Control and guidance of a hovering AUV pitching up or down

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***Abstract*—In this paper, we present an approach to control an autonomous underwater vehicle in the vertical and the horizontal planes while pitching down or up ($\theta = \pm\pi/2$). Such a capability is explored in MARES, a small-sized, torpedo-shaped hovering AUV with four degrees of freedom. Despite the fact that roll angle is not controllable, we find a guidance law that makes the vehicle reach any point in the horizontal plane while maintaining the vehicle in the vertical position.**

I. Introduction

Control of AUVs is a subject that has attracted the attention of several researchers over the last decades. The nonlinearity and the uncertainty of the corresponding mathematical models have originated several challenges in both control theory and design. Some examples can be found in [1], [2] or in [3] for an overview. Here, we approach the problem of stabilizing MARES in the vertical plane while pitching up or down. Under this condition, we further investigate the motion on the horizontal plane by determining a guidance law to enable the vehicle to reach any horizontal position.

Nontrivial motions of robots can be obtained by combining certain modes of operation. In this paper, we focus on the control of autonomous underwater vehicle (AUV) in an uncommon poses. We explore the control of a hovering AUV pitching up (down), i.e., with the nose pointing upwards (downwards) (referred to as vertical pose or vertical orientation throughout this paper). Confined horizontal areas and fast descent along the water column are scenarios that are related with the subject tackled in this paper.

A. The MARES AUV

The vehicle considered in this paper is MARES (Fig. 1), a small-sized, torpedo-shaped AUV with 1.5 meters of length and 20 centimeters of diameter. MARES was developed by the OceanSys group at INESC TEC, University of Porto. Besides the modularity feature, MARES differs from most of current AUVs since it has no fins and is capable of hovering without horizontal motion. The four thrusters placed on the hull provide four degrees of freedom (DOFs). Additionally, the vehicle can control all the DOFs independently as long as the actuator remain unsaturated. This feature enables the vehicle to decouple motion primitives such as the ones in the vertical and in the horizontal planes.

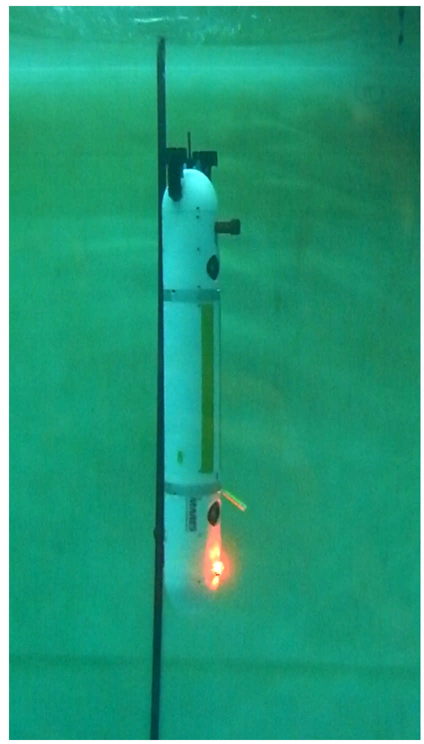

Fig. 1. The MARES Autonomous Underwater Vehicle pitching down

On board, a computer is responsible for generating the commands for the thruster and fuse the information from the sensors. The basic sensing equipment includes a depth sensor, an altimeter, an inertial measurement unit (IMU), a GPS for use on the surface and a transponder for long baseline (LBL) localization. Other sensors can occasionally be carried by taking advantage of the modular characteristic.

B. Vertical control

The two vertical thrusters of MARES make it possible to control heave independently of the surge motion. Besides the depth control, the pitch and yaw angles are also independently controllable. Only few AUVs hold this capability. Some examples can be encountered the Girona 500 [4], the ODIN III [5], [6] and the TriMARES [2]. The restoring moments, which are a consequence of non-coincident center of gravity and center of buoyancy, naturally stabilize MARES by "driving" the pitch and roll angles to zero. By properly actuating on the vertical thrusters, the vehicle can be controlled so that the pitch angle is different from its natural equilibrium point. Ultimately, the vehicle can even travel with composed motions with relatively large pitch angle while maintaining its depth constant. Such a feature is particularly appreciated in scenarios in which maneuverability or precise and even immobile positioning is required. Some examples can be found in intervention [4], archaeology [7] or inspection of underwater structures [2] using AUVs.

The approach presented in this paper takes advantage of the thruster configuration to control the vehicle in unusual modes of operation for most common AUVs. Motivated by scenarios that demand for large vertical velocity and/or for confined horizontal section, we have developed a control law that makes the vehicle position in the vertical, i.e., with the nose pointing either downwards or upwards ($\theta = -\pi/2$ or $\theta = \pi/2$). The stabilization of the vehicle is achieved both in motion or stationary. In fact, our control law stabilizes the vehicle independently of the heave motion. We use nonlinear control tools ([8] and [9] offer a broad coverage on the subject) to derive an appropriate control law and base our analysis on a dynamics model of MARES.

