# Chapter 15
# BHP Universality in Energy Sources

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## 15.1 Introduction

Modeling the time series of the prices of different energy sources is important in economics, finance and energy market, and it is essential in the management of large stock portfolios [5–7, 9, 18–20, 22–25]. In this paper we analyze the daily price of north american biofuel ethanol and the plant from which ethanol is produced, specifically, corn. Herein we present the results from two data sets, ethanol and corn daily prices, from May 2005 to August 2013. Let $Y(t)$ be the energy source (ES) price or adjusted close value at day $t$. We define the *ES daily return* on day $t$ by

$$r(t) = \frac{Y(t) - Y(t-1)}{Y(t-1)}$$

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We define the $\alpha$ *re-scaled ES daily positive returns* $r(t)^\alpha$, for $r(t) > 0$, that we call, after normalization, the $\alpha$ *positive fluctuations*. We define the $\alpha$ *re-scaled ES daily negative returns* $(-r(t))^\alpha$, for $r(t) < 0$, that we call, after normalization, the $\alpha$ *negative fluctuations*. We analyze separately the $\alpha$ positive and $\alpha$ negative daily fluctuations that can have different statistical and economic natures (see, for example, [1, 2, 16, 17, 21]). Our aim is to find the values of $\alpha$ that optimize the data collapse of the histogram of the $\alpha$ positive and negative fluctuations to the Bramwell-Holdsworth-Pinton (truncated BHP) probability density function (pdf) $f_{BHP}$ truncated to the support range of the data (see Bramwell et al. [3, 4]). Our approach is to apply the Kolmogorov-Smirnov (K-S) statistic test as a method to find the values of $\alpha$ that optimize the data collapse. The $\alpha$ values represent a new measure that allows the comparison between the intensity of gains and losses of market activity in different energy sources prices. Using this data collapse we do a change of variable that allows us to compute the analytical approximations of the pdf of the normalized positive and negative ES daily returns in terms of the BHP pdf $f_{BHP}$. Similar results have been observed for some other energy prices, exchange rates, commodity prices as well as different indices with different time scales (see [13–15]). Since the BHP probability density function appears in several other dissimilar phenomena (see, for example, [8, 10–12]), our result reveals a universal feature of the prices of energy sources. Furthermore, these results lead to the construction of a new qualitative and quantitative econophysics model for the stock market based on the two-dimensional spin model (2dXY) at criticality and a new stochastic differential equation model for the stock exchange market indices that provides a better understanding of several stock exchange crisis.

## 15.2 Energy Sources and BHP

### *15.2.1 Energy Source Positive Returns*

Let $T^+$ be the set of all days $t$ with positive returns, i.e.

$$T^+ = \{t : r(t) > 0\}.$$

Let $n^+$ be the cardinal of the set $T^+$. The $\alpha$ *re-scaled energy source daily positive returns* are the returns $r(t)^\alpha$ with $t \in T^+$. The *mean* $\mu_\alpha^+$ of the $\alpha$ re-scaled energy source daily positive returns is given by

$$\mu_\alpha^+ = \frac{1}{n^+} \sum_{t \in T^+} r(t)^\alpha \tag{15.1}$$

The *standard deviation* $\sigma_\alpha^+$ of the $\alpha$ re-scaled energy source daily positive returns is given by

$$\sigma_\alpha^+ = \sqrt{\frac{1}{n^+}\sum_{t\in T^+} r(t)^{2\alpha} - (\mu_\alpha^+)^2} \tag{15.2}$$

We define the $\alpha$ *positive fluctuations* by

$$r_\alpha^+(t) = \frac{r(t)^\alpha - \mu_\alpha^+}{\sigma_\alpha^+} \tag{15.3}$$

for every $t \in T^+$. Hence, the $\alpha$ *positive fluctuations* are the normalized $\alpha$ re-scaled energy source daily positive returns. Let $L_\alpha^+$ be the *smallest* $\alpha$ positive fluctuation, i.e.

