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**ACTIVE / REACTIVE DISPATCH IN MARKET ENVIRONMENT AND NODAL
ACTIVE / REACTIVE PRICE DETERMINATION**

Mário Helder Gomes

mgomes@ipt.pt

Departamento de Engenharia Electrotécnica
Escola Superior de Tecnologia de Tomar,
Quinta do Contador, Estrada da Serra,
2300 Tomar, Portugal

João Tomé Saraiva

jsaraiva@fe.up.pt

FEUP/DEEC – Fac. de Eng. da Univ. do Porto
INESC Porto – Instituto de Engenharia de Sistemas
e Computadores do Porto
Campus da FEUP, Rua Dr. Roberto Frias, 4200-
465 Porto, Portugal

SUMMARY

This paper describes a mathematical formulation for the active/reactive dispatch in electricity markets, addressing a number of technical and economic difficulties present in market models in force in several countries. Traditional market approaches are implemented considering a sequence of activities namely in terms of active power dispatch (via the pool centralized market and bilateral contracts) and ancillary services. However, these two problems are coupled in the sense that dispatching reactive power is not independent from active power scheduling of generators and so a particular reactive power output required by the ISO may be unfeasible in view of the active power dispatch. The proposed model admits that it is known the purely economic pool dispatch together with bilateral contracts and then it aims at dispatching reactive resources considering voltage and branch limit constraints and constraints reflecting the alternator capability curve. The search on the solution space uses an Sequential Linear Programming, SLP, approach and it is governed by the minimization of an objective function reflecting branch active losses, the cost of generator active power adjustment bids required to accommodate reactive power requirements or to eliminate branch congestion and also load active power adjustment bids. As a sub-result, this formulation also outputs nodal active and reactive marginal prices that can be useful to build tariff schemes. Finally, the paper includes a case study based on the IEEE 30 bus/41 branch system to illustrate the results obtained with this approach.

KEYWORDS

Electricity markets, System Operator, adjustment bids, alternator curve capability, integrated dispatch, active/reactive nodal marginal prices, SLP algorithm.

Contact author – J. Tomé Saraiva, fax +351.22.2094150, phone: +351.22.2094230, email: jsaraiva@fe.up.pt

1. Introduction

The implementation of market approaches in several countries and geographic areas has a number of common aspects as the unbundling of traditional vertically integrated utilities in a large number of players and agents dedicated to regulated activities, to market driven businesses and to technical coordination or regulatory activities. This restructuring typically lead to the adoption of competition in the extremes of the system, that is, in the generation and retailing activities implemented either in terms of bid-based forward markets, usually centralized in a day-ahead pool, or using bilateral contracts having different time horizons. The physical relation between generation and demand is ensure by network companies – both transmission and distribution wiring companies – that provide their services in a monopoly basis, just because it is unfeasible to duplicate networks in the same geographic area in order to introduce competition. The absence of competition in these areas and the imperfect competition in other ones, together with the trend to reduce the direct intervention of the governments and the greater accent on technical robust justifications, are the basic reasons explaining the advent of independent regulatory agencies.

In any case, several authors stress that electricity is not easily marketable since it is not a true commodity. In fact, electricity cannot be stored in large amounts (apart from hydro reservoirs) so that it cannot be sold when prices get higher. This means that it has to be generated at the same time it is consumed and that the market is physically established in transmission grids that impose technical constraints – Kirchoff Laws and limit constraints - to what market agents can eventually consider to be the most advantageous strategies. Apart from these problems, when talking about electricity markets one usually refers to some platform relating entities generating and consuming active power. However, ancillary services as reactive power and voltage control and several time span reserves are also crucial in order to maintain system security. The technical coordination of system operation is assigned to Independent System Operators, ISO, or Transmission System Operators, TSO, that receive information from the purely economic dispatch and technical information about bilateral contracts to evaluate the technical feasibility of this set of injections and to assign reactive generation and reserves.

In most cases, the active power dispatch – both by the bid-based pool and by bilateral contracts – and the ISO or TSO technical activities are performed in a temporal sequence of actions and studies. The adoption of a chronologic sequence of actions to perform the active dispatch, the reactive dispatch and the dispatch of reserves is a decoupled approach that is likely to cause several inefficiencies. In the first place, several power system components are constrained by technical limits depending both on active and reactive power. This is certainly the case of branch flow limits dependent on thermal limits associated to the current and to synchronous generators in which an operation point is characterized by an active / reactive pair of values. This means that bidding for active power and being dispatched by the Market Operator in the first place, immediately conditions the possible range that the reactive output can assume. This means that for security reasons, the System Operator can impose changes on the active dispatch of a generator just because the required reactive power would not be possible given the Market Operator dispatch. In other cases, weak transmission networks can contribute to give market power to agents connected to some particular locations. Just consider a generator connected to a node of a weakly meshed network. This generator has power over the market since it may know that it has to be used because it is required in terms of reactive support/voltage control. Therefore, it will eventually not bid on the active power market, waiting the System Operator to dispatch it due to the reactive support/voltage control requirement.