Further, we extend our approach to control the vehicle in the horizontal plane when it is pitching up or down. We exploit the periodic rotation of MARES along the x-axis to drive the vehicle to the desired position. Indeed, the actuation of the stern thrusters creates a moment along the x-axis (often undesired in normal operation [10]) that makes MARES roll. This effect is due to asymmetric stern propellers that, when actuating in steady state, make the roll stabilize in an angle different from zero when it is in the horizontal pose (i.e. pitch angle equal to zero). The stability is achieved because of a restoring moment induced by a non-null distance between the center of gravity and the center of buoyancy. However, such a moment along the x-axis no longer exists when the vehicle is in the vertical pose. Consequently, the roll dynamics model shows that the vehicle continuously rotates under constant actuation on the stern thrusters.

By taking advantage of this behavior, it is possible to drive the vehicle to any horizontal point. A method similar to the one derived in [11] will be used in this approach. Originally used in the context of efficient motion of Lagrangian profilers using tidal currents and different depth layers, the method is extended to the motion of MARES when it takes a vertical pose. Roughly speaking, MARES activates its through-hull thrusters only when they are aligned with the general direction (sector-of-sight) of the desired horizontal position.

C. Organization of the paper

The paper is organized as follows: The section II presents the general model for the dynamics and the kinematics of underwater vehicles. Some considerations on the kinematics and sequence of rotations are also delineated. In the section III, we present the derivation of the control law to stabilize MARES in the vertical pose with pitch up or down, while considering the possibility of controlling "independently" the heave velocity. This capability is then explored in the section IV to derive a guidance law for the horizontal position. We validate our approach with both simulation and experimental tests and present the results in the section V.

II. Kinematics, kinetics and reduced models

In this section, we present a mathematical model for MARES. Using the same notation as in [12], the kinematics and the kinetics expressions are respectively given by

$$\dot{\eta} = J(\eta)\nu, \tag{1}$$
$$\dot{\nu} = A(\nu)\nu + g(\eta) + T\tau, \tag{2}$$

where $\eta \in \mathbb{R}^6$ is the pose vector, $\nu \in \mathbb{R}^6$ is the velocity vector, $J \in \mathbb{R}^{6\times 6}$ is a matrix that maps the linear and angular velocities expressed in the body-fixed frame into the earth-fixed, inertial referential frame. The matrix $A \in \mathbb{R}^{6\times 6}$ results from the hydrodynamic forces applied on the body of the vehicle when it is moving at a velocity ν. The term $A(\nu)\nu$ constitute the effect of added mass, Coriolis, centriptal and viscous damping forces and moments. The vector $g \in \mathbb{R}^6$ includes the effects of the restoring forces and moments, while T maps the forces and moments created by the four thrusters, whose actuation forces are given in the vector $\tau \in \mathbb{R}^4$, in the body-fixed frame.

For the purpose of stabilizing MARES in the vertical pose, we reduce the order of the system by projecting the pose and the velocities in a subspace. We are interested in controlling the depth, and the pitch and yaw angles using the four controllable DOFs. Therefore, we define the reduced order system as follows:

$$\eta_z = P^{356}\eta, \tag{3}$$
$$\nu_l = P^{1356}\nu, \tag{4}$$

where

$$P^{356} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$
$$P^{1356} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

are projection matrices.

Using (1)-(2) and (3)-(4) we can write the projected system as follows:

$$\dot{\eta}_z = \bar{J}(\eta)\nu_l(t), \tag{5}$$
$$\dot{\nu}_l = \bar{A}(\nu)\nu_l(t) + \bar{g}(\eta) + \bar{T}\tau, \tag{6}$$

where $\bar{J}(\eta) = P^{356}J(\eta)P^{1356^\dagger}$, $\bar{A}(\nu) = P^{1356}A(\nu)P^{1356^\dagger}$, $\bar{g}(\eta) = P^{1356}g(\eta)$ and $\bar{T} = P^{1356}T$. Throughout the paper, the notation $(\cdot)^\dagger$ is used to denote the generalized inverse. In what follows, we assume that $\bar{T}$ has full rank.

