in electric drives. These controllers are based on simple, but effective, Proportional+Integral (PI) control and manipulate the converter duty cycle to regulate the converter input current. On the outer loop, similar linear controllers are employed to regulate the DC bus and SC voltage. The SC voltage controller is, in fact, an indirect control over the SC SOC, and generates the preferred set-points for the SC currents (note that they are just "preferred" set-points because the ultimate decision on the SC current is made by the CA layer). The CA is employed in the middle layer and it has the responsibility to manage the multiple objectives that can appear in current sharing strategy. Finally, as analyzed in Section I, the generation of SC SOC reference, q_{sc}^*, is a strategic variable in the EMS and constitutes the fourth and last layer. To simplify the notation, throughout the article, we consider the variables with the superscript $'*'$ as reference values that should be imposed by the control system. To make a clear distinction between the currents in the ES side and in the DC bus side, the nomenclature i' is employed in the later case.

A. Layer 4- SC SOC reference generator

The results obtained with the optimal non-causal allocation provided evidence of a small correlation between the SC SOC and the vehicle kinetic energy (see Figure 3). Based on this finding, one of the simplest strategies for generating the SC SOC reference, q_{sc}^*, consists in impose a linear relationship between the kinetic energy of the vehicle ($E_{c,car}$) and SC energy (E_{SC}):

$$\underbrace{\frac{1}{2}C_{sc}\left(\tilde{v}_{sc}\right)^2}_{E_{sc}} = \frac{1}{2}C_{sc}\left(\overline{v_{sc}}\right)^2 - \delta\underbrace{\frac{1}{2}Mv^2}_{E_{c,car}} \tag{7}$$

where $\overline{v_{sc}}$ is the SC nominal voltage and δ is a tuning parameter. Based on this relation, the SC SOC reference is given by:

$$q_{sc}^*(v) = \sqrt{1 - \delta \frac{Mv^2}{C_{sc}\overline{v_{sc}}^2}} \tag{8}$$

Even though this simple strategy for constructing q_{sc}^* hardly provide the optimum SC SOC value for all the possible vehicle operating points, the main idea behind the EMS structure, depicted in Figure 5, is to conjugate the SC SOC control with additional energetic objectives, imposed in the CA layer, making the EMS more robust to imperfections in q_{sc}^*.

B. Layer 1 and 3 - Current and Voltage Controllers

Layers 1 and 3 are composed by four linear PI controllers that can be designed using well known linear tuning rules to ensure stability and good transient behaviour. For the sake of brevity, these design guidelines are omitted in this article. In the interest of simplifying the voltage controller design, it was assumed that the output current of the power converters (i'_{bat}, i'_{sc}) are directly controlled. However, in practice, we only have direct control of the SC and battery current (i_{sc}, i_{bat}). To overcome this problem, we employed the input/output power balance in the DC/DC conversion (considering a lossless converter) to find a relation between the converter's input and output currents:

$$i_{sc} = \underbrace{\frac{v_{out}}{v_{sc}}}_{G_{sc}} i'_{sc} \qquad i_{bat} = \underbrace{\frac{v_{out}}{v_{bat}}}_{G_{bat}} i'_{bat} \tag{9}$$

This method, also known as the decoupling method [4], is used to connect Layers 2 and 1 (see Figure 5).

C. Layer 2- Control Allocation

In order to allocate the DC bus controller current request (i'_{out}) by the two redundant ES's considered in this work, i.e. the battery and SC, the Control Allocation framework [8] was employed. The main idea behind the CA formulation is to explore the ES's redundancy to always satisfy one "hard objective" (delivering i'_{out} to the DC bus) and find a suitable balance between two additional "soft objectives": a) the SC SOC control; and b) minimizing the ES's power losses. Depending on the operation mode of the vehicle, the two soft objectives can be assigned with different priorities. For instance, during fast transients, it may be more useful to give priority to the minimization of the ES's power losses, since, for short periods of time, it is not problematic to have small errors in the SC SOC tracking. On the other hand, when these fast transients are over (on the low frequency range), the SC SOC control should be given preference in order to drive the SC to the ideal SC SOC, q_{sc}^*.

Mathematically, the problem can be formulated as follows: find the DC bus currents $\mathbf{I}' = \left[i'_{bat}\ i'_{sc}\right]^T$, such that:

$$\begin{aligned} \min_{\mathbf{I}' \in \mathbb{R}^2} \quad & \mathbf{I}'^T \mathbf{W}_R \mathbf{I}' + W_{soc}(i_{sc,p} - \begin{bmatrix} 0 & G_{sc} \end{bmatrix} \mathbf{I})^2 \\ s.t. \quad & \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{I}' = i'_{out} \\ & \underline{\mathbf{I}}'(q_{bat}, q_{sc}) \leq \mathbf{I}' \leq \overline{\mathbf{I}}'(q_{bat}, q_{sc}) \end{aligned} \tag{10}$$

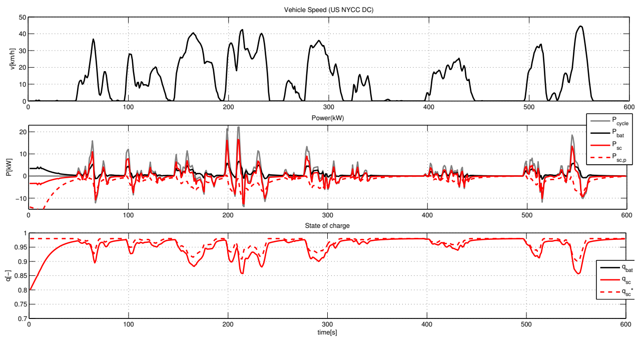


Figure 6. Power sharing with the Control Allocation approach during the NYCC driving cycle.

