ACTIVE/REACTIVE MARKET MODEL BASED ON ADJUSTMENT BIDS AND INTEGRATING CAPACITOR BANKS AND TRANSFORMER TAPS

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Abstract – This paper describes a mathematical model and the developed solution algorithm to solve an integrated active / reactive dispatch while retaining competitive aspects. The main drive for this research was the recognition that the introduction of competitive mechanisms lead to a certain extent to a decoupling between active and reactive power scheduling. Aiming at remarrying them, the developed approach includes an initial bid based uniform price active power auction run by the Market Operator, followed by a technical validity analysis run by the System Operator. If necessary, the System Operator uses adjustment bids to recover the technical feasibility of the dispatch. These bids are presented both by the demand and by generators meaning that demand can play an important role in increasing the liquidity of this specific market. This approach also includes capacitor banks and transformer taps leading to a combinatorial problem solved using Simulated Annealing. Finally, the paper includes results from a case study based on the IEEE 24 Bus Test System to illustrate the interest of this type of approaches.

Keywords – active/reactive dispatch, adjustment bids, mixed integer optimization problem, metaheuristics, active/reactive nodal prices.

1. INTRODUCTION

The implementation of market mechanisms in several countries lead to the development of pool models and bilateral contracts together with a set of technical actions assigned to Independent System Operators or Transmission System Operators, ISO/TSO's. This typically lead to a set of mechanisms basically related to active power while reactive power/voltage control and reserves are considered as ancillary services scheduled in subsequent phases. However, active and reactive powers are inherently coupled. This paper describes an active/reactive adjustment market model aiming at remarrying active and reactive powers by including the AC power flow equations, apparent power branch flow limits and the capability diagram of synchronous generators.

The developed approach includes two main steps as follows. In the first one, it is run an uniform price auction simulating a symmetric pool market based on generator and demand bids. Apart from that, we also consider bilateral contracts agreed between generators and eligible consumers. In a second step, the technical information related with the pool dispatch and with the bilateral contracts is conveyed to the ISO/TSO for technical analysis. This information is assumed by the ISO/TSO as a base dispatch that will have to be changed if there are violations of system constraints. This base dispatch has to be completed by allocating reactive power and, since the operation of several components is constrained jointly by active and reactive power, this pair of values can lead to infeasible operation conditions for branches, transformers or generators. In order to identify these changes, the ISO/TSO runs an optimization problem based on adjustment bids communicated both by generators and loads. Regarding the generators, the adjustment bids include the maximum percentage a generator can accept to change its initial schedule and the corresponding price. Regarding the demand, the adjustment bids include the maximum amount a load can accept to reduce its initial schedule together with a price. This is in line with the interruptible contracts already existing in several countries and contributes to turn the operation of the system more flexible while turning this market more liquid. Using these adjustment bids, the ISO/TSO solves an optimization problem aiming at minimizing the cost of network active losses plus the generator and demand adjustment costs related with the changes on the initially scheduled amounts to enforce reactive power/voltage support requirements and other system constraints. Therefore, this model includes:

- demand adjustment variables representing possible reductions of the load;
- generator active power adjustment variables decoupled in two terms. The first one represents the contribution of each generator to balance active losses and the second models the change on the initial schedule to enforce operation or security constraints or to allow a reactive support, given the generator capability curve.

This corresponds to a non linear optimization problem since it includes the AC power flow equations and branch apparent power limit constraints. To solve it, we used Sequential Linear Programming, SLP, implemented using the *lingprog* function of MATLAB. As a result, we obtain the final active and reactive generation, voltages and flows. To increase the realism of this model, we included binary variables to represent transformer taps and capacitor and inductance banks. The resulting mixed integer optimization problem was solved using Simulated Annealing, a well known metaheuristic that already gave good results when solving several combinatorial problems.

After this introduction, Section 2 details reactive power allocation approaches described in the literature, Section 3 details the mathematical formulation, Section 4 explains the adopted solution algorithm, and Section 5 includes results from a Case Studies based on the IEEE 24 Bus Test System. Finally, Section 6 draws the most relevant conclusions.

2. REACTIVE POWER ALLOCATION METHODS

The initial dispatch models corresponded to active power formulations to schedule generators according to their active power generation cost. It was only afterwards that dispatch models integrated information about network constraints and

then the complete AC power flow model. From a conceptual point of view, this move represented a trend to build more complete and realistic models. However, reactive power has always been much more complex to value because of its nature and also because active power costs are directly related with fuel prices. In any case, in recent years reactive power started to deserve closer attention recognizing its importance for power system security and voltage quality. In line with this interest, in the next paragraphs we provide an overview about reactive power cost allocation procedures described in the literature. In this scope, reference [1] describes an OPF model proposed in the scope of the implementation of market procedures in the UK. It is based on a reactive power cost curve assuming that a component of its payment was based on metered values and it includes the AC power flow equations, operation limits and voltage constraints and several reactive power controls as taps and capacitors.

One of the concerns of power system operators is related with the reduction of the number of changes of control devices or with the minimization of its cost. Reference [2] incorporates the cost incurred from adjusting control devices as it is argued that pure loss minimization strategies frequently lead to a large number of control changes. Similarly, [3] describes an OPF model that admits that the limits of some constraints have a soft nature in the sense that some degree of violation is allowed. This soft nature is modeled using Fuzzy Set concepts and the objective function of the problem also includes a term that penalizes strategies having an excessive number of control actions.

