Addressing the Impact of Demand and Generation Cost Uncertainties in the Operation of Power Systems

Bruno André Gomes*. João Tomé Saraiva**

**INESC Porto – Inst. de Eng. de Sistemas e Computadores do Porto, Campus da FEUP, Rua Dr. Roberto Frias, 4200-465 Porto Portugal (e-mail: bgomes@inescporto.pt).*
***Faculdade de Engenharia da Universidade do Porto & INESC Porto – Inst. de Eng. de Sistemas e Computadores do Porto, Campus da FEUP, Rua Dr. Roberto Frias,4200-465 Porto Portugal (e-mail:jsaraiva@fe.up.pt)*

Abstract: This paper describes a set of mathematical formulations designed to include uncertainties modeled by fuzzy numbers in DC OPF studies. These approaches enhance and generalize an initial formulation and solution algorithm described in several papers co-authored by the second author. The approaches described in this paper adopt multiparametric optimization techniques in order to translate to the results the uncertainties affecting loads, for one side, the generation costs, for another, and also both of them in a simultaneous way. These approaches can be very useful nowadays given the uncertainties and volatility affecting data required to run several studies. They can also be the basis for the computation of nodal short time marginal prices reflecting these uncertainties. This paper also includes results obtained from a Case Study based on the IEEE 24 bus test system.

Keywords: Uncertainties, fuzzy models, DC optimal power flow, multiparametric programming.

1. INTRODUCTION

Addressing uncertainties is not a new issue among the power systems community. During the 70's several probabilistic models were developed to address the behaviour of the demand and to reflect demand uncertainties in the results of power flow studies. In this scope, Borkowska (1974) and Allan et al. (1977) describe different approaches using convolution techniques and linearized models for the AC power flow problem. Given the inaccuracies of these linerarized models reported and analysed by Alan et al. (1981), Alan and Leite da Silva (1981) developed a model that uses several linearization points to build partial probabilistic distributions that are combined at the end in order to reduce the inaccuracies namely in the tails of the distributions. Finally, Leite da Silva et al. (1985) considers network outage data in the probabilistic power flow problem and Karakatsanis et al. (1994) integrates operation constraints in the problem as a way to increase its realism.

During the last two decade emerged a new class of approaches to model uncertainty. These new approaches use fuzzy models to represent the uncertainty affecting several variables and parameters. Differently from probabilistic models, fuzzy approaches are not based on the observation of a large number of events to derive probabilistic distributions, but rely on the experience of the users and, to a certain extent, reflect their subjectivity. Accordingly, fuzzy models represent a different kind of uncertainty inherent, for instance, to expressions of our language as "large", "more or less" or "approximately". Using these expressions do not result from the repetitive simulation of the same phenomenon but they express the past experience of the user and his subjective evaluation. The volatility of our world in which uncertainty became a structural element is a key aspect to understand the development of fuzzy models and their application to power systems.

In this scope, Miranda et al. (1990) details the first DC and AC fuzzy power flow models and Saraiva et al. (1994, 1996) describe approaches to model load uncertainties in DC OPF problems and on a Monte Carlo simulation to obtain estimates of the expected value of the Power Not Supplied.

In recent years, this research was resumed recognizing that the development of electricity markets, the volatility of fuel prices and the more intensive use of volatile primary resources (for instance, wind) place a new emphasis on uncertainties. In this scope, Gomes and Saraiva (2009) described an initial contribution to consider simultaneously uncertainties in fuel prices and in the demand. In that paper, we described the basic concepts behind the problem and presented a small illustrative example to highlight the results and their potential interest. In this paper, we are now detailing the complete mathematical formulation of the problem and the solution algorithms based on linear multiparametric programming techniques. The models developed in the 90's only addressed load uncertainties and would only run some parametric studies eventually leading to narrower membership functions when compared with the real ones. Using those models, we were unable to fully and adequately characterize the possible behavior of generators and branch flows in view of demand and cost uncertainties.

Apart from this Introduction, Section 2 presents basic concepts of Fuzzy Set Theory used along the paper. Sections 3 and 4 detail the mathematical models for the Fuzzy Optimal Power Flow Problem and the developed solution algorithms. Sections 5 presents a Case Study based on the IEEE 24 bus test system and Section 6 presents the conclusions.

