Optimal Control Formulations for the Unit Commitment Problem

Dalila B.M.M. Fontes and Fernando A.C.C. Fontes and Luís A.C. Roque

Abstract The Unit Commitment (UC) problem is a well-known combinatorial optimization problem arising in operations planning of power systems. It involves deciding both the scheduling of power units – when each unit should be turned on or off–, and the economic dispatch problem – how much power each of the on units should produce –, in order to meet power demand at minimum cost, while satisfying a set of operational and technological constraints. This problem is typically formulated as nonlinear mixed-integer programming problem and has been solved in the literature by a huge variety of optimization methods, ranging from exact methods (such as dynamic programming and branch-and-bound) to heuristic methods (genetic algorithms, simulated annealing, and particle swarm). Here, we discuss how the UC problem can be formulated with an optimal control model, describe previous discrete-time optimal control models, and propose a continuous-time optimal control model. The continuous-time optimal control formulation proposed has the advantage of involving only real-valued decision variables (controls) and enables extra degrees of freedom as well as more accuracy, since it allows to consider sets of demand data that are not sampled hourly.

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1 Introduction

In this work, we address the Unit Commitment (UC) problem using Optimal Control methodologies. Despite being an highly researched problem with dynamical and multi-period characteristics, it appears that it has not been addressed by optimal control methods before, except in [13].

A problem that must be solved frequently by a power utility is to economically determine a schedule of which units are to be used and how much each unit should produce in order to meet the forecasted demand, while satisfying operational and technological constraints, over a short time horizon [30, 31]. Good solutions are of most importance since they not only may provide substantial savings (tens to hundreds of millions of dollars) in operational and fuel costs but also maintain system reliability by keeping a proper spinning reserve[42]. Due to its combinatorial nature, multi-period characteristics, and nonlinearities, this problem is highly computational demanding and, thus, solving the UC problem for real sized systems is a hard optimization task: it is a NP-hard problem. The UC problem has been extensively studied in the literature. Several numerical optimization techniques, based both on exact and on approximate algorithms have been reported.

Several approaches based on exact methods have been used, such as dynamic programming, mixed-integer programming, benders decomposition, lagrangian relaxation and branch and bound methods, see e.g. [20, 8, 36, 3]. The main drawbacks of these traditional techniques are the large computational time and memory requirements for large complexity and dimensionality problems. Dynamic programming [20, 27] is a powerful and flexible methodology, however its suffers from the dimensionality problem, not only in computational time, but also in storage requirements. Recently a stochastic dynamic programming approach to schedule power plants was proposed [29]. In [3], a solution using lagrangian relaxation is proposed. However, the problem becomes too complex as the number of units increases and there are some difficulties in obtaining feasible solutions. Takriti [36] addresses the unit commitment problem by using mixed-integer programming which is a very hard task when the number of units increases since it requires large memory and leads to large computational time requirements. Other authors have proposed the use of mixed integer linear programming to solve the linearized versions of the problem, see e.g. [14, 39]. The branch-and-bound method proposed in [8] uses a linear function to represent the fuel consumption and a time-dependent start-up cost, but has a exponential growth in the computational time with problem dimension.

More recently, several metaheuristic methods such as evolutionary algorithms and hybrids of the them have been proposed, see e.g. [38, 10, 34, 7, 2]. These approaches have, in general, better performances than the traditional heuristics. The most commonly used metaheuristic methods are simulated annealing [26, 34], evolutionary programming [18, 28], memetic algorithms [38], particle swarm optimization [41], tabu search [25, 40], and genetic algorithms [19, 35, 9, 32]. For further discussion and comparison of these methodologies, with special focus on metaheuristic methods, and other issues related to the unit commitment problem see the very recent review by Saravanan et al [33].

Although the UC problem is a highly researched problem with dynamical and multi-period characteristics, it appears that it has not been addressed before by optimal control methods, except in [13] as mentioned previously. In this work, the authors have formulated the UC problem as a discrete mixed-integer optimal control problem, which has then been converted into one with only real-valued controls. Here, we discuss formulations of the UC problem as an Optimal Control (OC) model and propose a new optimal control modeling approach. The model derived is a continuous one and only involves real-valued decision variables (controls).