Rotation sequence

In this problem, we choose the sequence of rotation to be ZYZ (also known as 323) which is composed by a sequence of rotation along the z, the y and again the z-axis. The body-fixed referential frame is obtained by a sequence of three rotations of an inertial earth-fixed frame rotation: First, a rotation of an angle ϕ about the z-axis; second, a rotation of θ about the y-axis and; third, a rotation of an angle ψ about the z-axis. The J matrix results

$$J(\eta) = \begin{bmatrix} s\phi c\psi c\theta - s\phi s\psi & -c\psi s\phi - c\phi c\theta s\psi & c\phi s\theta & 0 & 0 & 0 \\ c\phi s\psi + c\psi c\theta s\phi & c\phi c\psi - s\psi c\theta s\phi & s\phi s\theta & 0 & 0 & 0 \\ -c\psi s\theta & -c\psi s\phi - c\phi c\theta s\psi & c\phi s\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & c\psi/t\theta & -s\psi/t\theta & 1 \\ 0 & 0 & 0 & s\psi & c\psi & 0 \\ 0 & 0 & 0 & -c\psi/s\theta & s\psi/s\theta & 0 \end{bmatrix}. \quad (7)$$

We note that this representation is not unique. Other sequences of rotations can also be applied [13]. However, this one is the most appropriate in the context of this work, as it is likely the most intuitive sequence of rotation and consequently allows for a simple definition of the reference angles. This representation is used throughout the following developments.

III. Pose stabilization

Our main goal in this work is to stabilize the vertical pose of the MARES AUV. We aim at stabilizing the vehicle at pitching angles of $\pi/2$ or $-\pi/2$ radians, an unusual pose for this type of vehicles.

A. Control law

Let $\eta_z^*(t)$ be a smooth desired pose vector of the vehicle and define the error vector as

$$\tilde{\eta}_z(t) = \eta_z(t) - \eta_z^*(t). \quad (8)$$

Similarly, let us define $\nu_l^*(t)$ as a desired velocity reference. We will see that this vector is constrained to lie in a manifold that depends on the orientation of the vehicle as we will see later on. For now, assume that it does not influence the stability of the system. We define the velocity error vector as follows:

$$\tilde{\nu}_l(t) = \nu_l(t) - \nu_l^*(t). \quad (9)$$

By noting that $\eta_z(t) = \tilde{\eta}_z(t) + \eta_z^*(t)$, $\nu_l(t) = \tilde{\nu}_l(t) + \nu_l^*(t)$ from (8) and (9), respectively, we can re-write the system (5)-(6) as

$$\dot{\tilde{\eta}}_z(t) = \chi_1(\eta, t) + \bar{J}(\eta)\tilde{\nu}_l(t), \quad (10)$$

$$\dot{\tilde{\nu}}_l(t) = \chi_2(\nu, \eta, t) + \bar{T}\tau, \quad (11)$$

where

$$\begin{aligned} \chi_1(\eta, t) &= -\eta_z^*(t) + \bar{J}(\eta)\nu_l^*(t) + \bar{J}(\eta)\tilde{\nu}_l(t), \\ \chi_2(\nu, \eta, t) &= \bar{A}(\nu)\tilde{\nu}_l(t) + \bar{A}(\nu)\nu_l^*(t) - \dot{\nu}_l^*(t) \\ &\quad + \bar{g}(\eta) + \bar{T}\tau. \end{aligned}$$

Based on the backstepping method (see, for example, [8]), we consider the virtual control law α, defined as

$$\alpha(\eta, t) = \bar{J}^\dagger(\eta)\big(- \chi_1(\eta, t) - K_\eta \tilde{\eta}_z(t)\big), \quad (12)$$

where $K_\eta \in \mathbb{R}^{3\times 3}$ is a positive definite gain matrix. Using this virtual control law, we rewrite (10):

$$\dot{\tilde{\eta}}_z(t) = -K_\eta \tilde{\eta}_z(t) + \bar{J}(\eta)\big(\tilde{\nu}_l(t) - \alpha(\eta, t)\big). \quad (13)$$

We want now to drive $\dot{\tilde{\eta}}_z(t)$ to zero. In this sense, $\tilde{\nu}_l(t)$ has to be (indirectly) controlled so that $\tilde{\eta}_z(t)\bar{J}(\eta)z_2 < 0$, being $z_2 = \tilde{\nu}_l(t) - \alpha(\eta, t)$ a new error variable. Again, we rewrite the system (10)-(11) as

$$\begin{aligned} \dot{\tilde{\eta}}_z(t) &= -K_\eta \tilde{\eta}_z(t) + \bar{J}(\eta)z_2, && (14) \\ \dot{z}_2 &= \dot{\tilde{\nu}}_l(t) - \dot{\alpha}(\eta, t) \\ &= \chi_2(\nu, \eta, t) - \dot{\alpha}(\eta, t) + \bar{T}\tau, && (15) \end{aligned}$$

and choose the control law

$$\tau = \bar{T}^{-1}\big(\chi_2(\nu, \eta, t) + \dot{\alpha}(\eta, t) - \bar{J}(\eta)^T\tilde{\eta}_z(t) - K_\nu z_2\big), \quad (16)$$

where $K_\nu \in \mathbb{R}^{4\times 4}$ is a positive definite gain matrix. The choice of this control law makes the system (14)-(15) result into

$$\dot{\tilde{\eta}}_z(t) = -K_\eta \tilde{\eta}_z(t) + \bar{J}(\eta)z_2, \quad (17)$$

