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A two-stage planning and control model toward Economically Adapted Power Distribution Systems using analytical hierarchy processes and fuzzy optimization

Gustavo Schweickardt [a,*], Vladimiro Miranda [b,1]

[a] Instituto de Economía Energética, Fundación Bariloche, Centro Atómico Bariloche – Pabellón 7, Av. Bustillo km 9500, 8400 Bariloche, Argentina
[b] INESC Porto, Instituto de Engenharia de Sistemas e Computadores do Porto and FEUP, Faculdade de Engenharia da Universidade do Porto, R. Dr. Roberto Frias, 378, 4200-465 Porto, Portugal

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ABSTRACT

This work presents a model to evaluate the Distribution System Dynamic De-adaptation respecting its planning for a given period of Tariff Control. The starting point for modeling is brought about by the results from a multi-criteria method based on Fuzzy Dynamic Programming and on Analytic Hierarchy Processes applied in a mid/short-term horizon (stage 1). Then, the decision-making activities using the Hierarchy Analytical Processes will allow defining, for a Control of System De-adaptation (stage 2), a Vector to evaluate the System Dynamic Adaptation. It is directly associated to an eventual series of inbalances that take place during its evolution.

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1. Introduction

The definition of an Economically Adapted Power Distribution System (EAPDS) (sometimes referred as Adapted Distribution Network) is a concept that Regulatory Authority, after the de-regulation of electricity industry, has coined and applied without both theoretical and methodological bases. Following the Neoclassical Economics Paradigm, only considering system expansion and operation costs, require that the total cost in some system horizon planning, becomes minimum. Any departure from this condition, once the planning is available, is judged as a de-adaptation. Therefore, all possible system de-adaptation will be static, ignoring the historical-evolutive nature of the system. This lack in the concept and in their operational definition, requires, of course, the previous knowledge of system planning. However, this planning is not enough to obtain a definition operationally useful, as measure to an economically de-adapted system. Fourth reasons, at least, can be observed for it: (a) the planning facing a multi-criteria optimization problem and several of such criteria, have not economic assessment (environmental cost, for example); (b) many or all control variables involved in the planning, have uncertainties, the nature of which is not stochastic and, then, its treatment for any classical optimization model is very difficult; (c) suppose that all criteria have an economic opportunity cost associated, and the system control variables are deterministic: still, the departure for evaluating the system de-adaptation will be static, and, in practice, evaluated at the beginning of tariff control period; (d) a uniform criteria to departure definition and its implementation do not exist (could be and over-cost in equipment, because the demand was not the forecasted/predicted, or another arbitrary criteria). Under conditions (c) and (d), the concept is applied, commonly, for Regulatory Authority in countries such as Argentina, Chile, Spain and Portugal.

This paper presents a formal solution model, in the framework to Fuzzy Dynamic Programming (FDP) and Analytic Hierarchy Processes (AHP) techniques, in two stages: the first stage, corresponding to the Distribution System Planning in the mid/short-term, considering available the long-term planning. The second stage, corresponding to the Control of System De-adaptation.

Planning FDP technique is based on the principles optimally proposed by [1] and [2].

Control of the System De-adaptation, required the introduction of the technique AHP [3,4] which completes the model by establishing a Priority Vector that is associated to the criteria (seen as system attributes) that define the aptitude for each alternative of solution. This combined model is intended for: (a) developing the theoretical aspects required to define and, operationally, introduce

* Corresponding author. Tel./fax: +54 2944 462500.
E-mail addresses: gustavoschweickardt@ciudad.com.ar (G. Schweickardt), vmiranda@inescporto.pt (V. Miranda).
[1] Tel.: +351 22 2094230; fax: +351 22 2094150.

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Nomenclature

Acronyms

AHP	Analytic Hierarchy Processes
EAPDS	Economically Adapted Power Distribution System
ESNQC	Energy Supplied in Non-Quality Conditions, vinculated to Out-of-Supply Voltage attribute
FDP	Fuzzy Dynamic Programming
FI	frequency of interruption, vinculated to Interruption Rate attribute
GC	Global Cost of Distribution System
GPL	Global Power Losses of Distribution System
NSE	Non-Supplied Energy
OSV	Out-of-Supply Voltage attribute
PDS	Power Distribution Planning

Symbols

Ai	ith variable-attribute of system
$\{Ai\}$	fuzzy set associated to ith variable-attribute
$a_{i\,ref}$	value of reference to Ai
a_i	value-instance of Ai
a_{ij}	generic element of Judgment Matrix to the set of atttributes **A**
CCA	Random Consistency Coefficient
E^k	state vector corresponding to system in the stage k of planning
e_i^k	ith state/component of E^k
IC_{Saaty}	Saaty's Consistency Index
I_D	De-Adaptation Index
$\boldsymbol{MP}_A$	Judgment Matrix to the set of attributes **A**
p_i	ith component of $\boldsymbol{V}^{Yager}$ corresponding to ith fuzzy set $\{Ai\}$
RC	Consistency Ratio related to IC_{Saaty} and CCA
R_D	Adaptation Ratio
$[T^*]$	resulting trajectory from FDP
u_i	departure variable of Ai
V_{Adap}	Dynamic Adaptation Vector for trajectory $[T^*]$
$\boldsymbol{V}_P$	Perron's eigenvector of Judgment Matrix. Priority Vector in the context of AHP
$\boldsymbol{V}^{Yager}$	Vector of Exponential Weighers (from Yager) applied on fuzzy set $\{Ai\}$
λ_P	Perron's eigenvalue. Maximum and Positive (real) eigenvalue of $\boldsymbol{MP}_A$
$\{D\}$	decision fuzzy set defined as the intersection of all $\{Ai\}$
$\mu_{\{D\}}$	membership function of $\{D\}$
$\Delta_{\mu_{\{D\}}}$	Acceptance Variation of Trajectory from present stage to horizon of planning
$\mu_{\{Ai\}}(u_i)$	membership function of $\{Ai\}$
$\mu_{\{D\}}[T^*]$	maximizing decision on fuzzy set $\{D\}$, corresponding to resulting trajectory $[T^*]$, referred as Intrinsic Risk of trajectory $[T^*]$
Θ_{Ext}	Extrinsic Robustness

into the problem of decision-making, the *intrinsic risk* associated to a certain satisfactory solution; and (b) giving a *formal treatment* to the concept of Adaptation (De-adaptation) of the system.

Therefore, the proposal model will be referred as (FDP + AHP) planning.

For discussion of some results, a Study Case is presented. The system under analysis is the power distribution system for the city of San Carlos de Bariloche, Province of Rio Negro, Argentina, for which a 10-year horizon has been considered, starting in 1998, for the long-term. The mid/short-term coincides with the 5-year tariff period 1998–2002.

2. Power distribution systems planning

As regards the objectives of the present work, such a planning is set forth through a two-stage strategy: the long-term (1–15 years) planning, that is focused on the spatial optimization of the PDS and, the mid/short-term (5–7 years) planning model that emphasizes the time optimization of the PDS.