These aspects illustrate how active and reactive power are married so that attempting to decouple their dispatch may lead to technical difficulties of prevent the adoption of the most adequate economic decisions. Therefore, having this concern in mind, this paper presents a formulation to solve the active / reactive power dispatch in a more integrated way while adopting a market frameworks and attempting to bring more realism to the whole process.

The developed model includes two main steps:

- the first one aims at relating the demand and the generation side of power systems. This is currently being done using symmetrical pool mechanisms as detailed in section 3.2;

in a second moment, the ISO or the TSO interprets the pool hourly active power schedules, the injections from bilateral contracts and the involved nodes as a base dispatch that, however, is not the final one. In the first place, the base dispatch has to be completed by allocating reactive power to the generators and capacitors. Secondly, since the operation of several components is constrained jointly by active and reactive power, this pair of values can lead to infeasible operation conditions for branches, transformers or generators so that the active / reactive dispatch has to be changed. This is performed by solving an optimization problem aiming at minimizing the cost of network active losses plus the eventual reduction of the revenues of the generators that have to alter their dispatch in order to fulfill a reactive power/voltage support requirement. This problem includes:

- adjustment variables affecting the active power dispatch either due to the need to enforce operation or security constraints or to turn feasible some amount of reactive support together with the active power dispatch, given the generator capability curve;
- variables representing the reactive output of generators and capacitors.

As a result, this problem provides a technically feasible active/reactive dispatch based on the initial active power market dispatch, together with nodal marginal prices both for active and reactive powers that can help in building consistent tariff systems. This model corresponds to an enhancement of the formulation detailed in [1] in the sense that they are now included variables representing the amount of active power assigned to each generator in order to balance active power losses. This will also enable the ISO or the TSO to allocate this power in a more transparent and technically robust way.

The proposed paper is organized as follows. After this introductory section, Section 2 addresses some available models to tariff reactive power as well as attempts to implement markets for ancillary services. In Section 3 we will describe the proposed model together with the adopted solution algorithm. Section 4 presents a case study to illustrate the proposed approach and to highlight its advantages. Finally, Section 5 draws the most relevant conclusions.

2. Reactive Power Dispatch Models

As referred before, most publications and models on electricity markets address active power issues and pay little attention to reactive power / voltage support. This fact is mainly due to the difficulty in pricing reactive power and on the belief that its economic impact is significantly reduced when compared with active power. It is also widely recognized that reactive power allocation has a stronger technical accent and that it can be efficiently addressed after getting an initial active power schedule. The importance of reactive power started to rise as in several countries, namely in Europe, the number and the power of wind parks having asynchronous generators rapidly increased. These wind parks are typically connected to remote weak distribution networks and are supplied by reactive power in order to create the generators induction magnetic field. As a result, distribution companies typically imposed connection requirements. For instance, in Portugal it is limited the possibility of having a reactive power demand larger than a percentage of the active output in peak and full hours so that wind parks had to install capacitors to prevent being billed for their reactive demand. In general, it is now recognized that active and reactive dispatches are strongly married and that the active power schedule may have to be changed so that a generator can comply with a reactive power requirement.

Recognizing the importance of reactive power several publications describe models to dispatch and to value it. In the first place, in reference [2] NGC proposed a reactive power optimization model under which individual generation units would offer Mvar piece-wise cost curves so that it was solved an optimization problem to minimize the sum of such costs by adequately changing the output of synchronous generators, capacitors and transformer taps. Reference [3] also aims at solving an OPF problem able to minimize different objective functions and subjected to power flow equations and security constraints. In this case, the model includes several types of controls as transformer taps and capacitors and it is adopted a fuzzy set based model in order to adequately model the idea that some constraints or limits have a soft nature.

In reference [4] the authors address the characteristics of reactive power/voltage support in terms of its local nature and of the conflicting objectives to be balanced in order to establish a price for this ancillary service. The authors also refer that the importance of an adequate reactive power/voltage support is much larger than the price usually assigned to it. Regarding the valuation of this service, they propose three approaches. The first one corresponds to compute operation costs and determine the part to allocate to the reactive service. The second implies imposing performance requirements and connection standards to consumers and generators inside a control area so that, if they are met, there is no reactive power charge. Finally, the last approach is related with the creation of local reactive power markets for each control area. It would then be necessary to compute nodal reactive power adjustment factors corresponding to multipliers to adjust generation and demand and reflecting the reactive power locational value. In line with this third approach, reference [5] proposes a two-stage model to optimise active and reactive power scheduling. Once the final dispatch is obtained it is possible to compute active and reactive nodal prices reflecting the impact of changes of active power in the active generation cost in the first case and the impact of reactive power changes in the branch active losses.

Reference [6] discusses how a cost could be assigned to reactive power, namely related with explicit and opportunity generation costs and also explicit costs from several transmission sources. In this approach, a generator opportunity cost is related with the profit of selling an amount of active power that is in fact not achieved because there is a reactive power requirement. Based on these ideas, it is formulated a reactive power dispatch problem solved by a successive linear programming approach.