$$\dot{z}_2 = -\bar{J}(\eta)^T\tilde{\eta}_z(t) - K_\nu z_2. \quad (18)$$

The subsequent proof of stability is based on the analysis of a Lyapunov function. For this purpose, we define the Lyapunov function candidate

$$V = \frac{1}{2}\tilde{\eta}_z(t)^T\tilde{\eta}_z(t) + \frac{1}{2}z_2^T z_2, \quad (19)$$

whose time derivative results

$$\begin{aligned} \dot{V} &= \tilde{\eta}_z(t)^T\dot{\tilde{\eta}}_z(t) + z_2^T\dot{z}_2 && (20) \\ &= \tilde{\eta}_z(t)^T K_\eta \tilde{\eta}_z(t) + \tilde{\eta}_z(t)^T\bar{J}(\eta)z_2 \\ &\quad -z_2^T\bar{J}(\eta)^T\tilde{\eta}_z(t) - z_2 K_\nu z_2 \\ &= -\tilde{\eta}_z(t)^T K_\eta \tilde{\eta}_z(t) - z_2^T K_\nu z_2 \\ &< 0, \ \forall\ \tilde{\eta}_z(t) \neq 0, z_2 \neq 0. \end{aligned}$$

The negative definiteness of the time derivative of the Lyapunov function ensures that the system is uniformly exponentially stable.

Note that $\nu_l^*(t)$ was assumed not to disturb the stability of the pose reference. Indeed, this desired velocity reference can not collide with the control objective for the pose. The following development gives the necessary condition to ensure the overall exponential stability of the system.

B. Constraint on the desired velocity

In this section, we first start illustrating the dependence of the controllable DOFs in the pose of the vehicle by showing their relationship.

We can note that

$$\begin{aligned} P^{356} &= \begin{bmatrix} P^3 & 0_{1\times3} \\ 0_{2\times3} & P^{56} \end{bmatrix} \text{and} \\ P^{356\dagger} &= P^{356^T} = \begin{bmatrix} P^{3^T} & 0_{3\times2} \\ 0_{3\times1} & P^{56^T} \end{bmatrix}, \\ P^{1356} &= \begin{bmatrix} P^{13} & 0_{2\times3} \\ 0_{2\times3} & P^{56} \end{bmatrix} \text{and} \\ P^{1356\dagger} &= P^{1356^T} = \begin{bmatrix} P^{13^T} & 0_{3\times2} \\ 0_{3\times2} & P^{56^T} \end{bmatrix}. \end{aligned}$$

where

$$P^3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad P^{13} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$P^{56} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Moreover, from [12], the matrix $J(\eta)$ can be written in the block diagonal form:

$$J(\eta) = \begin{bmatrix} J_1(\eta) & 0_{3\times3} \\ 0_{3\times3} & J_2(\eta) \end{bmatrix}. \tag{21}$$

Defining $\eta_z = [\eta_{z1}\ \eta_{z2}]^T = \bar{J}(\eta)\nu_l$, with $\nu_l = P^{1356}\nu$ and $\eta_{z1} = z, \eta_{z2} = [\theta\ \psi]^T$, we obtain

$$\begin{aligned} \begin{bmatrix} \dot{\eta}_{z1} \\ \dot{\eta}_{z2} \end{bmatrix} &= P^{356} J(\eta) P^{1356^\dagger} \nu_l \\ &= \begin{bmatrix} P^3 & 0_{1\times2} \\ 0_{2\times1} & P^{56} \end{bmatrix} \begin{bmatrix} J_1(\eta) & 0_{3\times3} \\ 0_{3\times3} & J_2(\eta) \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} P^{13^T} & 0_{3\times2} \\ 0_{3\times2} & P^{56^T} \end{bmatrix} \begin{bmatrix} \nu_{l1} \\ \nu_{l2} \end{bmatrix} \\ &= \begin{bmatrix} P^3 J_1(\eta) P^{13^T} & 0_{1\times2} \\ 0_{2\times2} & P^{56} J_2(\eta) P^{56^T} \end{bmatrix} \begin{bmatrix} \nu_{l1} \\ \nu_{l2} \end{bmatrix} \quad (22) \\ &= \begin{bmatrix} -c\psi s\theta & c\theta & 0 & 0 \\ 0 & 0 & c\psi & 0 \\ 0 & 0 & s\psi/s\theta & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ r \end{bmatrix}, \end{aligned}$$

where $\nu_{l1} = [u\ w]^T$ and $\nu_{l2} = [q\ r]^T$ are the vectors of linear and angular velocities, respectively.

One can easily see that for the vehicle pitching up or down ($\theta = \pm\pi/2$), the heave velocity has no influence on the vertical pose and can therefore be handled independently of the remaining DOFs. This simple example serve as a basis for the following developments in this section.