Several other publications as [4 - 9] describe opportunity costs as the costs incurred if a particular reactive power requirement imposes a change on a pre-scheduled amount of active power. This reduction in the active power revenue is then used in [4, 5] to price reactive power and in [6] to build reactive power cost curves. These curves are then used in an optimisation problem that includes the AC power flow equations and several operation limit constraints. In [7] the authors propose a reactive power bid structure considering the alternator capability curve and opportunity costs. This approach is further developed in [8, 9] with a formulation that takes into account the local nature of reactive power.

Reference [10] describes the features of reactive power to consider when designing a market. Among other aspects, the authors propose the creation of several reactive power markets due to its local nature leading to the concept of zonal reactive power marginal prices. These prices are also used in [11] and are obtained by solving an OPF problem subjected to the AC power equations and several operation constraints. Reference [12] proposes the use of nodal marginal reactive prices. The authors argue that these prices transmit more effective economic signals to network users and that they can be the basis to establish a reactive power market.

References [13, 14] describe several approaches to remunerate reactive power. In [13], reactive power is paid using a capacity and an utilization term and in [14] reactive costs are organized in explicit and implicit terms. Explicit costs include fix investment and variable maintenance and operation costs and implicit costs include opportunity costs.

References [15, 16] describe multi-objective problems to assign reactive power. Reference [15] minimizes transmission losses and a voltage stability index while [16] minimizes active operation costs, transmission losses, voltage deviations and reactive bid costs.

Finally, reference [17] describes the activity rules of the Spanish electricity market and the model run by the System Operator to allocate reactive power. In the first stage, it is identified a feasible solution redispatching generators and minimizing total costs. The second stage refines this solution tuning voltage control devices for each hour of the next day.

As it will be described in the next section, the developed approach has a clear resemblance with OPF models, detailed for instance in [18, 19]. Differently from the models in these references, the developed formulation is designed to be used by the System Operator after running the day-ahead market and aims at minimizing the cost of making adjustments on the initial generator schedules, if that is necessary from a technical point of view while allocating the power to balance active losses as well as the reactive support. This is based on adjustment bids made both by generators and loads in an attempt to introduce competitive mechanisms in this technical problem, while enlarging the role of the demand.

3. THE DEVELOPED MATHEMATICAL MODEL

3.1 Initial uniform price auction

Using generator and demand bids, the Market Operator solves the problem (1–4) to maximize the Social Welfare Function Z that corresponds to the area between the aggregated load and generator bid curves. These curves are built by organizing load/generation bids according to descending/ascending order of their bid prices. In the simplest version, these bids include (quantity, price) pairs.

$$\max \quad Z = \sum_{j=1}^{Nd} Cd_j . Pd_j - \sum_{i=1}^{Ng} Cg_i . Pg_i \tag{1}$$

$$\text{s.t.} \quad 0 \le Pd_j \le Pd_j^{bid} \tag{2}$$

$$0 \le Pg_i \le Pg_i^{bid} \tag{3}$$

$$\sum_{j=1}^{Nd} Pd_j = \sum_{i=1}^{Ng} Pg_i \tag{4}$$

In this problem, Cd_j and Cg_i are the buying and selling prices, Pd_j^{bid} and Pg_i^{bid} are the load and generation bid quantities, Pd_j and Pg_i are the load and generation at the final solution and Nd and Ng are the number of buying and selling bids. The objective function (1) is subjected to load limits (2), to generation limits (3) and to a load/supply balance equation (4). The outputs of this problem are the cleared generation and load bids, and the clearing price interpreted as the system short-term marginal price, ρ^{MO}.

3.2 Capability diagram of synchronous generators

The System Operator validates the Market Operator dispatch together with the Bilateral Contracts, allocates reactive power and balances active losses. Since active and reactive powers are coupled, it is important to adequately model the components that can impose changes on the

operation point due to these couplings. Regarding synchronous generators, Figure 1 represents the capability diagram defining the feasible operation points in the PQ plane. This diagram results from several constraints of the generator. In this figure, Curve 1, from Qg_i^{max} to $\underline{S}_1$, is the rotor field current limit, Curve 2, from $\underline{S}_1$ to $\underline{S}_2$, is the armature limit and Curve 3, from Qg_i^{min} to $\underline{S}_2$, models the stability limit. Curve 2 is usually approximated by a vertical line related with the mechanical power of the turbine and Curves 1 and 3 can be approximated by linear expressions formulated using Pg_i^{max}, Qg_i^{max}, Qg_i^{min}, Qg_i^{a} and Qg_i^{b}.

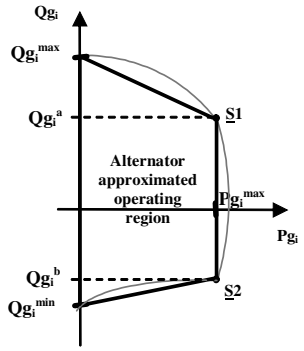

Figure 1: Capability diagram of a synchronous generator.

