# FUZZY FLOW SIMULATION IN GAS AND ELECTRIC NETWORKS

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## Abstract

Steady state fuzzy flow modelling in networks is proposed, in an unified approach, to take in account qualitative aspects or vagueness and uncertainty in data that have not probabilistic or random nature. An incremental approach is adopted, starting from a previously calculated feasible operating point. Based on concepts from fuzzy set theory, approximate possibility distributions for branch flows and node potentials are then evaluated. The formulation derived is identified with gaz and electric networks and may form the basis for network planning-aid-systems development.

## INTRODUCTION

Computer simulations of steady state power flows in electrical systems or gaz flows in gaz networks aim at estimating the values of node voltages or pressures and branch flows for known consumptions at load nodes and, eventually, known source node voltages or pressures.

The presentation of algorithms in the literature to solve this problem has always dealt with it from a deterministic point of view. However, it is recognized that data may have other nature, namely probabilistic. This would be the case if, for instance, a consumption would be known through a probabilistic distribution.

In other cases, data has a nature that is neither deterministic non probabilistic. In planning, for instance, the analysis of consumption trends depends on the evolution of economic activities and competition among several sources of energy which affect forecasts. There are, thus, scenarios to be considered and this makes it unrealistic to choose, in a deterministic sense, specific values for problem data.

Besides, the uncertainty found in planning judgements is greatly due incomplete human knowledge that cannot be represented by a probabilistic approach, because looking into the future is really not the same as considering a workframe of repetition of events. Nevertheless, judgements are produced based on human accumulated knowledge and experience and translated into linguistic declarations such as "Consumption in node 10 will be approximately 1 $m^3/s$" or "Load in node 5 has a domestic type diagram". Such declarations clearly have a vague nature of a third kind, possibilistic.

It is therefore important to use concepts and adopt techniques to capture the fuzzy nature that often characterizes human activity or human knowledge.

Fuzzy set theory (Zadeh 1965, 1978; Zimmermann 1985) gives an adequate workframe to model such kind of imprecision and has been applied with success to several domains and systems. In this paper, it will be used to obtain approximate possibility distributions of line or pipe flows evaluated from a fuzzy description of node consumptions. In the Appendix one may find some brief notes on basic concepts of fuzzy set theory.

Electrical and gaz networks have in common a set of features such as topology description and Kirchhoff laws. Both systems are usually described in such a way that the mathematical models are non linear. In order to deal with fuzzy quantities, they must be linearized so that flows can be directly related to node injections. Linearization of electrical system models has led to the well known DC model. Linearization of gaz systems implies the linearization of pipe flow equations.

In this paper, an unified approach is for the first time presented for both systems, based on the calculation of a sensitivity coefficient matrix relating branch flows to fuzzy power injections. The idea behind them, however, may be generalized to other distribution systems.

## DESCRIBING FUZZY LOADS

Loads will be described by fuzzy numbers, which come associated to a possibility distribution having the properties of a membership function.

A simple way of specifying a load is through a triangular or trapezoidal fuzzy number, such as in fig.1., as suggested in (Freska 1982).

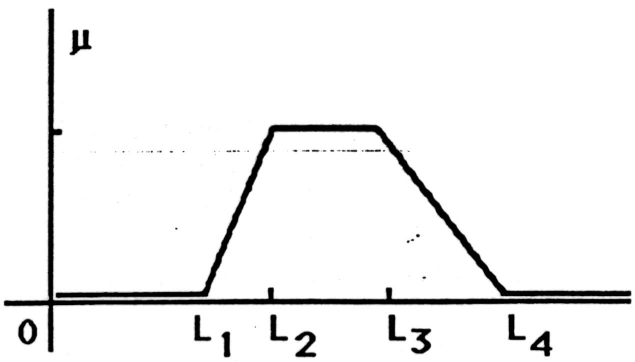



Fig.1 - Trapezoidal fuzzy number modelling a fuzzy load

The planning engineer must declare and interval $[L_2, L_3]$ where loads are considered typical possible values for the type of consumption and time step; and values $L_1$ and $L_4$, under and above which the load is estimated not possible to occur. Then, values in the intervals $[L_1, L_2]$ and $[L_3, L_4]$ are taken as possible representations of the load but not with the same strength as values in $[L_2, L_3]$. These conditions are translated by the values of the membership function depicted in fig.1.

