Contents

Chapter 1
Sectors and Routes in Solid Waste Collection

Ana M. Rodrigues and J. Soeiro Ferreira

Abstract Collecting and transporting solid waste is a constant problem for municipalities and populations in general. Waste management should take into account the preservation of the environment and the reduction of costs. The goal with this paper is to address a real-life solid waste problem. The case reveals some general and specific characteristics which are not rare, but are not widely addressed in the literature. Furthermore, new methods and models to deal with sectorization and routing are introduced, which can be extended to other applications. Sectorization and routing are tackled following a two-phase approach. In the first phase, a new method is described for sectorization based on electromagnetism and Coulomb's Law. The second phase addresses the routing problems in each sector. The paper addresses not only territorial division, but also the frequency with which waste is collected, which is a critical issue in these types of applications. Special characteristics related to the number and type of deposition points were also a motivation for this work. A new model for a Mixed Capacitated Arc Routing Problem with Limited Multi-Landfills is proposed and tested in real instances. The computational results achieved confirm the effectiveness of the entire approach.

1.1 Introduction

Collecting and transporting solid waste is a common problem for municipalities and populations in general. A good waste management program takes into account routing issues, not only because the economic benefits, but also to preserve the environ-

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ment. Better routes mean lower fuel consumption, which leads to lower emissions of CO_2. Collecting vehicles have an extremely high level of consumption, and reducing a few *km* on a daily route represents a significant reduction of costs by the end of the month.

The aim of this paper is to address a real-life situation, based on the region of Monção in the North of Portugal. The case study reveals some general and specific characteristics which are not rare, at least in the country.

Municipalities are so large that prior sectorization is convenient, which means dividing the territory into sectors. Electromagnetism and Coulomb's Law were used to divide the territory. Forces of attraction or repulsion were used to group the elementary regions (in Portugal these regions are called *freguesias*) into sectors. Besides considering the common facets of sectorization, this new method also addresses particular situations and requirements of *freguesias*, as in some of them waste is collected daily, while in others it is collected two or three times a week. The case study also features specific conditions related to the number and kind of deposition points. Two types of points are considered: landfills and transfer stations. Landfills can be used whenever a collecting vehicle needs to be emptied and they have no limitation regarding the number of visits. Transfer stations are points that temporarily receive waste. What happens is that sometimes transfer stations are small and cannot receive all the waste. As a consequence, the number of daily visits can be limited. To deal with these matters, a new problem and model are introduced: The Mixed Capacitated Arc Routing Problem with Limited Multi-Landfills (MCARP-*LML*). Sectorization and Routing are used to solve the case of solid waste collection.

The paper is organized as follows: Section 1.2 briefly reviews the literature on sectorization and applications. Section 1.3 provides an overview of routing, particularly arc routing problems with capacity restrictions. The real-case of waste collection in Monção is described in Section 1.4. Section 1.5 presents the solution approach proposed to solve the problem. Section 1.6 provides the computational results based on real instances from Monção. Finally, Section 1.7 provides some conclusions.

1.2 Sectorization: reasons and scope

Sectorization means dividing into sectors or parts, a procedure that occurs in many contexts and applications, usually to achieve some goal or to facilitate an activity. Most of the time, this division or partition aims at better organizing or simplifying a large problem into smaller sub-problems, or promoting groups with similar characteristics. The idea is different from the one of a clustering process in which, although the groups to be formed are composed of individuals with similar features, they must be as different as possible from each other, [23].

The suitability of sectorization is not new. One of the initial publications, [20], discusses the division of a territory applied to the definition of political districts.

More recently, the literature refers to various other applications, such as: dividing sales territories by different sellers, school districts, salt spreading operations in the winter, or collecting waste. Some applications are described below.

In order to evaluate the quality of the sectors (districts, parts) obtained after sectorization, it is convenient to use some general measures:

- Equilibrium - different districts must contain approximately the same "population" or "amount of work";
- Contiguity - each district or sector must be composed of just "one body", that is, it should be possible to "move" between any pair of points in a district without leaving the district;
- Compactness - it is a measure of "concentration", that is, "U" shapes and dispersed territory should be avoided; instead, round shapes that are more compact should be preferred.

Sectorization related to political districting has been studied for some decades. The main idea is dividing a certain territory into a given number of sectors (districts), based on the principle of one man - one vote. Votes are transformed into seats, and a correct partition of the territory cannot admit advantages and/or disadvantages for some sectors. Other measures are considered in political districting: the integrity of territories, the respect for the administrative subdivision of the territory, the existence of small communities, and the preservation of the minorities strength. In [43], the authors organized the literature related to political districting in groups according to the models and approaches used. Other references are [20], [14] or [6].

Sectorization is also used to design sales territories. Changes in the number of salesmen can not only justify a redefinition of territories, but also a better use of the existing potential or a better coverage of the territory. The main idea is defining boundaries within a territory to produce sectors. The common objective is to maximize the profit, dividing a certain sales force into a given number of smaller areas. The work [47] analyzes some approaches to maximize the profit described by different authors in the literature. Other references are: [21], [23] or [51].

School districting or redistricting consists of dividing a city (or a municipality) into smaller areas or neighborhoods that must be assigned to schools. A good sectorization makes it possible, for example, to minimize the total travel distance between student residences and schools, and/or to take into account racial balance or crossing arterial roads on the way home from school. The papers [49] and [7] present interesting results on school districting.