where $\mathbf{W}_R = \mathrm{diag}(G_{bat}^2 W_{bat}, G_{sc}^2 W_{sc})$ penalize the battery and SC use, W_{soc} penalizes deviations between the SC current i_{sc} and the preferred set-point, $i_{sc,p}$, generated by the SC voltage controller, and $\underline{\mathbf{I}}, \overline{\mathbf{I}}$ represents the lower and upper current limits for the ES's (note: $' \leq '$ is evaluated component-wise). The preferred set-point is a concept borrowed from the aeronautical applications, where $i_{sc,p}$ normally represents a set of actuator positions that minimize a given performance metric, for example the aerodynamic drag [17]. In the context of the EMS design, the preferred SC current, $i_{sc,p}$, is generated by the SC voltage controller and defined the SC current level that will bring the SOC SC to the (hypothetical) optimal value, q_{sc}^*. Moreover, it important to mention that, unlike the optimization problem discussed in the previous section, the problem (10) has a small number of variables (just 2), therefore is easily solved in real-time with numerical methods, like the active set method [18]. To take full advantage of the CA framework, the weights W_{bat}, W_{sc}, W_{soc} should be carefully selected in accordance with the vehicle operation mode, which was accomplished by employing fuzzy logic (see [19] for additional details).

D. Simulations Results

For validation of the proposed causal EMS, numerical simulations were carried out, following a similar setting as described in Section II-C. To illustrate the qualitative behaviour of the CA, Figure 6 shows the allocation results during the NYCC urban driving cycle. To start with, it is interesting to note the allocation values obtained when the vehicle is stopped and the SC have a significant SC SOC error (which happens between $0-60$s): during this transient, the SC voltage controller is requesting a very high power (> 10kW) to charge the SC ($P_{sc,p}$), but, due to the penalization of Joule losses taken in the CA formulation, the final allocated power is substantially less than requested one. Another point worth to be highlighted, are the periods when the vehicle is subject to strong accelerations (e.g. around $200s$), where it can be seen that the SC provide a quick and effective response to the peak power requests, limiting the battery stress. A similar trend is observed during vehicle decelerations. It is also interesting to note the flexibility introduced by the CA approach during transient and steady-state operation; during the transient acceleration/deceleration operations, small tracking errors arise in q_{sc} regulation, as a result of the trade-off between the SC SOC tracking and the power losses minimization that the CA is managing. However, these small errors are not problematic, and are corrected when the transients are extinguished.

A quantitative comparison between the causal CA and three other allocation strategies, namely 1) "pure battery" configuration; 2) non-causal optimal control, discussed in the previous section; and 3) SC SOC control ($W_{bat} = W_{sc} = 0$), is presented in Figure 4 for different urban driving cycles. These results show that, when the CA is active, the energy losses and the battery peak power is substantially less than the allocation based on SC SOC control, being very close to the optimal non-causal solution.

IV. Conclusion

In this article, two design strategies were proposed to manage the hybridization of SCs and batteries. The first approach, based on non-causal optimization, offered an effective tool to derive the power allocation that minimizes the energy losses over a given driving cycle. A more practical approach is then introduced, inspired on the Control Allocation principle, providing a causal control law that results from a trade-off between the SC SOC control and the power losses minimization. For urban driving cycles, this causal allocation strategy produced results very close to the non-causal optimal solution, but without needing the advance knowledge of the vehicle driving cycle.

Acknowledgment

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Appendix A
EMS parameters (see Table II)

Table II
Vehicle, batteries and supercaps parameters

Variable	Symbol	Value	Units
Vehicle total mass	M	850	kg
Rolling resistor coefficient	f_r	0.013	-
aerodynamic drag coefficient	C_d	0.35	-
vehicle frontal area	A_f	1.8	m^2
Battery: 13x Saft NHE-10-100 (Ni-MH) [20]			
Battery mass	m_{bat}	241.8	kg
Battery nominal voltage	v_{bat}	156	V
Battery nominal capacity	$Q_{0,bat}$	100	A.h
Battery SOC limits	$[\underline{q}_{bat}, \overline{q}_{bat}]$	$[0.2, 1]$	-
Battery open-circuit min. voltage	κ_{bat}	157.3	V
Battery open-circuit gain	m_{bat}	16.60	V/-
Battery internal resistance	R_{bat}	159	mΩ
Supercapacitor: 3x Maxwell BMOD0083 P048 [21]			
SC mass	m_{sc}	33	kg
SC nominal voltage	v_{sc}	145.8	V
SC Capacitance	C_{sc}	27.7	F
SC nominal capacity	$Q_{0,sc}$	1.122	A.h
SC SOC limits	$[\underline{q}_{sc}, \overline{q}_{sc}]$	$[0.5, 1]$	-
SC open-circuit min. voltage	κ_{sc}	0	V
SC open-circuit gain	m_{sc}	145.8	V/-
SC internal resistance	R_{sc}	31	mΩ

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