The main contributions of the proposed modeling approach are twofold. Firstly, since it allows decisions to be taken at any time moment, and not only a specific points in time (usually, hourly), it may render better solutions. It should be noticed that the proposed approach allows for decisions about unit commitment/decommitment and about power production variation at any moment in time. Secondly, it no longer forces utilities to treat demand variations as instantaneous, i.e. time steps. In addition, if one chooses to use the approximated hourly data, as usual in the literature, the solution strategies (both regarding unit commitment/decommitment and power production) of the proposed model will approximate the discrete-time solutions since actions are only required to be taken hourly.

The remaining of this article is organized as follows. In Section 2, the UC problem is described and its mathematical programming formulation is given. The mixed-integer optimal control formulation and the variable time transformation that allows for rewriting it with only real-valued controls are given in Section 3. In Section 4, we provide a detailed description of the continuous optimal control model including only real-valued controls, which is proposed here. Finally, in Section 5 we draw some conclusions and discuss future work.

2 The Unit Commitment Problem

The Unit Commitment Problem involves both the scheduling of power units (i.e., the decision when each unit is turned on or turned off along a predefined time horizon), and the economic dispatch problem (the problem of deciding how much each unit that is on should produce). The scheduling of the units is an integer programming problem and the economic dispatch problem is a nonlinear (real-valued) programming problem. The UC problem is then as a nonlinear, non-convex and mixed integer optimization problem [9]. The objective of the UC problem is the minimization of the total operating costs over the scheduling horizon while satisfying the system demand, the spinning reserve requirements, and other generation constraints such as capacity limits, ramp rate limits, and minimum up/down times.

The objective function is expressed as the sum of the fuel, start-up, and shut-down costs.

2.1 Mixed-Integer Mathematical Programming Model

The model has two types of decision variables. Binary decision variables $u_j(t)$, which are either set to 1, meaning that unit j is committed at time t; or otherwise are set to zero. Real-valued variables $y_j(t)$, which indicate the amount of power produced by unit j at time t. For the sake of simplicity, we also define the auxiliary variables $T_j^{on/off}(t)$, which represent the number of time periods for which unit j has been continuously on-line/off-line until time t.

Objective Function:
The objective function has three cost components: generation costs, start-up costs, and shut-down costs. The generation costs, also known as the fuel costs, are conventionally given by the following quadratic cost function.

$$F_j(y_j(t)) = a_j \cdot (y_j(t))^2 + b_j \cdot y_j(t) + c_j, \tag{1}$$

where a_j, b_j, c_j are the cost coefficients of unit j.

The start-up costs, that depend on the number of time periods during which the unit has been off, are given by

$$S_j(t) = \begin{cases} S_{H,j}, & \texttt{if } T_{min,j}^{off} \le T_j^{off}(t) \le T_{min,j}^{off} + T_{c,j}, \\ S_{C,j}, & \texttt{if } T_j^{off}(t) > T_{min,j}^{off} + T_{c,j}, \end{cases} \tag{2}$$

where $S_{H,j}$ and $S_{C,j}$ are, respectively, the hot and cold start-up costs of unit j and $T_{min,j}^{on/off}$ is the minimum uptime/downtime of unit j. The shut-down costs S_{dj} for each unit, whenever considered in the literature, are not time dependent.

Therefore, the cost incurred with an optimal scheduling is given by the minimization of the total costs for the whole planning period,
Minimize

$$\sum_{t=1}^{T} \left(\sum_{j=1}^{N} \left\{ F_j(y_j(t)) \cdot u_j(t) + S_j(t) \cdot (1 - u_j(t-1)) \cdot u_j(t) \right\} + S_{dj} \cdot (1 - u_j(t)) \cdot u_j(t-1) \right). \tag{3}$$

Constraints:
As said before, there are two types of constraints: the operational constraints and the technological constraints. The first set of constrains can be further divided into unit output range limit (equation (4)), maximum output variation, i.e. ramp rate constraints (equation (5)), and minimum number of time periods that a unit must be continuously in each status (on-line or off-line) (equations (6) and (7)); while the

second set of constraints can be divided into load requirements (equation (8)) and spinning reserve requirements (equation (9)).