Sectorization can also be used to partition a certain region into smaller areas to ensure an optimal allocation of health services. Optimal hospital districting, considering demand and capacity measured in number of hospital beds, is mentioned in [41]. The work in [4] presents a problem linked to home health care. Another reference, [37], deals with the partitioning of a territory into a number of areas where at least one source of a certain social service must be present.

Sectorization (districting or redistricting) is also present in many other situations: the definition of police command boundaries is tackled in [9]; medical emergency systems in highways are addressed in [22]; cost minimization in jail systems, taking into account existing and possible new jails, are described in [36]; the authors of [5] deal with urban emergency services such as fire services; sector design related to patrolling operations by vessels of a maritime agency is described in [34]; in [15], the authors determine an overall service area and the necessary transit mobile repair units in emergency situations.

Sectors and waste collection

Sectorization has also been applied to Solid Waste Collection. In [19], the authors propose to create balanced sectors according to the daily collection time. Firstly, the region is divided into a predetermined number of sectors. Then, and for each sector, collector vehicle routes are built so that the time spent in each sector is limited in the pre-defined range $[T_{min}, T_{max}]$, and as closely as possible to a "target" $T^* \in [T_{min}, T_{max}]$. The number of connected components in each sector is also reduced. The work [35] reports a situation in which streets without direction restrictions are grouped into sectors. The amount of waste to be collected in each sector should not exceed the vehicle's capacity. The objective is to minimize the total cost associated with revisited streets or streets where collection is not required. Work levels are used in [29] to solve sectoring problems in urban waste collection. Two vertices belong to level n if the shortest route between them has n intermediate nodes. A feasible solution for sectoring rejects solutions with very high work levels. The authors in [39] introduce three methods to deal with municipal waste collection: two different two-phase methods, where phase 1 is dedicated to creating the sectors, and phase 2 consists of determining the trips in each sector, and a third method based on best insertion. Evaluation criteria such as imbalance, diameter and dispersion measures are used to compare the algorithms.

1.3 Routing: overview and applications

General routing problems are addressed differently in the literature. Using graph language, there are not only routes associated with arcs/edges, but also routes associated with vertices, and more general routes associated with both arcs/edges and vertices. Arc Routing Problems (ARP) are related to the determination of a least cost traversal of the set (or subset) of edges, on a graph $G = (V, E)$, where V is the set of vertices and E is the set of edges. This graph can be undirected, directed (using the term arc instead of edge), or mixed (with edges and arcs). A cost $c_{ij} \geq 0$ is associated with each edge $(i, j), i \neq j$. If $(i, j) \notin E$, then $c_{ij} = \infty$. These problems appear in a large variety of practical contexts ([11], [10], [38] and [46]), such as mail delivery, delivery of telephone books, garbage collection, street sweepers, salt gritting, inspection of streets for maintenance, meter reading, snow removal, internet routing, cutting process manufacturing, or printed circuit board manufacturing. Node Routing Problems (NRP) are analogous problems but they focus on vertices (or nodes) instead of edges. An NRP can be transformed into an ARP and vice versa, [2]. Well-known examples are the Travelling Salesman Problem (TSP), the Asymmetric TSP and the Vehicle Routing Problem (VRP). ARP and NRP are special cases of a broader class of problems named General Routing Problems (GRP). The GRP, introduced by [40], look for the minimum cost tour on a graph $G = (V, E)$, where a subset of edges E_R ($E_R \subseteq E$) and a subset of vertices V_R ($V_R \subseteq V$) are mandatory.

Despite the importance of the NRP, more broadly referred in the literature, this paper only addresses arc routing.

According to the characteristics of the problems, ARP can be generally classified into four independent/distinct major classes: Windy, Capacitated, Hierarchical and Other problems. The latter includes all the problems that do not fit the other three classes. Capacitated ARP (CARP) emerges when capacity restrictions are introduced into the closed paths of each vehicle (or postman).

CARP can be defined in a graph $G = (V, E)$, undirected, directed or mixed, with a subset E_R ($E_R \subseteq E$) of required edges and a special vertex, v_0, called depot. A demand $d_e > 0$ is associated with each required edge $e \in E_R$. A fleet of n vehicles (not necessarily identical), each one with capacity $Q_i > 0, i = 1, ..., n$, is available. The objective is to find a minimum cost set of vehicle routes, which must start and end at the depot, such that each required edge is serviced exactly once, and the sum of demands of the serviced edges, in each route, does not exceed the vehicle's capacity, [30] and [33]. The CARP is an NP-Hard problem and it was introduced by [17]. A recent survey of CARP and its variants is provided by [50].

The CARP is also a generalization of the Capacitated Chinese Postman Problem (CCPP), where demand is positive in all edges, [12].

The Mixed CARP (MCARP) is a CARP with arcs and edges, that is, with directed and undirected links, respectively. The Periodic CARP (PCARP), defined by [25], is motivated by the need to assign a daily service to the edges, in real applications. It is an extension of the CARP to multiple periods, [8]. The final objective is to minimize the required fleet and the total cost of the trips in the selected multi-period horizon.

The Location ARP (LARP), initially called Arc Oriented Location Routing by [31], is another extension of ARP. LARP simultaneously deals with location and arc routing decisions. The work in [42] addresses applications in distribution systems, while [32] contains a survey and suggestions for future research.

CARP with Refill Points (CARP-RP) are LARP with two different types of vehicles: the servicing vehicle and the refilling vehicle. The refilling vehicle can meet the first one at any place to refill it. [1] presents a practical application of this problem: "road network maintenance, where the road markings have to be painted or repainted every year." In the Sectoring ARP (SARP), the network is partitioned into a given number K of sectors. The aim is to solve K MCARP and to minimize the total duration of the trips over all sectors, [39]. Special applications are associated with waste collection in large urban areas.