2. GENERAL ASPECTS ON FUZZY SETS

According to Zimmerman (1992), a fuzzy set is defined as a set of ordered pairs (1) in which x_1 is an element of the universe X under analysis and $\mu_{\tilde{A}}(x_1)$ is the membership degree of x_1 to the fuzzy set. Typically, $\mu_{\tilde{A}}(x_1)$ takes values in $[0.0;1.0]$ and it reflects the degree of compatibility of the elements of X with the proposition defining the fuzzy set. These degrees can be seen as a function $\mu_{\tilde{A}}(x)$ that assigns a membership degree to each element x.

$$\tilde{A} = \{(x_1; \mu_{\tilde{A}}(x_1)), x_1 \in X\} \quad (1)$$

Fuzzy numbers are a particular class of fuzzy sets. A fuzzy set $\tilde{A}$ is as a fuzzy number if it is a convex fuzzy set defined on the real line R such that its membership function is piecewise continuous. As an example, Figure 1 represents the membership function of a trapezoidal fuzzy number in which the membership degree is maximum in $[A_2; A_3]$ and it decreases from 1.0 to 0.0 from A_2 to A_1 and from A_3 to A_4. This representation can be used to model the uncertainty associated with the interval of values $[A_2; A_3]$. Regarding this type of numbers and given their particular shape, they are uniquely defined by the values of A_1, A_2, A_3 and A_4. These numbers are then usually notated as $(A_1; A_2; A_3; A_4)$.

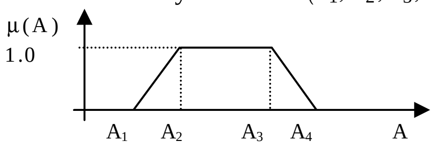

Fig. 1. Illustration of a trapezoidal fuzzy number.

An α-level set or an α-cut of a fuzzy set $\tilde{A}$ is defined in X as the hard set A_α obtained from $\tilde{A}$ for each $\alpha \in [0.0;1.0]$ according to (2). As a result of this definition, the 0.0-cut of the fuzzy number in Figure 1 is $[A_1; A_4]$ and the 1.0-cut is given by $[A_2; A_3]$. On the other hand, the central value of $\tilde{A}$ corresponds to the mean value of the 1.0-cut. Considering again the trapezoidal fuzzy number in Figure 1, the corresponding central value, A^{ctr}, is given by (3).

$$A_\alpha = \{x_1 \in X : \mu_{\tilde{A}}(x_1) \geq \alpha\} \quad (2)$$

$$A^{ctr} = (A_2 + A_3)/2 \quad (3)$$

Let us finally consider two generic fuzzy sets, $\tilde{A}$ and $\tilde{B}$, each of them defined according to (1). Their union, $\tilde{A} \cup \tilde{B}$, and their intersection, $\tilde{A} \cap \tilde{B}$, are two fuzzy sets and their membership functions are defined by (4) and (5). Regarding the union, this means that the membership degree of an element x_1 in $\tilde{A} \cup \tilde{B}$ corresponds to the maximum of the membership degrees of x_1 in $\tilde{A}$ and $\tilde{B}$. For the intersection, the minimum operator applies instead of the maximum.

$$\mu_{\tilde{A} \cup \tilde{B}}(x_1) = \max(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_1)) \quad (4)$$

$$\mu_{\tilde{A} \cap \tilde{B}}(x_1) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_1)) \quad (5)$$

3. MATHEMATICAL FORMULATION

A FOPF study was originally defined by Saraiva et al. (1994) as an optimization problem aiming at identifying the best generation strategy driven by a generation cost function if at least one load is represented by a fuzzy number. The FOPF approach originally adopted the DC model to represent the operation of the network. In this problem, load uncertainties are represented by fuzzy numbers from which we can extract the central values as defined Section 2. These values are used to perform the crisp DC-OPF study (6-10) to identify an initial feasible and optimal solution.

$$\min f = \sum c_k . Pg_k + G.\sum PNS_k \quad (6)$$

$$\text{subj. } \sum Pg_k + \sum PNS_k = \sum Pl_k^{ctr} \quad (7)$$

$$Pg_k^{min} \leq Pg_k \leq Pg_k^{max} \quad (8)$$

$$PNS_k \leq Pl_k^{ctr} \quad (9)$$

$$P_b^{min} \leq \sum a_{bk}.(Pg_k + PNS_k - Pl_k^{ctr}) \leq P_b^{max} \quad (10)$$

This problem minimizes the generation cost given that Pg_k is the generation in bus k with a cost c_k, PNS_k is the Power Not Supplied in bus k with cost G, Pg_k^{min}, Pg_k^{max}, P_b^{min} and P_b^{max} are the minimum and maximum generation in bus k and the minimum and maximum flow in branch b. On the other hand, a_{bk} is the DC sensitivity coefficient of the flow in branch b regarding the injected power in bus k and Pl_k^{ctr} is the central value of the load in node k.