In reference [7] the authors recognize there are contradictory objectives to achieve when dispatching reactive power. Therefore, they aim at minimizing active power losses and maximizing the voltage stability margin, by changing operation and control variables and considering AC power flow equations. The solution of this problem is accomplished by adopting a successive linear programming approach. Each linearized problem is then solved by a Multi-objective Fuzzy Linear Programming Approach to deal with the two contradictory objectives.

Following the ideas applied by NGC after 1998, in reference [8] the authors present a linear programming based security constrained reactive optimal power flow model to allocate reactive support on a competitive basis. The objective function aims at minimizing total annual costs of reactive power capability (reflecting bidding prices for reactive power) and utilization costs (reflecting utilization costs related with Mvarh price curve). The problem includes the AC power flow equations, reactive control and voltage limits while enforcing a number of contingency constraints.

References [9], [10] and [11] propose a structure for the reactive power bids based on the synchronous generator capability curve. According to this diagram, there are operation regions in which the generator has no need to alter its active output to meet a reactive requirement, while there are areas in which meeting a reactive requirement implies a change on the active output. In this case, the revenue of the generator on the active power market gets reduced and so it must be compensated for this opportunity cost. Using this idea, the authors formulate an optimisation problem to minimize active losses subjected to power flow constraints and to constraints related with the referred capability curve.

In references [12] and [13] the authors identify the costs of reactive power providers and discuss methods to allocate them to grid users. In the first of them, these costs are organized in explicit ones (fixed costs, and variable maintenance and operation costs) and implicit ones (losses or profits due to reactive support) and the payments should be structured in a capacity term to remunerate fixed costs and an usage fee reflecting variable costs. In the second one, the authors consider that the dominant component on the cost structure of the reactive power ancillary service is related with the opportunity costs corresponding to the lost revenues of a generator in selling active power because of a reactive power/voltage support requirement. With a different point of view, in reference [14] the authors consider that minimizing branch active losses within the reactive power-scheduling problem may lead to a excessive number of operations of control devices turning the implementation of these strategies difficult in real time. Therefore, they propose modelling the cost of adjusting these devices namely transformer taps and capacitor banks. This leads to a new objective function that is constrained by the AC power flow equations, voltage and generator reactive power limits and available taps and capacitor outputs. A genetic algorithm solves the resulting mixed integer non-linear problem.

Finally, recognizing the coupling between active and reactive power and the difficulty in dealing with system constraints, reference [15] details the activities of the Spanish TSO to validate the dispatch from the Market Operator while ensuring branch and voltage limits. The authors propose getting a final solution in two stages. In the first one, they aim at obtaining a feasible solution re-dispatching generators and minimizing the total system cost. Once this is guaranteed, voltage control resources are fine tuned for each hour of the next day by running an LP based OPF minimizing transmission losses.

3. Proposed Approach

3.1. General aspects

In this section we describe an approach to allocate reactive power in coupled way with active power while retaining competitive market aspects. The first results of this research are reported in [1], and the original model is now enhanced by including variables representing the power allocated to each generator to balance active losses. This approach is based in two steps. In the first one, it is run a bid based uniform price auction to obtain an active power schedule together with the technical information available from bilateral contracts. Considering this set of injections as a base operation point of the system, the ISO or the TSO will then schedule reactive power while enforcing technical constraints related namely with branch flow limits, voltage limits and with the capability curve of synchronous generators. This scheduled is obtained by solving an optimization problem aiming at minimizing the cost of active losses plus adjustment costs that may be required to allow a generator to accomplish a reactive power output or to curtail load to enforce voltage magnitude and branch limit constraints.

3.2. Symmetrical Pool Model

As referred before, the first step corresponds to run a bid based uniform price auction. This auction is performed for each hour of the next day and, if considering Simple Bids, there are no inter-temporal constraints leading to 24 hourly independent auctions. In this simple version, this auction is based in buying and selling bids each one represented by pairs power/price. The result of the auction can be obtained by solving the problem (1) to (4) admitting there are N_d buying and N_g selling players. In this formulation, x_{si} , c_{si} and s_i are the amount sold by player i , the selling price proposed by player i and the maximum amount player i can provide, and x_{bj} , c_{bj} and b_j are the amount bought by player j , the price offered by player j and the maximum amount player j admits to buy. This simple bid approach can be turned more realistic structuring generator bids in a set of blocks, that is, in a set of pairs power/price as a way to better follow the generator cost curves. A second change is related with the integration of coupling constraints related, for instance, with ramps. These constraints imply that the 24 hourly auctions are coupled so that the problem has to be solved in an integrated way.

$$\max z = \sum_{j=1}^{N_d} c_{bj} \cdot x_{bj} - \sum_{i=1}^{N_g} c_{si} \cdot x_{si} \quad (1)$$

$$\text{subj } x_{bj} \leq b_j \quad (2)$$

$$x_{si} \leq s_i \quad (3)$$

$$\sum_{j=1}^{N_d} x_{bj} = \sum_{i=1}^{N_g} x_{si} \quad (4)$$

3.3. Alternator Capability Curve

The ISO or the TSO has to evaluate the technical feasibility of the pool schedule together with the injections from bilateral contracts. One of the technical aspects to be evaluated is related with the capability curve in Figure 1 characterizing the operation of synchronous generators. This diagram results from several curves representing different operation aspects. In the first place, Curve 1 between $Q_{g_i}^{\max}$ and \underline{s}_1 represents the rotor field current limit. Secondly, Curve 2 between \underline{s}_1 and \underline{s}_2 is the armature limit and finally Curve 3, the arc between $Q_{g_i}^{\min}$ and \underline{s}_2 , represents the stability limit.