$$L_\alpha^+ = \min_{t\in T^+}\{r_\alpha^+(t)\}.$$

Let $R_\alpha^+$ be the *largest* $\alpha$ positive fluctuation, i.e.

$$R_\alpha^+ = \max_{t\in T^+}\{r_\alpha^+(t)\}.$$

We denote by $F_{\alpha,+}$ the *probability distribution of the $\alpha$ positive fluctuations*. Let the *truncated BHP probability distribution* $F_{BHP,\alpha,+}$ be given by

$$F_{BHP,\alpha,+}(x) = \frac{F_{BHP}(x)}{F_{BHP}(R_\alpha^+) - F_{BHP}(L_\alpha^+)}$$

where $F_{BHP}$ is the BHP probability distribution (see Bramwell et al. [4] and [14]). We apply the K-S statistic test to the null hypothesis claiming that the probability distributions $F_{\alpha,+}$ and $F_{BHP,\alpha,+}$ are equal. The Kolmogorov-Smirnov *P value* $P_\alpha^+$ is plotted in Fig. 15.1. We observe that $\alpha_{BHP}^+$ is the point where the $P$ value $P_{\alpha_{BHP}^+}^+$ attains its maximum.
It is well-known that the Kolmogorov-Smirnov $P$ value $P_\alpha^+$ decreases with the distance

$$D_{\alpha,+} = \|F_{\alpha,+} - F_{BHP,\alpha,+}\|$$

between $F_{\alpha,+}$ and $F_{BHP,\alpha,+}$. In Fig. 15.2, we plot

$$D_{\alpha_{BHP}^+,+}(x) = \left|F_{\alpha_{BHP}^+,+}(x) - F_{BHP,\alpha_{BHP}^+,+}(x)\right|$$

and we observe that $D_{\alpha_{BHP}^+,+}(x)$ for ethanol and corn attains its maximum value for the $\alpha^+$ positive fluctuations below the mean of the probability distribution.

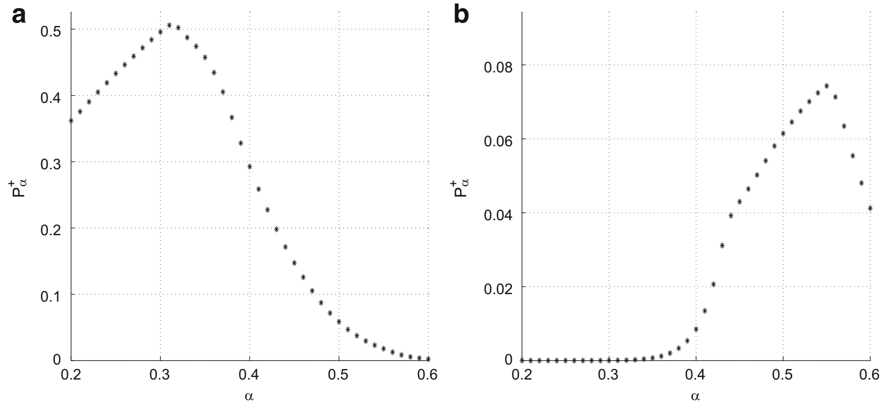



**Fig. 15.1** The Kolmogorov-Smirnov $P$ value $P_\alpha^+$ for values of $\alpha$ in the range $[0.2, 0.6]$, for daily returns. (**a**) Ethanol. (**b**) Corn