After solving the problem (6–10), we get a feasible and optimal solution for a particular load combination, that is, for the set of central load values. To reflect load uncertainties in the problem, we use one parameter Δ_k for each load so that we formulate a multiparametric problem as (11-13). In this problem, b and b' are the vectors of the right hand side terms of the constraints considering that some of them are independent and some others depend on the parameters Δ_k.

$$\min f = c^t.\tilde{X} \quad (11)$$

$$\text{subj } A.\tilde{X} = b + b'(\Delta_k) \quad (12)$$

$$\Delta_{k1} \leq \Delta_k \leq \Delta_{k4} \quad (13)$$

When considering these parameters, the solution obtained for the problem (6–10) can eventually get unfeasible in some areas of the hyper-volume defined by (13). If this happens, the algorithm detailed in Saraiva et al. (1994) identifies vertices of this hyper-volume according to some rules. Once these vertices are identified, it is run a parametric analysis for each of them leading to partial membership functions of generations, branch flows, voltage phases and PNS. The final results are obtained applying the Fuzzy Union Operator to these partial results.

In a condensed way, this problem can be written as the multiparametric formulation given by (14-15). In this problem, the cost vector c includes crisp costs admitting they are not affected by uncertainties, $\tilde{X}$ represents the output variables, that is generations, branch flows, voltage phases and PNS and $\tilde{b}$ is the vector of independent terms in which we admit there is at least one trapezoidal fuzzy number.

$$\min \quad f = c^t.\tilde{X} \quad (14)$$

$$\text{subj. } A.\tilde{X} \leq \tilde{b} \quad (15)$$

As mentioned above, the outlined solution algorithm transforms this problem into a set of parametric linear optimization problems. In this paper, apart from solving this problem using multiparametric optimization techniques, we are also enlarging this original concept to consider not only load uncertainties, but also generation cost uncertainties and both of them in a simultaneous way. Accordingly, if we want to consider generation cost uncertainties, we get the condensed model (16-17). In this case, we assume that loads are given by crisp values and that $\tilde{c}$ includes at least one trapezoidal fuzzy number to represent a generation cost.

$$\min \quad f = \tilde{c}^t.\tilde{X} \quad (16)$$

$$\text{subj.} \quad A.\tilde{X} \le b \quad (17)$$

Finally, if we consider in a simultaneous way load and generation cost uncertainties we obtain the problem (18-19).

$$\min \quad f = \tilde{c}^t.\tilde{X} \quad (18)$$

$$\text{subj.} \quad A.\tilde{X} \le \tilde{b} \quad (19)$$

4. SOLUTION ALGORITHM

4.1 Overview

As mentioned before, linear multiparametric programming provides a systematic way to analyze the effect of changing several parameters in the optimal solution of a linear mathematical programming problem. In this application, we used the algorithms described in Gal (1979) to address this problem. They are based on the Simplex method and integrate into the Simplex tableau parameters to model the uncertainties. As a result, the optimality and feasibility conditions are generalized so that they become function of the parameters Φ_k (modelling cost uncertainties) and Δ_k (modelling load uncertainties). The two major steps in these algorithms can be stated as follows:

1. Initially, substitute uncertain fuzzy data by the respective central values and solve the resulting crisp linear optimization problem. In this step, Φ_k and Δ_k are zero;
2. Once a feasible and optimal solution is identified for the above problem, we find other optimal and feasible basis provided they are valid in one region of the uncertainty space. Under these conditions, these regions are called critical regions. This process is conducted by pivoting over all the optimal and feasible identified regions.

