Advanced Models and Algorithms for Demand Participation in Electricity Markets

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***Abstract*—This paper proposes a novel Energy Aggregator model responsible for managing the flexibility of low voltage customers in order to reduce electricity costs. The flexibility of the customers is represented by the availability of their controllable loads to reduce/increase power consumption. The flexible loads are managed according to the customers' preferences and the technical limitations of the flexible loads. The Energy Aggregator model developed includes an algorithm designed to manage customers' flexibility in quasi-real-time, with the objective of minimizing the deviations from the energy bought by the aggregator in the market. A scenario with 30 households located in a semi-urban area is used to illustrate the application of the algorithm and validate the proposed approach.**

Keywords—Aggregators, Controllable Loads, Demand Response, Electricity Markets, Physically-Based Load Models

I. INTRODUCTION

Nowadays, retailers buy electricity from the market without direct participation of the retail-customers. This leads to inefficiency and non-competitiveness in electricity markets [1], since it does not incentivize the customers to change their consumption as a function of electricity prices.

Even though electricity costs vary substantially from hour to hour within a single day, most customers buy electricity based on flat or time-of-use rates that are set well in advance of actual use. Low voltage customers should be given the chance to choose the most suitable tariff for their needs, either real-time (hourly), flat or time-of-use prices.

On the one hand, allowing customers to respond to the variability of electricity prices would improve economic efficiency, operational security, and reduce environmental impacts from electricity generation sector [1, 2]. On the other hand, real-time pricing also has potential to bring benefits to the demand side, since low voltage customers can optimally adjust their energy consumption by participating into demand response programs and reduce their electricity bill [3]. In fact, demand response is identified as a key functionality in smart grids, which will not only turn markets more efficient and competitive, but also improve energy efficiency and increase global reliability of power systems [4, 5].

To minimize the electricity bill, consumers engaged in price-based demand response would be able to respond to time varying electricity prices and shift their consumptions to periods with lower electricity prices.

In [6] an optimal and automatic energy consumption scheduling framework for operating appliances in a household is proposed, while considering the trade-off between minimum electricity payment and maximum consumer's utility. In order to address a similar problem, an optimization model to manage appliances with the objective of adjusting their consumption in response to real-time (hourly) prices was developed in [7]. In [8] a different approach is presented, where low voltage customers are assumed to have a contract with an Aggregator in order to have access to the market. This Aggregator is responsible for sending price-volume signals, which consist in specifying a monetary reward (price) if power consumption during certain hours of the day is below a specified threshold (volume). Based on these signals, the end-user manages its appliances to maximize the monetary reward, taking into account its own preferences and needs.

In this paper, a novel Energy Aggregator (EA) model for domestic clients is proposed. The EA is responsible for grouping and managing the consumption of low voltage clients, in order to reduce electricity costs. With this purpose, the EA manages the flexibility of each client, by controlling the flexible loads available inside its household.

The proposed framework includes the development of an algorithm designed to manage the clients' flexibility in quasi-real-time (shorts periods of time), in order to minimize the deviations from the electricity bought by the EA in the day-ahead spot market.

The referred algorithm uses thermostatically controlled loads (such as electric water heaters, air conditioners and refrigerators) as well as electric vehicles (EV) to simulate load control actions. Physically-based load models are used to simulate the thermostatically controlled loads. Regarding EV, assumptions on their power consumption and state-of-charge control conditions are considered.

A case study of 30 households located in a semi-urban area of Porto (Portugal) was used in order to test the feasibility of the algorithm developed.

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II. ENERGY AGGREGATOR FRAMEWORK

The EA is an entity responsible for grouping and managing the consumption of low voltage clients (households), with the objective of reducing electricity costs.

In the previous day, the EA buys electricity in the spot-market, according to the flexibility of their clients. Based on the electricity bought, the EA manages the flexibility of the households in quasi-real time, in order to minimize the deviations between the real and forecasted consumption.

In order to manage the clients' consumption, the EA needs to communicate with a local controller installed at the customer's site – the Home Energy Manager (HEM). The main functionalities of the HEM are monitoring the consumption of flexible and inflexible loads and controlling the consumption of flexible loads, according to control actions performed by the EA. The flexible loads are managed by the HEM, taking into account the consumer preferences and their technical restrictions. The HEM framework is presented in Fig.1.