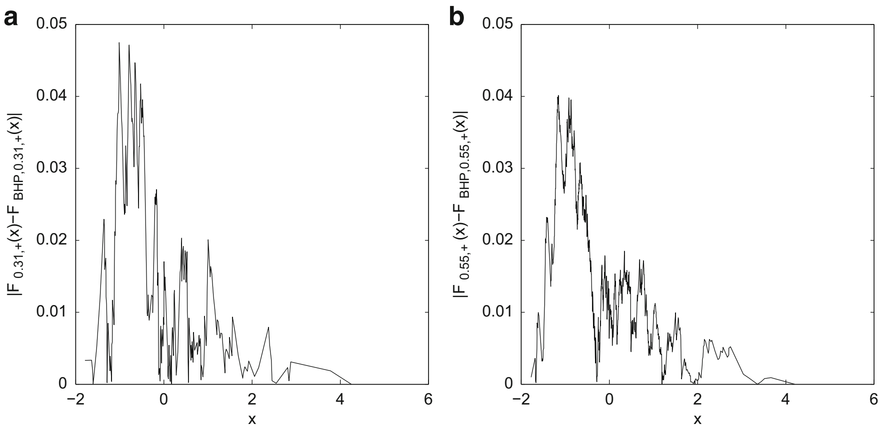



**Fig. 15.2** The map $D_{\alpha^+_{BHP},+}(x) = |F_{\alpha^+_{BHP},+}(x) - F_{BHP,\alpha^+_{BHP},+}(x)|$, for daily returns. (**a**) Ethanol. (**b**) Corn

In Fig. 15.3, we show the data collapse of the histogram $f_{\alpha^+_{BHP},+}$ of the $\alpha^+_{BHP}$ positive fluctuations to the truncated BHP pdf $f_{BHP,\alpha^+_{BHP},+}$.

We assume that the probability distribution of the $\alpha^+_{BHP}$ positive fluctuations $r^+_{\alpha^+_{BHP}}(t)$ is approximated by $F_{BHP,\alpha^+_{BHP},+}$. The pdf of the energy source positive returns $r(t)$ is approximated by (see [14])

$$f_{BHP,ES,+}(x) = \frac{\alpha^+_{BHP} x^{\alpha^+_{BHP}-1} f_{BHP}\left(\left(x^{\alpha^+_{BHP}} - \mu^+_{\alpha^+_{BHP}}\right)/\sigma^+_{\alpha^+_{BHP}}\right)}{\sigma^+_{\alpha^+_{BHP}}\left(F_{BHP}\left(R^+_{\alpha^+_{BHP}}\right) - F_{BHP}\left(L^+_{\alpha^+_{BHP}}\right)\right)}.$$

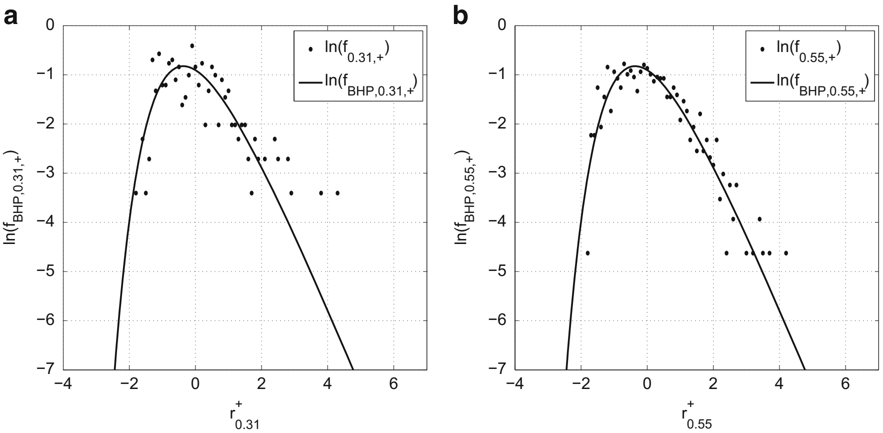



**Fig. 15.3** The histogram of the $\alpha^{+}_{BHP}$ fluctuations with the truncated BHP pdf $f_{BHP,\alpha,+}$ on top, in the semi-log scale, for daily returns. (**a**) Ethanol. (**b**) Corn

Hence, we get

$$f_{BHP,Ethanol,+}(x) = 1.02x^{-0.69} f_{BHP}(10.40x^{0.31} - 3.17).$$

and

$$f_{BHP,Corn,+}(x) = 5.53x^{-0.45} f_{BHP}(20.53x^{0.55} - 1.77).$$

If $\alpha < \alpha^{'}$ then the probability density of the returns near zero scale with order $x^{\alpha-1}$ and $x^{\alpha'-1}$ near zero. Hence the returns near probability zero have higher probability for the density with $\alpha$ than with $\alpha^{'}$. Therefore the exponent $\alpha$ is a new measure of intensity of the market.