For the long-term planning, the system optimization has two aims: (a) to minimize the costs associated to building substations and network lines, and their operative costs (Global Costs of the PDS) as well; to minimize the non-quality (that is, the absence of good quality) of electrical energy, pondered through the Non-Supplied Energy (NSE) under contingency.

The two main constraints are: (a) radiality, and (b) coherence in equipment selection and combinable variants, which ensure the coherence among the equipment of the PDS, by defining states between consecutive stages. Other three constraints, regarded implicitly included, are: (a) restriction on demand supply; (b) restriction on the limits of power supply capacity of substations, and on feeders capacity; and (c) restriction of maximum permissible voltage drops. With these considerations, the goal of the strategy is to achieve a set of solution trajectories admitted as satisfactory, on which the system will be capable to evolve. Restriction (b) allows flexing the choice (combinability) of variants to conform various trajectories on a time-horizon where the uncertainty has decreased. Then, in the short/mid-term planning, multiple objectives/restrictions will take place, whose merits will be evaluated in order to define the best trajectories for the mid or short-term.

3. Long-term planning: A suggested approach from evolutionary programming

This work supposes that the long-term planning is available. However, under a hierarchy planning basic principle, the best short/mid-term planning must be driven for the best long-term planning. Therefore, suggestions about the model used for the long-term planning must be made. Classical approaches are frequently used, such as those presented in [5] for substations, and in [6] for conductors sizing. This work suggests a multi-objective model for optimal system expansion in the long-term, by following a pseudo-dynamic multi-stage, and facing the uncertainties of its environments. Specifically, it must focus both on the power lines (conductors sizing) on the system substations. The suggested technique, is an approach from evolutionary programming, in particular, using Genetic Algorithms, as coherent alternative facing the type of uncertainties (non-stochastic) inherent to system. See Refs. [7] and [8].

4. Stage 1. Mid/short-term planning: an application of fuzzy dynamic programming

4.1. Main elements of the model

With the FDP technique, the fuzzy sets, associated to the n-attributes with which the aptitude of the system is defined, are mapped on the same fuzzy space of decision-making. To this aim, such sets are built upon the departures from a given reference value of the evaluated magnitude.

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For a given attribute *Ai*, whose value for a given state (instance) is a_i, the departure with respect to certain reference value $a_{i\,ref}$, from which the associated fuzzy set is constructed, will be:

$$u_i = \frac{|a_i - a_{i\,ref}|}{a_{i\,ref}} \tag{1}$$

and the corresponding fuzzy set, {*Ai*}, whose membership function is $\mu_{\{Ai\}}(u_i)$, is defined as:

$$\{Ai\} = \sum_{i=1}^{n} \mu_{\{Ai\}}(u_i)/u_i; \quad u \in U \tag{2}$$

following the notation for fuzzy sets [2], where each variable u belongs to discurse universe U.

Once the fuzzy sets associated to the departures have been defined, the next step is to introduce the AHP technique for which the judgment matrix $\boldsymbol{MP}_A$ is conformed. If **A** is the set of attributes of system, then the $\boldsymbol{MP}_A$ elements, a_{ij}, result from a pairwise comparison of preferences for all attributes in the set **A**. By following the *dominance scale* proposed by [3], where a_{ij} is an integer in the interval [1,10] (a_{ji} is a rational number equal to $1/a_{ij}$), $\boldsymbol{MP}_A$ is defined as:

$$\boldsymbol{MP}_A = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \vdots & 1 \end{pmatrix} \text{ with } \begin{cases} a_{ij} > 0 & \forall i,j = \{1 \ldots n\} \\ a_{ji} = \frac{1}{a_{ij}} \end{cases} \tag{3}$$

Perron's Theorem [9] warrants, for such a matrix, the existence of a *dominant*, *positive eigenvalue* λ_P and its corresponding *eigenvector* $\boldsymbol{V}_P$, also with *positive* components. Both elements have fundamental importance for this model. The following inequality will be true:

$$\lambda_P \geqslant n \tag{4}$$

and only if matrix $\boldsymbol{MP}_A$ shows *consistent judgements*, $\lambda_P = n$.

The *consistency condition* [4], is expressed on the elements of $\boldsymbol{MP}_A$ as follows:

$$a_{ik} = a_{ij} x a_{jk}; \quad \forall i,j,k = 1 \ldots n \tag{5}$$

Condition (5) is introduced in the form of a Saaty's Consistency Index to ponder the *transitivity on preferences between the system attributes*, that had been established for the planner in $\boldsymbol{MP}_A$:

$$IC_{Saaty} = \left(\frac{\lambda_P - n}{n-1}\right) \tag{6}$$

Index IC_{Saaty} may seem to be strict, if the number of attributes is greater than 5 (five). In this way, the *Consistency Condition* gets reformulated by using a *Random Consistency Coefficient*, $CCA > 1$, if $n_A \geqslant 6$ [4], which can be stated as:

$$RC = \frac{IC_{Saaty}}{CCA} \leqslant 0.1 \tag{7}$$

RC: *Consistency Ratio*.

Associated to Perron's *eigenvalue*, λ_P, Perron's *eigenvector*, $\boldsymbol{V}_P$, satisfies the *Hierarchical Composition Principle* [4], which is defined as (from the *eigenvector* (**V**) – *eigenvalue* (c) expression, for a squared matrix):

$$\boldsymbol{MP}_A \boldsymbol{x} \boldsymbol{V} = c \times \boldsymbol{V} \tag{8}$$

if $c = \lambda_P$, and, consequently, $\boldsymbol{V} = \boldsymbol{V}_P$.

$\boldsymbol{V}_P$ results to be the *Priority Vector on the preferences established between the system attributes*. Therefore, if its form is changed to Yager's form [10], by *normalizing* and *multiplying* it by the *order of the Judgment Matrix*, a *Vector of Exponential Weighers*, $\boldsymbol{V}^{Yager}$ will be obtained. This vector will be applied to the fuzzy sets associated to the various attributes in order to modify their importance in the fuzzy optimization.

4.2. Fuzzy dynamic programming and formal model for mid/short-term

Fig. 1, present one step in the transition between the stage k to $k-1$ in a *forward dynamic* [1]. The criteria followed for each transition (each state $(j, k-1)$ to state $(1,k)$) are the *Max{Min{ ... }}* of the membership function value of each system attribute $(1 \ldots n)$, $\mu_{\{Ai\}}(u_i)$[2]. Considering the *Vector of Exponential Weighers*, $\boldsymbol{V}^{Yager}$, calculated as explained above, each $\mu_{\{Ai\}}(u_i)$ will be pondered as the preferences established in the Judgment Matrix $\boldsymbol{MP}_A$. Such exponential weight, expressed as $[\mu_{\{Ai\}}(u_i)]^{p_i}$, where p_i is the *i*th component of $\boldsymbol{V}^{Yager}$, has an effect of *contraction* or *expansion* on the corresponding fuzzy set, according to the value of p_i: if $p_i > 1$, then there will be *contraction*, and the fuzzy set $[\mu_{\{Ai\}}(u_i)]^{p_i}$ will be given a higher importance than $\mu_{\{Ai\}}(u_i)$ in the *Max{Min{ ... }}* optimization criteria. The opposite occurs when $p_i < 1$, where there will be *expansion* of fuzzy set.