$$Ymin_j \cdot u_j(t) \le y_j(t) \le Ymax_j \cdot u_j(t), \text{ for } t \in \{1,...,T\} \text{ and } j \in \{1,...,N\}. \quad (4)$$

$$-\Delta_j^{dn} \le y_j(t) - y_j(t-1) \le \Delta_j^{up}, \text{ for } t \in \{1,...,T\} \text{ and } j \in \{1,...,N\}. \quad (5)$$

$$T_j^{on}(t) \ge T_{min,j}^{on}, \text{ for each time } t \text{ in which unit } j \text{ is turned off and } j \in \{1,...,N\}. \quad (6)$$

$$T_j^{off}(t) \ge T_{min,j}^{off}, \text{ for each time } t \text{ in which unit } j \text{ is turned on and } j \in \{1,...,N\}. \quad (7)$$

$$\sum_{j=1}^{N} y_j(t) \cdot u_j(t) \ge D(t), t \in \{1,...,T\}. \quad (8)$$

$$\sum_{j=1}^{N} Ymax_j \cdot u_j(t) \ge R(t) + D(t), t \in \{1,...,T\}. \quad (9)$$

The parameters used in the above equations are defined as follows:
T: Number of time periods (hours) of the scheduling time horizon;
N: Number of generation units;
$\mathbf{R(t)}$: System spinning reserve requirements at time t, in $[MW]$;
$\mathbf{D(t)}$: Load demand at time t, in $[MW]$;
$\mathbf{Ymin_j}$: Minimum generation limit of unit j, in $[MW]$;
$\mathbf{Ymax_j}$: Maximum generation limit of unit j, in $[MW]$;
$\mathbf{T_{c,j}}$: Cold start time of unit j, in $[hours]$;
$\mathbf{T_{min,j}^{on/off}}$: Minimum uptime/downtime of unit j, in $[hours]$;
$\mathbf{T_{j,0}^{on}}$: Initial state of unit j at time 0, time since the last status switch off/on, in $[hours]$;
$\mathbf{T_{j,0}^{off}}$: Initial state of unit j at time 0, time since the last status switch on/off, in $[hours]$;
$\Delta^{\mathbf{dn/up_j}}$: Maximum allowed output level decrease/increase in consecutive periods for unit j, in $[MW]$.

3 Discrete-Time Optimal Control Approach

In this section, we describe the work in [13], where a mixed-integer optimal control model (OCM) is proposed to the UC problem. Although it is possible to address optimal control problems (OCPs) with discrete control sets (see, e.g., [11, 24]), it is computationally demanding. Thus, it was proposed to convert this model into another OCM with only real-valued controls. The conversion process requires the use of a novel variable time transformation that was able to address adequately several discrete-valued control variables arising in the original problem formulation. Finally, The transformed real OCM was transcribed into a nonlinear programming problem to be solved by a nonlinear optimization solver.

3.1 Discrete-Time Mixed-Integer Optimal Control Model

The mixed-integer optimal control model has two types of decision/control variables. On the one hand, binary control variables $u_j(t)$, which are either set to 1, meaning that unit j is committed at time t; or otherwise set to zero. On the other hand, real-valued variables $\Delta_j(t)$, which enable to control, by increasing or decreasing, the power produced by unit j at time t. We consider two types of state variables: variables $y_j(t)$, which represent the power generated by unit j at time t and variables $T_j^{on/off}(t)$, which represent the number of time periods for which unit j has been continuously on-line/off-line until time t. For convenience, let us also define the index sets: $\mathbb{T} := \{1,\dots,T\}$ and $\mathbb{J} := \{1,2,\dots,N\}$. The parameters related to the problem data are as defined in the previous section. The UC problem can now be formulated as a mixed-integer optimal control model.

Objective Function:
Minimize

$$\sum_{t=1}^{T}\left(\sum_{j=1}^{N}\left\{F_j(y_j(t))u_j(t)+S_j(t)(1-u_j(t-1))u_j(t)+S_{dj}\cdot(1-u_j(t))\cdot u_j(t-1)\right\}\right) \tag{10}$$

where the costs are as before.