3.3 Demand and generator adjustment bids

If there are violated constraints, the System Operator activates an adjustment market to alter the initial schedule and to regain feasibility. In this approach we admitted that both generators and loads can submit adjustment bids to the TSO. This increases its flexibility to operate the system, when enlarging the number of possible solutions, as well as increasing the liquidity of this market. Regarding the generators, the adjustment bids indicate the maximum amount of active power they admit changing their initial active power schedule. These bids include a percentage vg_i^{tol} regarding the initially scheduled active power and the adjustment price Cg_i^A. Loads can also present adjustment bids including the maximum percentage reduction of the initially allocated active power together with the corresponding adjustment price, Cd_i^A. Finally, it should be referred that the formulation includes adjustment bids both from agents cleared at the day-ahead market and from bilateral contracts. This is necessary because all these agents use the same grid and so the technical evaluation should be conducted for the whole set of scheduled powers. In any case, it should be realized that the bilateral contract price is confidential and the developed formulation only requires adjustment bids, not the initial private negotiated price.

3.4 Mathematical formulation

The System Operator uses the information regarding the adjustment bids to determine the less costly strategy while enforcing technical and security constraints. This strategy is identified by solving an optimization problem in which generators and loads are organized in three sets as follows:

- S_1 – generators and loads that bided to the Market Operator and that were scheduled;
- S_2 – generators and loads that bided to the Market Operator but that were not scheduled;
- S_3 – generators and loads having Bilateral Contracts.

Generator adjustment variables are decoupled in two terms:

- ΔPg_i^L represents the generator i adjustment to contribute to balance active losses. The index i takes values in sets S_1 and S_3. It can also take values in S_2 if the System Operator asks a generator to balance losses, although it was not originally scheduled;
- ΔPg_i^A variable represents the generator i positive or negative adjustment required to enforce technical or security constraints as branch flow or voltage limits. The index i can take values in sets S_1, S_2 and S_3;

Decoupling generator adjustments this way enables to access the amount of power each generator is asked to contribute to balance losses so that this service can be paid at the market day-ahead price. Finally, the load adjustments are non positive eventually leading to load reductions. They are represented by ΔPd_j^A variables and the index j takes values in S_1 and S_3. Using this notation, the System Operator solves the problem (5-20) to conduct the referred technical validation study.

$$\text{Min } Z = \rho^{MO}.\sum_{S1\cup S2\cup S3}\Delta Pg_i^L + \sum_{S1\cup S2}|\Delta Pg_i^A|\times Cg_i^A + \sum_{S1}|\Delta Pd_j^A|\times Cd_j^A + \sum_{S3}|\Delta Pg_i^A|\times Cg_i^A + \sum_{S3}|\Delta Pd_j^A|\times Cd_j^A \quad (5)$$

$$\text{s.t. } \Delta V_i^{min} \le \Delta V_i \le \Delta V_i^{max} \quad \text{for all nodes} \quad (6)$$

$$\Delta\theta_{ij}^{min} \le \Delta\theta_{ij} \le \Delta\theta_{ij}^{max} \quad \text{for all nodes} \quad (7)$$

$$0 \le \Delta Pg_i^L \le \Delta Pg_i^{L\,max} \quad i\in S1\cup S2\cup S3 \quad (8)$$

$$-\frac{vg_i^{tol}}{100}\times Pg_i \le \Delta Pg_i^A \le \frac{vg_i^{tol}}{100}\times Pg_i \quad i\in S1\cup S3 \quad (9)$$

$$0 \le \Delta Pg_i^A \le \frac{vg_i^A}{100}\times Pg_i^{max} \quad i\in S2 \quad (10)$$

$$\Delta Pg_i^{min} \le \Delta Pg_i^A + \Delta Pg_i^L \le \Delta Pg_i^{max} \quad i\in S1\cup S2\cup S3 \quad (11)$$

$$-Pd_j \le \Delta Pd_j^A \le 0 \quad i\in S1\cup S3 \quad (12)$$

$$Qg_i + \Delta Qg_i \le Qg_i^{max} - \frac{Qg_i^{max} - Qg_i^a}{Pg_i^{max}}.(Pg_i + \Delta Pg_i^A + \Delta Pg_i^L) \quad i\in S1\cup S2\cup S3 \quad (13)$$

$$Qg_i + \Delta Qg_i \ge Qg_i^{min} + \frac{Qg_i^b - Qg_i^{min}}{Pg_i^{max}}.(Pg_i + \Delta Pg_i^A + \Delta Pg_i^L) \quad i\in S1\cup S2\cup S3 \quad (14)$$

$$\Delta S_{ij}^{min} \le \Delta S_{ij}(\Delta V,\Delta\theta) \le \Delta S_{ij}^{max} \quad \text{for all branches} \quad (15)$$

$$\sum_{k=1}^{Nb}\Delta P_k^{loss}(\Delta V,\Delta\theta) = \sum_{S1\cup S2\cup S3}\Delta Pg_i^L \quad (16)$$

$$\Delta P_i^{inj}(\Delta V,\Delta\theta) = (\Delta Pg_i^A + \Delta Pg_i^L) - \Delta Pd_i^A \quad \text{for all nodes} \quad (17)$$

$$\Delta Q_i^{inj}(\Delta V,\Delta\theta) = \Delta Qg_i - \Delta Qd_i \quad \text{for all nodes} \quad (18)$$

$$\sum_{S1\cup S2}\Delta Pg_i^A = \sum_{S1}\Delta Pd_j^A \quad (19)$$

$$\sum_{S3}\Delta Pg_i^A = \sum_{S3}\Delta Pd_j^A \quad (20)$$

The objective function (5) includes five terms. The first one models the active power allocated to the generators to

balance active losses multiplied by the Market Operator day-ahead marginal price. The use of this price to value losses is a market design option since one could have used for instance the adjustment price of each generator adjustment bid. The second and third terms represent the adjustment costs of the generators and loads scheduled by the Market Operator and the fourth and the fifth terms are the adjustment costs of the generators and loads having Bilateral Contracts.