From a mathematical point of view, this can be formulated as follows. Let us admit that B is an optimal and feasible basis, ρ is the index of the corresponding set of basic variables, A_{nb} is the matrix with the columns of the non-basic variables in the Simplex tableau, c_b is the cost vector of the basic variables and c_{nb} is the cost vector of the non-basic variables. Assuming a linear minimization problem, the optimality and the feasibility conditions are written as (20) and (21), considering that parameters Φ_k and Δ_k model generation cost and load uncertainties.

$$c_{nb}^T(\Phi_k) - c_b^T.B_\rho^{-1}.A_{nb} = (c + c'(\Phi_k)) - c_b^T.B_\rho^{-1}.A_{nb} \ge 0 \quad (20)$$

$$B_\rho^{-1}.b(\Delta_k) = B_\rho^{-1}(b + b'(\Delta_k)) \ge 0 \quad (21)$$

For right hand side parameterization, the initial solution gets unfeasible if at least one variable becomes negative. In this case, a critical region is a region in the uncertainty space where the basis matrix B remains optimal and feasible and so it is defined by (21). When conducting cost parameterization studies, the initial solution can lose optimality if the reduced costs of the non-basic variables become negative and so the optimality condition corresponds to (20).

When running multiparametric studies, we can state the objective as follows - find all possible optimal solutions, their corresponding optimal values and critical regions so that these regions cover the entire uncertainty space. A critical region is thus defined as a closed nonempty polyhedron, associated to a set of linear inequalities in Δ, Φ, or in both, for which there is a feasible and optimal basis of the optimization problem. This set of constraints corresponds to an equivalent set of non-redundant constraints that are identified using a non-redundant test over all of them.

From an algorithmic point of view, this means that starting at the optimal and feasible solution of the initial crisp optimization problem we characterize the associated region of the uncertainty space where the basis of this initial problem is optimal and feasible using the set of constraints (20) and/or (21). Then, we identify the neighbouring regions or neighbouring basis. Two optimal and feasible basis, B_1 and B_2, are neighbours if and only if one goes from B_1 to B_2 doing one dual pivot step for a right hand side parametric problem, one primal pivot step for cost parameterization or one step of each type when parameterizing both generation costs and loads. After building the set of critical regions covering the entire uncertainty space, each region is analysed to obtain the possible behaviour of each output variable.

4.2 Treatment of load uncertainties

To consider load uncertainties, the algorithm starts with the solution of the initial DC-OPF problem (6-10), to get an optimal and feasible solution associated with the central values of the load fuzzy numbers. In the second step, we integrate in the optimization problem one parameter Δ_k per uncertain load building the multiparametric problem (22-26).

Then, using the feasibility condition (21), we identify a set of non-redundant constraints to define new neighbour critical regions. If there are no non-redundant constraints, the process stops. If they exist, we run a dual pivoting over the initial basis to move to a new region. This is repeated until no non-redundant constraints exist or all identified regions are already known. When this process finishes, the entire uncertainty space is covered by critical regions, each of them associated to a feasible and optimal basis.

$$\min f = \sum c_k.Pg_k + G.\sum PNS_k \quad (22)$$

$$\text{subj.} \sum Pg_k + \sum PNS_k = \sum Pl_k^{ctr} + \sum \Delta_k \quad (23)$$

$$Pg_k^{min} \le Pg_k \le Pg_k^{max} \quad (24)$$

$$PNS_k \le Pl_k^{ctr} + \Delta_k \quad (25)$$

$$P_b^{min} \le \sum a_{bk}.(Pg_k + PNS_k - Pl_k^{ctr} - \Delta_k) \le P_b^{max} \quad (26)$$

Once all critical regions are identified, we proceed to get the results of the output variables of the optimization problem, that is generations, branch flows, voltage phases and eventually PNS. In order to do this, let us recall that the optimization problem is linear and so the possible behavior of generations, branch flows, voltage phases and PNS is given by linear expressions. To characterize each of these variables for a given membership level, we must evaluate its range of variation minimizing and maximizing its linear expression subjected to the set of constraints defining the critical region under analysis. For a given level of uncertainty, these minimization and maximization linear problems lead to the possible range of values that a given output variable can take inside the region under analysis. In order to build the membership function of this output variable in that region, this minimization/maximization process should be repeated for a number of uncertainty levels in which the uncertain data is discretized. In order to illustrate this procedure let us consider Figure 2 that represents the 0.0 and the 1.0 membership levels for a two load system affected by uncertainty. In this Figure line "a" represents a non-redundant constraint, for instance related with a branch flow limit, and point O represents the solution of the initial crisp problem, considering that the parameters Δ_1 and Δ_2 associated to the two uncertain loads are zero. The dashed lines delimit the i^{th}-cut or, in other words, the range of values of the uncertainties at the i^{th} level of the data membership functions.

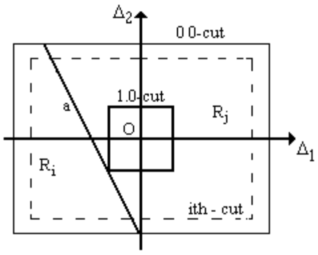

Fig. 2. Illustrative critical regions for a two load system.