The state dynamics:
The state dynamics in this model are as follows:

The production of each unit, at time t, depends of the amount produced in the previous time period and is limited by the maximum allowed decrease and increase of the output that can occur during one time period:

$$y_j(t) = [y_j(t-1)+\Delta_j(t)]\,.u_j(t), \text{ for } t\in\mathbb{T} \text{ and } j\in\mathbb{J}. \tag{11}$$

The number of time periods for which unit j has been continuously on-line until time t is given by

$$T_j^{on}(t) = \left[T_j^{on}(t-1)+1\right].u_j(t), \text{ for } t \in \mathbb{T} \text{ and } j \in \mathbb{J}. \tag{12}$$

The number of time periods for which unit j has been continuously off-line until time t is given by

$$T_j^{off}(t) = \left[T_j^{off}(t-1)+1\right].(1-u_j(t)), \text{ for } t \in \mathbb{T} \text{ and } j \in \mathbb{J}. \tag{13}$$

Pathwise Constraints:
The constraints are as before, except for the ramp rate constraints, and thus they are given by equation (4) and equations (6) to (9). The ramp rate constraints, which were given by equation (5) are now handled by the control constraints.

$$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right], \text{ for } t \in \mathbb{T} \text{ and } j \in \mathbb{J}. \tag{14}$$

3.2 The Variable Time Transformation Method

The idea here is to develop a variable time transformation in order to convert the mixed-integer OCM into an OCM with only real-valued controls. The transformation of a mixed-integer optimal control problem into a problem with only real-valued controls is not new, nor is new the general idea of a variable time transformation method. See the classical reference [17] and also [37, 22, 23, 1, 21]. See also the recent work [15] for a discussion on several variable time transformation methods.

Consider, for each unit j, a non-decreasing real-valued function $t \mapsto \tau_j(t)$. Consider also a set of values $\bar{\tau}_1, \bar{\tau}_2, \ldots$ such that when $\tau_j(t) = \bar{\tau}_k$ for odd k we have a transition from off to on for unit j, and when $\tau_j(t) = \bar{\tau}_k$ for even k we have a transition from on to off. So, we consider that unit j is:

- on if $\tau_j(t) \in [\bar{\tau}_1, \bar{\tau}_2) \cup [\bar{\tau}_3, \bar{\tau}_4) \cup \ldots \cup [\bar{\tau}_{2k-1}, \bar{\tau}_{2k})$;
- off if $\tau_j(t) \in [0, \bar{\tau}_1) \cup [\bar{\tau}_2, \bar{\tau}_3) \cup \ldots \cup [\bar{\tau}_{2k}, \bar{\tau}_{2k+1})$.

It might help to interpret τ_j to be a transformed time scale and that the values $\bar{\tau}_1, \bar{\tau}_2, \ldots$ are switching "times" in the transformed time scale. We can consider, without loss of generality, that the values $\bar{\tau}_k$ are equidistant. Nevertheless, in real time t, the distance between two events $\bar{\tau}_k$ and $\bar{\tau}_{k+1}$ can be stretched or shrunk to any non-negative value, including zero, depending on the shape of the function $t \mapsto \tau_j(t)$.

To simplify the exposition, and without loss of generality, let us consider that $\bar{\tau}_k - \bar{\tau}_{k-1}$ is constant and equal to 1, for all $k = 1, 2, \ldots$. In such case, unit j is:

- on if $\tau_j(t) \in [1,2) \cup [3,4) \cup \ldots \cup [2k-1, 2k)$;

- off if $\tau_j(t) \in [0,1) \cup [2,3) \cup \ldots \cup [2k, 2k+1)$.

Now, consider the controls

$$w(t) \in [0,1], \quad t = 0,1,\ldots,T-1,$$

that represent the increment from $\tau(t)$ to $\tau(t+1)$ such that

$$\tau(t) = \tau_0 + \sum_{k=0}^{t-1} w(k)$$

or

$$w(t) = \tau(t+1) - \tau(t), \quad \text{with } \tau(0) = \tau_0.$$

3.3 The Optimal Control Model with real-valued controls

We recall the index set $\mathbb{J}$ and redefine $\mathbb{T}$ to be more consistent with usual discrete-time control formulations.

$\mathbb{T} := \{0,\ldots,T-1\}$ and $\mathbb{J} := \{1,2,\ldots,N\}$.

In the same spirit, we redefine the control $\Delta_j(t)$ for $t \in \{0,\ldots,T-1\}$ to be the amount of power generation incremented or decremented for the next time period (rather than comparatively to the previous period).

Note that the controls are all real-valued and comprise:
$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right]$,
$w_j(t) \in [0,1]$.