This problem includes the following constraints:

- voltage limits and phase difference limits expressed by constraints (6) and (7);
- limits imposed to the generator adjustments to balance active losses, represented by (8);
- limits on the generator adjustments required to enforce security or operation constraints. These limits are modelled by (9) for the generators already scheduled or by (10) for the generators not initially scheduled;
- limits imposed to the total generator adjustments (11);
- limits imposed to the demand adjustments (12). As default, they can vary between zero and the total amount of the load;
- constraints (13) and (14) representing linearized versions of Curves 1 and 3 as referred in Section 3.2;
- bounds on the apparent branch power flows (15). These constraints are expressed in terms of a lineraized expression for the apparent flow in terms of the voltages and phases in the extreme nodes of each branch;
- constraint (16) imposing that the sum of the generator adjustments to balance active losses is equal to sum of the active loss variations obtained by linearized expressions in terms of the voltages and phases in the extreme nodes of each branch;
- constraints (17) and (18) corresponding to linearized versions of the AC active and reactive injected power equations expressed in terms of the voltage and phase in the system nodes;
- finally, constraints (19) and (20) impose that the two trading systems – Market Operator and Bilateral Contracts – are closed and balanced. For instance, this means that a Market Operator generator reduction has to be balanced by another Market Operator scheduled agent. In this case, this means increasing another Marker Operator generator or reducing a Market Operator load. If these constraints are not considered, the System Operator has more flexibility in enforcing security of technical constraints but it is possible that the final solution displays large unbalances between the referred two trading mechanisms.

3.5 Integration of discrete components

As referred, the model (5-20) was enhanced including transformer taps and capacitor and inductance banks. For each transformer, (21) and (22) indicate the admissible values of the transformer taps either admitting that the controlled node is in the primary or in the secondary. On the other hand, (23) and (24) indicate the available steps of the capacitor and inductance banks. When considering these elements, it should be noted that constraints (15-18) depend on the values of these variables since they are formulated using elements of the network admittance matrix.

$$\alpha_i \in \left\{\alpha_i^{min},...,1,...,\alpha_i^{max}\right\} \quad \text{all transf.} \quad (21)$$

$$\beta_i \in \left\{\beta_i^{min},...,1,...,\beta_i^{max}\right\} \quad \text{all transf.} \quad (22)$$

$$\tau_i^{cap} \in \left\{0,1,...,\tau_i^{cap,max}\right\} \quad \text{all capacitor banks} \quad (23)$$

$$\tau_i^{ind} \in \left\{0,1,...,\tau_i^{ind,max}\right\} \quad \text{all reactor banks} \quad (24)$$

4. SOLUTION ALGORITHM

4.1 General approach

The solution approach of problem (5-24) is presented in Figure 2. Blocks A1, A2 and A3 represent the input data, that is the network data, the bids submitted to the Market Operator and the Bilateral Contract injections. Then, in Block B, the Market Operator solves the problem (1-4) to get the cleared generation and demand bids and the day-ahead marginal price. This schedule together with the Bilateral Contracts is conveyed to the System Operator in order to conduct in Block C the technical validation study, to allocate active losses and reactive power. Block C is organized in four sub-blocks. In Block E it is solved the continuous problem (5-20) using the SLP algorithm detailed in Section 4.2. As a result, we get a feasible operation point admitting that all transformer taps are at their nominal position and the capacitor and inductance banks are disconnected. This solution is used in Block F to initialize the Simulated Annealing algorithm that addresses the integer problem given that we are now trying to select the most adequate positions of the transformer taps, capacitor and reactor banks. As a result, we get the most adequate combination of the discrete variables and in Block G we use them to update the admittance matrix. Finally, in Block H we run again the SLP approach to get the final solution and to compute the nodal active and reactive power marginal prices.