The Stochastic CARP (SARP) is a stochastic version of the CARP with random demand on the arcs, [13]. Real applications may be found in waste collection.

CARP with time dependent service costs are a variant of CARP in which the cost of servicing some arcs depends on the time the service starts, [48]. The applications include spreading grit on icy roads, and the timing of interventions is crucial.

Another CARP is the Extended CARP (ECARP)([27]), a problem that includes extensions, such as mixed multigraph with edges and arcs and parallel links, deadheading and collection costs per link, prohibited turns and turn penalties, and an upper limit on the cost of any trip.

In the CARP with Intermediate Facilities (CARPIF), vehicles may unload or replenish at intermediate facilities, which are a subset of the vertices. Garbage collection with visits to dump sites or incinerators is a possible application. An equivalent situation is the one in which the vehicle makes deliveries instead of collections, and vehicles must replenish to meet the demand, [16].

The CARP with Unit Demand (CARPUD) is a particular CARP where all required edges present unit demand, [3].

A Multi-objective version of the CARP is presented in [28], and its goalis twofold: minimizing the total duration of the trips, and reducing the duration of the longest trip. The work presented in [26] gives the example of waste management companies interested in both balancing the trips and minimizing their total durations.

The VRP is a capacitated version of the TSP, as the CARP is the capacitated version of the Rural Postman Problem, [2]. One approach to the CARP is the transformation into a VRP.

1.4 Real case study

Collecting and transporting solid waste is a difficult and complicated problem in modern societies. In this task, there are often many different specificities and constraints that need to be taken into account.

The case study presented in this paper is based on a municipality in the North of Portugal, Monção, which has some specific characteristics that are not rare, at least in the country.

This is an outcome of the work that the authors did on waste collection problems in Monção. The municipality of Monção is a region with 220 km^2 and a population of about 20,000 inhabitants. This municipality is a combination of rural and urban areas, and the population is divided into 33 small regions, called *freguesias* in Portuguese (see the map in Figure 1.1). This is a region with a strong component of emigrants and in the summer the population increases. As a consequence, the amount of waste produced increases as well. In the “more rural” regions, waste must be collected 2 or 3 times a week (depending on the season), while in “more urban”, regions waste must be collected every day.

There are two deposition points in Monção: the landfill of Valença and the transfer station of Messegães. The transfer station of Messegães also receives waste from other municipalities. Inhabitants also use this transfer station to deposit large objects

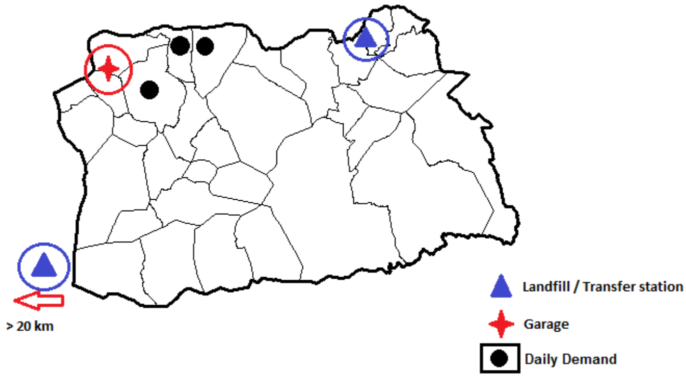

Fig. 1.1 All the *freguesias* of Monção, the garage, waste disposal points and *freguesias* with daily demand

such as old furniture. Generally, this transfer station only allows a single visit a day per vehicle. The municipality has around 1,600 containers of different types, from simple trash bags to more modern and large containers. An estimation was made of the time a container is collected, which depends on its type. The average speed of the vehicle was also estimated (30 km/h). The volume of each container was calculated considering the fact that, on average, containers are not completely full and vehicles have a system that compresses the waste collected. This paper only considers the case of one vehicle but in [45] a model with two types of vehicles is presented. Vehicles with different capacities, different costs and other specific characteristics such as dedicated containers.

To summarize, the challenge posed involves:

- a vehicle with a limited capacity;
- a heterogeneous group of containers (different capacities, different time to collect);
- one-way streets;
- different collecting frequencies (every day or 2 or 3 times a week);
- the garage is not a deposition point (when the vehicles start and end at the garage, vehicles are empty);
- some deposition points can present limitations in terms of the number of visits.

1.5 Two-phase approach

A two phase approach has been developed to solve the Monção case study. The process starts with a sectorization phase where the territory of that municipality is divided into smaller sectors to be considered in the next phase (circuits). Sectorization deals with points: each elementary unit represents the amount of waste produced by a community. The second phase of the approach deals with route planning. A route will emerge for each sector obtained during the first phase. The routing process is focused on the requirement of serving the link (edge or arc). There are several collecting points along the link and, in some cases, the collection is door to door. Routes must meet some specific constraints, such as the transfer station's limitation in terms of the amount of waste received. If the resulting routes are not "good enough" new sectors will be produced, and, after that, new routes will be obtained.

1.5.1 Sectors - Solution based on electromagnetism

There are two essential reasons for building sectors right from the start: firstly, the municipality of Monção is so large that it is not possible to collect all the waste in just one circuit, and for that reason waste must be collect throughout the week. The second reason is related to the frequency of collection. Different *freguesias* present different demands. Throughout most of the year, there are *freguesias* where the waste must be collected twice a week, and others where the collection occurs daily. The approach proposed considers the frequency of collection as an input to the sectorization phase.