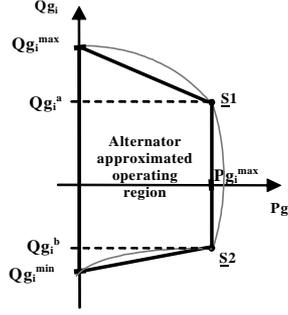


Fig. 1. Alternator capability curve.

3.4. Generator and Load Adjustment Bids

According to Figure 1, the operation of an alternator is characterized by a point in the PQ plan so that once it is scheduled in the active power market, its possible reactive output depends on the feasibility region delimited by its capability curve. Therefore, a generator may have to reduce its active output due to a particular reactive power requirement, for instance to enforce the voltage magnitude in a particular area. This reduction has to be balanced by other generators or by load shedding.

In terms of the expected revenues this can lead to an opportunity cost. This generator expected to obtain a revenue in the active power market but, due to a reactive power requirement, this revenue gets reduced. This is the basis to design a reactive power market in which generators communicate not only pairs of active power/prices but also adjustment bids to use if necessary. The ISO or TSO can use these bids not only to allocate reactive support but also to enforce system constraints if there is any violation. In this sense, they can be interpreted as resources to turn the operation of the system feasible. They integrate the acceptable maximum variation, v_g^{tol} , that the initial schedule of a generator can suffer together with an adjustment price, C_g^{adj} . If a generator was not dispatched in the market, its maximum possible adjustment can correspond to a percentage on its installed capacity.

3.5. Mathematical Model

The optimization problem to run to allocate reactive support and to make adjustments in the initial active schedule, if required, is formulated by (5-18), admitting a system with N_g generators, N_d loads, and N_b branches. The objective function (5) aims at minimizing the sum of three terms. The first term is cost of active losses represented by the contribution of each generator to balance losses, $\Delta P_{g_i}^{\text{loss}}$, admitting this power is valued at the marginal price obtained in the initial auction, ρ^{MO} . The second and the third terms correspond to the adjustment costs due to variations of the initially scheduled generation and load and are expressed as products of adjusted quantities by adjustment prices.

$$\text{Min } Z = \sum_{i=1}^{N_g} (\Delta P_{g_i}^{\text{loss}} \times \rho^{\text{MO}}) + \sum_{i=1}^{N_g} |\Delta P_{g_i}^{\text{adj}}| \times C_{g_i}^{\text{adj}} + \sum_{j=1}^{N_d} |\Delta P_d^{\text{adj}}| \times C_d^{\text{adj}} \quad (5)$$

$$\text{subj } \Delta V_i^{\text{min}} \leq \Delta V_i \leq \Delta V_i^{\text{max}} \quad (6)$$

$$\Delta \theta_{ij}^{\text{min}} \leq \Delta \theta_{ij} \leq \Delta \theta_{ij}^{\text{max}} \quad (7)$$

$$0 \leq \Delta P_{g_i}^{\text{loss}} \leq \Delta P_{g_i}^{\text{max}} \quad (8)$$

$$-\frac{v_i^{\text{tol}}}{100} \times P_{g_i}^{\text{MO}} \leq \Delta P_{g_i}^{\text{adj}} \leq \frac{v_i^{\text{tol}}}{100} \times P_{g_i}^{\text{MO}} \quad (9)$$

$$0 \leq \Delta P_{g_i}^{\text{adj}} \leq \frac{v_i^{\text{tol}}}{100} \times P_{g_i}^{\text{max}} \quad (10)$$

$$\Delta P_{g_i}^{\text{min}} \leq \Delta P_{g_i}^{\text{adj}} + \Delta P_{g_i}^{\text{loss}} \leq \Delta P_{g_i}^{\text{max}} \quad (11)$$

$$-P_d^{\text{MO}} \leq \Delta P_d^{\text{adj}} \leq 0 \quad (12)$$

$$Q_{g_i} \leq Q_{g_i}^{\max} - \frac{Q_{g_i}^{\max} - Q_{g_i}^a}{P_{g_i}^{\max}} (P_{g_i}^{\text{MO}} + \Delta P_{g_i}^{\text{adj}} + \Delta P_{g_i}^{\text{loss}}) \quad (13)$$

$$Q_{g_i} \geq Q_{g_i}^{\min} + \frac{Q_{g_i}^b - Q_{g_i}^{\min}}{P_{g_i}^{\max}} (P_{g_i}^{\text{MO}} + \Delta P_{g_i}^{\text{adj}} + \Delta P_{g_i}^{\text{loss}}) \quad (14)$$

$$\Delta S_{ij}^{\min} \leq \Delta S_{ij}(\Delta V, \Delta \theta) \leq \Delta S_{ij}^{\max} \quad (15)$$