One wants that $\nu_l^*(t)$ has no influence in the vertical pose stabilization. Therefore, the following expression must be verified:

$$\bar{J}(\eta)\nu_l^*(t) = \mathbf{0},\ \forall\ \eta(t). \tag{23}$$

We are particularly interested in setting the linear velocity reference to control the horizontal motion. Obviously, the angular velocity references must remain unchanged in order not to disturb the orientation stability. We define $\nu_{l1}^*(t) = [u^*(t)\ w^*(t)]^T$ and $\nu_{l2}^*(t) = [q^*(t)\ r^*(t)]^T$ ($\nu_l^*(t) = [\nu_{l1}^*(t)\ \nu_{l2}^*(t)]^T$) and focus our interest on the linear velocity dynamics.

The linear velocity subvector of the equation above thus becomes (see (22))

$$P^3 J_1(\eta) P^{13^T} \nu_{l1}^*(t)^T = \begin{bmatrix} -c\psi s\theta & c\theta \end{bmatrix} \begin{bmatrix} u^*(t) \\ w^*(t) \end{bmatrix} = 0. \tag{24}$$

In order to satisfy this equation, we must verify

$$u^*(t) = \frac{w^*(t)}{t\theta c\psi}, \tag{25}$$

to ensure the stability of the control law in (16).

IV. Horizontal guidance

So far in this paper, we assumed that $\bar{T}$ is full rank, i.e., from a practical point of view this means that the considered DOFs (surge, heave, pitch and yaw) are controllable. Now, we extend our study to the roll dynamics and on how this can be used to guide MARES in the horizontal plane.

A. Roll dynamics

The rotation of propellers induce a moment when generating the desired lift. This is a natural consequence of their inclination. A practical way to cancel such a moment is to use symmetric inclinations of propellers, for symmetrically located thrusters with respect to the center of gravity (CG). In this case, for the same force, the propellers rotate in opposite directions, thus cancelling each other moments. However, in this work we assume that the inclinations of the stern thrusters are the same, thus inducing a moment on the x-axis.

In our previous works (see, for example [14], [15]), this effect was neglected as its influence is residual compared to the moment generated by non-coincident vertical position of the center of buoyancy (CB) and the CG. Nevertheless, this restoring moment no longer exists when the vehicle is pitching up or down ($\pm\pi/2$) and the moment induced by the stern thrusters does influence the roll dynamics and can not be neglected. Actually, its presence is used here to drive the vehicle to an horizontal position.

Mathematically, the roll dynamics can be written as a scalar differential equation

$$\dot{p} = f_p(\dot{\nu}, \nu) - d_p(p)p - g_p(\eta) + (f_\tau(\tau_s))^T \tau_s, \tag{26}$$

where $f_p(\dot{\nu}, \nu)$ is a function containing the added mass, Coriolis, centripetal and cross-related viscous damping effects. The scalar function $d_p(p)$ represents the direct viscous damping effect, while $g_p(\eta)$ is the restoring moment. We assume that there exists a positive definite vector $f_\tau(\tau_s) : \mathbb{R}^2 \to \mathbb{R}^2$ that relates the moment generated by the thruster with the actual lift force of the stern thrusters that compose the entries of the vector $\tau_s \in \mathbb{R}^2$. Note that the function $g_p(\eta)$ is positive definite for all non-null p and verifies $d_p(p) = 0$ if and only if $p = 0$. We assume that $f_\tau(\tau_s) = \mathbf{0}$ if and only if $\tau_s = \mathbf{0}$. These facts are used in the forthcoming analysis.

At equilibrium (see section III) with $\theta = \pm\pi/2$, we can prove that $f_p(\dot{\nu}, \nu) \approx 0$ and $g_p(\eta) = 0$ for the case of MARES. Hence the roll dynamics results

$$\dot{p} \approx -d_p(p)p + (f_\tau(\tau_s))^T \tau_s, \tag{27}$$

which, for a constant τ_s, gives $d_p(p)p \approx (f_\tau(\tau_s))^T \tau_s$ in steady state. This means that the roll angular velocity is different from zero in steady state, when $\theta = \pm\pi/2$.

This result is particularly interesting since a constant actuation on the stern thrusters implies that the vehicle will keep rotating along the x-axis. Note that, under a given condition, it is not necessary the vehicle to be moving to have a constant actuation on the stern thrusters. Indeed, a positive (or negative) buoyancy would make the vehicle actuate on the stern thrusters to maintain the depth constant.

B. Guidance

We address now the problem of guiding the vehicle to a given desired static position in the horizontal plane, when the vehicle is pitching up or down. For the purpose, we define $\eta_h(t) = [x\ y]^T$ the horizontal position of MARES and $\eta_h^*(t) = [x^*\ \ y^*]^T$ to be a constant horizontal position reference. The error vector is defined as

$$\tilde{\eta}_h = \begin{bmatrix} \tilde{x}(t) \\ \tilde{y}(t) \end{bmatrix} = \eta_h(t) - \eta_h^*(t). \tag{28}$$

Now, our guidance objective is to drive the error vector to zero.