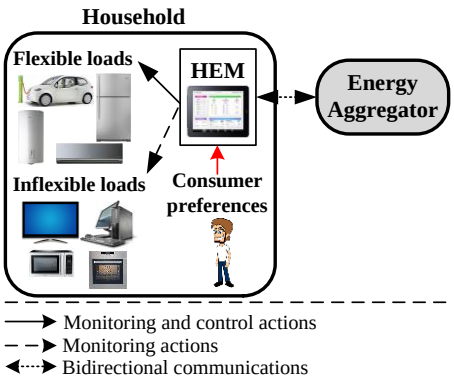

Fig. 1. Energy aggregator' hierarchical management structure

The feasibility of the proposed framework was evaluated using a procedure divided in three phases (Fig. 2).

The first phase consists in defining the availability of households to reduce/increase consumption, for the time step (t+1) – these calculation are performed by the HEM. The flexibility is calculated according to the consumer preferences and the technical constraints of the flexible loads. Additionally, the HEM forecasts the consumption of the inflexible loads.

Afterwards, in the second phase, this information is communicated to the EA that may use it to adjust the consumption of the households, in order to minimize the deviations from the electricity bought in the day-ahead spot market. For this purpose, an algorithm based on the optimization problem presented in section IV B is used. The second phase ends when the EA communicates to all HEM the new operating point of the households.

In the third phase, based on the information sent by the EA, the consumption of each flexible load in time step (t+1) is defined by the HEM. For this purpose, an optimization algorithm is used, which takes into account the availability of each flexible load. In order to enforce these power values, the HEM sends control signals to the controllable loads. This cycle is successively repeated.

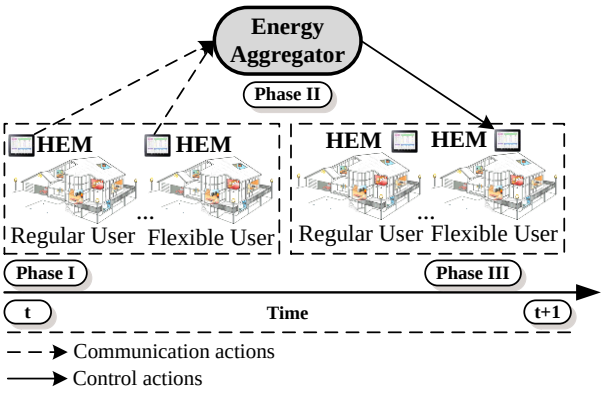

Fig. 2. Management algorithm framework

The mode of operation described was named "Flexible Operation". Besides this mode, the EA also can work as a traditional retailer ("Regular Operation"), where the clients consume without being subject to any control action. In regular operation, the EA only monitors the clients' consumption.

III. REGULAR OPERATION

In Regular operation, the clients consume electricity without being subject to any control action from the EA.

The behaviour of the domestic loads is defined according to the type of load considered: flexible or inflexible. The consumption of the inflexible loads is estimated, taking into account the customers' habits and the average electricity consumption. In this work, loads such as dishwashers, washers, computers, dryers, televisions, electric stoves and lighting are considered inflexible. Conversely, thermostatically controlled loads such as Electric Water Heaters (EWH), Air Conditioning (AC) and refrigerators are considered to be flexible. They were modelled through physically-based load models that define the operating state of the appliances according to their physical characteristics, external temperatures and internal temperatures defined by the household owner. EV were also assumed to be flexible loads.

A. *Modellization of the flexible loads*

The model used for the AC was based on an inverter system, since it is more efficient than traditional AC and can regulate the temperature with fewer oscillations. The inverter AC regulates the temperature (θ_t) of the room by a thermostat and an inverter that speed up or slows down the compressor, in order to reach the temperature set point (θ^s). In regular operation, the behaviour of the inverter AC for cooling is described by the following mathematical model, developed according to the thermal equation presented in [9].