According to this Figure, the uncertainty space is covered by two critical regions, R_i and R_j. When solving the initial optimization problem associated to point O, we identify R_j and then, doing a dual pivoting, we move to R_i. When all the uncertainty space is covered, we determine the widest possible behaviour of the output variables. To do this, let us consider a variable v written in terms of the parameters Δ_1 and Δ_2, that is, $v(\Delta_1,\Delta_2)$. To get the possible widest behaviour of $v(\Delta_1,\Delta_2)$, we minimize and maximize the expression of v subjected to the constraints defining each region and the ranges of the parameters Δ_1 and Δ_2 in that level of uncertainty. This means solving the problem (27-30).

$$\min/\max f = v(\Delta_1,\Delta_2) \quad (27)$$

$$\text{subj} \quad k_{1i}.\Delta_1 + k_{2i}.\Delta_2 \le b_i \quad (28)$$

$$\Delta_1^{\min\ i^{th}-cut} \le \Delta_1 \le \Delta_1^{\text{máx}\ i^{th}-cut} \quad (29)$$

$$\Delta_2^{\min\ i^{th}-cut} \le \Delta_2 \le \Delta_2^{\text{máx}\ i^{th}-cut} \quad (30)$$

In this problem, (28) represents the non-redundant constraints defining the region under analysis where k_{1i} and k_{2i} are real numbers and constraints (29) and (30) define the range of the uncertainty parameters Δ_1 and Δ_2 in the i^{th} level. The objective function of this problem is the expression of the output variable $v(\Delta_1,\Delta_2)$, that is a generation, a branch flow, a voltage phase or the PNS in a given bus.

After minimizing and maximizing $v(\Delta_1,\Delta_2)$ for a number of uncertainty levels, it is possible to build the membership function of v in this critical region. In fact, for each uncertainty level, we get the minimum and the maximum values that each variable v, that is, we obtain the interval of variation of v in that uncertainty level. If this procedure is repeated for a number of uncertainty levels we are then able to construct the membership function of that variable in that critical region, because each interval of variation is associated to a value in $[0.0;1.0]$ corresponding to the uncertainty level under analysis. Once all critical regions are analyzed, the final membership function of a generic variable v is obtained taking the Fuzzy Union of the partial membership functions obtained for v. As mentioned in Section 2, the Fuzzy Union is modelled by the maximum operator, meaning that the membership degree of an element x in the fuzzy union is given by the maximum of the membership degrees of x in the original sets. Adopting this operator, guarantees that the final result for v displays the widest possible behaviour.

4.3 Treatment of generation cost uncertainties

If at least one generation cost is modelled by a fuzzy number, then we are addressing the condensed problem (16-17). Using a solution strategy similar to the one in Section 4.2, we run in the first place a crisp optimization problem that is obtained substituting the fuzzy costs by their central values. Then, we include the parameters Φ_k modelling cost uncertainties leading to a multiparametric problem. Starting with the solution of the initial deterministic DC-OPF problem, the non-redundant constraints are then established using the optimality condition (20) in terms of the cost vector parameters. From this initial basis, one moves to new critical regions performing a primal pivoting over the initial basis. This is repeated over all the new regions until no non-redundant constraints exist or all identified critical regions correspond to already known ones. When this process ends, all the uncertainty space is covered meaning that the union of all critical regions corresponds to the entire uncertainty space. To get the final results, it is important to mention that when parameterizing cost function coefficients we are not changing the optimal solution, provided that the basis is the same. This means that the value of the output variables remain constant inside each critical region. A change on the output values will only occur when we move from one critical region to another one. Therefore, it is immediate to build the partial membership function of the output variables inside each region because the value of each generation, branch flow, voltage phase or PNS is fixed inside it. After analyzing all regions, the final membership function of each output variable is obtained aggregating its partial results using the Fuzzy Union Operator to get its widest possible behaviour.

4.4 Simultaneous load and generation cost uncertainties

Finally, let us admit that we want to model, simultaneously, generator cost and load uncertainties, which means addressing the condensed problem (18-19). Using an algorithm similar to the one in Section 4.2, we start running the initial deterministic DC-OPF problem substituting fuzzy loads and fuzzy cost coefficients by their central values, Pl_k^{ctr} and c_k^{ctr}, respectively. Using this optimal and feasible solution, we build a multiparametric problem both on the objective function and on the constraints using the load and the generation cost parameters Δ_k and Φ_k, respectively.