For convenience, we make a change of coordinates to the polar coordinate system and write the equivalent equation in (28) as

$$\begin{cases} \rho(t) = ||\tilde{\eta}_h|| \\ \gamma(t) = \angle(\tilde{\eta}_h) = atan2(\tilde{y}(t), \tilde{x}(t)) \end{cases}, \tag{29}$$

where $\angle(\tilde{\eta}_h)$ is the angle of the vector $\tilde{\eta}_h$ with respect to the x-axis, given in the reference frame. The function $atan2(\cdot, cdot)$, whose counter-domain is the interval $]-\pi, \pi]$, stands for the variation of the $atan(\cdot)$ function. Our control objective now becomes driving the distance from the vehicle to the reference $\rho(t)$ to zero.

With the vehicle in the vertical pose, the orientation of the body-fixed z-axis expressed in the horizontal plane of the inertial reference frame is given by $\phi(t)$. In order to find an appropriate control law, we derive the system (29) with respect to time:

$$\begin{cases} \dot{\rho}(t) = \frac{\dot{\tilde{\eta}}_h(t)^T \tilde{\eta}_h}{\rho(t)} \\ \dot{\gamma}(t) = \frac{\partial}{\partial t} atan2(\tilde{y}(t), \tilde{x}(t)) \end{cases},$$

which, after algebraic manipulation, results

$$\begin{cases} \dot{\rho}(t) = w(t)\cos(\tilde{\phi}(t)) \\ \dot{\gamma}(t) = \frac{1}{\rho(t)}\left(w(t)\sin(\tilde{\phi}(t))\right) \end{cases}, \tag{30}$$

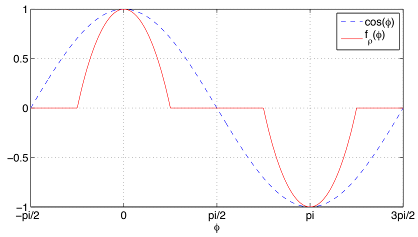

Fig. 2. The function $f_\rho(\tilde{\phi}(t))$

where $\tilde{\phi}(t) = \gamma(t) - \phi(t)$ and we recall that w is the heave velocity, which can be handled "independently" of the remaining DOFs when the vehicle pose verifies $\theta = \pm\pi/2$. We aim at finding a suitable guidance law to drive the vehicle towards its reference point by using $w(t)$ as the input. Implicitly, the derivation of the control law in section III, suggests that the heave velocity reference $w^*(t)$ must be at least C^1. Hence, we propose the following guidance law:

$$w^*(t) = -sat(\rho(t), \beta) \cdot f_\rho(\tilde{\phi}(t)), \tag{31}$$

where $\beta > 0$ and $sat(\cdot, \cdot)$ being the saturation function defined as

$$sat(\rho(t), \beta) = \begin{cases} \rho(t), & \text{if} |\rho(t)| \leq \beta \\ \beta \frac{\rho(t)}{||\rho(t)||}, & \text{otherwise} \end{cases}.$$

The continuous function $f_\rho(\tilde{\phi}(t))$ (illustrated in Fig. 2), reminiscent of the sector-of-sight, is given by

$$f_\rho(\tilde{\phi}(t)) = \begin{cases} \frac{\cos(2\tilde{\phi}(t))}{cos(\tilde{\phi}(t))}, & \text{if} \cos(2\tilde{\phi}(t)) > 0 \\ 0, & \text{otherwise} \end{cases}. \tag{32}$$

Hence, when $w(t) = w^*(t)$, the time derivative of the distance to the desired position is given by

$$\dot{\rho}(t) = \begin{cases} -sat(\rho(t), \beta)\cos\left(2\tilde{\phi}(t)\right), \text{if} \cos\left(2\tilde{\phi}(t)\right) > 0 \\ 0, \text{ else} \end{cases} \tag{33}$$

Note that $\dot{\rho}(t)$ is negative semi-definite and negative definite under the condition $\cos\left(2\tilde{\phi}(t)\right) > 0$. Since we assume that $\phi(t)$ is periodical, we only need to ensure that the difference $\tilde{\phi}(t) = \gamma(t) - \phi(t)$ does not remain constant under the condition $\cos\left(2\tilde{\phi}(t)\right) < 0$. If this condition is verified, from (31) we can see that $w(t) = 0$ (see (32)) and hence, from (30) we know that $\dot{\gamma}(t) = 0$. We can conclude that the difference $\tilde{\phi}(t)$ is not constant if the inequality $\cos\left(2\tilde{\phi}(t)\right) < 0$ holds, since $\phi(t)$ is periodical. We can therefore conclude that the position error vector $\tilde{\eta}_h$ converges to zero.