This representation can be looked at as a translation of a sentence or a linguistic declaration of the kind "**load may occur between $L_1$ and $L_4$ but it is likely to be between $L_2$ and $L_3$**".

Qualifying these values with a further linguistic declaration such as "**there is little uncertainty in this estimate**" is also possible and a new fuzzy representation will be obtained by composing the basic one with the fuzzy description of linguistic modifiers, in a process that has been proposed with several variations - see (Miranda 1985, 1991a), for example.

This feature becomes very useful if one aims at incorporating some kind of network performance assessment in a decision aid environment, namely for planning purposes.

## FUZZY FLOWS

### Data and Results

Let's admit a system based on a network, described by a branch node incidence matrix **T**, with source and load nodes. We will have, as data for a network with n nodes:

- known specified node potentials at $n_1$ source nodes, in vector $P_s$;
- known loads represented by fuzzy numbers at $(n-n_1)$, nodes, in vector $L_l$;
- known coefficients $K_i$ relating flow in branch i with potential levels at both ends, forming a diagonal matrix **K**.

We aim at assessing:

- branch flow possibility distributions for each of the on lines of the network - vector **f**;
- potential value possibility distributions at $(n-n_1)$ load nodes, derived from a vector $P_l$
- possibility distributions of flow injections at n1 source nodes, in vector $L_s$.

By any method, we are supposed to have already calculated a deterministic point with line flows $f_o$, node pressures $P_o$ and node injections $L_{so}$. This point is calculated on the basis of the central values with possibility degree 1 of the fuzzy numbers describing loads.

In the following paragraphs, we will derive an incremental formulation that will then be used to provide the fuzzy equations needed. Therefore, a deviation $\Delta L$ in loads from their specified value will lead to

$$f = f_o + \Delta f \;\; ; \;\; P = P_o + \Delta P \;\; ; \;\; L_s = L_{so} + \Delta L_s$$

### Linear branch flows

Linearization of branch flows is a needed step in order to deal with fuzzy quantities. In general, a linear branch flow equation in line i-j is described by

$$f_{ij} = K_{ij}^{-1}(P_i - P_j) \tag{1}$$

for $P_i > P_j$.

A linearization around $f_o$ is, for the set of m lines,

$$f = f_o + C\, T^t\, \Delta P \tag{2}$$

or $$f = f_o + \Delta f$$

where $C_{ij} = K_{ij}^{-1}$ is a constant characteristic of branch ij and:

- C is a diagonal matrix (m*m);
- $\Delta P = P - P_o$ is the vector of the variation of potential between the line extremes;
- $T : [T_{ij}]_{(n*m)}$ is the branch node incidence matrix.

### Fuzzy Flow Equations

For the calculated point $f_o$, 1st Kirchhoff law states that

$$T f_o = L_o$$

or $$\left[\, T_s \;\vdots\; T_l \,\right] f_o = [\, L_{so} \;\; L_{lo} \,]^t$$

For point $f_o + \Delta f$, we have $T f_o + T\Delta f = L_o + \Delta L$ which gives $T\Delta f = \Delta L$.

Substituting $\Delta f$, one gets

$$T C T^t \Delta P = \Delta L$$

$Y = TCT^t$ is a square matrix that can be partitioned separating source and load nodes to give

$$\begin{bmatrix} Y_{ss} & Y_{sl} \\ Y_{ls} & Y_{ll} \end{bmatrix} \begin{bmatrix} 0 \\ \Delta P_l \end{bmatrix} = \begin{bmatrix} \Delta L_s \\ \Delta L_l \end{bmatrix} \tag{3}$$

and therefore

$$\Delta P_l = Y_{ll}^{-1}\, \Delta L \tag{4}$$

whenever $Y_{ll}$ is non singular.