Given that we are simultaneously parameterizing the cost and the right hand side vectors, the identification of non-redundant constraints associated with critical regions is done using both the optimality (20) and the feasibility (21) conditions. This means that each critical region is now defined by linear constraints both on Δ_k and on Φ_k.

Departing from the initial optimal and feasible basis, we can find two kinds of neighbour basis. The first one is obtained applying the feasibility condition (21) and the second results from the optimality condition (20). This process is repeated for each new identified basis until no non-redundant constraints exist or all the identified critical regions correspond to already known ones. The final step corresponds to build the membership function of each output variable. As for the algorithm described in Section 4.2, in each critical region an output variable is given by a linear expression. Therefore, to identify its possible widest behaviour inside each critical region we minimize and maximize its linear expression subjected to the constraints defining that region as in problem (27-30) but now in terms of both Φ_k and Δ_k. These partial functions are finally aggregated using the Fuzzy Union Operator, as justified at the end of Section 4.2.

5. CASE STUDY USING THE IEEE 24 BUS SYSTEM

5.1 System data

The developed algorithms were tested using the IEEE 24 bus/38 branch test system firstly described in 1979. In this Case Study the load was increased to 4060.05 MW regarding the original load data and the total installed generation capacity is 5226.0 MW. Table 2 has the capacity and the central value of the generation costs and Table 2 contains the load central values. The capacity of the transformers is 400 MW, branches 1 to 6 and 8 to 13 have a capacity of 175 MW and the capacity of the branches 18 to 38 was set at 500 MW.

Table 1. Generation installed capacity (MW) and cost (€/MWh).

Bus/gen	Capacity	Cost	Bus/gen	Capacity	Cost
1 / 1	40.0	30.0	16 / 1	310.0	55.0
1 / 2	40.0	32.0	19 / 1	800.0	87.0
1 / 3	152.0	40.0	21 / 1	800.0	80.0
1 / 4	152.0	43.0	22 / 1	100.0	15.0
2 / 1	40.0	36.0	22 / 2	100.0	17.0
2 / 2	40.0	38.0	22 / 3	100.0	19.0
2 / 3	152.0	41.0	22 / 4	100.0	15.0
2 / 4	152.0	42.0	22 / 5	100.0	17.0
7 / 1	150.0	45.0	22 / 6	100.0	25.0
7 / 2	200.0	43.0	23 / 1	200.0	50.0
13 / 1	250.0	61.0	23 / 2	50.0	49.0
13 / 2	394.0	62.0	23 / 3	310.0	47.0
13 / 3	394.0	67.0	--	--	--

Table 2. Load central values (MW).

Bus	Load	Bus	Load	Bus	Load
1	220.48	9	385.82	17	0.00
2	270.80	10	216.49	18	226.76
3	3.94	11	40.00	19	265.53
4	32.67	12	10.00	20	103.92
5	105.94	13	162.45	21	50.00
6	187.65	14	262.88	22	10.00
7	218.77	15	650.36	23	0.00
8	398.09	16	225.50	24	12.00

5.2 Results only considering load uncertainties

In this case we considered trapezoidal fuzzy numbers to model loads. The 0.0 level of uncertainty of these numbers ranges from ±10% of their central values and at the 1.0 level the uncertainty ranges from ±5% of their central values. These uncertainty ranges correspond to typical values adopted in similar studies and if they become larger the number of basis to identify to cover the entire uncertainty space would increase, thus increasing the computational time. Having selected these ranges, it is possible to obtain the four numeric values mentioned in Section 2 to characterize each trapezoidal fuzzy number in terms of the central value. Figure 3 presents the membership functions of generators 19/1 and 21/1 and of the flow in branches 11-13, 15-21 and 16-19.

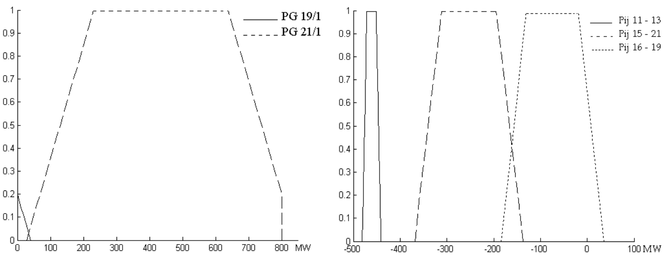

Fig. 3. Membership function of generators 19/1 and 21/1 and of the branch flows on branches connecting buses 11-13, 15-21 and 16-19.