If the *static optimization* (between two stages only) presented in Fig. 1, is extended for each state $[\forall e_j^{k-1} \in E^{k-1}]$ and each state $[\forall e_i^k \in E^k]$, where E^{k-1} (E^k) is the state vector for stage $k-1$ (k), the corresponding *fuzzy optimization* (formal model for mid/short-term planning) is stated as:

$$\left[\forall e_i^k \in E^k\right]; \quad k = 1,2 \ldots N:$$

$$\mu^*_{\{D\}}(i,k) = Max\left\{ Min_{\left[\forall e_j^{k-1} \in E^{k-1}\right]} \left\{ \begin{array}{l} \left[\mu_{\{A1\}}(j,k-1);(i,k)\right]^{p_1}; \\ \left[\mu_{\{A2\}}(j,k-1);(i,k)\right]^{p_2}; \cdots; \\ \left[\mu_{\{An\}}(j,k-1);(i,k)\right]^{p_n}; \mu^*_{\{D\}}(j,k-1) \end{array} \right\} \right\} \tag{9}$$

subject to:

(1) $\boldsymbol{MP}_A$ invariant in $[T^*]$ (the preferences between the system attributes, cannot be changed)
(2) $\mu_{\{D\}}[T^*] \geqslant [1 - \Theta_{Ext}]$

where *the membership value* $\mu^*_{\{D\}}(i,k)$ *is maximized in the decision-making fuzzy set*, {*D*}, for every state $[\forall e_i^k \in E^k]$, on the horizon of **N** stages; $p_1, p_2, \ldots, p_n$ are the *Yager Weighers* of vector $\boldsymbol{V}^{Yager}$; $[T^*]$ is the resulting Trajectory and $\mu_{\{D\}}[T^*] = \mu^*_{\{D\}}(i = 1, k = n)$ in the unique state *i*, for the last stage $k = n$, is the value obtained from the recursive application of (9). $\mu_{\{D\}}$ will be referred as *Intrinsic Risk* of $[T^*]$. Constraint (1) on invariability in $\boldsymbol{MP}_A$ imposes that the Yag-

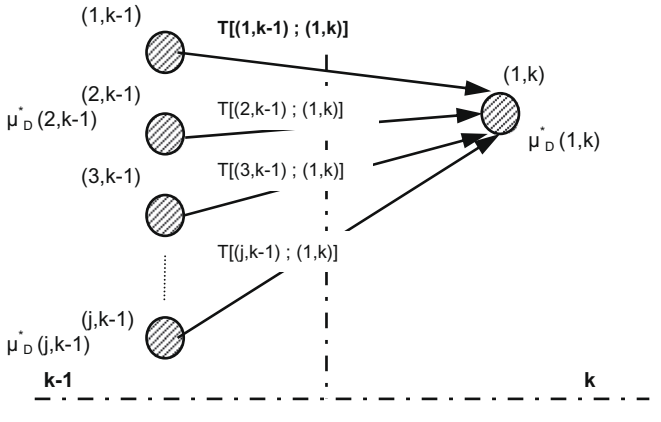

Fig. 1. Transition between stage $k-1$ to stage k in the FDP *forward*.

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er Weighers remain unaltered on the resulting Trajectory [T^*]. Constraint (2) will be called the *Extrinsic Risk Constraint* [11,12]. Is externally fixed by planner by mean a value, Θ_{Ext}, between [0,1], *as a complement to the less-acceptable membership of fuzzy set* {*D*}. The parameter Θ_{Ext} is called *Extrinsic Robustness*. A *high level* of *robustness* implies a *low level* of *risk*, but a *high level* of *uncertainties* [13].

The *link*:

$$\mu^*_{\{D\}}(j,k-1) \rightarrow \mu^*_{\{D\}}(i,k) \tag{10}$$

constitutes the *dynamic condition of the fuzzy optimization process.*

5. Stage 2. Control of system de-adaptation: the Dynamic Adaptation Vector

There are some theoretical concepts which required explanation, before its introduction in the second stage of proposal model and are developed in the following paragraphs.

5.1. The De-Adaptation Index: First component of Dynamic Adaptation Vector

The Dynamic Adaptation Vector is proposed in the second stage of model, in order to evaluate the de-adaptation of the real system (or *present state* system) respecting the projected system. It *has two components*: *the first one* is related to the variation that the system may present as regards its aptitude attributes, as a consequence of change in the preferences among such attributes. This work, is called the *De-Adaptation Index*, and is defined as:

$$I_D^{[k,Pre]} = \frac{\left(\frac{\lambda_P^{[k,Pre]} - n}{n-1}\right)}{CCA} \tag{11}$$

where k is the *present stage* of evaluation; $\lambda_P^{[k,Pre]}$ is Perron's eigenvalue for the *present judgments matrix*, $MP_A^{[k,Pre]}$. Its construction is proposed as follows, satisfying *symmetrical condition* in (3):

$$MP_A^{[k,Pre]} = \begin{pmatrix} [1] & \cdots & \cdots & \left[\frac{p_n^{[Proj]}}{p_1^{[k,Pre]}}\right] \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \left[\frac{p_1^{[Proj]}}{p_n^{[k,Pre]}}\right] & \cdots & \cdots & [1] \end{pmatrix} \tag{12}$$

where *Pre* refers to the *present stage* [*k, Pre*] and *Proj* the *projected stage* [*Proj*]. Depending on the stage of evaluation, *k*, the *present* V^{Yager} will be changed, as well as $\boldsymbol{MP}_A$ (expression (12)); then:

$$V^{Yager[k,Pre]} = \begin{pmatrix} p_1^{[Proj]} \times \frac{\log\{\mu_{A1[k,Proj]}\}}{\log\{\mu_{A1[k,Pre]}\}} \\ p_2^{[Proj]} \times \frac{\log\{\mu_{A2[k,Proj]}\}}{\log\{\mu_{A2[k,Pre]}\}} \\ \vdots \\ p_n^{[Proj]} \times \frac{\log\{\mu_{An[k,Proj]}\}}{\log\{\mu_{An[k,Pre]}\}} \end{pmatrix} \tag{13}$$

results of the *present Weighers Vector* (in stage *k*), obtained from the *Principle of Invariance on the Weighed Aptitude*, are formulated as follows (notice that *projected* μ_{Ai}, depends on the stage *k*):

$$\left[\mu_{Ai[k,Pre]}\right]^{\left[p_i^{[k,Pre]}\right]} = \left[\mu_{Ai[k,Proj]}\right]^{\left[p_i^{[Proj]}\right]}; \forall i[1\ldots n] \in A \tag{14}$$

which will have to satisfy the *present* system as regards the projected one. If $\mu_{Ai[k,Pre]}$ result 1, then *any* $p_i^{[Proj]}$ can satisfy (14). In this case, the product $[p_i^{[Proj]} \times \mu_{Ai[k,Proj]}]$ it is adopted as $p_i^{[k,Pre]}$. Other particular case is present when $\mu_{Ai[k,Proj]} = 1$, then $p_i^{[k,Pre]} = p_i^{[Proj]}$.