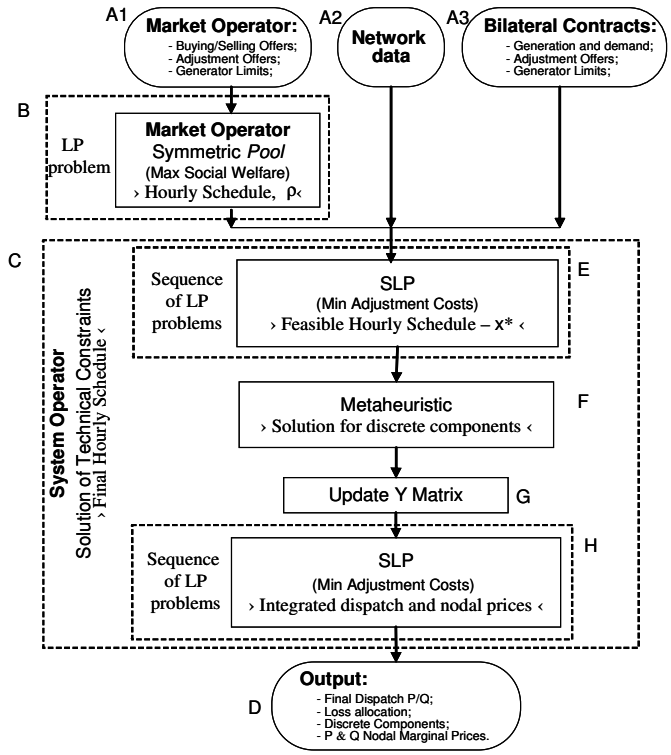

Figure 2 - Flowchart of the simplified solution algorithm.

4.2 Solution of the continuous problem

The optimization problem (5-20) was solved using Sequential Linear Programming, SLP, implemented using the *linprog* function of MATLAB. After solving the problem (1-4), the cleared demands and generations and the system marginal price together with the information about Bilateral Contracts is sent to the System Operator. In other to allocate active losses and reactive power as well as to enforce security or system constraints, the System Operator runs an initial AC Power Flow study and then linearizes constraints (15-18) around this operation point. After formulating the linear problem (5-20), the MATLAB *linprog* function is used to solve it and to calculate the generation, demand, voltage and phase deviations. These values are then used to update the previous operation point so that we can now run a new AC Power Flow problem to get a new linearization point. When the referred deviations are smaller than specified tolerances this iterative process finishes and we get the solution of the original non-linear continuous problem.

4.3. Solution of the combinatorial problem

After solving the continuous problem in Block E, we get an initial and feasible solution of the discrete problem that will be enhanced using a metaheuristic technique. In this application we used Simulated Annealing for its simplicity, although any other metaheuristic as genetic algorithms or particle swarm based techniques could also have been used instead as well as any other mixed integer solver package.

Given the discrete nature of the problem, starting at solution x^k, for k=0, we sample a new one by selecting a component – a transformer, a capacitor bank or an inductance bank - on which we will change the value of its discrete variable. For instance, if a transformer is sampled, we will then sample a move of its tap up or down, admitting that its current value is not in any of the extreme positions. Having identified x^{k+1}, we evaluate these two solutions using an evaluation function, EF, composed by the objective function (5) plus penalties on the violated constraints. This means we compute $EF(x^k)$ and $EF(x^{k+1})$.

If (25) holds, and since we are dealing with a minimization problem, than x^{k+1} is better than x^k and so the solution x^{k+1} is accepted. If it does not hold, that is if x^{k+1} is worse than x^k and since we are addressing a minimization problem, it is computed the probability of accepting worse solutions by (26) and it is sampled a number p in [0.0;1.0]. If (27) holds, then x^{k+1} is accepted, although it is worse than x^k. This mechanism corresponds to relax in a transitory way the optimality rule introducing more diversity on the search procedure eventually allowing the iterative process to escape from local minima.

$$EF(x^{k+1}) < EF(x^k) \tag{25}$$

$$p(x^k) = e^{\frac{EF(x^k) - EF(x^{k+1})}{T(k)}} \tag{26}$$

$$p < p(x^k) \tag{27}$$

Simulated Annealing has a clear analogy with the cooling process of thermodynamic systems and so one of the conditions to get a final low energy system corresponds to slowly reduce the temperature. In (26), the temperature at iteration k, $T(k)$, is assigned a larger value at the beginning and then it is reduced along the iterate process so that it becomes increasingly difficult to accept worse solutions. After running a large number of iterations, this means that the algorithm identified a promising area of the solution space where the solution will be improved and from where one doesn't want to go away. Typically, they are run a number of iterations for each temperature level and then $T(k)$ is reduced by a cooling factor smaller but usually close to 1.0. When they are run a specified number of iterations without improving the Evaluation Function, EF, at least by a minimum percentage one can consider that it was obtained a satisfactory solution and so the iterative process finishes.

4.4. Computation of nodal marginal prices

Nodal marginal prices correspond to the derivative of the objective function of an optimization problem regarding a variation of the demand at a given node and at a specified instant. Typically, they are computed for active power as a subresult of the solution of the optimization problem given their conceptual close relation with dual variables. When considering a discrete problem as the one we are addressing, this definition must be considered carefully since the problem is not continuous. In this case, to compute active and reactive power nodal marginal prices we will have to admit that the combination of values of the discrete variables in the final solution of the Simulated Annealing will not change even if there are small changes on the active or reactive demand. If that is the case, the dual variables of (17-20) obtained in Block H can be used to compute these prices.