We denote that $x^{\alpha^{+}_{BHP}-1}$ is the intensity term of $f_{BHP,ES,+}(x)$ at zero.

### *15.2.2 Energy Source Daily Negative Returns*

Let $T^{-}$ be the set of all days $t$ with negative returns, i.e.

$$T^{-} = \{t : r(t) < 0\}.$$

Let $n^{-}$ be the cardinal of the set $T^{-}$.

The $\alpha$ *re-scaled energy source daily negative returns* are the returns $(-r(t))^{\alpha}$ with $t \in T^{-}$. We note that $-r(t)$ is positive. The *mean* $\mu^{-}_{\alpha}$ of the $\alpha$ re-scaled energy source daily negative returns is given by

$$\mu_\alpha^- = \frac{1}{n^-} \sum_{t \in T^-} (-r(t))^\alpha \tag{15.4}$$

The *standard deviation* $\sigma_\alpha^-$ of the $\alpha$ re-scaled energy source daily negative returns is given by

$$\sigma_\alpha^- = \sqrt{\frac{1}{n^-} \sum_{t \in T^-} (-r(t))^{2\alpha} - (\mu_\alpha^-)^2} \tag{15.5}$$

We define the $\alpha$ *negative fluctuations* by

$$r_\alpha^-(t) = \frac{(-r(t))^\alpha - \mu_\alpha^-}{\sigma_\alpha^-} \tag{15.6}$$

for every $t \in T^-$. Hence, the $\alpha$ *negative fluctuations* are the normalized $\alpha$ re-scaled energy source daily negative returns. Let $L_\alpha^-$ be the *smallest* $\alpha$ negative fluctuation, i.e.

$$L_\alpha^- = \min_{t \in T^-} \{r_\alpha^-(t)\}.$$

Let $R_\alpha^-$ be the *largest* $\alpha$ negative fluctuation, i.e.

$$R_\alpha^- = \max_{t \in T^-} \{r_\alpha^-(t)\}.$$

We denote by $F_{\alpha,-}$ the *probability distribution of the* $\alpha$ *negative fluctuations*. Let the *truncated BHP probability distribution* $F_{BHP,\alpha,-}$ be given by

$$F_{BHP,\alpha,-}(x) = \frac{F_{BHP}(x)}{F_{BHP}(R_\alpha^-) - F_{BHP}(L_\alpha^-)}$$

where $F_{BHP}$ is the BHP probability distribution. We apply the K-S statistic test to the null hypothesis claiming that the probability distributions $F_{\alpha,-}$ and $F_{BHP,\alpha,-}$ are equal. The Kolmogorov-Smirnov *P value* $P_\alpha^-$ is plotted in Fig. 15.4. Hence, we observe that $\alpha_{BHP}^-$ is the point where the $P$ value $P_{\alpha_{BHP}^-}^-$ attains its maximum.
It is well-known that the Kolmogorov-Smirnov $P$ value $P_\alpha^-$ decreases with the distance

$$D_{\alpha,-} = \| F_{\alpha,-} - F_{BHP,\alpha,-} \|$$

between $F_{\alpha,-}$ and $F_{BHP,\alpha,-}$. In Fig. 15.5, we plot

$$D_{\alpha_{BHP}^-,-}(x) = \left| F_{\alpha_{BHP}^-,-}(x) - F_{BHP,\alpha_{BHP}^-,-}(x) \right|$$

and we observe that $D_{\alpha_{BHP}^-,-}(x)$ attains its maximum value, in ethanol and corn prices, for the $\alpha_{BHP}^-$ negative fluctuations below the mean of the probability