In (14), each system attribute value for the present stage should be surveyed, and their membership in the associated fuzzy sets should be attained.

The proposal construction in Eq. (12), is based on the following Consistent Judgment Matrix property: *if* $p_1, p_2, \ldots, p_n$ *are the Yager Weighers of vector* $\boldsymbol{V}^{Yager}$, *defined externally, then the matrix:*

$$\boldsymbol{MP} = \begin{pmatrix} 1 & \frac{p_2}{p_1} & \cdots & \frac{p_n}{p_1} \\ \frac{p_1}{p_2} & 1 & \cdots & \frac{p_n}{p_2} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{p_1}{p_n} & \frac{p_2}{p_n} & \cdots & 1 \end{pmatrix} \tag{15}$$

is a consistent matrix, and associated to Perron's eigenvalue, $\lambda_P = n$.

Such property can be easily demonstrated, applying the consistency condition (5).

As a result from *randomness*, from (12), a judgements matrix could be attained which is more consistent than the one established in the planning. In this case the resulting Yager's Vector will be the one that will prove the coherence of the process. In general, the following setting cases will take place:

(a) $\boldsymbol{MP}_A^{[k,Pre]}$ turns to be *consistent*, but the vector $V^{Yager[k,Pre]}$ *prioritizes* the attributes rather inconsistently to the eyes of the planner/regulator. The case will then be one of a *consistent de-adaptation*: $(\lambda_P^{[k,Pre]} \approx n; V^{Yager}[k,Pre] \neq V^{Yager}[Proj])$; (b) The situation $(\lambda_P^{[k,Pre]} \approx n; V^{Yager}[k,Pre] \neq V^{Yager}[Proj])$ with *consistent* $V^{Yager[k,Pre]}$ constitutes a *consistent adaptation* of the system. It will be a *perfect one* if $(\lambda_P^{[k,Pre]} \approx n; V^{Yager}[k,Pre] = V^{Yager}[Proj])$; (c) Finally, the case $(\lambda_P^{[k,Pre]} \gg n; V^{Yager}[k,Pre] \neq V^{Yager}[Proj])$ with *non-consistent* $V^{Yager}[k,Pre]$ constitutes a *non-consistent de-adaptation* respecting the projected system and the rationality of its agents.

The *adaptation condition relative to a change of judgments or Adaptation Ratio* in the *present stage k*, with respect to the *projected stage*, *proj*, will be expressed as:

$$\begin{aligned} R_D^{[k,Pre]} &= Max\left\{0; \left[I_D^{[k,Pre]} - RC^{[Proj]}\right]\right\} \Rightarrow R_D^{[k,Pre]} \\ &= 0 \rightarrow \ \textit{Perfect adaptation} \end{aligned} \tag{16}$$

5.2. Acceptance variation: Second component of Dynamic Adaptation Vector

The *second component of Dynamic Adaptation Vector*, referred as the *Acceptance Variation* of the *present* stage, is related *to the possibility that the system continues evolving from such a state according to a trajectory with a risk smaller or equal to the extrinsic one.* That is, the system should meet constraint 2) of model (9).

The condition is formulated as:

$$\Delta_{\mu_{\{D\}}}^{[k,Pre]} = Max\left\{0; \frac{\left\{\Theta_{\{D\}}^{*[k,Pre]} - \Theta_{Ext}\right\}}{\Theta_{Ext}}\right\} \tag{17}$$

where

$$\Theta_{\{D\}}^{*[k,Pre]} = \left[1 - \mu_{\{D\}}^{*[k,Pre]}\right] \tag{18}$$

To this, the application of (17) should be *re-stated* by employing $MP_A^{[k,Pre]}$ into Condition (1) of model (9), thus setting forth a new optimization for the period under consideration.

The Adaptation Condition for the system, respecting the extrinsic risk on the new evolution trajectory, is expressed as:

$$\Delta_{\mu_{\{D\}}}^{[k,Pre]} = 0 \tag{19}$$

Therefore, the *Dynamic Adaptation Vector* for a given state *e* on stage *k* (present stage) can be expressed as:

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$$V_{Adap}^{[k,Pre]} = \begin{bmatrix} R_D^{[k,Pre]} \\ \Delta_{\mu_{\{D\}}}^{[k,Pre]} \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \textit{Perfect Adaptation Condition} \quad (20)$$

The proposed Adaptation Vector is totally a dynamic parameter because its definition is based on the *historical-evolutionary nature of the system*. The *dynamic adaptation* does not need to continue with the same evolution trajectory, neither does it require that the comparing states have the same aptitude, a necessary condition for a *static adaptation*.

6. Simulation – A study case

6.1. Stage 1: planning. System attributes and parameters from judgment matrix and FDP

It is first assumed that the long term of the strategy planning has already been solved through the above-suggested techniques in Section 3. The mid-term planning, in this case, coincident with control tariff period (5 years), has been solved through the model (9). In the Appendix the equations and tables of results obtained are presented. Five criteria, seen as attributes in the optimization problem, are considered for the PDS: Global Cost (GC); Non-Supplied Energy (NSE); Interruption Rate (FI); Out-of-Supply Voltage (OSV) and Global Power Losses (GPL). The membership functions are exponential (decreasing), but another form-characteristic may be proposed. One cannot fully describe in the paper all the details of the example, but most relevant steps, following this introduction (helped with Tables), illustrate the results obtained.

Step 1: Definition of Judgment Matrix $\boldsymbol{MP}_A$ in the planning (or projected) stage:

$\boldsymbol{MP}_A$	GC	NSE	FI	OSV	GPL
GC	1	3	3	2	1
NSE	1/3	1	1	2/3	1/3
FI	1/3	1	1	2/3	1/3
OSV	1/2	3/2	3/2	1	1/2
GPL	1	3	3	2	1

(21)

Step 2: The following *extrinsic robustness* $\Theta_{Ext} = 0.18$ has been fixed externally to bind the *set (in general) of satisfactory trajectories in the evolution of the system with an extrinsic risk*

$$1 - \Theta_{Ext} = 1 - 0.18 = 0.82 \quad (22)$$

Step 3: Perron's eigenvalue of $\boldsymbol{MP}_A$ and Vector of Yager Weighers: In this simulation, $\boldsymbol{MP}_A$ have consistent judments (preferences between attributes) and *Perron's eigenvalue* result:

$$\lambda_P^{[Proj]} = 5 \quad (23)$$

and the Transpose Vector of Yager Weighers:

$$V_T^{Yager[Proj]} = [0.5 \quad 1.5 \quad 1.5 \quad 1 \quad 0.5] \quad (24)$$

Step 4: Application of Fuzzy Optimization Model (9): In general there will be an *Evolution Trajectory Set* T_i^*, that belongs to the set bounded by Θ_{Ext}. It ensures the membership condition (constraint (2) of model (9)):

$$\mu_{\{D\}}[T_i^*] \geqslant [1 - \Theta_{Ext}] \quad (25)$$

Table 1
Penalties in *Urban* areas **ESNQC [$/kWh]–Rural [MV], [LV]**.