Let us assume that γ_k^P and γ_k^Q are the dual variables of (17) and (18) for each node k, and $\gamma^{P,MO}$ and $\gamma^{P,BC}$ are the dual variables of the balance constraints (19) and (20). Regarding active power, one should recall there are two contractual systems – Market Operator and Bilateral Contracts – and so they are obtained two nodal prices, $\rho_k^{P,MO}$ and $\rho_k^{P,BC}$, one for each of these systems, given by (28) and (29). Regarding reactive power, the marginal price of node k is given by (30). Finally, if closing conditions (19) and (20) are not considered, that is, if the market design allows cross-adjustments between the two contractual systems, then the active power nodal marginal prices given by (28) and (29) will not include $\gamma^{P,MO}$ and $\gamma^{P,BC}$.

$$\rho_k^{P,MO} = \gamma_k^P + \gamma^{P,MO} \tag{28}$$

$$\rho_k^{P,BC} = \gamma_k^P + \gamma^{P,BC} \tag{29}$$

$$\rho_k^Q = \gamma_k^Q \tag{30}$$

5. CASE STUDY – IEEE 24 BUS TEST SYSTEM

5.1 System data

We will now present results obtained using the IEEE 24 Bus Test System to illustrate the developed approach. The branch data is available from [20] and Table 1 includes the generation bids in terms of (quantity, price) admitting that each generator organizes its bid at most in three blocks. Table 2 contains the information regarding the capability diagram of each generator together with the adjustment bids, that is

the percentage vg_i^{tol}, and the adjustment price, Cg_i^A. Table 3 details the buying bids transmitted to the Market Operator, that is the pairs (quantity, price), reactive power and adjustment price. Finally, Table 4 contains the generators and loads having Bilateral Contracts and the load adjustment price and Table 5 contains the data of the capability diagram of these generators.

Bus	Pg_1^{bid} MW	Cg_1^{bid} €/MWh	Pg_2^{bid} MW	Cg_2^{bid} €/MWh	Pg_3^{bid} MW	Cg_3^{bid} €/MWh
1	94.0	35.0	150.0	42.0	192.0	47.0
2	96.0	37.0	154.0	41.5	192.0	48.0
7	150.0	14.0	285.0	27.5	300.0	38.5
13	300.0	15.0	460.0	24.0	591.0	37.5
15	80.0	13.0	145.0	26.0	215.0	36.0
16	110.0	20.0	155.0	34.5	---	---
18	250.0	34.0	350.0	38.0	400.0	46.0
21	150.0	26.0	300.0	35.5	400.0	45.0
22	150.0	11.0	205.0	21.0	300.0	37.0
23	300.0	15.0	470.0	24.0	660.0	39.0

Table 1 – Bidding blocks of the generators that bided to the MO.

bus	Pg_i^{max} MW	Qg_i^{max} Mvar	Qg_i^{a} Mvar	Qg_i^{b} Mvar	Qg_i^{min} Mvar	vg_i^{tol} %	Cg_i^A €/MWh
1	192.0	80.0	65.0	-40.0	-50.0	40.0	110.0
2	192.0	80.0	65.0	-40.0	-50.0	40.0	115.0
7	300.0	180.0	150.0	0.0	0.0	40.0	120.0
13	591.0	240.0	160.0	0.0	0.0	40.0	105.0
15	215.0	110.0	90.0	-35.0	-50.0	40.0	100.0
16	155.0	80.0	70.0	-45.0	-50.0	40.0	112.0
18	400.0	200.0	150.0	-40.0	-50.0	40.0	130.0
21	400.0	200.0	150.0	-35.0	-50.0	40.0	160.0
22	300.0	96.0	70.0	-40.0	-60.0	40.0	103.0
23	660.0	310.0	205.0	-95.0	-125.0	40.0	118.0

Table 2 – Capability diagram points of the generators that bided to the MO.

bus	Pd_j^{bid} MW	Cd_j^{bid} €/MWh	Qd_j Mvar	Cd_j^A €/MWh
1	108.0	66.0	21.93	295.0
2	97.0	54.0	19.70	290.0
3	180.0	41.5	36.55	289.0
4	74.0	35.0	15.03	288.0
5	71.0	68.0	14.42	296.0
6	136.0	37.5	27.62	300.0
7	125.0	51.0	25.38	285.0
8	171.0	34.5	34.72	295.0
9	175.0	53.0	35.54	296.0
10	195.0	43.0	39.60	288.0
13	265.0	38.5	53.81	287.0
14	194.0	61.0	39.39	305.0
15	317.0	64.0	64.37	301.0
16	100.0	57.0	20.31	294.0
18	333.0	60.0	67.62	296.0
19	181.0	34.0	36.75	298.0
20	128.0	56.0	25.99	291.0

Table 3 – Demand and adjustment bids for the loads that bided to the MO.

				Generators i having bilateral contract with load j					
Load bus	Pd_j MW	Qd_j Mvar	Cd_j^A €/MWh	bus	Pg_i MW	bus	Pg_i MW	bus	Pg_i MW
1	10.0	2.03	299.0	15	10.0	---	---	---	---
2	20.0	4.06	288.0	7	20.0	---	---	---	---
4	15.0	3.76	285.0	22	15.0	---	---	---	---
5	72.0	14.62	296.0	7	22.0	21	25.0	22	25.0
7	30.0	7.52	290.0	13	30.0	---	---	---	---
9	16.0	3.25	280.0	15	16.0	---	---	---	---
10	16.0	2.28	294.0	16	16.0	---	---	---	---
13	45.0	13.13	292.0	13	18.0	16	15.0	22	12.0
14	10.0	2.51	289.0	7	10.0	---	---	---	---
16	10.0	2.03	287.0	21	10.0	---	---	---	---
18	38.0	9.52	276.0	13	13.0	18	25.0	---	---
19	35.0	10.21	299.0	18	35.0	---	---	---	---
20	44.0	8.93	286.0	16	20.0	21	24.0	---	---