The sectorization process presented in this paper was inspired by electromagnetism. When regarding the containers over a map, those belonging to the same sector should demonstrate some kind of "attraction"; containers in different sectors should present some kind of "repulsion". Quite simply, this is the idea behind the approach and its connection to electromagnetism and Coulomb's Law, which establishes a relation of force between two point charges, [44].

Adaptations and extensions of the concepts of attraction and repulsion will be conducted to be applied in sectorization.

Coulomb's Law states that:

"The force between a given pair of charges is inversely proportional to the square of the distance between them. [...] is directly proportional to the *quantity* of one charge multiplied by the *quantity* of the other," (in [24]).

Suppose there are two electrical charged points with charges q_1 and q_2, which are at a distance of d_{12}. The force $\overrightarrow{F}$ between the two charges presents an intensity given by:

$$\overrightarrow{F} = k \cdot \frac{|q_1| \cdot |q_2|}{d_{12}^2} \cdot \hat{r}_{12}. \tag{1.1}$$

k represents the Constant of Coulomb and corresponds to $8.99 \times 10^9 Nm^2.C^{-2}$ and $\hat{r}_{12}$ is unit vector.

The force is along the straight line joining the charges. If they have the same sign, the electrostatic force between them is repulsive; if they have different signs, the force between them is attractive.

1.5.1.1 Attraction produces sectors

Inspired by electromagnetism, a system composed of n collecting points is seen as n "charged points" over a map. The position of each point (latitude and longitude) is known and the respective "charge" corresponds to the amount of waste to collect at that point.

Following Coulomb's Law, represented in (1.1), a symmetric matrix A of attractions between each pair of n collection points was calculated. Initially, each pair of points was always supposed to have charges with different signals, that is, the relation between two points is always attractive, and never repulsive. Furthermore, the constant k was set to be equal to 1.

The vector length that represents the "force of attraction" between two points $I = (x_i, y_i)$ and $J = (x_j, y_j)$ with charges q_i and q_j, respectively, is calculated using the quotient between the product of the charges and the squared distance from I to J, represented in (1.2).

$$a_{ij} = ||\overrightarrow{a_{ij}}|| = ||\overrightarrow{a_{ji}}|| = \frac{q_i \cdot q_j}{d_{ij}^2} \tag{1.2}$$

where $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ is the Euclidean distance between points I and J.

After calculating the value of the attraction between each pair of points, that is, after constructing the matrix A, it is possible to find the pair with the greatest "admissible attraction." "Admissible" because capacity restrictions must be validated, which means that the sum of the charges of the two points with maximum attraction cannot be greater than the capacity available for the sector. Suppose that Q represents the maximum charge (amount of waste) that can be collected, and I and J are two generic points with charges q_i and q_j, respectively. The objective here is to find the pair (I, J), such that $arg\ max\{a_{ij} \in A : q_i + q_j \leq Q\}$.

After that, the two selected points will join at a new point $C = (x_C, y_C)$, with charge q_C as illustrated in Figure 1.2.

Distances k_i and k_j represented in Figure 1.2 are equal to $k_i = \frac{q_j}{q_i+q_j} \cdot d_{ij}$ and $k_j = (1 - \frac{q_j}{q_i+q_j}) \cdot d_{ij} = \frac{q_i}{q_i+q_j} \cdot d_{ij}$.

Unless the charges q_i and q_j are exactly the same, point C is not at the same distance between I an J. C is always closer to the point with higher charge.

After the first iteration, the resulting charge usually attains a larger value. This means that, in the second iteration, that charge will be given an unfair preference because of its dimension, to be presented in the next fusion. Therefore, when equation (1.2) is used to calculate attractions between charges, the result will be quite unbalanced. Charges will increase until the capacity restriction is not violated.

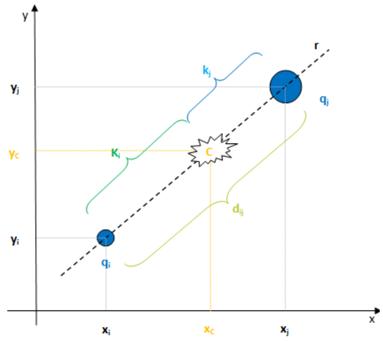

Fig. 1.2 Union of points I e J in a new point C

To deal with this difficulty, that is, the resulting "extreme force" of attraction, a few changes are proposed for the first equation (1.2). This means that, the exponent of the denominator will increase iteration after iteration. The resulting attraction matrix is A^d, now defined by expression 1.3:

$$||\overrightarrow{a^d_{ij}}|| = ||\overrightarrow{a^d_{ji}}|| = \frac{q_i \cdot q_j}{d_{ij}^{2+NIP-NS}} \tag{1.3}$$

NIP represents the Number of Initial Points and NS is the current Number of Sectors. In the beginning, $NS = NIP$ after the first iteration $NS = NIP - 1$, and then $NS = NIP - 2$, and so on, until the desired number of sectors is obtained.

This change in the matrix of attractions not only prevents unbalanced sectors, but also increases the compactness and the contiguity of the resulting sectors.

In the specific case of waste collection, another change was done to "open" the distribution of the set of points to collect. It was convenient to take into account some other points (landfills or transfer stations).These will have fixed charges and exert an attractive force on the resulting charge, which is proportional to the point's receiving capacity.