$0.05 \leqslant \Delta u^u < 0.06 \rightarrow 0.013$	$0.12 \leqslant \Delta u^u < 0.13 \rightarrow 0.300$
$0.06 \leqslant \Delta u^u < 0.07 \rightarrow 0.026$	$0.13 \leqslant \Delta u^u < 0.14 \rightarrow 0.700$
$0.07 \leqslant \Delta u^u < 0.08 \rightarrow 0.039$	$0.14 \leqslant \Delta u^u < 0.15 \rightarrow 1.100$
$0.08 \leqslant \Delta u^u < 0.08 \rightarrow 0.052$	$0.15 \leqslant \Delta u^u < 0.16 \rightarrow 1.400$
$0.09 \leqslant \Delta u^u < 0.10 \rightarrow 0.070$	$0.16 \leqslant \Delta u^u < 0.18 \rightarrow 1.800$
$0.10 \leqslant \Delta u^u < 0.11 \rightarrow 0.086$	$\Delta u^u \geqslant 0.18 \rightarrow 2.000$
$0.11 \leqslant \Delta u^u < 0.12 \rightarrow 0.100$	

Table 2
Penalties in *Rural* areas.

$0.10 \leqslant \Delta u^u < 0.11 \rightarrow 0.025$
$0.11 \leqslant \Delta u^u < 0.12 \rightarrow 0.050$
$0.12 \leqslant \Delta u^u < 0.13 \rightarrow 0.075$
$0.13 \leqslant \Delta u^u < 0.14 \rightarrow 0.100$
$0.14 \leqslant \Delta u^u < 0.15 \rightarrow 0.300$
$0.15 \leqslant \Delta u^u < 0.16 \rightarrow 0.700$
$0.16 \leqslant \Delta u^u < 0.18 \rightarrow 1.400$
$\Delta u^u \geqslant 0.18 \rightarrow 2.000$

For this simulation, it is considering only one trajectory (corresponding to the maximum value of membership in the fuzzy decision set, $\{D\}$). Applying the equations presented in the Appendix, with the parameters in the membership function for each fuzzy set, $\mu_{Ai}(u_i)$: $\alpha = 0.5$; $\beta = 0.2$; $\delta = 0.2$; $\gamma = 0.2$; $\rho = 0.7$, the results shown in Tables 1–8, are obtained. Then, the Evaluation Trajectory of the system for all $N = 6$ stages of the considered period has attained a maximum membership value

$$\mu_{\{D\}}[T^*] = 0.9181 \geqslant [1 - \Theta_{Ext}] \quad (26)$$

thus satisfying constraint (2) in (9), with (25).

6.2. Stage 2: control. Computing system dynamic de-adaptation

The period spanning 1998–2002 was considered for the mid/short-term planning. When surveying the system data for year 2001 (k = IV), the *present stage* is established (see Table 7). In such a Trajectory, the Dynamic Adaptation Vector of (20) will be calculated. For this, the membership of the associated fuzzy sets is computed from the magnitudes evaluated for each system attribute in (k = IV).

By applying condition (14) and considering the projected values for each attribute in such a stage, the new Transpose Vector of Yager's Weighers is attained (expression (13)). Then:

Step 1: Vector of Yager Weighers of $\boldsymbol{MP}_A^{[k=IV,Pre]}$:

$$V_T^{Yager[k=IV,Pre]} = [0.1694 \quad 1.5 \quad 1.5 \quad 0.1342 \quad 0.3077] \quad (27)$$

Step 2: Judgment Matrix for the present state, $\boldsymbol{MP}_A^{[k=IV,Pre]}$, and *Perron's eigenvalue*:From (12), $\boldsymbol{MP}_A^{[k=IV,Pre]}$ is obtained. Its Perron's eigenvalue results:

$$\lambda_P^{[k=IV,Pre]} = 9.0084 \quad (28)$$

Step 3: *Adaptation Ratio*: For five attributes (Table 5), there is a Random Consistency Coefficient:

$$CCA(5) = \mathbf{0.9} \quad (29)$$

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Table 3
Transition Values of Attributes in Planning Stages with FDP. $\mathbf{e_i}$:states in stage K-1; $\mathbf{e_j}$ **:states in stage K; GC [k$]; NSE [MWh]; GPL [KW].Other Attributes are addimentional. Stage I: 1998; Stage II: 1999; Stage III: 2000; Stage IV: 2001; Stage V: 2002; Stage VI: 2003 (begin of year)**.