$$\sum_{k=1}^{N_b} \Delta P_{ij}^{\text{loss}}(\Delta V, \Delta \theta) = \sum_{i=1}^{N_g} \Delta P_{g_i}^{\text{loss}} \quad (16)$$

$$\Delta P_i^{\text{inj}}(\Delta V_i, \Delta \theta_i) = (\Delta P_{g_i}^{\text{adj}} + \Delta P_{g_i}^{\text{loss}}) - \Delta P d_i^{\text{adj}} \quad (17)$$

$$Q_i^{\text{inj}}(V_i, \theta_i) = Q_{g_i} - Q d_i \quad (18)$$

The above objective function is constrained by voltage limits (6) and phase differences (7). The active output of a generator i can change regarding the initial one due to a contribution to balance losses, $\Delta P_{g_i}^{\text{loss}}$, or to an adjustment required to enforce a system constraint, $\Delta P_{g_i}^{\text{adj}}$. In this sense, constraint (8) imposes a limit to the contribution of each generator to balance losses, constraints (9) and (10) impose limits to the technical or operational adjustment for generators scheduled or not yet scheduled and (11) imposes limits for the addition of these two types of generation changes. Constraint (12) represents the possible load shedding and constraints (13) and (14) represent the upper and lower curves of the capability diagram of each generator, that is, curves 1 and 3 referred in 3.3. Constraint (15) represents the minimum and maximum limits of the apparent power in branch i - j . Constraint (16) represents the balance between total system losses and sum of losses allocated to generators and constraints (17) and (18) correspond to the active and reactive power injection equations.

3.6. Solution Algorithm

In line with the ideas in [16], a non-linear optimization problem can be addressed by solving in a successive way linearized problems. In each iteration of the Successive Linear Programming approach, SLP, all non-linear expressions are linearized and the variables of the problem are constrained to a specified range around the linearization point so that the approximation remains valid.

In order to apply this approach to the active / reactive dispatch problem (5-18), we considered that the Market Operator runs a uniform price bid-based auction (1-4). Then, the ISO or the TSO runs an SLP problem to obtain the active / reactive dispatch, guaranteeing its technical feasibility, and the active/reactive nodal marginal prices. The adopted solution algorithm is organized as follows:

- once the uniform price bid-based auction is completed, it is run an AC power flow to characterize the current operation point;
- using this operation point as a linearization point, branch flow apparent power and active losses expressions are linearized. This means linearizing constraints (15) to (18) in Section 3.5;
- as this linearization is complete, the linearized problem is fully built and it can now be solved;
- afterwards it is evaluated the convergence of the iterative process by computing the changes of voltages, phases and of the objective function in successive iterations. When these differences are smaller than specified tolerances, we get the solution for the non-linear problem;
- if convergence was not yet obtained, we update generations, loads, voltages and phases considering the adjustment values obtained from the linearized problem. This means we can run a new AC power flow to fully characterize the new operation point.

4. Case Study

4.1. Network Data

The formulation described in Section 3 was tested using the IEEE 30 bus/41 branch Test System sketched in Figure 2. In the simulations we used 0.94 and 1.07 pu for the voltage limits, node 1 as phase reference and 100 MVA as power base. Table I includes the generator bidding blocks and the points of the approximated generator capability curves, Table II details the generator adjustment bids and Table

III includes load data. In nodes 5, 8, 11 and 13 there are synchronous compensators varying from -40 to 40 Mvar in node 5, -10 to 40 Mvar in node 8 and -6 to 24 Mvar in nodes 11 and 13.

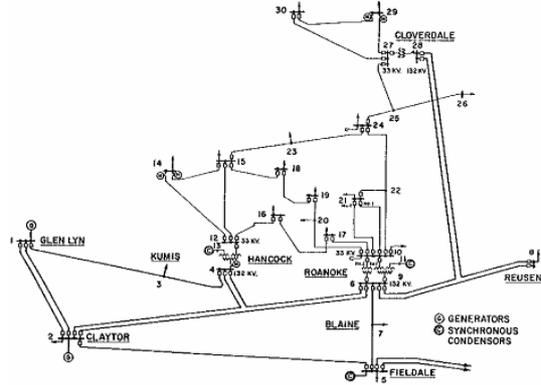


Fig. 2. IEEE 30 bus/41 branch test system.

TABLE I - STRUCTURE OF THE GENERATOR SELLING BIDS (PAIRS QUANTITY/PRICE) AND POINTS OF THE CAPABILITY CURVE.

bus	$P_{gi}^{bid 1}$ (MW)	C_{gi}^1 (€/MWh)	$P_{gi}^{bid 2}$ (MW)	C_{gi}^2 (€/MWh)	P_{gi}^{max} (MW)	Q_{gi}^{max} (Mvar)	Q_{gi}^a (Mvar)	Q_{gi}^b (Mvar)	Q_{gi}^{min} (Mvar)
1	260.2	22.5	300	32.8	300	85	45	-40	-80
2	40	26	70	35	70	50	50	-40	-40