Figure 1.3 exemplifies what has been said. Suppose the points with maximum attraction are P_1 and P_2, with charges q_1 and q_2, respectively. P represents the resulting point with charge $q_t = q_1 + q_2$. Suppose also that there are two landfills with different receiving capacities. The capacity of each landfill is represented by its charge: landfill A_1 with charge qA_1 and A_2 with qA_2 and $qA_1 > qA_2$. Influenced by the resulting force exerted by deposition points, the point P will be displaced to the position P'.

1.5.1.2 Repulsions for admissibility

Attractive forces were operated in the previous section. The comparison with electromagnetism will be enhanced by including "repulsive forces". In fact, the follow-

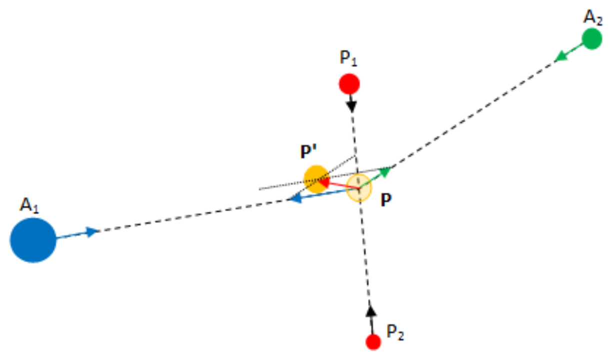

Fig. 1.3 The action of the resultant force on point P

ing situations only make sense in this framework if repulsive forces are assumed. The first situation refers to the case in which different containers cannot be collected by the same vehicle. Suppose that two containers are close. One of the containers requires a crane to be collected (a large vehicle) and the other is in a narrow street, and therefore a smaller vehicle is mandatory. Even though both containers are close, they cannot be included in the same sector. Another situation is related to the collection frequency.

Generally, different locations in the same municipality require different frequencies of collection. Frequency is usually related to population density, and also to the proximity to public spaces, such as schools or hospitals.

Consider, for instance, a simple situation where two regions A and B must be collected. Suppose that in region A the collection is done every day, and in region B the collection is done three times a week. Sundays are dedicated to regions with daily collection (region A is an example). Table 1.1 characterizes a weekly work schedule composed of two circuits C_1 and C_2 repeated until the end of the week. This means that the planning is done for two days and repeated three times. Remember that region B must be collected three times a week and, obviously, the collection must be spread throughout the week.

Table 1.1 Weekly schedule

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Circuit	C_1	C_2	C_1	C_2	C_1	C_2

Region A, where waste is collected every day, must belong to circuits C_1 and C_2, and region B will be in C_1 or in C_2. For instance, if region B is included in circuit C_1, then waste will be collected on Mondays, Wednesdays and Fridays.

When planning for two days (sector 1 with circuit C_1 and sector 2 with circuit C_2), an "exact copy" will be made of containers of region A. Therefore, when the process of sectorization is initiated, it is necessary to guarantee that the same container (the original and the copy) will not belong to the same sector. Moreover, a container and its copy must repulse each other, thus preventing the same container to be collected twice in the same day.

1.5.1.3 Different levels of attraction and repulsion

The dichotomy attraction vs repulsion may become too strict, and for that reason it is necessary to make improvements and adaptations to reality. Imagine another situation comprising two locations separated by a river, a small Euclidian distance - the two locations seem to be close but in terms of the road network, the distance between them may be quite large. Consequently, it is not right to prevent those two locations from belonging to the same sector, but a negative weight should be associated with this "mix".

Other intermediate situations were considered besides the dichotomous *attraction vs repulsion*. The justification to create more levels is not just geographical. Broader situations could be incorporated, such as a decision-maker who might say: "although it is not required, we prefer waste collection in regions A and B to be done on the same day (or on different days)."

When this kind of information (geographical or past experiences) is accompanying pairs of points I and J, a symmetric matrix S (with null diagonal) is generated, $S = [s_{ij}], i, j = 1, ..., n$, where n is the number of points in the system,

$$s_{ij} = \begin{cases} -1 & \text{, points } i \text{ and } j \text{ will be in different sectors} \\ -0,5 & \text{, points } i \text{ and } j \text{ have some repulse} \\ 0 & \text{, points } i \text{ and } j \text{ are independent} \\ 0,5 & \text{, points } i \text{ and } j \text{ have some affinity} \\ 1 & \text{, points } i \text{ and } j \text{ have much affinity} \end{cases}$$

If the intention is to add this new information to the matrix of attraction A^d presented before, then the new matrix A' is obtained, as defined in 1.4.

$$||\overrightarrow{a'_{ij}}|| = ||\overrightarrow{a'_{ji}}|| = \frac{q_i \cdot q_j}{d_{ij}^{2+NIP-NS}} \cdot (s_{ij} + 1) \tag{1.4}$$

Example

To illustrate the idea, consider the following example (see Figure 1.4) with five points and four sectors to be generated.

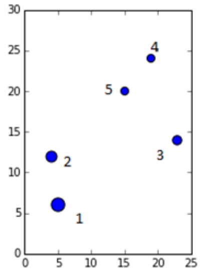

Fig. 1.4 Five points to form four sectors

In practice, each point represents a container (or group of containers). The position of each point and the quantity to be collected are known, Table 1.2.