Table 4 – Bilateral contracts (BC) involving load j and generator i.

bus	Pg_i^{max} MW	Qg_i^{max} Mvar	Qg_i^{a} Mvar	Qg_i^{b} Mvar	Qg_i^{min} Mvar	vg_i^{tol} %	Cg_i^A €/MWh
7	100.0	65.0	50.0	-40.0	-60.0	40.0	108.0
13	191.0	95.0	50.0	-50.0	-80.0	40.0	103.0
15	115.0	70.0	40.0	-25.0	-50.0	40.0	111.0
16	55.0	30.0	20.0	-15.0	-30.0	40.0	113.0
18	100.0	65.0	30.0	-20.0	-60.0	40.0	101.0
21	100.0	50.0	40.0	-20.0	-50.0	40.0	98.0
22	80.0	55.0	40.0	-25.0	-40.0	40.0	107.0

Table 5 – Capability diagram points and adjustment bids of BC generators.

Apart from this data, voltage limits were set at 0.94 pu and 1.06 pu and it was not specified any maximum load curtailment limit meaning that load curtailment can be as large as required to enforce technical or security constraints. The system has a synchronous compensator in node 14 (-50.0 to 200.0 Mvar), a capacitor bank on node 14 (steps 20.0, 10.0, 10.0, 3.0, 3.0 Mvar) and an inductance bank in node 6 (steps 40.0, 20.0, 20.0, 10.0, 10.0 Mvar). Finally, there are transformers between nodes 3-24 and 10-12 (taps: -10%,-7.5%,-5%,-2.5%,0,+2.5%,+5.0%,+7.5%,+10%) and 9-11, 9-12 and 10-11 (taps: -5%,-2.5%,0,+2.5%, +5.0%).

5.2 Initial Market Operator dispatch

As detailed in Section 3.1, the Market Operator runs the optimization problem (1-4) to obtain the purely economic schedule presented in Table 6. The total cleared power was 2424.0 MW and the system marginal price, ρ^{MO}, is 36.0 €/MWh. This dispatch together with the generations and loads related to Bilateral Contracts (in Table 4) will now be conveyed to the System Operator for technical validation.

bus i	Pg_i MW	Pd_i MW	bus i	Pg_i MW	Pd_i MW
1	94.0	108.0	13	460.0	265.0
2	0.0	97.0	14	---	194.0
3	---	180.0	15	205.0	317.0
4	---	0.0	16	155.0	100.0
5	---	71.0	18	250.0	333.0
6	---	136.0	19	---	0.0
7	285.0	125.0	20	---	128.0
8	---	0.0	21	300.0	---
9	---	175.0	22	205.0	---
10	---	195.0	23	470.0	---

Table 6 – Market Operator initial dispatch.

5.3 Case 1 - Results without congestion

In the first simulation, we solved the problem (5-24) using the algorithm described in Section 4 considering branch limits as large as necessary so that there are no congested branches. In this case, the generation adjustment variables related with the enforcement of constraints are zero and so the generator outputs will only be altered to balance active losses, that is, only ΔPg_i^L variables will be non zero.

	MO generators				BC generators		
Bus i	ΔPg_i^L MW	ΔPg_i^A MW	Pg_i^F MW	Qg_i^F Mvar	ΔPg_i^A MW	Pg_i^F MW	Qg_i^F Mvar
1	0	0	94.00	41.24	---	---	---
2	48.73	0	48.73	-0.28	---	---	---
7	0	0	285.00	39.53	0	52.00	-20.01
13	0	0	460.00	106.84	0	61.00	14.48
15	0	0	205.00	90.93	0	26.00	63.22
16	0	0	155.00	45.08	0	51.00	6.58
18	0	0	250.00	66.02	0	60.00	3.29
21	0	0	300.00	-24.34	0	59.00	-21.68
22	0	0	205.00	-27.37	0	52.00	-13.34
23	0	0	470.00	-13.26	---	---	---

Table 7 – Case 1 - Final dispatch of the generators.

According to Table 7, the active losses (48.73 MW) are totally balanced by the Market Operator generator in node 2. The final value of the objective function (5) is 1754.28 €/h and it results from the product of the losses by the system marginal price of 36.0 €/MWh. Finally, the synchronous compensator in node 14 generates 80.6 Mvar, the inductance bank in node 6 has all steps on except the fifth, the capacitor in bus 14 has the steps 1 and 2 on, the transformers at nodes 3-24, 9-11, 9-12 and 10-12 were set at the voltage ratio of +2.5%, +5.0%, -5.0% and +5.0%, and the transformer at nodes 10-11 is at the nominal position.

5.4 Case 2 - Results with congestion

In this Case, the branch 7-8 limit was reduced from 200.0 MVA to 150.0 MVA. Solving again the problem (5-24), it is now necessary to adjust generators not only to balance active losses but also to enforce system constraints. According to Table 8, active losses are once again balanced by the Market Operator generator in node 2. The generators in node 7 reduce their output and these reductions are balanced by Market Operator generators, on one side, and by Bilateral Contract generators, on the other, due to constraints (19) and (20). The final value of the objective function is 8471.65, €/h, much larger than the one from Case 1, due to the costs related with the second and fourth terms of (5). Finally, the synchronous compensator in node 14 generates 48.4 Mvar, the inductance and capacitor banks in nodes 6 and 14 have all steps on, the transformers connected between nodes 9-11, 9-12, 10-11 and 10-12 were set at the voltage ratio of +5.0%, -2.5%, -2.5% and +5.0%, and the transformer at nodes 3-24 is at the nominal position.