TABLE II - GENERATOR ADJUSTMENT BIDS (MAXIMUM ADJUSTMENT IN % AND ADJUSTMENT COST)

bus	v_{gi}^{tol} (%)	C_{gi}^{adj} (€/MWh)	bus	v_{gi}^{tol} (%)	C_{gi}^{adj} (€/MWh)
1	50	45	2	52	48

TABLE III - DEMAND BIDS (ACTIVE AND REACTIVE POWER, BID PRICE FOR THE MARKET OPERATOR AND ADJUSTMENT PRICE)

bus	P_{dj}^{bid}	Q_{dj}	C_{dj}	C_{dj}^{adj}	bus	P_{dj}^{bid}	Q_{dj}	C_{dj}	C_{dj}^{adj}
	(MW)	(Mvar)	(€/MWh)	(€/MWh)		(MW)	(Mvar)	(€/MWh)	(€/MWh)
2	21.7	12.88	30	70	17	9	5.81	29	77
3	2.4	1.23	28	75	18	3.2	0.93	31	79
4	7.6	1.54	27	80	19	9.5	3.45	32	74
5	94.2	19.13	31	73	20	2.2	0.72	34	72
7	22.8	11.04	29	74	21	17.5	11.30	28.1	81
8	30	29.76	27	69	23	3.2	1.64	30	83
10	5.8	1.91	26.5	80	24	8.7	6.75	33	74
12	11.2	7.53	27.3	75	26	3.5	2.26	27	76
14	6.2	1.55	28.2	82	29	2.4	0.87	29	77
15	8.2	2.39	27	86	30	10.6	1.51	28	80
16	3.5	1.79	26.4	75					

4.2. Market Operator Dispatch

In the first place, the Market Operator ran the auction according to the model (1-4). Table V details the results in terms of accepted generation and demand bids. The market clearing price was 26.0 €/MWh.

TABLE IV – RESULTS OF THE MARKET OPERATOR UNIFORM PRICE BID BASE AUCTION.

bus	P_{gi}^{MO} (MW)	P_{di}^{MO} (MW)									
1	260.2	-	8	-	30.0	17	-	9.0	23	-	3.2
2	23.2	21.7	10	-	5.8	18	-	3.2	24	-	8.7
3	-	2.4	12	-	11.2	19	-	9.5	26	-	3.5
4	-	7.6	14	-	6.2	20	-	2.2	29	-	2.4
5	-	94.2	15	-	8.2	21	-	17.5	30	-	10.6
7	-	22.8	17	-	9.0						

4.3. Case 1 results

In the first simulation, there were no congested branches, no load shedding and no changes in the active power initially scheduled to the generators 1 and 2 required by some particular reactive outputs. Table V details voltages and phases as well as the final active and reactive generations and loads.

TABLE V – CASE 1 – FINAL DISPATCH RESULTS INCLUDING GENERATION ADJUSTMENT TO BALANCE BRANCH LOSSES.

bus	V_i	θ_i	$\Delta P_{g_i}^{\text{loss}}$	$P_{g_i}^{\text{Final}}$	$Q_{g_i}^{\text{Final}}$	$P_{d_i}^{\text{Final}}$	$Q_{d_i}^{\text{Final}}$	bus	V_i	θ_i	$P_{d_i}^{\text{Final}}$	$Q_{d_i}^{\text{Final}}$
	(pu)	(deg)	(MW)	(MW)	(Mvar)	(MW)	(Mvar)		(pu)	(deg)	(MW)	(Mvar)
1	1.07	0.00	0.000	260.201	-17.625	-	-	16	1.00	-15.60	3.500	1.793
2	1.06	-5.20	16.817	40.020	50.000	21.700	12.876	17	1.00	-15.87	9.000	5.813
3	1.04	-7.33	-	-	-	2.400	1.230	18	0.99	-16.67	3.200	0.933
4	1.03	-9.03	-	-	-	7.600	1.543	19	0.98	-16.82	9.500	3.448
5	1.02	-13.79	-	-	38.443	94.200	19.128	20	0.99	-16.59	2.200	0.723
6	1.02	-10.73	-	-	-	-	-	21	0.99	-16.11	17.500	11.304
7	1.01	-12.50	-	-	-	22.800	11.043	22	0.99	-16.09	-	-
8	1.02	-11.47	-	-	40.000	30.000	29.755	23	0.98	-16.37	3.200	1.639
9	1.02	-13.91	-	-	-	-	-	24	0.97	-16.40	8.700	6.752
10	1.00	-15.64	-	-	-	5.800	1.906	25	0.98	-16.07	-	-
11	1.07	-13.90	-	-	24.000	-	-	26	0.96	-16.53	3.500	2.261
12	1.02	-15.11	-	-	-	11.200	7.526	27	0.99	-15.57	-	-
13	1.05	-15.10	-	-	24.000	-	-	28	1.02	-11.34	-	-
14	1.00	-16.04	-	-	-	6.200	1.554	29	0.97	-16.91	2.400	0.871
15	1.00	-16.07	-	-	-	8.200	2.392	30	0.96	-17.87	10.600	1.510

According to the results in Table V, the change in the active generation in node 2 is required because it is necessary to balance branch active losses. These are fully compensated in node 2 because this is the most advantageous place from the point of view of minimizing losses. Just recall that, according to the objective function (5), branch active losses are valued at the uniform price coming from the Market Operator run and they do not depend on the adjustment prices offered by each generator.