$$\theta_{t+1} = \theta_t - \frac{\Delta t}{CR}(\theta_t - \theta_t^o + RP_t\eta + w_t) \tag{1}$$

$$P^{Var} = \frac{(\theta_{t+1}-\theta^s)C}{\Delta t \cdot \eta} - \frac{\theta_{t+1}-\theta_t^o+w_t}{R\cdot\eta} \tag{2}$$

$$P_{t+1} = \begin{cases} P^{Max}, & if\ P^{Var} > P^{Max} \\ P^{Min}, & if\ P^{Var} < P^{Min} \\ P^{Var}, & otherwise \end{cases} \tag{3}$$

The index t defines time and $\Delta t(h)$ the time elapsed per step. C and R are the thermal capacitance $(kWh/°C)$ and thermal resistance $(°C/kW)$ of the room and P_t (kW) is the electric power of the inverter AC, which is the thermal power divided by its coefficient of performance (η). θ_t^o (°C) is the outdoor temperature. w_t is a noise process accounting for all heat gain and loss not modelled explicitly, which results from opening and closing doors, solar gains, and operation of other loads. The parameters $P^{Max}$and $P^{Min}(kW)$ represent the maximum and minimum electric power of the inverter AC, while $P^{Var}(kW)$ corresponds to the rates of power that the inverter AC can assume to reach the temperature set point.

Similar thermal equations were used to characterize other types of thermal appliances, namely refrigerators and EWH. However, instead of operating in continuous mode, these appliances regulate their internal temperature through a thermostat and relay actuator that determines whether the compressor is switched on $(m = 1)$ or off $(m = 0)$.

Regarding the refrigerator, its behaviour is described by the following mathematical model, developed according to the thermal equation presented in [10].

$$\theta_{t+1} = \varepsilon\theta_t + (1-\varepsilon)\left(\theta^i - m_t RP\eta\right) \; with \; \varepsilon = e^{-\frac{\Delta t}{RC}} \tag{4}$$

$$m_{t+1} = \begin{cases} 1, & if \; \theta_t > \theta^s + \frac{\delta}{2} \\ 0, & if \; \theta_t < \theta^s - \frac{\delta}{2} \\ m_t, & otherwise \end{cases} \tag{5}$$

Instead of considering the outdoor temperature as in (1), this model uses the indoor temperature (θ^i) of the room.

The behaviour of the EWH is simulated by a mathematical model similar to the refrigerator. Instead of the thermal equation (6), the evolution of the EWH is simulated by an equation proposed in [11] by Chong and Debs.

Concerning the EV, the charging behaviour was characterized by a mathematical model that takes into account assumptions regarding the power $P_t(kW)$ required to be supplied and state-of-charge (SOC) control conditions. Instead of thermal capacitance considered in thermal loads, the EV model includes the battery capacity $-BC$ (kWh). Furthermore, the parameter η is added to the model, in order to represent the efficiency of the EV charging process.

$$SOC_{t+1} = SOC_t + \frac{\eta P_t}{BC}\,\Delta t \tag{6}$$

$$P^{Var} = \frac{\left(SOC^{Max} - SOC_{t+1}\right)BC}{\eta \Delta t} \tag{7}$$

$$P_{t+1} = \begin{cases} P^{Max}, & if \; P^{Var} > P^{Max} \\ P^{Min}, & if \; P^{Var} < P^{Min} \\ P^{Var}, & otherwise \end{cases} \tag{8}$$

It is assumed that all EV will charge until they reach the maximum state-of-charge (SOC^{Max}) or are disconnected from the grid by their owners.

B. Modellization of the inflexible loads

As previously referred, the behaviour of inflexible loads is estimated according to the customers' habits (sleeping periods, occupancy patterns and scheduling programs) and average electricity consumption. Others aspects, such as the number of occupants and daylight hours are also taken into account.

IV. FLEXIBLE OPERATION

In flexible operation, the EA manages the clients' flexibility to minimize the deviations from the electricity bought in the market. The objective of this management strategy is reducing electricity costs to the EA and consequently to the end users.

The flexible operation was simulated through a management algorithm presented in section II. This algorithm is composed of three phases that will be described in the following subsections.

A. Phase 1

The first phase consists in defining the availability of the clients to reduce/increase consumption and forecasting the consumption of the inflexible loads. These calculations are performed by the HEM for time step (t+1).

The flexibility of the customers to reduce/increase consumption is calculated taking into account the clients' preferences and the technical limitations of the flexible loads.