	MO generators				BC generators		
Bus i	ΔPg_i^L MW	ΔPg_i^A MW	Pg_i^F MW	Qg_i^F Mvar	ΔPg_i^A MW	Pg_i^F MW	Qg_i^F Mvar
1	0	0	94.00	45.39	---	---	---
2	42.78	0	47.28	11.02	---	---	---
7	0	-11.27	273.73	48.14	-20.80	31.20	-16.48
13	0	0.41	460.41	107.33	0	61.00	13.42
15	0	10.00	215.00	90.00	0	26.00	63.22
16	0	0	155.00	52.56	0	51.00	9.88
18	0	0	250.00	71.37	0	60.00	4.38
21	0	0	300.00	-29.41	20.80	79.80	-19.40
22	0	0.86	205.86	-27.18	0	52.00	-13.42
23	0	0	470.00	18.91	---	---	---

Table 8 – Case 2 - Final dispatch of the generators.

5.5 Nodal marginal active/reactive power prices

Figure 3 displays the profiles of the active and reactive power nodal marginal prices. For both cases, there are two active power prices for each node since there are two trading mechanisms using the same network that have to remain balanced due to constraints (19) and (20). Regarding Case 1, one should recall there is no congestion and Figure 3 shows that the Market Operator subsystem nodal marginal prices are very homogeneous varying around 100.0 €/MWh. This value is in line with the less expensive generation adjustment bid inside this system that was submitted by generator 15. For some other nodes, the prices are a bit above this value if a load increase has a positive impact on active losses, while for some others they can be a bit below 100.0 €/MWh if the impact on losses is negative. For the Bilateral Contract subsystem, the less expensive generation adjustment bid was submitted by generator in bus 21, 98.0 €/MWh. Figure 3 indicates that the active power nodal marginal prices for the Bilateral Contract subsystem are in line with this value. The impact on active losses explain small variations of the prices above or below 98.0 €/MWh.

In Case 2, branch 7-8 is congested at 150 MVA leading to the negative active power prices for node 7. Regarding, for instance, the Market Operator subsystem, this congestion means that the solution obtained for Case 1 has to be changed reducing the active power generated at node 7. According to Table 2, the adjustment price in node 7 is 120.0 €/MWh meaning that the second term of the objective function (5) will now be the product by 120.0 €/MWh due to the generation adjustment. In order to compute the nodal price, we will now reason in terms of a load increase of 1 MW in node 7. It is clear that this increase will be directly balanced by the generator in node 7, which means that the adjustment of this generator is reduced by 1 MW. This also reduces the objective function (5) by 120 €/MWh explaining the negative price of -120.0 €/MWh. For the Bilateral Contract subsystem a similar situation occurs explaining the negative price also obtained for node 7.

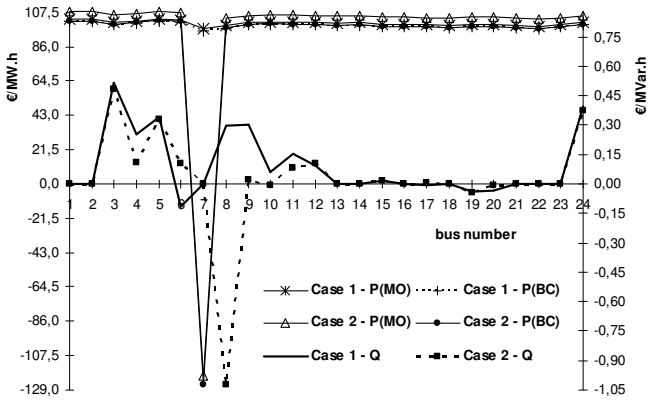

Figure 3 – Profiles of nodal marginal prices of active/reactive power.

Finally, the reactive power marginal prices for Cases 1 and 2 are also shown in Figure 3. In some nodes these prices are zero indicating there are reactive power devices on those nodes that directly accommodate reactive power increases (nodes 1, 2, 7, 13, 14, 16, 18, 21, 22 and 23). On some others, the reactive power price is positive reflecting a positive impact on active losses from increasing the reactive demand while in some others it is negative because a reactive power increase reduces active losses.

6. CONCLUSIONS

This paper describes an integrated formulation for the active/reactive power dispatch retaining market mechanisms. It is based on an initial uniform price based auction and on the schedules from Bilateral Contracts that are then analysed by the System Operation to check system and security constraints, to allocate reactive power and active losses. The main features of this approach result from the possibility of activating an adjustment market based on bids submitted both by generators and loads, the potentially more intervenient role played by the demand, the computation of the active power allocated to each generator to balance active losses turning it possible to price this service in a more accurate way and the computation of active and reactive nodal marginal prices. As a whole, this kind of approaches can play an important role in the activity of system operators,

remarrying active and reactive power and contributing to preserve a clear market driven basis.

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