Define the sets of time periods:

$$I_j^{on} := \{t \in \mathbb{T} : \tau_j(t) \in [2k-1, 2k), k \geq 1\},$$
$$I_j^{off} := \mathbb{T} \setminus I_j^{on},$$
$$I_j^{off>on} := \{t \in \mathbb{T} : \tau_j(t) \geq 2k+1, \tau_j(t-1) < 2k+1, k \geq 0\},$$
$$I_j^{on>off} := \{t \in \mathbb{T} : \tau_j(t) \geq 2k, \tau_j(t-1) < 2k, k \geq 1\}\}.$$

Finally, the unit commotment problem can be formulated as an optimal control problem, as follows:

Minimize

$$\sum_{j=1}^{N} \left(\sum_{t \in I_j^{on}} F_j(y_j(t)) + \sum_{t \in I_j^{off>on}} S_j(t) + \sum_{t \in I_j^{on>off}} S_{dj}(t) \right), \tag{15}$$

subject to the dynamic constraints

$$\tau_j(t+1) = \tau_j(t) + w_j(t) \quad j \in \mathbb{J},\ t \in \mathbb{T}, \tag{16}$$

$$T_j^{on}(t+1) = \begin{cases} T_j^{on}(t)+1 & j \in \mathbb{J},\ t \in I_j^{on}, \\ 0 & j \in \mathbb{J},\ t \in I_j^{off}, \end{cases} \tag{17}$$

$$T_j^{off}(t+1) = \begin{cases} T_j^{off}(t)+1 & j \in \mathbb{J},\ t \in I_j^{off}, \\ 0 & j \in \mathbb{J},\ t \in I_j^{on}, \end{cases} \tag{18}$$

$$y_j(t+1) = \begin{cases} y_j(t)+\Delta_j(t) & j \in \mathbb{J},\ t \in I_j^{on}, \\ 0 & j \in \mathbb{J},\ t \in I_j^{off}, \end{cases} \tag{19}$$

the initial state constraints

$$T_j^{on}(0) = T_{j,0}^{on} \quad \text{(given)}, \tag{20}$$

$$T_j^{off}(0) = T_{j,0}^{off} \quad \text{(given)}, \tag{21}$$

$$\tau_j(0) = \begin{cases} 0 \text{ if } T_{j,0}^{on} = 0 \\ 1 \text{ if } T_{j,0}^{on} > 0, \end{cases} \tag{22}$$

$$y_j(0) = \begin{cases} 0 & \text{if } T_{j,0}^{on} = 0 \\ y_{j,0} \in [Ymin_j, Ymax_j] & \text{if } T_{j,0}^{on} > 0, \end{cases} \tag{23}$$

the control constraints

$$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up}\right], \tag{24}$$

$$w_j(t) \in [0,1], \tag{25}$$

and the pathwise state constraints

$$y_j(t) \in [Ymin_j, Ymax_j] \quad j \in \mathbb{J},\ t \in I_j^{on}, \tag{26}$$

$$\sum_{j\in\mathbb{J}} y_j(t) \geq D(t) \quad t = 1,2,\ldots,T, \tag{27}$$

$$\sum_{j\in\mathbb{J}} Ymax_j(t) \geq R(t) + D(t) \quad t = 1,2,\ldots,T, \tag{28}$$

where $Ymax_j(t) = Ymax_j$ if $t \in I_j^{on}, Ymax_j(t) = 0$ otherwise

$$y_j(t) \in [Y_{min_j}, \max\{Y_{min_j}, \Delta_j^{up}\}] \quad j \in \mathbb{J}, t \in I_j^{off>on}, \tag{29}$$

$$T_j^{on}(t-1) \geq T_{min,j}^{on} \quad j \in \mathbb{J},\ t \in I_j^{on>off}, \tag{30}$$

$$T_j^{off}(t-1) \geq T_{min,j}^{off} \quad j \in \mathbb{J},\ t \in I_j^{off>on}. \tag{31}$$

3.4 Conversion into a Nonlinear Programming Problem

To construct the nonlinear programming problem (NLP), we start by defining the optimization variable x containing both the control and state variables. That is

$$x = [\Delta, w, \tau, T^{on}, T^{off}, y]$$

with dimension $(6T+1)\times N)$.

(We could have considered just the controls Δ, w together with the free initial state $y(0)$. An option which, despite having the advantage of a lower dimensional decision variable, is known to frequently have robustness problems, specially in optimal control problems with pathwise state constraints such as ours. For further discussion see e.g. Betts [4].)

The objective function should be rewritten in terms of x: Minimize $J(x)$ over x.