The figure on the right includes membership functions of three branch flows – 11-13, 15-21 and 16-19. As a general indication, branch flows also depict a fuzzy behaviour since they reflect fuzzy generations and loads. It is particularly interesting the membership function of the flow in branch 16-19. According to this figure, as a consequence of the specified load uncertainties, the power flow in this branch can be reverted. This means that for some generation strategies associated to some load patterns, the flow in this branch goes from bus 16 to 19 while for some other strategies this flow reverts and goes from bus 19 to 16.

In order to evaluate the impact of congested branches, the maximum capacity of the two branches connecting buses 15 and 21 was reduced from 500 MW to 350 MW. The results in Figure 4 indicate that for some load combinations these branches get congested. This figure also indicates that now the maximum output of generator 21/1 doesn't reach the corresponding limit of 800.0 MW. On the other hand, the maximum output of generator 19/1 is now larger than the value in Figure 3. These changes on generators 21/1 and 19/1 indicate that when considering larger load uncertainties, the flow limit of branches 15-21 is reached prior than the thermal limit of 800.0 MW of the generator 21/1.

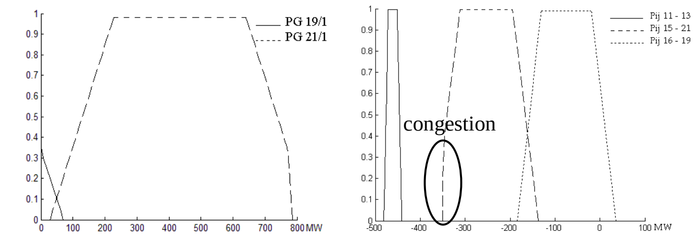

Fig. 4. Membership function of generators 19/1 and 21/1 and of the branch flows on branches connecting buses 11-13, 15-21 and 16-19.

5.3 *Results using both load and cost uncertainties*

For illustration purposes, we admitted that loads are modelled by trapezoidal fuzzy numbers ranging from ±5% and ±2.5% of their central values at the 0.0 and at the 1.0 uncertainty levels. We also considered the fuzzy generation costs given by (31) to (36). These values were selected to represent costs of coal, natural gas and oil, as well as the volatility of their price in international markets. Using this data, Figure 5 presents the membership functions of generators 13/3, 19/1 and 21/1 and of the flows in branches 11-13, 15-21 and 16-19. This figure indicates that for some combinations of the uncertainties there is congestion on branches 11-13 and 16-19 preventing generators 19/1 and 13/3 from having larger outputs.

$$\tilde{CP}_{G1/1} = (26.0\,;27.5\,;32.5\,;34.0)\text{ €/MWh} \qquad (31)$$

$$\tilde{CP}_{G2/1} = (33.0\,;34.5\,;37.5\,;39.0)\text{ €/MWh} \qquad (32)$$

$$\tilde{CP}_{G7/1} = (42.0\,;43.5\,;46.5\,;48.0)\text{ €/MWh} \qquad (33)$$

$$\tilde{CP}_{G19/1} = (74.0\,;82.0\,;92.0\,;100.0)\text{ €/MWh} \qquad (34)$$

$$\tilde{CP}_{G22/2} = (14.0\,;15.5\,;18.5\,;20.0)\text{ €/MWh} \qquad (35)$$

$$\tilde{CP}_{G23/2} = (46.0\,;47.5\,;50.5\,;52.0)\text{ €/MWh} \qquad (36)$$

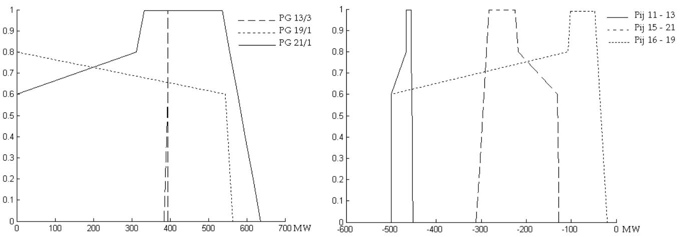

Fig. 5. Membership function of generators 13/3, 19/1 and 21/1 and of the flows on branches connecting buses 11-13, 15-21 and 16-19.

In order to test the algorithm considering more stressed operation conditions, thus leading to an increased number of active constraints, the maximum capacity of the branches 15-21 was reduced from 500 MW to 300 MW. Figure 6 presents the membership functions of generators 13/3, 19/1 and 21/1 and of the flows on branches 11-13, 15-21 and 16-19.