The mathematical model that represents the flexibility to increase/reduce load of the inverter AC $\left(F_{t+1}^{Max}/F_{t+1}^{Min} - kW\right)$ is described by the following equations:

$$Np^{Max} = \frac{\left(\theta_{t+1} - \theta^{Min}\right)C}{\eta \cdot \Delta t} - \frac{\theta_{t+1} - \theta_t^o}{\eta \cdot R} \tag{9}$$

$$Np^{Min} = \frac{\left(\theta_{t+1} - \theta^{Max}\right)C}{\eta \cdot \Delta t} - \frac{\theta_{t+1} - \theta_t^o}{\eta \cdot R} \tag{10}$$

$$F_{t+1}^{Max} = \begin{cases} P^{Max}, & if \; Np^{Max} > P^{Max} \\ P^{Min}, & if \; Np^{Max} < P^{Min} \\ Np^{Max}, & otherwise \end{cases} \tag{11}$$

$$F_{t+1}^{Min} = \begin{cases} P^{Max}, & if \; Np^{Min} > P^{Max} \\ P^{Min}, & if \; Np^{Min} < P^{Min} \\ Np^{Min}, & otherwise \end{cases} \tag{12}$$

The parameters θ^{Max} and θ^{Min} (°C) represent the interval of thermal comfort predefined by the consumer, in order to give flexibility to the HEM to reduce/increase load. Np^{Max} and Np^{Min} (kW) correspond to the necessary power that the appliance needs to match the temperatures θ^{Min} and θ^{Max}, respectively. Thus, the maximum and minimum flexibility is represent by F_{t+1}^{Max} and F_{t+1}^{Min}.

The availability of the refrigerator, EWH and EV to reduce/increase load were calculated by mathematical models presented by J.P. Iria and F.J. Soares in [12].

Finally, it is necessary to sum the minimum and maximum flexibility of all flexible loads $\left(\sum F_{t+1}^{Min}, \sum F_{t+1}^{Max}\right)$ to define the overall availability of the client. This information, together with the forecasted inflexible load, will be afterwards communicated to the EA.

B. Phase 2

In the second phase, the EA defines the consumption of each household to minimize the deviations from the electricity bought in the day-ahead market. Based on the information communicated by each HEM, the EA defines the flexible load

of each customer for time step $(t+1)$, through the optimization algorithm formulated below:

$$Min \left| \frac{EBE_{t+1}}{\Delta t} - \left[IL_{t+1} + \sum_{i=1}^{nc} CFL_{t+1,i} + \sum_{j=1}^{nd} \left(\pi_{t+1,j} \left(DFL_{t+1,j}^{Max} - DFL_{t+1,j}^{Min} \right) + DFL_{t+1,j}^{Min} \right) \right] \right| \quad (13)$$

$$Subject\ to$$

$$CFL_{t+1,i}^{Min} \leq CFL_{t+1,i} \leq CFL_{t+1,i}^{Max} \qquad i = 1, \dots . nc \quad (14)$$

$$\pi_{t+1,j} \in \{0,1\} \qquad j = 1, \dots . nd \quad (15)$$

This problem is solved differently according to the dominant type of flexible loads that households have: "discrete" or "continuous". Households are considered to be "discrete households" when their availability to reduce/increase load results mainly from discrete loads (refrigerator/EWH). Conversely, "continuous households" are those whose availability results mainly from continuous loads (EV/Inverter AC). The indexes j and i define the "discrete households" and the "continuous households", respectively. In turn nd and nc set the number of "discrete" and "continuous" households. The parameter $EBE_{t+1}(kWh)$ represents the energy bought in the market by the EA for the following time interval, $\Delta t\,(h)$. IL_{t+1} (kW) represents the inflexible load forecasted for all households. The variables $CFL_{t+1,i}\,(kW)$ and $\pi_{t+1,j}\,(0\ or\ 1)$ are the decision variables of the optimization problem, which define the amount of flexible load that each "continuous" and "discrete" household will consume, respectively. When $\pi_{t+1,j}$ is equal to 1, it means that the discrete household will consume at their maximum flexible power ($DFL_{t+1,j}^{Max}$). Otherwise, the discrete household will consume at their minimum flexible power ($DFL_{t+1,j}^{Min}$). The variable $CFL_{t+1,i}$ can assume continuous values in the interval $\left[CFL_{t+1,i}^{Min}, CFL_{t+1,i}^{Max}\right]$. The constraints (18) and (19) represent the availability of the households to increase/reduce load in the next period.