4.4. Case 2 results

In this second simulation, the thermal limit of branch 1-2 was reduced from 200 MVA to 150 MVA leading to a congestion in this branch once one considers the initial Market Operator dispatch in Table IV. As a result, the outputs of the generators in nodes 1 and 2 are adjusted not only to balance active losses (fully allocated to node 2, as in Case 1) but also to eliminate the congestion in branch 1-2. According to the results in Table VI, this is handled reducing the output of generator 1 by 27.329 MW, increasing the generation in node 2 by 12.064 and shedding the load in node 2 by 15.265 MW. The final generation in node 2 comes therefore from the initial one assigned by the Market Operator (23.2 MW), increased by 12.064 MW from the adjustment bid and by 16.197 MW to balance active losses, thus leading to 51.461 MW.

TABLE VI – FINAL DISPATCH INCLUDING GENERATION AND DEMAND ADJUSTMENTS TO SOLVE CONGESTION AND TO BALANCE LOSSES.

bus	V_i	θ_i	$\Delta P_{g_i}^{\text{adj}}$	$\Delta P_{g_i}^{\text{loss}}$	$P_{g_i}^{\text{Final}}$	$Q_{g_i}^{\text{Final}}$	$\Delta P_{d_i}^{\text{adj}}$	$P_{d_i}^{\text{Final}}$	$Q_{d_i}^{\text{Final}}$	bus	V_i	θ_i	$P_{d_i}^{\text{Final}}$	$Q_{d_i}^{\text{Final}}$
	(pu)	(deg)	(MW)	(MW)	(MW)	(Mvar)	(MW)	(MW)	(Mvar)		(pu)	(deg)	(MW)	(Mvar)
1	1.07	0.00	-27.329	0	232.854	13.976	-	-	-	16	0.99	-15.37	3.50	1.793
2	1.04	-4.37	12.064	16.197	51.461	12.821	-15.265	6.418	3.808	17	0.98	-15.64	9.00	5.813
3	1.02	-6.97	-	-	-	-	0	2.40	1.230	18	0.97	-16.48	3.20	0.933
4	1.01	-8.59	-	-	-	-	0	7.60	1.543	19	0.96	-16.64	9.50	3.448
5	1.00	-13.31	-	-	-	33.844	0	94.20	19.128	20	0.97	-16.39	2.20	0.723
6	1.00	-10.28	-	-	-	-	-	-	-	21	0.97	-15.88	17.50	11.304
7	0.99	-12.06	-	-	-	-	0	22.80	11.043	22	0.97	-15.86	-	-
8	1.00	-11.05	-	-	-	40.000	0	30.00	29.755	23	0.96	-16.17	3.20	1.639
9	1.00	-13.60	-	-	-	-	-	-	-	24	0.96	-16.19	8.70	6.752
10	0.98	-15.40	-	-	-	-	0	5.80	1.906	25	0.96	-15.84	-	-
11	1.05	-13.60	-	-	-	24.000	0	-	-	26	0.94	-16.32	3.50	2.261
12	1.00	-14.87	-	-	-	-	0	11.20	7.526	27	0.97	-15.32	-	-
13	1.03	-14.87	-	-	-	23.642	0	-	-	28	1.00	-10.92	-	-
14	0.98	-15.83	-	-	-	-	0	6.20	1.554	29	0.95	-16.71	2.40	0.871
15	0.98	-15.87	-	-	-	-	0	8.20	2.392	30	0.94	-17.71	10.60	1.510

4.5. Active and Reactive Nodal Marginal Prices

As a sub result of the optimization problem detailed in Section 3, we can also obtain the active and reactive nodal marginal prices, as presented in Figure 3 for the two simulations just described. In Simulation 1, both active and reactive nodal prices are quite flat reflecting the absence of congested branches. The active price in node 1 directly reflects the adjustment cost of generator 1 and then tends to

increase reaching the maximum value of 49.559 €/MW.h in node 30, thus reflecting the increasing impact of the cost of losses in buses further way from node 1.

In Simulation 2, the congestion in branch 1-2 originates large oscillations both in active and reactive prices. The negative active price in node 1 is due to the fact that a load increase in this node is directly compensated by generator 1, thus reducing its negative adjustment. This means that the value of the objective function gets reduced by 45 €/MW.h, that is, its variation is negative. If the load increases in node 2, it happens that generator 2 has already completely exhausted its maximum adjustment of 52 % of the initially allocated power of 23.2 MW. This means that if the load in node 2 increases, it will be curtailed and the cost of load shedding in node 2 in 70.0 €/MW.h. Regarding the active price in node 3, increasing the load in node 3 contributes to reduce the flow in branch 1-2 and this makes room to reduce the load shedding in node 2. The price of 19,875 €/MW.h in node 3 results from a combination of -70.0 €/MW.h (coming from reducing the load shedding in node 2) and 45.0 €/MW.h (coming the adjustment of the generator in node 1). For the remaining nodes, the active prices follow in general the ones from Simulation 1, except for nodes 5 and 7, although they are a bit more reduced. This small reduction is explained by a similar reason to one presented for the behaviour of the price in node 3, although increasing the load in nodes 4 to 30 (except nodes 5 and 7) does not relieve the congestion in branch 1-2 as increasing the load in node 3.