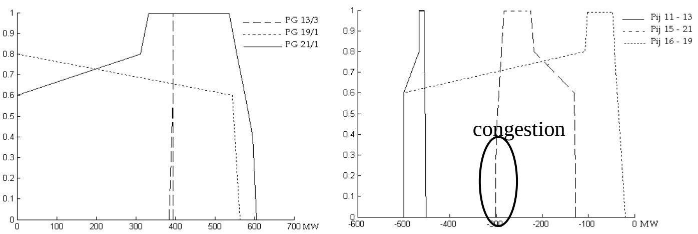

Fig. 6. Membership function of generators 13/3, 19/1 and 21/1 and of the branches flows on branches 11-13, 15-21 and 16-19.

From this figure it is clear that due to the new transmission capacity limit, the maximum generation of generator 21/1 becomes smaller. The minimum flow on branch 15-21 gets reduced to –300 MW. This value coincides with the thermal limit imposed to this branch meaning that the reduction of this limit from 500 to 300 MW already mentioned has an impact on the final results that were obtained in this case.

6. CONCLUSIONS

In this paper we described multiparametric linear problems to model the operation conditions of power systems admitting that at least one load and/or one generation cost are modelled by fuzzy numbers. This is novel since in the literature there were publications describing DC OPF problems admitting the parameterization of the right hand side vector, but not the simultaneous parameterization of the cost and of the right hand side vectors of optimization problems.

Apart from this conceptual novelty, given the enlarged volatility of our world (for instance, the volatility of the price of some primary resources and the volatility of the availability of some others, as wind and solar resources) it is important to internalize these uncertainties in power system analysis. The models and algorithms described in this paper aim at contributing to give a step forward in this direction.

REFERENCES

Borkowska, B. (1974). Probabilistic Load Flow, *IEEE Transactions on PAS*, vol. PAS-93 (3), 752-759.

Allan, R., Al-Shakarchi, M. (1977). Probabilistic Techniques in a.c. Load Flow Analysis. *Proceedings of the IEE*, vol. 124 (2), 154-160.

Allan, R., Leite da Silva, A., Burchett, R. (1981). Evaluation Methods and Accuracy in Probabilistic Load Flow Solutions. *IEEE Transactions on PAS*, vol. PAS-100 (5), 2539-2546.

Allan, R., Leite da Silva, A. (1981). Probabilistic Load Flow Using Multilinearisations. *IEE Proceedings*, vol. 128 (C.5), 280-287.

Leite da Silva, A., Allan, R., Soares, S., Arienti, V. (1985). Probabilistic Load Flow Considering Network Outages. *IEE Proceedings*, vol. 132 (C.3), 139-145.

Karakatsanis, T., Hatziargyriou, N. (1994). Probabilistic Constrained Load Flow Based on Sensitivity Analysis. *IEEE Transactions on Power Systems*, vol. 9 (4), 1853-1860.

Miranda, V., Matos, M.A., Saraiva, J.T. (1990). Fuzzy Load Flow – New Algorithms Incorporating Uncertain Generation and Load Representation. *Proceedings of the 10th PSCC*, Butterworths, London.

Saraiva, J.T., Miranda, V., Pinto, L.M. (1994). Impact on Some Planning Decisions From a Fuzzy Modelling of Power Systems. *IEEE Transactions on Power Systems*, vol. 9 (2), 819-825.

Saraiva, J.T., Miranda, V., Pinto, L.M. (1996). Generation / Transmission Power System Reliability Evaluation by Monte-Carlo Simulation Assuming a Fuzzy Load Description. *IEEE Transactions on Power Systems*, vol. 11 (2), 690-695.

Gomes, B.A., Saraiva, J.T. (2009), Demand and Generation Cost Uncertainty Modelling in Power System Optimization Studies, *Electric Power Systems Research*, vol. 79 (6), 959-972.

Zimmermann, H.-J. (1992). *Fuzzy Set Theory – and Its Applications*, 2nd ed, Kluwer Academic Publishers, London.

Gal, T. (1979). *Postoptimal Analysis, Parametric Programming and Related Topics*, McGraw Hill Book Company, Boston.

Task Force of Application Probabilistic Methods Subcommittee (1979). IEEE Reliability Test System. *IEEE Transactions on PAS*, vol. PAS-98 (6), 2047 – 2054.