Finally, it is important to mention that it was necessary to change the formulation of the optimization problem, in order to solve it through Mixed Integer Programming techniques. The required changes were made using the method described in [13].

C. Phase 3

The third phase consists in defining the consumption of each controllable load, based on the flexible consumption set by the EA for the household. In order to do that, the HEM runs an optimization algorithm that minimizes the deviations between the flexible consumption of the household and the consumption of the flexible loads. The optimization algorithm used is described in [12].

The consumption values are enforced by the HEM through control signals sent to the flexible loads. The control signals specify the operation states (m_t) and power rates (P_t) of the flexible loads. The behaviour of the refrigerator is simulated by the thermal equation (4), according to the operation state set by the HEM. The behaviour of the inverter AC and EV is set by equations (1) and (6), based on the power rates defined by the HEM. The evolution of the EWH is simulated through the thermal equation proposed in [11], based on the operation state defined by the HEM.

V. Case Study

A. Description

The EA framework and algorithm were tested in a scenario with 30 households located in a semi-urban area in Portugal. Two summer weekdays were considered (July 26th and 27th of 2013 were used) to analyse and compare the economic impact of regular operation vs. flexible operation.

The households' consumption is mainly influenced by the owners' habits and the energy use of the domestic loads. In relation to the consumers' habits, different occupancy, bath and sleeping patterns were considered.

Concerning the loads, two types were analysed: flexible and inflexible. The inflexible loads consumption was estimated taking into account typical energy uses [14] and consumers' habits. The behaviour of the flexible loads was defined by load models characterized by the following parameters.

TABLE I. Flexible loads parameters

Parameters	Inverter AC	EWH	Refrigerator	EV
$P(kW)$	0.24 - 1.10	1.5 – 2.0	0.07-0.12	0.0 - 3.68
$C(kWh/°C)$	1.05 - 4.55	0.058 - 0.116	0.03-0.07	-
$R(°C/kW)$	2.86 - 14.29	-	92.6-163.0	-
$\alpha(kW/°C)$	-	0.0009-0.0012	-	-
η	3.0 – 3.5	-	3.0 – 3.5	0.87 - 0.94
$BC(kW/h)$	-	-	-	18 - 26
$\theta^w(°C)$	-	20	-	-
$\theta^d(°C)$	-	55 - 60	-	-
$\theta^i(°C)$	-	20	20	-
$v\ (kW/°C)$	-	0.21-0.35[a]	-	-
Initial SOC	-	-	-	0.15 - 0.25

[a] Correspond to hot water consumption values between 3 and 5 l/min

B. Comfort Levels Adopted

The operation of the thermostatically controlled appliances implies the adoption of comfort levels defined by the owners. In regular operation, the levels of comfort are established by the temperature set point and dead band (Table II).

TABLE II. Levels of thermal comfort in regular operation

Appliances	Temperature set point (°C)	Dead band (°C)
Inverter AC	20 - 21	-
EWH	55 or 60	0.5
Refrigerator	5 - 6	0.5

TABLE III. Intervals of thermal comfort in flexible operation

Appliances	Minimum temperature (°C)	Maximum temperature (°C)
Inverter AC	1-3 below set point	1-3 above set point
EWH	55	70
Refrigerator	1-2 below set point	1-2 above set point

In flexible operation, the users define an interval of thermal comfort for each thermostatically controlled load, in order to provide flexibility to the EA (Table III).

Regarding the EV, as referred previously, it was assumed that owners require the battery fully charged at the time of disconnection.

C. *Electricity Prices and Outdoor Temperature*

For the purpose of this work, the energy bought by the EA in the day-ahead market was roughly estimated based on the electricity prices for the Iberian market, on the outdoor temperature and on forecasts of the flexible load available in the clients' households. An optimization problem was then formulated, using this data as inputs, with the objective of defining the EA's bids that would minimize the price paid for the energy.