Using (3) in (2), one gets

$$f = f_o + C\, T_l^t\, Y_{ll}^{\cdot}\;\; \Delta L_l \tag{5}$$

Using (3) in $Y_{sl}\, \Delta P_l = \Delta L_s$ , one gets

$$\Delta L_s = Y_{sl}\; Y_{ll}^{-1}\; \Delta L_l \qquad (6)$$

In equations (4), (5) and (6), if the deviations $\Delta L$ are replaced by fuzzy numbers $\underline{\Delta L}$ with their associated membership functions, one gets fuzzy numbers as a result, after performing calculations according to the properties of fuzzy operations. Fuzzy numbers $\underline{\Delta L}$ must be taken as uncertainties around $L_o$ values, and the results accordingly.

We recall that the addition of fuzzy numbers is obtained by max-min convolution, which becomes very efficient for triangular or trapezoidal membership functions. For other shapes (descriptions) of fuzzy data, the concept of $\alpha$-cuts can be used with simplicity.

## Interpretation in Terms of Electrical Networks

The model above can readily be recognized as the DC load flow model for electrical networks, if loads L and flows f are given in MW and the node potentials P are taken as the vector $\Theta$ of voltage angles measured in radians. There will be a reference node for which the angle is fixed at zero and therefore matrix $Y_{ss}$ will have only one element. Matrix $Y_{ll}$ is identifiable with the commonly designated matrix **B**, and the branch coefficients $K_{ij}$ stand for branch reactances $X_{ij}$.

Fuzzy load flows (DC and AC) have been recently proposed in Power System Engineering (Miranda 1990, 1991b).

Obviously, similar interpretations could be presented for networks described by a system such as I = YE, where loads are given under the form of currents (although it is not the usual way in Power Systems). It can easily be shown that this is valid either for DC and for AC networks; in the latter case, the equations on complex current and voltage values need only to be transformed into a system of 2n real equations, and the complex fuzzy loads must have independent fuzzy descriptions of their real and imaginary parts (giving a sort of rectangular fuzzy complex number).

## Fuzzy Gaz Flow

In general, the pipe flow equation in line i-j is described by

$$(q_{ij})^a = K_{ij}^{-1}(P_i - P_j)$$

for $P_i > P_j$, where "a" depends on operating pressure and $P_i$, $P_j$ are, in general, the square of actual node pressure values (usually measured in bar).

A linearization through the origin and $q_o$ is

$$q_{ij} = q_o + (q_o)^{a-1}\, K_{ij}^{-1}\, [(P_i - P_j) - (P_{io} - P_{jo})]$$

So, in this case, when building a system similar to (2), one will have $C_{ij} = (q_o)^{a-1}\, K_{ij}^{-1}$.

Fuzzy gaz flow equations are then (5) and (6), where flows q are identified with vector f and are measured, for instance, in $m^3/h$. In this case, one can have a number $n_l$ of source nodes, with fixed pressure values, and equation (6) becomes important, as it regulates the amount of gaz injection at the source nodes.

A special important case appears in gaz networks: the presence of a compressor station. A compressor type node must be split into two nodes, one of load type and one of source type such as in fig. 2

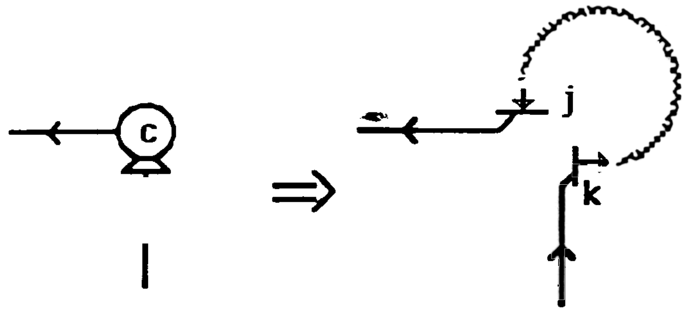



Fig. 2 - Representation of split compressor node

The following conditions are imposed:

$$P_j = P_{\text{specified}}$$
$$L_j + L_k = 0$$

Equation (3) becomes partitioned into

$$\begin{bmatrix} Y_{ss} & Y_{sl} & Y_{sla} \\ Y_{sas} & Y_{sal} & Y_{sala} \\ Y_{ls} & Y_{ll} & Y_{lla} \\ Y_{las} & Y_{lal} & Y_{lala} \end{bmatrix} \begin{bmatrix} 0 \\ \hline \Delta P_l \\ \hline \Delta P_{la} \end{bmatrix} = \begin{bmatrix} \Delta L_s \\ \Delta L_{sa} \\ \Delta L_l \\ \Delta L_{la} \end{bmatrix}$$

where matrices indexed "sa" correspond to compressor source type nodes and "la" to compressor load type nodes.

Taking in account that $\Delta L_{sa} + \Delta L_{la} = 0$ , by adding lines we can get

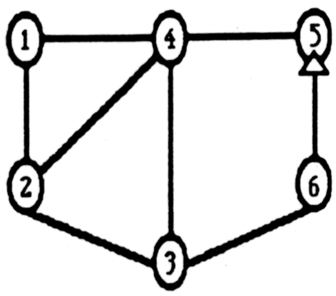
$$\begin{bmatrix} Y_{ss} & Y_{sl} & Y_{sla} \\ Y_{sas} & Y_{sal} & Y_{sala} \\ Y_{ls} & Y_{ll} & Y_{lla} \\ Y_{las}+Y_{sas} & Y_{lal}+Y_{sal} & Y_{lala}+Y_{sala} \end{bmatrix} \begin{bmatrix} 0 \\ \hline \Delta P_l \\ \hline \Delta P_{la} \end{bmatrix} = \begin{bmatrix} \Delta L_s \\ \Delta L_{sa} \\ \Delta L_l \\ 0 \end{bmatrix}$$

From this system one can derive expressions similar to (4), (5) and (6) that give $[\Delta P_l \mid \Delta P_{la}]^t$ , [f] (or [q]) and $[\,\Delta L_s \mid \Delta L_{sa}\,]^t$ .

## 4. EXAMPLES

### 4.1. Gaz networks

In this section a small numerical example is presented to illustrate the kind of results obtained with the model described above. In fig. 3 one may find a six node/eight branch meshed network with one source node and one compressor station. Data for a steady state gaz flow simulation (Osiadacz 1988) are given in that figure and on the following tables 1 and 2.




N nodes : 6
N branches: 8

Length units: m
Press. " : bar
Flow " : $m^3/h$

Sp. Grav. of gaz : 0.0589
Comp. factor : 1
Cin. visc. : 0.135 $m^2/s$
Temp. : 288 K
Air press. : 1 bar
Pipe efic.: 1

N. of source node: 3
N. of compressor node: 5

Node 5 (comp. station) will be split into nodes 5 (creating branch 5-4) and 7 (creating branch 7-4).

Figure 3 - Scheme and data of the example

**BRANCH DATA**

| N. | Node ns. | diam | length |
|---|---|---|---|
| 1 | 3 - 2 | 0.1 | 500 |
| 2 | 6 - 3 | 0.15 | 1000 |
| 3 | 3 - 4 | 0.12 | 800 |
| 4 | 6 - 5 | 0.09 | 300 |
| 5 | 5 - 4 | 0.1 | 1000 |
| 6 | 1 - 4 | 0.1 | 500 |
| 7 | 2 - 4 | 0.15 | 800 |
| 8 | 2 - 1 | 0.09 | 300 |

**NODE DATA**

| N. | Type | Flow (load) | Pressure |
|---|---|---|---|
| 1 | LOAD | 5000 | - |
| 2 | LOAD | 3000 | - |
| 3 | SOURCE | - | 25 |
| 4 | LOAD | 50000 | - |
| 5 | COMPR. | - | 25 on 5-4 side |
| 6 | LOAD | 3000 | - |