Table 1.2 Position and quantities of the five points

Point	(x,y)	*Quantity*
1	(5,6)	300
2	(4,12)	200
3	(23,14)	140
4	(19,24)	100
5	(15,20)	100

Suppose also that for some reason points 1 and 2 repulse each other ($s_{12} = -1$), and that all other points are independent of each other (if $(i,j) \neq (1,2)$ and $(i,j) \neq (2,1)$ then $s_{ij} = 0$). The quantity in each sector must be not greater than 500. $NIP = 5$ and, in the first iteration, $NS = NIP$. The matrix of attractions A' is

$$A' = \begin{bmatrix} 0 & 0 & 108 & 58 & 123 \\ 0 & 0 & 77 & 54 & 108 \\ 108 & 77 & 0 & 121 & 140 \\ 58 & 54 & 121 & 0 & \mathbf{313} \\ 123 & 108 & 140 & \mathbf{313} & 0 \end{bmatrix}.$$

The application of the proposed sectorization method results in the attraction of points 4 and 5, see Figure 1.5, given the quantities, distances and repulsion between points 1 and 2.

⋆

Every time two points are joined in the same sector, the size of matrix A^d decreases and a new matrix must be calculated. After two points are united, say I and J, the relations between the newly obtained point C and all the others must be defined, taking into account the previous relations between I and J, and the others.

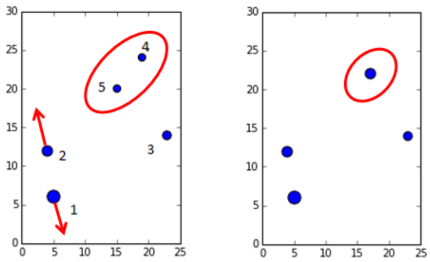

Fig. 1.5 Repulsion between points 1 and 2 and the resulting attraction between points 4 and 5

Define the commutative operation $\Delta : M \times M \to M$, considering $M = \{-1; -0,5; 0; 0,5; 1\}$ and Δ, as expressed in Table 1.3.

Table 1.3 Δ Operation

Δ	-1	$-0,5$	0	$0,5$	1
-1	-1	-1	-1	-1	-1
$-0,5$	-1	-0,5	-0,5	-0,5	-0,5
0	-1	-0,5	0	0,5	1
$0,5$	-1	-0,5	0,5	0,5	0,5
1	-1	-0,5	1	0,5	1

As an example, suppose that point I presents a relation of absolute repulsion relatively to point G, and point J does not attract or repulse point G. In that case, J and G are absolutely independent. When the new point C is created by joining I and J, the resulting relation between C and G will be of repulsion ($s_{cg} = s_{gc} = -1$).

It should be noted that the relations of repulsion should be the exception and not the rule.

1.5.1.4 Evaluating sectors

The resulting sectors are evaluated taking into consideration three characteristics described at the beginning of this paper.

Equilibrium

A coefficient of variation in the amount of waste (CV_q) is considered to evaluate the balance. It is calculated as follows, for a group of k sectors with quantities $q_i, i = 1, \ldots, k$:

$$CV_q = \frac{s'_q}{\overline{q}} \tag{1.5}$$

where $\overline{q} = \frac{\sum_{i=1}^{k} q_i}{k}$ and $s'_q = \sqrt{\frac{1}{k-1} \cdot \sum_{i=1}^{k} (q_i - \overline{q})^2}$

Hence, in terms of the quantity of waste collected, balanced sectors should have a CV_q as close to zero as possible.

Compactness

The compactness d_i of each sector i, here perceived as a measure of concentration of waste to be collected in each sector, is defined by (1.6):

$$d_i = \frac{\sum_j q_{ij}}{dist(o_i, p_i)} \tag{1.6}$$

where q_{ij} represents the charge (or amount of waste) of the point j in sector i, and $dist(o_i, p_i)$ is the distance (Euclidean) between the centroid of the sector i, o_i, and the point of the same sector, p_i, that is farthest from o_i.

Higher values of d_i represent higher values of compactness, which means a "higher density" in sector i.This measure does not guarantee *extreme compactness*, but prevents spread out sectors.

In the same sectorization process, it is not desirable to have one sector with a high concentration (compactness), while others present poor values. Therefore, similarly to the balance analysis, compactness is evaluated using the coefficient of variation CV_d defined in 1.7.

$$CV_d = \frac{s'_d}{\overline{d}} \tag{1.7}$$

where $\bar{d} = \frac{\sum_{i=1}^{K} d_i}{K}$ and $s'_d = \sqrt{\frac{1}{K-1} \cdot \sum_{i=1}^{K} (d_i - \overline{d})^2}$.

A good sectorization must have a CV_d close to zero.

Contiguity

Consider the original graph $G = (V, E)$, with $|V| = N$, where K sectors must be constructed ($K < N$). The evaluation of the contiguity of the K sectors is calculated using the adjacency matrices obtained from the k subgraphs $G'_i = (V'_i, E'_i)$ ($i = 1, ..., K$), where V'_i and E'_i represent the set of vertices and the set of edges of subgraph G_i', respectively. The number of vertices of each sector i is represented by: $|V_i'| = n_i, i = 1,...,K$.

For each subgraph, G_i' also considers the symmetric matrix given by $M^i = [m^i_{wj}]_{w,j=1,...,n_i}$ with main diagonal zero, where

$$m^i_{wj} = \begin{cases} 1 \text{ if in sector i exists a walk between w and j} \\ 0 \qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$M^i = \begin{bmatrix} 0 & m^i_{12} & m^i_{13} & ... & m^i_{1n_i} \\ m^i_{21} & 0 & m^i_{23} & ... & m^i_{2n_i} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m^i_{n_i1} & m^i_{n_i2} & m^i_{n_i3} & ... & 0 \end{bmatrix}.$$

If for all $j \in \{1,...,n_i\}$, $\sum_{w=1}^{n_i} m^i_{wj} = n_i - 1$ or for all $w \in \{1,...,n_i\}$ the condition $\sum_{j=1}^{n_i} m^i_{wj} = n_i - 1$ is met, which is equivalent, then sector i is contiguous.