The electricity prices used in the optimization problem are presented in Fig. 3. These values are real prices from the Iberian electricity market that were collected for July 26th and 27th of 2013.The measured outdoor temperature for the same days is also presented in Fig. 3.

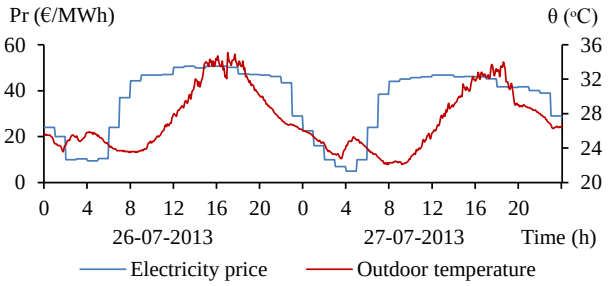

Fig. 3. Price of electricity in the Iberian Market [15] and outdoor temperature

The output of the optimization problem, *i.e.* the energy bought by the EA in the day-ahead market, is represented by the green line in Fig. 4.

VI. RESULTS

The results obtained from the simulations performed with the 30 households operating in regular and flexible operation are presented in this section. An economic analysis is performed and an overall comparison between regular and flexible operation is made. Two situations are analysed: the aggregated behaviour of the 30 households and the behaviour of a single household. In all the simulations performed, a time step of 5 min was considered.

A. *Thirty Households*

Based on customers' flexibility, the EA buys electricity in day-ahead spot market. In the present day, the EA manages the flexibility of the clients in quasi-real time, in order to minimize the deviations from the electricity bought in the market (flexible operation). This behaviour is shown in Fig. 4, where the flexible operation curve follows the evolution of the energy bought by the EA. Two periods (4-6 and 27-30 hours) can be identified, where the EA does not have enough flexibility to fulfil the curve of the energy bought by the EA.

Table IV presents, in a summarized form, the main results obtained from the simulations performed. The flexible operation allowed the EA to reduce electricity cost by 15.6% and energy consumption by 3.5%, in comparison with the regular operation. The decrease in energy consumption is due to the curve of the energy bought by the EA not be strictly followed. In addition, as shown in Fig. 4, the flexible operation strategy also allows reducing the peak load of the aggregated households from 101.8 to 76.2 kW (around 25%).

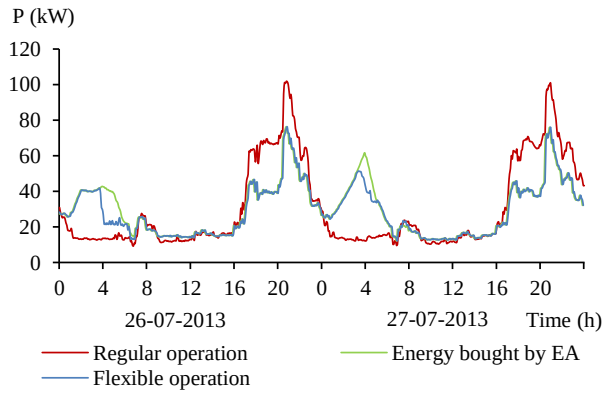

Fig. 4. Evolution of the energy bought by EA and the energy consumed by the 30 households in Regular operation and Flexible operation

TABLE IV. EA GLOBAL RESULTS

	Cost (€)	Energy (kWh)	Average cost (€/kWh)
Energy bought by EA	48.2	1446	0.033
Regular operation	57.1	1446	0.039
Flexible operation	47.8	1395	0.034

B. *Single Household*

The HEM manages the flexible loads, according to the demand actions performed by the EA.

The consumption values are enforced by the HEM, through control signals sent to the flexible loads. In turn, the inflexible loads behave independently of the control actions.

The evolution of the energy consumed by one single household in regular and flexible operation is presented in Fig. 5. Four periods can be identified: two (17-24 and 40-48 hours) where the HEM reduces the consumption of the flexible loads to the minimum and two (2-6 and 24-30 hours) where the HEM increases the consumption to the maximum.