To facilitate the optimization algorithm, we separate the constraints that are simple variable bounds, linear equalities, linear inequalities and the remaining:

- upper/lower bounds: equations (24)-(26);
- linear equalities: equation (16);
- linear inequalities: equation (27);
- nonlinear equalities: equations (17)-(19); and
- nonlinear inequalities: equations (28)-(31).

Note that equations (20)-(23) are not implemented as constraints since the initial values of these state variables are considered as parameters and not variables.

With these considerations the problem is formulated as the following NLP

$$\begin{aligned}
&\text{Minimize}_{x\in\mathbb{R}^{(6T+1)\times N}} J(x)\\
&\text{subject to}\\
&\quad LB \le x \le UB\\
&\quad A_{eq}x = b_{eq}\\
&\quad A_{ineq}x \le b_{ineq}\\
&\quad g(x) = 0\\
&\quad h(x) \le 0.
\end{aligned}$$

More specifically

Minimize over x

$$J(x) = \sum_{j=1}^{N}\left(\sum_{t\in I_j^{on}} F_j(y_j(t)) + \sum_{t\in I_j^{off>on}} S_j(t) + \sum_{t\in I_j^{on>off}} S_{dj}(t)\right).$$

Subject to

- lower bounds:
  $\Delta_j(t) \geq -\Delta_j^{dn}$, for $t \in \mathbb{T}$ and $j \in \mathbb{J}$,
  $w_j(t) \geq 0, \quad j \in \mathbb{J},\ t \in \mathbb{T};$
  $\tau_j(t) \geq 0, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $T_j^{on}(t) \geq 0, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $T_j^{off}(t) \geq 0, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $y_j(t) \geq 0, \quad j \in \mathbb{J},\ t \in \mathbb{T},$

- upper bounds:
  $\Delta_j(t) \leq \Delta_j^{up}, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $w_j(t) \leq 1, \quad j \in \mathbb{J},\ t \in \mathbb{T};$
  $\tau_j(t) \leq T, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $T_j^{on}(t) \geq 2T, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $T_j^{off}(t) \geq 2T, \quad j \in \mathbb{J},\ t \in \mathbb{T},$
  $y_j(t) \leq Ymax_j, \quad j \in \mathbb{J},\ t \in \mathbb{T},$

- linear equalities:
  $\tau_j(t+1) - \tau_j(t) - w_j(t) = 0 \quad j \in \mathbb{J},\ t \in \mathbb{T};$

- linear inequalities:
  $\sum_{j \in \mathbb{J}} y_j(t) - D(t) \geq 0 \quad t \in \mathbb{T};$

- nonlinear equalities:
  $T_j^{on}(t+1) = \begin{cases} T_j^{on}(t) + 1 & \text{if } j \in \mathbb{J},\ t \in I_j^{on}, \\ 0 & \text{if } j \in \mathbb{J},\ t \in I_j^{off}, \end{cases}$
  $T_j^{off}(t+1) = \begin{cases} T_j^{off}(t) + 1 & \text{if } j \in \mathbb{J},\ t \in I_j^{off}, \\ 0 & \text{if } j \in \mathbb{J},\ t \in I_j^{on}, \end{cases}$
  $y_j(t+1) = \begin{cases} y_j(t) + \Delta_j(t) & \text{if } j \in \mathbb{J},\ t \in I_j^{on}, \\ 0 & \text{if } j \in \mathbb{J},\ t \in I_j^{off}, \end{cases}$
  and
- nonlinear inequalities:
  $y_j(t) \geq Ymin_j \quad j \in \mathbb{J},\ t \in I_j^{on},$
  $\sum_{j \in \mathbb{J}} Ymax_j(t) - R(t) - D(t) \geq 0 \quad t \in \mathbb{T},$
  $y_j(t) - Y_{min_j} \geq 0 \quad j \in \mathbb{J}, t \in I_j^{off>on},$
  $y_j(t) - \max\{Y_{min_j}, \Delta_j^{up}\} \leq 0 \quad j \in \mathbb{J}, t \in I_j^{off>on},$
  $T_j^{on}(t-1) - T_{min,j}^{on} \geq 0 \quad j \in \mathbb{J},\ t \in I_j^{on>off},$
  $T_j^{off}(t-1) - T_{min,j}^{off} \geq 0 \quad j \in \mathbb{J},\ t \in I_j^{off>on}.$

Of course, since this (real-valued) NLP is a problem that originally was a MINLP, it is still a very hard problem. Namely, it is a nonconvex problem and standard NLP solvers will find just a local, not necessarily global, optimum. Nevertheless, this is very useful since it can be embedded, as a local search optimizer, into a global search heuristic method.