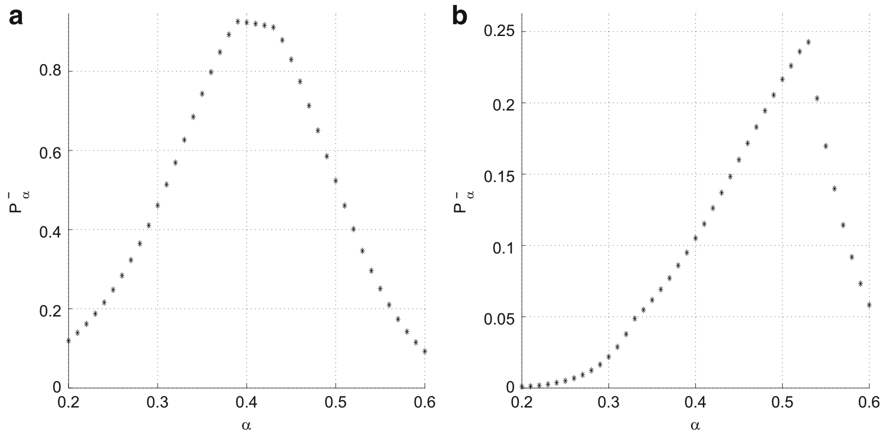



**Fig. 15.4** The Kolmogorov-Smirnov $P$ value $P_\alpha^-$ for values of $\alpha$ in the range $[0.2, 0.6]$, for daily returns. (**a**) Ethanol. (**b**) Corn

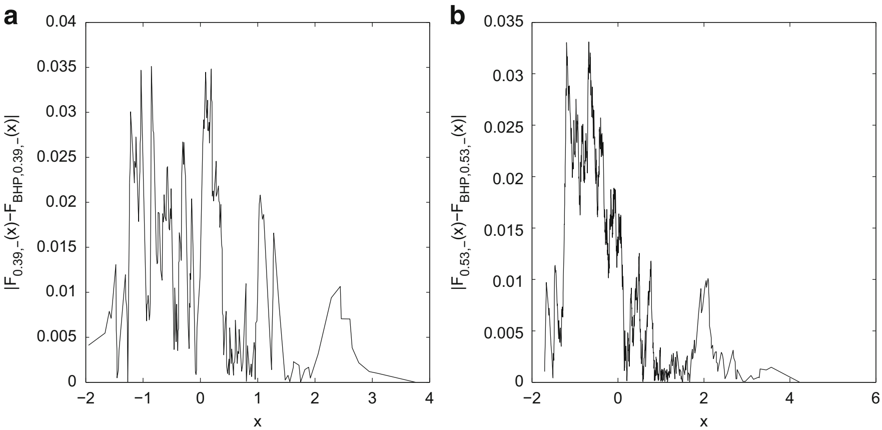



**Fig. 15.5** The map $D_{\alpha^-_{BHP}}, -(x) = |F_{\alpha^-_{BHP},-}(x) - F_{BHP,\alpha^-_{BHP},-}(x)|$, for daily returns. (**a**) Ethanol. (**b**) Corn

distribution. In Fig. 15.6, we show the data collapse of the histogram $f_{\alpha^-_{BHP},-}$ of the $\alpha^-_{BHP}$ negative fluctuations to the truncated BHP pdf $f_{BHP,\alpha^-_{BHP},-}$.

We assume that the probability distribution of the $\alpha^-_{BHP}$ negative fluctuations $r_{\alpha^-_{BHP}}(t)$ is approximated by $F_{BHP,\alpha^-_{BHP},-}$. The pdf of the energy source daily (symmetric) negative returns $-r(t)$, with $T \in T^-$, is approximated by (see [14])

$$f_{BHP,Energysource,-}(x) = \frac{\alpha^-_{BHP} x^{\alpha^-_{BHP}1} f_{BHP}\left(\left(x^{\alpha^-_{BHP}} - \mu^-_{\alpha^-_{BHP}}\right)/\sigma^-_{\alpha^-_{BHP}}\right)}{\sigma^-_{\alpha^-_{BHP}}\left(F_{BHP}\left(R^-_{\alpha^-_{BHP}}\right) - F_{BHP}\left(L^-_{\alpha^-_{BHP}}\right)\right)}.$$