The next expression (1.8) for c_i is used to measure contiguity (c_i) in each sector I:

$$c_i = \frac{\sum_{j=1}^{n_i} (\sum_{w=1}^{n_i} m^i_{wj})}{n_i(n_i - 1)} \tag{1.8}$$

For every sector i, $0 \leq c_i \leq 1$.

This is not enough to characterize the "level" of contiguity. The quality of the sectors must combine the contiguity of all sectors produced. The weighted average of *isolated contiguities* is used to evaluate the resulting contiguity.

$$\bar{c} = \frac{\sum_{i=1}^{K} c_i \cdot n_i}{N}. \tag{1.9}$$

$\bar{c}$ is a value that is always between zero and one.

From the perspective of contiguity, a good sectorization must have a $\bar{c}$ value as close to one as possible.

1.5.2 Routes - Traveling in sectors

A vehicle leaves the garage, and returns empty at the end of a work day. After the vehicle is filled, it is emptied in special points (landfills or transfer stations) that are available for this purpose. Each vehicle is responsible for collecting waste in each sector. The model for the arc routing problem linked to the case study is presented in [45].

Moreover, some deposition points may have limitations regarding the number of visits received daily. The limitations are mainly due to the small size of the existing facilities. The deposition in different points may represent different costs for the vehicle. Landfills and transfer stations are not simple "crossing points". Each time a vehicle enters a deposition point is emptied.

1.5.2.1 Arc Routing model with Limited Multi-Landfills

The Mixed Capacitated Arc Routing Problem with Limited Multi-Landfills (MCARP-*LML*) is an MCARP with multiple landfills ([44] and [45]) some of which with a limited number of visits for waste disposal. The objective is to minimize the cost of travels between the landfill/transfer station and the garage or depot, while the demand is met without exceeding the capacity of the vehicle. This model is based on the work by [18] for the MCARP. The main differences between those two models is that in the model presented by [18] landfill and garage are represented by the same point. As a consequence, the landfill is unique and has no limitations regarding the number of empties. In the model used here, the number of landfills (which are distinct from the garage) is greater or equal to one. As previously stated, only the case with one vehicle is addressed. The situation with different vehicles is presented by the authors in [45].

1.5.2.2 Evaluating routes

The quality of routes is once again evaluated considering the equilibrium between the elements in the group. In this case, the time spent collecting waste in each sector must be as similar as possible. Considering K routes (K sectors), $r_i, i = 1, ..., K$ represents the time to collect sector i, the coefficient of variation CV_r is defined in (1.10) as:

$$CV_r = \frac{s'_r}{\bar{r}} \tag{1.10}$$

where $\bar{r} = \frac{\sum_{i=1}^{K} r_i}{K}$ and $s'_r = \sqrt{\frac{1}{K-1} \cdot \sum_{i=1}^{K} (r_i - \bar{r})^2}$.

A good set of routes must have a CV_r close to 0.

1.6 Computational results

This section presents the computational results for the real case study described in this paper. The results were obtained using the CPLEX 12.6 (IBM ILOG CPLEX Optimization Studio) solver in an Intel Core i7 2.00 GHz computer with Turbo Boost up to 3.1GHz computer, and 4.00 GB of RAM. In the most common situation throughout the year, three *freguesias* must be collected every day, and the others twice a week. Two circuits are designed for each day, except Sundays. The week is divided into three parts: the first part includes Monday, Tuesday and Wednesday; the second is a copy of the first and includes Thursday, Friday and Saturday; finally, the third part, which corresponds to Sunday, is only dedicated to the three regions (*freguesias*) with daily demand.

The electromagnetism-based approach was initially applied and the results are depicted in Figure 1.6. Euclidean distances between elementary units (*freguesias*) of the municipality of Monção were used.

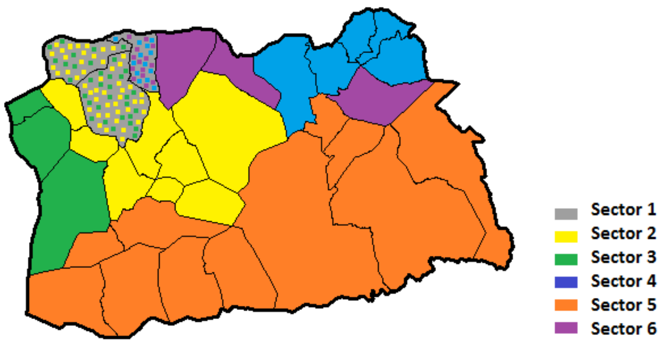

Fig. 1.6 Different colors identify diverse sectors of the municipality of Monção

Table 1.4 reveals the values obtained for the *three measures of quality*

Table 1.4 Measures of sector's *quality*

Equilibrium	Compactness	Contiguity
0.1744	0.2712	0.6667

A possible schedule is presented in Table 1.5 considering the three parts of the week characterized before.

One route was determined for each of the six sectors, and only one vehicle was considered. The results are presented in Table 1.6, where

Table 1.5 Weekly schedule (S_i represents Sector i)

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
S_1	S_4	S_6	S_1	S_4	S_6	S_1
S_5	S_2	S_3	S_5	S_2	S_3	-

$\#T$ - is the number of trips.
$\#V$ - represents the number of vertices.
$\#R$ - is the number of required edges (or arcs).
$\#NR$ - is the number of non-required edges (or arcs).