Table 1 - Branch and node data

**FUZZY LOAD DATA**

| Node | a1 | a2 | a3 | a4 |
|---|---|---|---|---|
| 1 | 4500 | 4750 | 5250 | 5500 |
| 2 | 2700 | 2850 | 3150 | 3300 |
| 4 | 45000 | 47500 | 52500 | 55000 |
| 6 | 2700 | 2850 | 3150 | 3300 |

Table 2 - Fuzzy loads, assuming trapezoidal possibility distributions

**NODE PRESSURES**

| Node | Pressure |
|---|---|
| | 21.4 |
| 2 | 21.71 |
| 3 | 25 (sp.) |
| 4 | 21.41 |
| 5 | 22.54 |
| 6 | 24.32 |
| 7 | 25 (sp.) |

**BRANCH FLOWS**

| N. | Node ns. | flow |
|---|---|---|
| 1 | 3 - 2 | 19183 |
| 2 | 6 - 3 | -16775 |
| 3 | 3 - 4 | 25042 |
| 4 | 6 - 5 | 13775 |
| 5 | 5 - 4 | 13775 |
| 6 | 1 - 4 | -12 |
| 7 | 2 - 4 | 11195 |
| 8 | 2 - 1 | 4988 |

Table 3 - Branch and node results of steady state deterministic gaz flow simulation

A fuzzy load flow was also run according to the model described above. Loads were represented by trapezoidal possibility distributions, whose data are given in table 4. In table 5 one may find the results under the form of approximate trapezoidal distributions for node pressures and pipe flows.

**BRANCH FLOW POSSIBILITY DISTRIBUTIONS**

| Br. | q1 | q2 | q3 | q4 | q min | q max |
|---|---|---|---|---|---|---|
| 1 | 17265 | 18224 | 20142 | 21101 | 17265 | 21101 |
| 2 | -18452 | -17614 | -15936 | -15097 | -18452 | -15097 |
| 3 | 22538 | 23790 | 26294 | 27546 | 22538 | 27546 |
| 4 | 12397 | 13086 | 14464 | 15152 | 12397 | 15152 |
| 5 | 12395 | 13085 | 14465 | 15155 | 12397 | 15152 |
| 6 | -1019 | -515 | 491 | 995 | -930 | 922 |
| 7 | 9821 | 10508 | 11882 | 12569 | 9819 | 12569 |
| 8 | 4377 | 4682 | 5294 | 5600 | 4378 | 5599 |

Table 4 - Fuzzy gaz flow results compared with Monte Carlo results for maximum and minimum values of pressures and flows

In order to evaluate the margin of error of the linearized model, a Monte Carlo simulation was also run with a Newton type method (Osiadacz 1988) aiming at calculating the minimum and maximum possible values of pipe flows and node pressures, and compare them with the extreme values of the possibility distributions obtained. This exercise shows that:

a) The accuracy obtained for pipe flows is remarkable;
b) The extreme possible values of node pressures are slightly underestimated (the trapezoidal approximation is narrower than the true one);
c) With only one iteration of the fuzzy model a far more significative amount of information was obtained than with hundreds of steady state classical iterative deterministic algorithms.

These results deserve highlighting although one must realize that they were obtained for a small academic example. Our experience has shown that the conclusions remain valid for larger networks.

**4.2. Electric systems**

The methodology described above was used to study some scenarios for the expansion of the 60 kV electrical distribution system of Oporto, Portugal. The simplified network is shown in Fig. 2.