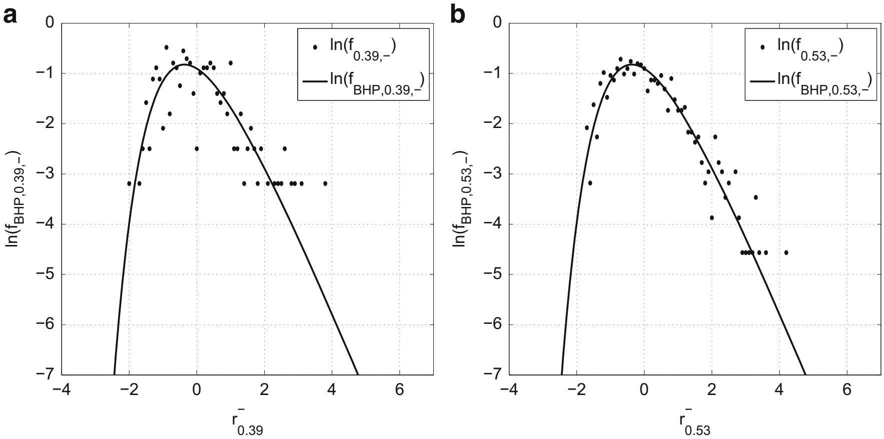



**Fig. 15.6** The histogram of the $\alpha^-_{BHP}$ fluctuations with the truncated BHP pdf $f_{BHP,\alpha,-}$ on top, in the semi-log scale, for daily returns. (**a**) Ethanol. (**b**) Corn

Hence, we get

$$f_{BHP,Ethanol,-}(x) = 1.67x^{-0.61} f_{BHP}(10.72x^{0.39} - 2.51)$$
$$f_{BHP,Corn,-}(x) = 5.02x^{-0.47} f_{BHP}(20.07x^{0.53} - 2.12)$$

We denote that $x^{\alpha^-_{BHP}-1}$ is the intensity term of $f_{BHP,ES,-}(x)$ at zero.

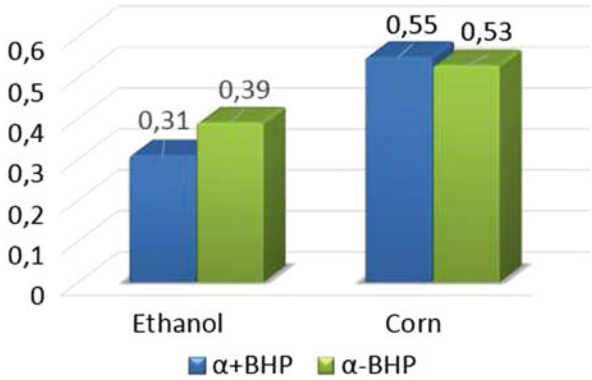
## *15.2.3 Ethanol and Corn Prices Gains and Losses*

In the presented data analysis all the $P$ values are always higher that 0.01, which can indicate universality of the values in these energy source prices. For daily returns, in ethanol and corn prices, we observe that $P^-_{BHP} > P^+_{BHP}$.

Considering the exponent $\alpha$ as new measure of intensity of the market, we observe that in daily returns, in the period of 8 years considered, in ethanol prices, $\alpha^+_{BHP} < \alpha^-_{BHP}$ which can indicate that the market was more intense or more active in the losses than in the gains.

A smaller value of $\alpha$ causes a higher blowup of the returns near zero and the fact that, in corn daily prices, $\alpha^+_{BHP} > \alpha^-_{BHP}$, shows that the market was less intense or less active in losses (Fig. 15.7).

**Fig. 15.7** Values of $\alpha^{+}_{BHP}$ and $\alpha^{-}_{BHP}$, for daily returns




## 15.3 Conclusions

We computed the analytical approximations of the pdf of the distinct normalized daily positive and negative returns for specific energy sources, in terms of the truncated BHP pdf. We showed that the data collapse of the histogram of the positive and negative returns supports our proposed theoretical pdfs. We presented a measure of the intensity of gains and losses of different energy sources prices.

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