Remark 1: The choice of the function $f_\rho(\tilde{\phi}(t))$ is not restricted to the one presented here. In fact, the function must only be derivable and verify two conditions: 1) $f_\rho(\tilde{\phi}(t)) = 0$ if $\tilde{\phi}(t) = \pm\pi/2$ and; 2) $f_\rho(\tilde{\phi}(t))\cos(\tilde{\phi}(t)) \geq 0$ for all $\tilde{\phi}(t) \neq \pm\pi/2$ and verify $f_\rho(\tilde{\phi}(t))\cos(\tilde{\phi}(t)) > 0$ for, at least, an interval $\Omega \subset [-\pi, \pi] \setminus \{\pm\pi/2\}$

Remark 2: In the guidance law in (31) we introduced the saturation function for practical reasons as any real actuator have a limited force and consequently limits the heave velocity.

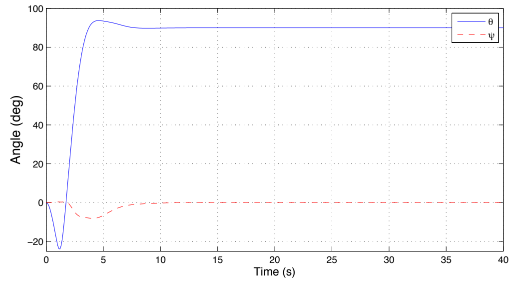

Fig. 3. Pitch and yaw angles

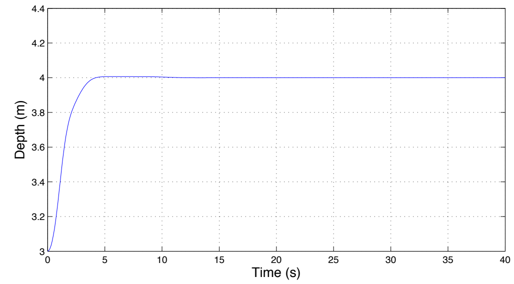

Fig. 4. Depth

The constant β plays the role of an upper bound on the absolute value of the heave velocity w.

V. Results

To illustrate and validate our approach to stabilize MARES in the vertical pose and guide it to an horizontal position, hereinafter we present simulated and experimental data. The first ones show the simulation of a six-DOFs model of MARES stabilized in the vertical pose and horizontally guided to a reference point. The experimental results in a tank validate our control law to stabilize MARES in the vertical pose.

A. Simulation

We start showing the results of the simulation of stabilizing the vertical pose of MARES only. For this purpose, the gain matrices were set such that $K_\eta = diag([1, 1, 1])$ $K_\nu = diag([1, 10, 1, 1])$, where $diag(\cdot)$ denotes a diagonal matrix with diagonal entries equal to the arguments. The Fig. 3 shows the evolution of the pitch (θ) and yaw (ψ) angles for a desired pose defined by the vector $\eta_z^*(t) = [4\ \pi/2\ 0]^T$. The depth is shown in the Fig. 4. The initial condition were set to $\eta(0) = [0\ 0\ 3\ 0\ 0\ 0]^T$ and $\nu(0) = \mathbf{0}$.

The Fig. 3-4 show that the control law in (16) stabilizes MARES in the vertical pose in less than ten seconds. Note that the pitch angle starts decreasing to negative values and posteriorly starts increasing to the assigned reference. This is the result of a relatively large gain for the depth. The control law makes the heave velocity be large in the first instants thus influencing the pitch dynamics. Although not included, simulation results using smaller values for the gain corresponding to the depth error show that the decrease of the θ angle becomes smaller during the initial instants.

The Fig. 5 shows the trajectory of the vehicle for the application of the guidance law derived in the section IV-B. The desired horizontal position was set to $\eta_h^*(t) = [10\ 10]^T$. The initial conditions remained the same as the ones above. The simulation shows successful tracking of the target.

B. Experiments

In order to test our approach, we have implemented the control algorithm in the MARES AUV and carried out tests in a tank. The current control software interprets and sequentially executes mission scripts where maneuvers and parameters (desired depth, angles, duration, etc.) are defined. The controllers have been implemented in the MARES on-board computer. Although MARES is composed by several sensors, in this paper only depth and inertial measurement unit (IMU) have been used for measuring relevant data to control MARES in the vertical pose. From the IMU, angles and angular rates have been read and directly used to feedback the controller. In practice, we expect angle errors below few degrees (typically less than 4 degrees). Depth measurement are fairly precise with errors in the order of millimeters.

The gain matrices used in the control law (16) were $K_\eta = diag([2\ 0.7\ 1])$ and $K_\nu = diag([0.01\ 0.01\ 1\ 1])$. The mission script included a sequence of three maneuvers that are implicitly defined by the desired pose: 1) $\eta_z^* = [z^*\ \theta^*\ \psi^*] = [1\ \pi/2\ 0]$ during the first 30 seconds; 2) $\eta_z^* = [2.5\ \pi/2\ 0]$ for the subsequent 60 seconds and; 3) $\eta_z^* = [2.5\ -\pi/2\ 0]$ for the final 60 seconds. To ensure that there is no abrupt variation on the pose references, their values were linearly smoothed over time.