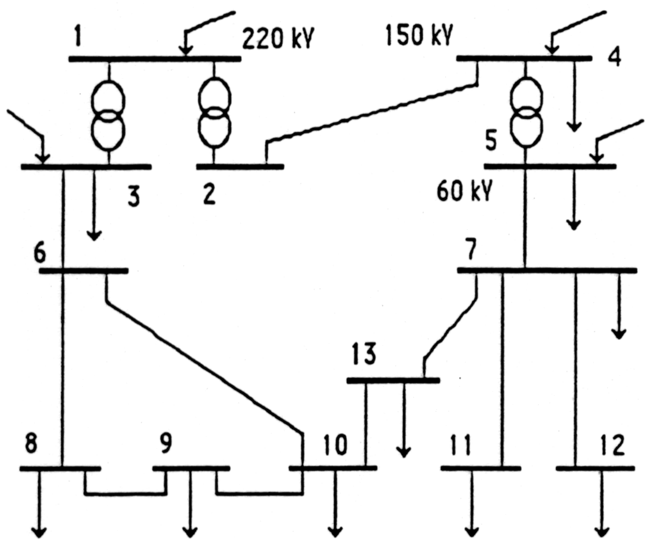



Figure 2 Oporto simplified 60 kV distribution network

The network data such as line and transformer characteristics, specified voltages in the Slack and PV buses, load possibility distributions in all buses, possibility distributions of generated active power in PV and PQ buses and generated reactive power in PQ buses (a 500 MVA power base was considered) can be found in reference (Miranda 1990). These distributions were assumed as trapezoidal fuzzy numbers.

The models included in that reference consist of a DC fuzzy load flow as described above, and also of an AC model that allows the calculation also of possibility distributions for node voltages and angles, losses, reactive power flows and line currents.

The results for the possibility distributions of active (and reactive) power flows are assumed as keeping a trapezoidal shape which stands as a fairly good approximation in the general case. The error in this approach has been estimated through Monte Carlo/fuzzy simulations and has been presented in (Saraiva 1991). As in the case of gaz flows, the trapezoidal distributions are good approximations of the actual possibility distributions.

One's attention may be drawn to line 10-13, where the possibility of power flow in either direction is clearly detected by the correspondent possibility distribution but not by the deterministic extreme case analysis (corresponding to the values obtained through two deterministic load flows, using the Newton-Raphson method, considering the maxima and minima load values and generated powers of each possibility distribution). Such a situation, which had also been detected in the gaz network simulation, in pipe 6, is exemplified in Fig. 4 corresponding to active power flow in line 10-13.

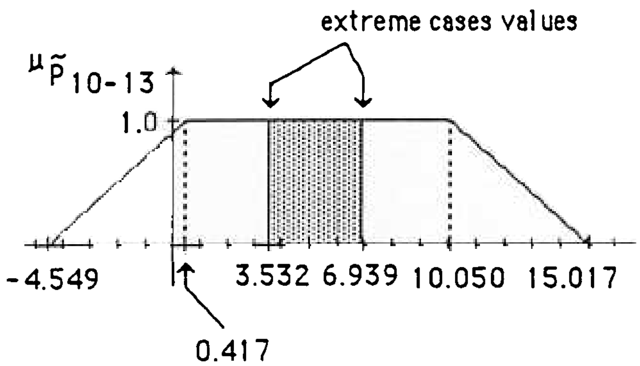



Figure 4 Possibility distribution of active power flow in line 10-13, and the extreme case analysis results.

From the analysis of these results and the results obtained by the authors for other case studies, one may in general conclude that:

- The possibility distributions calculated cover in general a wider range of values than the interval delimited by the values obtained with the two deterministic load flow studies using maxima and minima loads and generated powers.

- This methodology reveals the possibility of power flow occurring in either direction at some lines (due to different combinations of loads and generated powers), which could remain undetected if only some deterministic scenarios were studied.

## 4. CONCLUSIONS

Whenever uncertainty in data is recognized which has not a probabilistic nature, fuzzy set theory concepts can be used to incorporate qualitative aspects in the mathematical models of the systems under analysis. Such is the case of the steady state network flow problem, for which we have presented a way of approximately deducing node potentials and branch flows from load data affected by imprecision.

Furthermore, we have shown that the fuzzy flow formulation derived can easily be identified with linearizations of electrical or gaz systems, and by obvious reasons with other network systems as well.

The translation of qualitative linguistic declarations such as "more or less" or "almost", when qualifying consumption values, can be achieved by means of fuzzy numbers, and several techniques have been proposed in order to capture the meaning of such expressions. Their discussion is not the aim of this paper.