K-1	e_i	K	e_j	GC	NSE	FI	OSV	GPL
I	1	II	1	128.01	17050.17	0.73	0.0453	185.40
I	1	II	2	153.31	7930.59	1.19	0.0477	210.45
I	1	II	3	166.80	6919.35	1.02	0.0362	150.04
I	1	II	4	177.72	14994.58	0.81	0.0403	178.90
I	1	II	5	189.37	9144.42	0.50	0.0393	190.65
II	1	III	1	107.89	9042.98	0.89	0.0473	150.00
II	1	III	2	111.46	11656.20	0.98	0.0392	178.65
II	1	III	3	139.19	17224.94	1.02	0.0389	145.66
II	1	III	4	230.77	10196.59	0.89	0.0375	201.54
II	2	III	1	141.37	13443.41	1.46	0.0417	186.54
II	2	III	2	117.29	13117.08	0.64	0.0403	170.65
II	2	III	3	150.31	9708.09	1.65	0.0457	164.30
II	2	III	4	328.77	9083.66	0.66	0.0399	180.43
II	3	III	1	172.33	17377.21	1.43	0.0405	193.45
II	3	III	2	124.42	10094.92	0.94	0.0436	158.69
II	3	III	3	204.09	9761.17	1.49	0.0404	193.21
II	3	III	4	225.18	16849.79	1.31	0.0373	154.68
II	4	III	1	133.43	10722.77	1.01	0.0402	180.35
II	4	III	2	121.05	18284.90	0.57	0.0394	175.34
II	4	III	3	186.81	18279.32	1.38	0.0410	160.04
II	4	III	4	302.12	11088.98	1.60	0.0483	165.98
II	5	III	1	98.95	18324.34	0.90	0.0474	190.02
II	5	III	2	158.29	17852.25	0.79	0.0416	201.54
II	5	III	3	190.68	14236.06	1.31	0.0448	178.54
II	5	III	4	213.78	8451.13	1.62	0.0386	164.66
III	1	IV	1	75.97	7059.08	1.24	0.0451	130.02
III	1	IV	2	104.08	16356.60	1.56	0.0471	125.43
III	1	IV	3	137.75	16652.26	1.32	0.0456	150.08
III	1	IV	4	170.10	7119.08	1.04	0.0516	135.98
III	2	IV	1	110.66	14618.64	1.60	0.0506	115.64
III	2	IV	2	103.46	17963.78	1.57	0.0468	105.66
III	2	IV	3	130.95	8184.89	0.97	0.0512	121.45
III	2	IV	4	162.53	11248.05	0.80	0.0481	140.08
III	3	IV	1	136.32	17822.66	1.07	0.0480	109.12
III	3	IV	2	107.65	8612.47	0.87	0.0491	135.08
III	3	IV	3	128.15	16485.70	0.93	0.0521	127.41
III	3	IV	4	129.30	15456.34	1.48	0.0530	140.09
III	4	IV	1	128.53	15233.89	1.33	0.0491	112.10
III	4	IV	2	129.27	10873.81	1.45	0.0441	101.23
III	4	IV	3	103.13	16075.44	0.67	0.0461	120.09
III	4	IV	4	171.22	17967.88	1.11	0.0471	123.80
IV	1	V	1	125.69	18489.06	1.39	0.0565	165.05
IV	1	V	2	213.77	17949.82	2.14	0.0590	170.09
IV	1	V	3	140.57	9343.41	0.87	0.0548	163.23
IV	2	V	1	236.58	16607.82	0.89	0.0615	155.44
IV	2	V	2	154.60	8797.17	1.62	0.0515	151.02
IV	2	V	3	169.42	20180.27	1.93	0.0556	155.43
IV	3	V	1	125.61	17531.08	1.31	0.0546	180.98
IV	3	V	2	165.38	18050.65	1.94	0.0539	185.03
IV	3	V	3	155.81	16699.96	1.43	0.0535	170.41
IV	4	V	1	182.86	19675.06	1.25	0.0576	165.79
IV	4	V	2	193.92	17445.31	0.94	0.0601	159.65
IV	4	V	3	161.59	8552.54	1.50	0.0577	180.10
V	1	VI	1	126.79	11404.11	1.56	0.0643	289.14
V	2	VI	1	191.56	8429.04	1.70	0.0583	245.35
V	3	VI	1	188.30	21474.51	2.21	0.0610	268.57

Table 4
Reference Values of Attributes in Planning Stages with FDP.

Stage	NSE Ref [MWh]	FI Ref	$I_{OSV}Ref$	$Min\ I_{OSV}Ref\ Ur$	Min $I_{OSV}Ref\ Ru$
II	11826	0.8	0.0381	0.0205	0.0175
III	12483	1.1	0.0362	0.0215	0.0146
IV	12921	1.1	0.0401	0.0228	0.0173
V	13578	1.5	0.0441	0.0244	0.0197
VI	14673	1.5	0.0515	0.0261	0.0253

Table 5
Random Consistency Coefficient, CCA(n).

1	2	3	4	5	6	7	8	9
0	0	0.58	0.9	0.9	1.12	1.24	1.41	1.45

The *Consistency Ratio* corresponding to the *projected system* (for the entire horizon under study) becomes:

$$RC^{Proj} = 0,\ \textit{because the system exhibits perfectly consistent preferences} \tag{30}$$

and, considering (32). the De-Adaptation Index for the present stage [k = IV], is (from (11)):

$$I_D^{[k=IV,Pre]} = \frac{\left(\frac{9.0084-5}{5-1}\right)}{0.9} = 1.1134 \tag{31}$$

Then, the Adaptation Ratio

$$R_D^{[k=IV,Pre]} = Max\{0;[1.1134-0]\} = 1.1134 \tag{32}$$

It is not thus complied with the adaptation condition relative to a change in judgment (20), and, from Section 5.1, the system could exhibit (it must be judged by the regulatory authority) a non-consistent de-adaptation.

Step 4: *Acceptance Variation*: The adopted model (FDP + AHP) is again applied by adopting: (a) the *present stage* as the *initial (only one state)* [k = IV], (b) the present Vector of Yager's Weighers, and (c) a dynamics on a horizon of $(k-N)$ *remainder* stages *governed by the same mid/short-term planning*. Hence, the following membership value is attained (see Table 8):

$$\mu_{\{D\}}^{*[k=IV,Pre]} = 0.7758 \rightarrow \Theta_{\{D\}}^{*[k=IV,Pre]} = 1 - 0.7758 = 0.2242\ (\text{from } (18)) \tag{33}$$

and the *Acceptance Variation*, *the second component* of the *Dynamic Adaptation Vector*, results to be:

$$\Delta_{\mu_{\{D\}}}^{[k=IV,Pre]} = Max\left\{0;\frac{\{0.2242-0.18\}}{0.18}\right\} = 0.2455 \tag{34}$$

Therefore, from (19), the adaptation condition of the system, respecting the extrinsic risk on the new evolution trajectory, is not satisfied.

Finally, in order to judge about the possible *de-adaptation* of the system, the Dynamic Adaptation Vector can be used:

Table 6
Optimal Trajectory in Planning Stages with FDP.

Optimal State	$\mu_D[T^*]$	$\mu^*_{u_{GC}}$	GC [k$]	$\mu^*_{u_{NSE}}$	NSE [MWh]	$\mu^*_{u_{FI}}$	FI	$\mu^*_{u_{OSV}}$	OSV	$\mu^*_{u_{GPL}}$	GPL [kW]
[1, I]	1.000	1.0000	0.000	1.0000	0.00000	1.0000	0.000	1.0000	0.0000	1.0000	0.000
[4, II]	0.9228	1.0000	177.720	0.9228	14994.580	0.9963	0.810	0.9774	0.0403	1.0000	178.90
[1, III]	0.9228	0.9233	311.150	1.0000	10722.770	1.0000	1.010	0.9996	0.0402	0.9754	359.25
[4, IV]	0.9228	0.9850	481.250	1.0000	7119.080	1.0000	1.040	0.9665	0.0516	0.9470	495.230
[2, V]	0.9181	0.9634	675.170	0.9181	17445.310	1.0000	0.940	0.9669	0.0601	0.9403	654.880
[1, VI]	**0.9181**	0.9313	866.730	1.0000	8429.040	0.9608	1.700	0.9998	0.0583	1.0000	900.230

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Table 7
Values of Attributes and Membership Function (without Yager's Weighers) in Present State, Stage k=IV.

Present State at Stage	$\mu^*_{u_{GC}}$	GC [k$]	$\mu^*_{u_{NSE}}$	NSE [MWh]	$\mu^*_{u_{FI}}$	FI	$\mu^*_{u_{OSV}}$	OSV	$\mu^*_{u_{GPL}}$	GPL [kW]
Ref-Min [IV]		467.123		12921.000		1.100		0.0401		459.485
[IV]	0.9146	550.5	0.9791	14280.550	0.9837	1.190	0.7758	0.091	0.8378	575.65

Table 8
Optimal Trajectory in Re-Planning Stages with FDP.