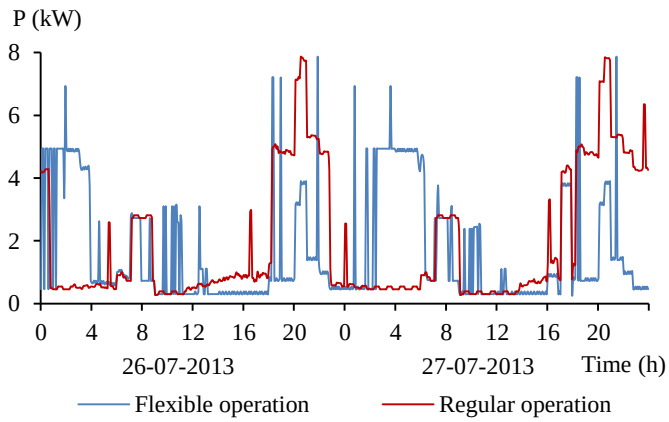

Fig. 5. Evolution of energy consumed by one single household in Regular and Flexible operation

The control actions performed by the EA and enforced by the HEM have impact in the behaviour of the flexible loads. The influence of these control actions in the refrigerator is shown in Fig. 6.

It is important to emphasize that the control actions are limited by the intervals of thermal comfort predefined by the user (3 and 7 ℃) and the characteristics of the refrigerator.

In regular operation, the behaviour of the refrigerator is conditioned by the temperature set point predefined by the user (5 ℃) and the characteristics of the refrigerator (Fig. 7).

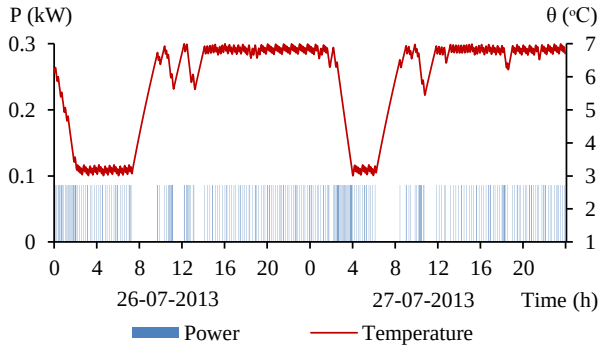

Fig. 6. Evolution of the refrigerator, in Flexible operation

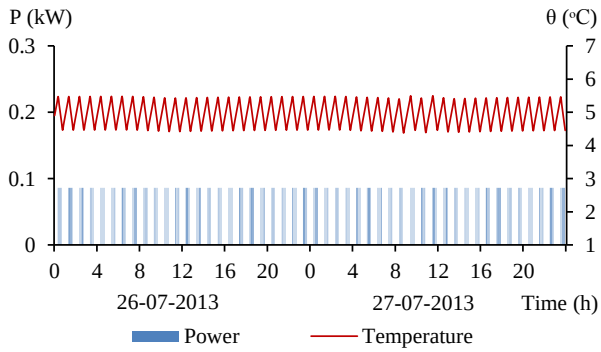

Fig. 7. Evolution of the refrigerator, in Regular operation

VII. Conclusion

Electric power systems operation and planning have been undergoing several changes in the past few years due to the smart grid advent. Deeper changes are expected in the near future, when smart grid solutions get mature and start being deployed in large scale. In this context, demand response is expected to play an important role as it incentivizes end-use customers to adjust their normal consumption patterns in response to changes in the electricity prices over time.

This paper presented a new aggregator model that allows incorporating demand response policies in order to decrease the electricity cost of retailing companies, while contributing to make electric power systems more reliable and efficient. The proposed approach included optimization algorithms for the EA and low voltage clients (implemented at the customers' premises in the HEM) aiming at managing the flexibility of the several resources available at the domestic level.

A test case with 30 households was used to validate the approach developed. By analysing the results obtained from the simulations, it may be concluded that the proposed aggregator model can be used to implement effective demand response schemes and use the clients' flexibility in the market to purchase electricity at lower prices. For a scenario with 30 households, the flexible operation strategy (demand response model) allowed the EA to reduce electricity cost by 15.6% and energy consumption by 3.5%, in comparison with the regular operation without any control.

Other issues remain open such as a more detailed quantification of the economic benefits that the EA can achieve in the market by using demand response and how these benefits should be shared with the end-use customers. Moreover, the way how demand response can be used to solve technical problems in the grids, such as poor voltage profiles or branch overloading, need also to be addressed in the future.

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