The fuzzy flow model presented has the potential to be included in Decision Support Systems for planning purposes, whenever a friendly linguistic interface can be built. Calculations are not heavy and the max-min convolution of possibility distributions has not the complexity of traditional convolution of, for instance, probability distributions.

Deterministic flow analysis, however useful may be, can result very incomplete in planning studies unless a very, very large number of scenarios is studied. Instead, a single fuzzy flow study produces a larger amount of significant information.

The adoption of a fuzzy approach such as the one derived in this paper has the advantage of explicitly recognizing incomplete knowledge on the problems dealt with. It seems a rather more honest approach than adopting simple deterministic values to represent data and results that, after all, are uncertain. We believe that it is always better to have results that express uncertainty than evaluating a situation with "exact"numbers that hide reality and may lead to less good decisions or, at least, less informed ones.

## APPENDIX

Fuzzy set theory can be understood as an extension of n-valued logic when the number of the admissible logic values tends to infinity. For example, given an universe X and a subset $X_1$ of X, the membership value of an element $x_1$ to $X_1$ belongs to:

- {0, 1} in the Boolean logic.
- {0, 0.5, 1} in the trivalued Lukasiewicz's logic.
- [0, 1] if normalized fuzzy sets are considered.

Therefore, a fuzzy set $\tilde{A}$ is characterized by a membership function $u_{\tilde{A}}(x)$ associating an element $x_1$ to its compatibility degree to $X_1$. In this sense, the transition between full and no membership is gradual rather than abrupt.

$$\tilde{A} = \{(x_1, u_{\tilde{A}}(x_1)), x1 \in X1 \} \quad \text{(A1)}$$

An $\alpha$-cut of a fuzzy set $\tilde{A}$ can be defined as the crisp set $A_\alpha$ of elements whose membership value to $\tilde{A}$ is not inferior to $\alpha$.

$$A_\alpha = \{x_1 \in X1 : u_{\tilde{A}}(x_1) \geq \alpha\} \quad \text{(A2)}$$

## REFERENCES


Freska, C., 1982, "Linguistic description of human judgements in expert systems and in the soft sciences", in Approximate reasoning in Decision Analysis - M.M.Gupta & E.Sanchez edt., North Holland.

Miranda,V., Matos, M., 1989, "A Holistic Approach in Multicriteria Decision Aid", in Improving Decision Making in Organizations, A. G. Lockett Edt., Lecture Notes in Economics and Mathematical Sciences, Springer Verlag, Berlin

Miranda,V., Matos, M., Saraiva, J. T., 1990, "Fuzzy Load Flow - New Algorithms Incorporating Uncertain Generation and Load Representation" in Proceedings of the 10th PSCC, Butterworths, London, 1990.

Miranda,V., 1991a, Using fuzzy reliability indices in a decision aid environment for establishing interconnection and switching location policies" in Proceedings of CIRED 91 (IEE/AIM), Liège, Belgium, 1991

Miranda,V., Saraiva, J. T., 1991b, "Fuzzy Modelling of Power System Optimal Load Flow", PICA 91, Baltimore, USA, May 1991; selected for publication in IEEE Transactions on Power Systems.

Osiadacz, A., Pienkosz, K., 1988, Methods of steady state simulation for gas networks - Int. J. Systems Sciences, vol 19-7,1311-1321.

Saraiva, J. T., Miranda,V., Matos, M., 1991, "Generation and load uncertainties incorporated in load flow studies, in Proceedings of MELECON 91 (IEEE-8), Ljubliana, Yugoslavia, 1991

Zadeh, L.A., 1965,"Fuzzy sets", Information and Control, 8 p. 338-353.

Zadeh,L., 1978, "Fuzzy sets a a basis for a theory of possibility", Fuzzy Sets and Systems 1, p. 3-28.

Zimmermann, H.J., 1985, Fuzzy Set Theory - and its applications, Int. Series Manag. Sc./Op. Res., Kluwer-Nijhoff Publishing, Boston