Optimal State	$\mu_D[T^*]$	$\mu^*_{u_{GC}}$	GC [k$]	$\mu^*_{u_{NSE}}$	NSE [MWh]	$\mu^*_{u_{FI}}$	FI	$\mu^*_{u_{OSV}}$	OSV	$\mu^*_{u_{GPL}}$	GPL [kW]
[4, IV]	0.7758	0.9850	550.5	0.9689	14280.550	0.9757	1.190	0.7758	0.091	0.9470	575.65
[1, V]	0.7758	0.7804	785.48	0.8614	19345.540	0.9367	1.340	0.90452	0.190	0.7854	974.71
[1, VI]	**0.7758**	0.8633	957.36	0.8206	21435.72	0.9089	1.450	0.98742	0.0590	0.7944	950.54

$$V_{Adap}^{[k=IV,Pre]} = \begin{bmatrix} R_D^{[k=IV,Pre]} \\ \Delta^{[k=IV,Pre]}_{\mu_{\{D\}}} \end{bmatrix} = \begin{bmatrix} 1.1134 \\ 0.2455 \end{bmatrix} \tag{35}$$

The new trajectory starts from a state/stage whose relative judgments *are not adapted* (the *Adaptation Ratio* is greater than 0), and decision-making level (acceptance) that *does not meet the risk threshold restriction* (the *Acceptance Variation of the present State* is greater than 0 too). In addition to the above, there still remains the point of evaluating whether the Judgment Matrix meets a logic permitted by the regulator. This could be evaluated alternatively by comparing the Yager's Vectors corresponding to the projected and present stages (associated to $\boldsymbol{MP}_A^{[k=IV,Pre]}$) in the stage $k = IV$.

7. Conclusions

(1) The traditional concept of an Economically Adapted System is based on the Neoclassic economics paradigm. It is related only to the *productive efficiency*, which implies the expansion and operation of system with a minimum cost. It assumes as well a permanent equilibrium, which is a reason for establishing that such efficiency must take place even in the future, regardless that the planning decisions are made in the present. It ignores the uncertainties, or it renders them a stochastic nature, which they do not necessarily show to have. The proposed model implies, in such a context, *a change of paradigm*.

(2) In this *alternative paradigm*, featured by the *fundamental uncertainty* of Keynes [14] and *driven by the risk*, it is rather preferred to have a set of good solutions (evolution trajectories for the system) than an 'optimal' solution that is based on an inexistent equilibrium.

(3) In this context, a new model to evaluate the Dynamic Deadaptation of a Distribution System with respect to its planning for a given period of tariff control, is presented. It is applied in the simulation (study case), considering five system attributes (as merit measures) observed by regulation of Argentina. As result, the system evolves as the most satisfactory trajectory (*intrinsic risk*). If the *extrinsic risk* is based on opportunity costs of those attributes, and the constraint (2) of model is satisfied, then such trajectory guarantees the system adaptation.

(4) In terms of *system adaptation* for a certain present stage k, $V^{Yager[k,Pre]}$ can be reconstructed. However, the equation for the hierarchic composition principle: $\boldsymbol{MP}_A^{[k,Pre]} \times V^{Yager[k,Pre]} = \lambda_P^{[k,Pre]} \times V^{Yager[k,Pre]}$ does not have a unique solution in $(\boldsymbol{MP}_A^{[k,Pre]}, \lambda_P^{[k,Pre]})$. The approach proposed in Eq. (12), is aimed at finding a solution that also incorporates the randomness as an adaptive element of the system. In this way, and confronting the *instrumentalism* that is featured by the dominant Neoclassic paradigm, this work has tried to incorporate a greater *realism* into the methodology.

(5) The evaluation of the static adaptation of the above point is complemented by a dynamic evaluation, by taking into account the *Adaptation Variation* in the present stage. It introduces the disturbance that the future system trajectory may experience as a consequence of its history.

Appendix A. Definition of departures from Eq. (1) and membership function of fuzzy set for each system attribute

In all descriptions, the Regulatory Authority is the Authority of Argentina, Province of Rio Negro. All departure variables and fuzzy sets of attributes are proposed in this work, and there not exist proposal, in the regulation, about this matter from this authority. The sub-indication $[j,k]$ refer the state j in the stage k.

A.1. Global cost

$$\begin{array}{ll} \textit{Departure } u_{GC} & \textit{Membership Function of Fuzzy Set} \\ u_{GC[j,k]} = \frac{AC_{v[j,k]} - Min\{AC_{v[j,k]}\}_{k-1}}{Min\{AC_{v[j,k]}\}_{k-1}} & \mu_{u_{GC[j,k]}} = e^{-\alpha \times u_{GC[j,k]}} \end{array} \tag{A.1}$$

where $0 < \alpha \leqslant 1$, and $AC_{v^{[j,k]}}$ is the annual cost in investment (choice *v*of equipment) plus system's maintenance in $[j,k]$. $Min\{AC_{v^{[j,k]}}\}_{k-1}$ is the minimum cost of transition, between stages $(k-1,k)$ applying a classic dynamic programming approach, as a reference *GC* for each state.

A.2. Non-supplied energy

$$\begin{array}{ll} \textit{Departure } u_{NSE} & \textit{Membership Function of Fuzzy Set} \\ u_{NSE[j,k]} = Max\left\{0; \frac{(NSE_{[j,k]} - NSE^{Ref}_{[k]})}{NSE^{Ref}_{[k]}}\right\} & \mu_{u_{NSE[j,k]}} = e^{-\beta \times u_{NSE[j,k]}} \end{array} \tag{A.2}$$

where $NSE^{Ref}_{[k]}$ is a external limit for stage k, of *NSE*, and $0 < \beta \leqslant 1$. The limit $NSE^{Ref}_{[k]}$ is imposed for Regulatory Authority, as function of demand in each stage k.

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A.3. Interruption rate

Departure u_{FI} *Membership Function of Fuzzy Set*

$$u_{FI[j,k]} = Max\left\{0; \frac{\left(FI_{[j,k]} - FI^{Ref}_{[k]}\right)}{FI^{Ref}_{[k]}}\right\} \quad \mu_{u_{FI[j,k]}} = e^{-\delta \times u_{FI[j,k]}} \tag{A.3}$$

$$\text{with}: FI_{[j,k]} = \sum_{i \in NInt}\left(\frac{Q^{os}_{CT_{i[j,k]}}}{Q_{CT_{Inst_{[j,k]}}}}\right) \tag{A.4}$$

where $FI^{Ref}_{[k]}$ is a external limit for stage k, of FI, and $0 < \delta \leqslant 1$; $Q^{os}_{CT_{i[j,k]}}$ is the number of substations MV/LV out of service under interruption i, in the state j, stage k; and $Q_{CT_{Inst_{[j,k]}}}$ is a number of total installed substations MV/LV in $[j,k]$; $NInt$ is a total number of interruptions in $[j,k]$. The limit $FI^{Ref}_{[k]}$ is imposed for the Regulatory Authority.