The results of the mission are show in the Fig. 6-8. The Fig. 6 shows the pitch and the yaw angles, the Fig. 7 presents the evolution of the roll angle, while the Fig. 8 shows the depth. Despite the natural disturbances, the control law ensure stability of MARES.

We can note that the pitch angle suffers a relatively large oscillation before stabilizing around $\theta^* = \pi/2$. This is due to

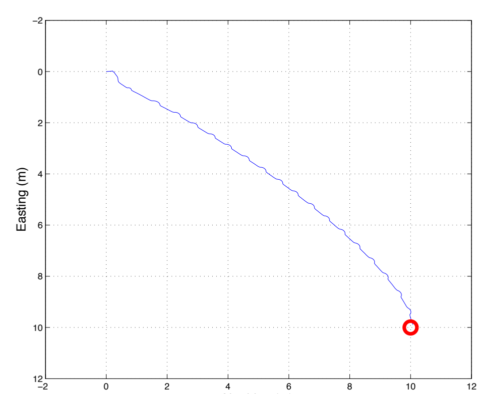

Fig. 5. Horizontal trajectory

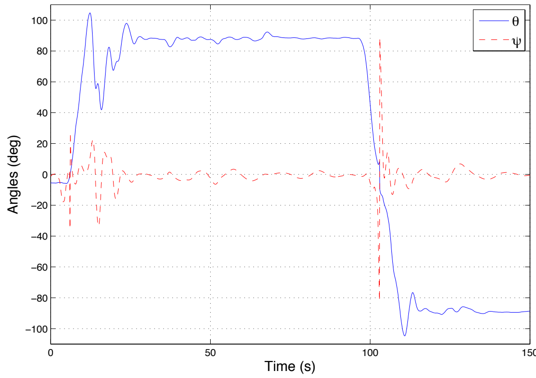

Fig. 6. Pitch and yaw angles

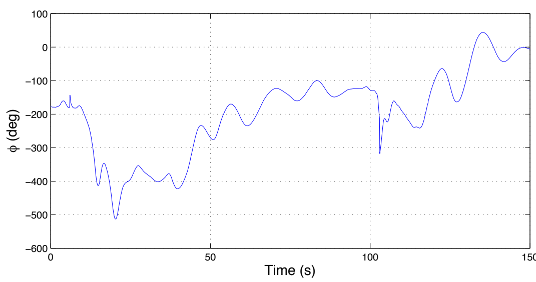

Fig. 7. Roll angle

the restoring moment that made MARES rapidly rotate around the body-fixed x-axis when pitch was greater than $\pi/2$. The evolution of the roll angle can be checked in the Fig. 7 for the corresponding interval. We can also note that the discontinuity at time $t \approx 100$ on θ and ψ is caused by the singularity at $\theta = \pi/2$ (see section II). The small deviation of θ (typically less than three degrees) is mainly induced by uncertainties on the mathematical model of MARES used in the control law. On the contrary, we believe that the oscillation in ψ is caused by a small offset in the measurements.

As expected, a positive trend on ϕ (roll angle) is visible when the vehicle stabilizes at $\theta = \pm\pi/2$. Recall from section IV-A, that the actuation on the stern thrusters make the vehicle rotate about the x-axis. Indeed, the stabilization of the depth implies non-null actuation since the slightly positive buoyancy of MARES creates a force that lead it to the surface. The oscillation of the roll angle is caused by the moment about the x-axis created by a non-null yaw.

VI. Conclusions

We presented a control law to stabilize MARES in the vertical pose. Although the derivation was based on this specific vehicle, we believe that it can be easily adapted to others with similar characteristics. Based on the nonlinear control theory, and more specifically on backstepping, we derived a control law that ensures exponential stability for the vertical pose. In order to explore the capabilities of MARES, we also have determined a control law that enables

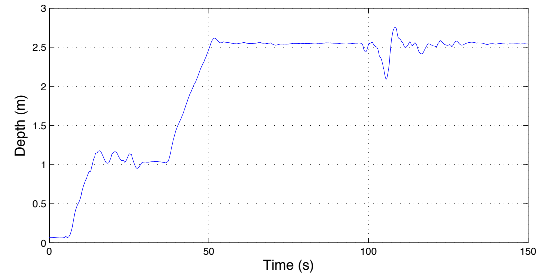

Fig. 8. Depth

the vehicle to reach any horizontal reference by using a sector-of-sight-like controller. We have verified mathematically that both guidance and control law makes the vehicle converge to the desired pose. The approach shows to be effective and both simulations and experiments validated our approach, originating very encouraging results.

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