The values in the last three columns reflect the $Gap(\%)$, the computational time in seconds and the objective function (seconds).

Table 1.6 Characteristics of the six routes

ID [a]	$\#T$	$\#V$	$\#R$	$\#NR$	$Gap(\%)$	$Comp.Time(sec)$	OF
S_1	4	183	145	121	0	92.11	36111.448
S_2	4	200	165	124	0	2642.08	43619.712
S_3	3	136	110	82	0	36.23	27867.984
S_4	4	189	151	153	0	4877.91	46561.504
S_5	4	206	161	133	0.28	$3h$ [b]	50937.096
S_6	4	157	114	126	0.04	$3h$ [b]	39376.232

[a] Instances are available in

$$http://www.inescporto.pt/\sim amr/Limited\_Multi\_Landfills/RealCase/1Vehicle/$$

[b] After 3 hours running

When (only) Euclidean distances between *freguesias* are considered, just by looking at the map the real resulting sectors do not seem to be the most suitable. In fact, there are situations where even though two *freguesias* are close, there is no link (or road) between them. Figure 1.7 shows all the possible direct links (green lines) between two *freguesias*.

The initial signs of attraction/repulsion between two *freguesias* were redefined taking into consideration the reality of road connections and distances. The resulting sectorization is presented in Figure 1.8.

The new sectors obtained feature a better equilibrium when the amount of waste and better contiguity are taken into account, as confirmed in Table 1.7.

Table 1.7 Measures of sector's quality

Equilibrium	Compactness	Contiguity
0.1558	0.3224	0.8130

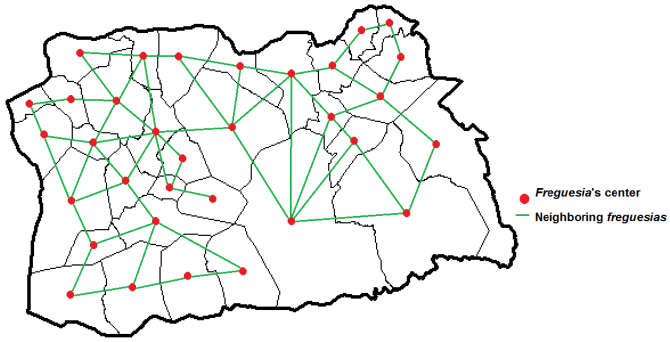

Fig. 1.7 Linked *freguesias*

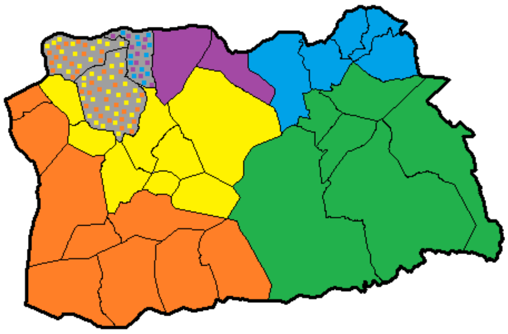

Fig. 1.8 Different colors identify different sectors of the municipality of Monção (scenario 2)

The resulting planning is provided in Table 1.8.

Table 1.8 Weekly schedule (S_i represents Sector i)

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
S_1	S_2	S_4	S_1	S_2	S_4	S_2
S_3	S_6	S_5	S_3	S_6	S_5	-

Taking into consideration the new sectors, new routes are calculated and presented in Table 1.9.

After analyzing and comparing the two scenarios in terms of routes (results displayed in Tables 1.6 and 1.9), it was possible to confirm that the variation coefficient decreased from 0.2009 to 0.1038 when the information regarding the topology of the territory was considered.

Table 1.9 Characteristics of the new six routes (scenario 2)

ID [a]	*#T*	*#V*	*#R*	*#NR*	$Gap(\%)$	$Comp.Time(sec)$	*OF*
S_1	4	157	106	114	0	582.05	37401.512
S_2	4	183	145	121	0	92.11	36111.448
S_3	4	200	165	124	0	2642.08	43619.712
S_4	4	189	151	153	0	4877.91	46561.504
S_5	4	207	162	166	3.78	$3h$ [b]	43924.176
S_6	3	143	117	89	0	429.70	38296.704

[a] Instances are available in

$$http://www.inescporto.pt/\sim amr/Limited\_Multi\_Landfills/RealCase/1Vehicle/$$

[b] After 3 hours running

1.7 Conclusions

This paper deals simultaneously with sectorization and routing problems. It is based on a real-life waste collection problem in Monção, Portugal, but the authors believe that the methods developed can be extended to other applications. That is exactly the case of the new method for sectorization, driven by electromagnetism and Coulomb's Law. The first phase of the approach to the real-life problem was dedicated to sectorization. Not only was the division of the territory considered, but also the frequency with which waste is collected, a critical issue in these types of applications. The new method was able to address the situation by considering geographical information and several quality measures, such as equilibrium, contiguity, and compactness. Additionally, the method allows decision-makers to define levels of "attraction and/or repulsion", meeting their expectations of controlling the results. Special characteristics related to the number and type of deposition points also served as motivation for this work. The second part addresses the routing problems in each sector. The new model $MCARP-LML$ - Mixed Capacitated Arc Routing Problem with Limited Multi-Landfills, based on a model from the literature, was proposed and tested in real instances. The results obtained confirmed the effectiveness of the entire approach.

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