Regarding the reactive power, the prices in Simulation 2 are in general higher than in Simulation 1 with a spike in node 3. Increasing the reactive load in node 3 the apparent power flow in line 1-2 is increased and so the load shedding in node 2 gets larger thus leading to a larger reactive price. In the remaining nodes, except in nodes 5 and 13, this effect is also present but it is not so strong as for node 3. In nodes 5 and 13 the synchronous compensators in these nodes are not in the limit. Therefore, increasing the reactive load in these nodes is directly supplied by these devices thus having no impact in the objective function, namely in branch losses.

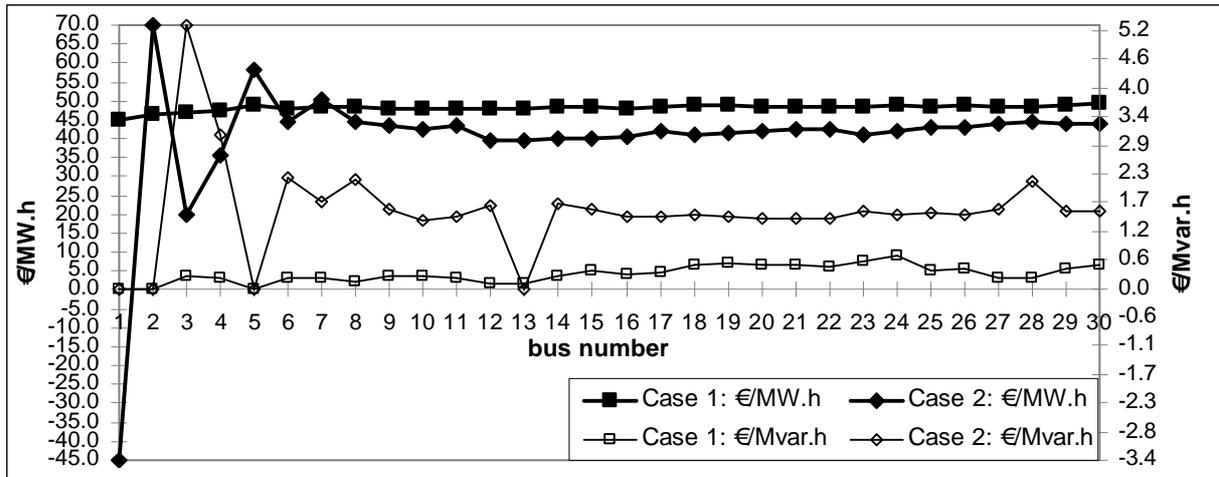


Fig. 3. Profiles of the active and reactive Nodal Marginal Prices.

5. Conclusions

In this paper we reported a second set of results from a model enhancing a previous one detailed in [1]. This model has the crucial advantage of bringing realism to the active power/reactive power scheduling under competitive electricity markets since it explicitly recognizes that active and reactive power are married either in terms of the synchronous generator capability curves, the apparent power used to formulate branch flow limit constraints and also by explicitly including active and reactive power AC power flow equations. This formulation also allows us to adequately evaluate generator opportunity costs coming from changes in the active power schedule required to a reactive output. Together with the minimization of active losses, this brings the possibility of computing active and reactive nodal prices reflecting the impact in the objective function from changes in the active and reactive demand. The final operation point is feasible since several constraints are enforced, leading to a clarification of the role of the ISO or TSO while retaining competitive aspects in the formulation since it is based on adjustment

bids. Finally, the model allows one to assign to each generator the active power required to compensate branch losses. This can be the basis of a more transparent and technically robust approach to allocate and to remunerate this service. As a whole, this model remarries the active power centralized pool market with the reactive power scheduling, thus formulating in a novel and more integrated way the ancillary services allocation specifically in what concerns reactive power/voltage control support.

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BIOGRAPHIES

Mário Helder Rodrigues Gomes was born in Mozambique in 1972. He received his licentiate and MSc degrees from Faculdade de Engenharia da Universidade do Porto (FEUP), Portugal, in 1997 and 2000 in Electrical and Computer Engineering. In 1997 he joined the Polytechnic Institute of Tomar. He currently has a grant from the PRODED government financed program to complete his PhD at FEUP.

João Tomé Saraiva was born in Porto, Portugal in 1962. In 1987, 1993 and 2002 he got his MSc, PhD, and Agregado degrees in Electrical and Computer Engineering from the Faculdade de Engenharia da Universidade do Porto, Portugal, where he is currently Professor. In 1985 he joined INESC Porto where he is senior researcher and collaborated in several EU financed projects, in projects related with the development of DMS systems, quality in power systems, and tariffs due for the use of transmission and distribution networks. Several of these projects were developed under consultancy contracts with the Portuguese Electricity Regulatory Agency.