A.4. Out-of-supply voltage

Departure u_{OSV} *Membership Function of Fuzzy Set*

$$u_{OSV[j,k]} = Max\left\{0; \frac{\left(IOSV_{[j,k]} - IOSV^{Ref}_{[k]}\right)}{IOSV^{Ref}_{[k]}}\right\} \quad \mu_{u_{OSV[j,k]}} = e^{-\gamma \times u_{OSV[j,k]}} \tag{A.5}$$

where $0 < \gamma \leqslant 1$ and $IOSV^{Ref}_{[k]}$ is an index limit for stage k, which construction require the following steps:

(a) Exist a set of penalties imposed for the Regulatory Authority to Energy Supplied in Non-Quality Conditions (ESNQC). This penalty, depends on two factors: (a) Density of Distribution (Urban [Ur] or Rural [Ru] areas) and (b) maximum voltage drop in the feeders $\left(\Delta u^u = \left|\frac{u - u_{nom}}{u_{nom}}\right|\right)$. Tables 1 and 2 show penalties corresponding, depending of limits in the factor (b), (Δu^u), for each distribution area (Urban and Rural), which have an Energy Supplied E^{Total} in $[j,k][Ur/Ru]$.

(b) With this penalties, it construct an auxiliary index, which to weight the energy supplied estimated ($EfOSV^{Ref,i}_{[j,k]Ru}$, $EfOSV^{Ref,i}_{[j,k]Ur}$) in all nodes, i, in each area (Urban, $nNOSV_{[j,k]Ur}$ – Rural, $nNOSV_{j,k]Ru}$) (see Tables 1 and 2), on an average assessment factor: $\left(\frac{\$p_{[FirstD][Ru/Ur]}}{\$p_{[FirstD]Ru} + \$p_{[FirstD]Ur}}\right)$ depending on the penalty applied to first step (departure of Δu^u) and the total energy supplied. Then:

$$IOSV^{Ref}_{[j,k][Ru/Ur]} = \left[\frac{\sum_{i=1}^{nNOSV_{[k,j][Ru/Ur]}} EfOSV^{Ref,i}_{[j,k][Ru/Ur]}}{E^{Total}_{[j,k][Ru/Ur]}}\right] \times \left[\frac{\$p_{[FirstD][Ru/Ur]}}{\$p_{[FirstD]Ru} + \$p_{[FirstD]Ur}}\right] \tag{A.6}$$

(c) Adding the minimum value obtained from both auxiliary index, in each stage k, it obtained a reference composite index:

$$IOSV^{Ref}_{[k]} = Min\left\{IOSV^{Ref}_{[k,j]Ur}\right\}_{\forall j \in k} + Min\left\{IOSV^{Ref}_{[k,j]Ru}\right\}_{\forall j \in k} \tag{A.7}$$

(d) Then, the index $IOSV_{[j,k]}$ can be calculated to each departure of Δu^u, with the number of nodes corresponding, through Eq. (A.6) (for separate, Urban and Rural and applying the corresponding penalty):

$$IOSV_{[j,k][Ru/Ur]} = \frac{\sum_{i=1}^{nNOSV_{[k,j][Ru/Ur]}}\left[EfOSV^{Ref,i}_{[j,k][Ru/Ur]} \times \$p_{[\Delta u^u]^i[Ru/Ur]}\right]}{E^{Total}_{[j,k][Ru/Ur]} \times \left[\$p_{[FirstD]Ru} + \$p_{[FirstD]Ur}\right]} \tag{A.8}$$

Then, adding the results, the departure Index is compound as:

$$IOSV_{[j,k]} = IOSV_{[j,k]_{Ur}} + IOSV_{[j,k]_{Ru}} \tag{A.9}$$

The number of nodes $nNOSV_{[k,s]Ur/Ru}$ is estimated by means an AC power flow in the sub-system MV (13.2 [kV]) in study case. Therefore, each substation MV/LV (13.2/0.38 [kV]), become an evaluation node. In addition with this Index, are accepted voltage violations, without penalties, only in the first departure of Δu^u for both areas [Ur/Ru]. This is based on that the first step of penalties, is imposed only as reference and, in practice, non-penalty is applied. The substations MV/LV under evaluation, have a typical loads curve. From this, the corresponding ESNQC is estimated for each node.

A.5. Global power loss

Departure u_{GPL} *Membership Function of Fuzzy Set*

$$u_{GPL[j,k]} = \frac{Ploss_{v[j,k]} - Min\left\{Ploss_{v[j,k]}\right\}_{k-1}}{Min\left\{Ploss_{v[j,k]}\right\}_{k-1}} \quad \mu_{u_{GPL[j,k]}} = e^{-\rho \times u_{GPL[j,k]}} \tag{A.10}$$

where $0 < \rho \leqslant 1$, and $Min\{Ploss_{v[j,k]}\}_{k-1}$ are minimal power losses of transition, between stages $(k-1,k)$, obtained from a classic dynamic programming approach, as a reference *Ploss*, for each state.

References

[1] Bellman R, Dreyfus E. Applied dynamic programming. Princeton University Press; 1962.
[2] Bellman R, Zadeh L. Decision-making in a fuzzy environment. Manage Sci 1970;17:141–64.
[3] Saaty TL. A scaling method for priorities in hierarchical structures. J Math Psychol 1977;15:234–81.
[4] Saaty TL. The analytic hierarchy processes. New York: McGraw-Hill; 1980.
[5] El-Fouly THM, Zeineldin HH, El-Saadany EF, Salama MMA. A new optimization model for distribution substation stiting, sizing, and timing. Int J Electr Power Energy Syst 2008;30(5):308–15.
[6] Kaur D, Sharma J. Optimal conductor sizing in radial distribution system planning. Int J Electr Power Energy Syst 2008;30(4):261–71.
[7] Miranda V, Srinivasan D, Proença LM. Evolutionary computation in power systems. Int J Electr Power Energy Syst 1998;20(2):89–98.
[8] Carvalho PMS, Ferreira LAFM, Lobo FG, Barruncho LMF. Optimal distribution expansion planning under uncertainty by evolutionary decision convergence. Int J Electr Power Energy Syst 1998;20(2):125–9.
[9] Lax PD. Linear algebra. New York: Wiley Interscience; 1997. p. 196 [chapter 16].
[10] Yager RR. Multiple objective decision making using fuzzy sets. Int J Man–Mach Stud 1977;9:53–64.
[11] Schweickardt G, Andreoni A, Miranda V. Evaluation of risk bounded distribution expansion cost with a fuzzy dynamic programming approach. In: Proceedings of the 14th PSCC'02. Sevilla, Spain; 2002.
[12] Schweickardt G, Miranda V. A fuzzy dynamic programming approach for evaluation of expansion distribution cost in uncertainty environments. J LAAR 2007;37(4):227–34.
[13] Miranda V, Proença LM. Why risk analysis outperforms probabilistic choice as the effective decision support paradigm for power systems. IEEE Trans Power Syst 1998;13(3).
[14] Lavoie M. Foundations of PostKeynesian economic analysis. Edward Elgar Publishing; 1992.

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