4 Continuous-Time Optimal Control Approach

In this section, we develop a continuous-time optimal control formulation for the unit commitment problem that uses only real-valued decision variables.

To introduce the ideas and concepts used in this formulation let us start by analysing a specific and simple situation.

Consider a generation unit for which the minimum time it must be consecutively on is 2 hours ($T_{min}^{on} = 2$) and the minimum time it must be consecutively off is 3 hours ($T_{min}^{off} = 3$). Furthermore, consider also the unit to be initially off-line. Let the unit be turned off and turned on as soon as the elapsed time reaches T_{min}^{on} and T_{min}^{off}, respectively. Such a strategy corresponds to the unit having the maximum number of status switches. Thus, for a 24h period, we would obtain a profile as given in Fig. 1.

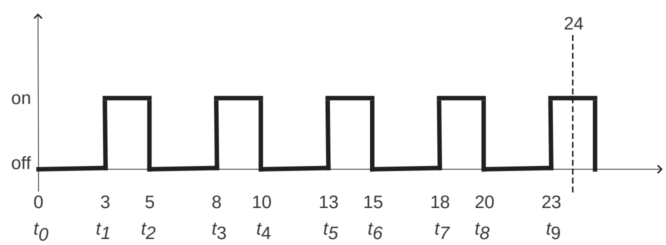

Fig. 1 Unit status, when the status switching strategy is as often as possible.

For the example just described, the times at which status switching occurs are given by

$$t_{i+1} = \begin{cases} t_i + T_{min,j}^{on}, & \text{if } i \text{ is odd,} \\ t_i + T_{min,j}^{off}, & \text{if } i \text{ is even.} \end{cases}$$

All other feasible status switching strategies can be obtained from the one just described by stretching any number of time intervals $[t_i, t_{i+1})$ with $i = 1, \ldots, S$, where S - the maximum number of status switches that can occur within the 24-hours scheduling period is given by

$$S = 1 + 2 * \left(24 \text{ DIV } (T_{min,j}^{on} + T_{min,j}^{off}) \right),$$

where DIV denotes integer division.

The stretching magnitude α_i in the time interval $[t_i, t_{i+1})$ is bounded from below by 1, since the interval is initially defined as small as possible; and from above by $[1, (24 - t_i)/T_{min}]$, where T_{min} is set to $T^{on}_{min,j}$ or $T^{off}_{min,j}$ depending on wether i is odd or even, respectively, which allows for reaching the end of the scheduling period. It should be noticed that all switches occur at times $t_i \leq 24 - T_{min}$ with T_{min} as defined.

Using a convenient selection of the α_i's we can generate any admissible switching profile. For example, choosing $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_9] = [1,2,1,2,1,2,1,1,1,1]$ leads to the profile given in Fig. 2.

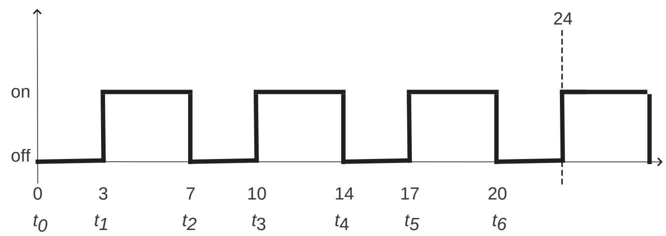

Fig. 2 Status of unit obtained with $\alpha = [\alpha_0, \alpha_1, \ldots, \alpha_9] = [1,2,1,2,1,2,1,1,1,1]$.

Therefore, in any situation the computation of the switching times is given by

$$t_{i+1} = \begin{cases} t_i + \alpha_i T^{on}_{min,j} & \text{if } i \text{ is odd,} \\ t_i + \alpha_i T^{off}_{min,j} & \text{if } i \text{ is even.} \end{cases}$$

4.1 Formulation

Let us define some parameters before introducing the formulation. When considering several units, the maximum number of switches is not the same for all units since they may have different limits on the number of periods that must elapse before a switch is possible. The same is true for the maximum magnitude of the stretch. Therefore and in order to have one single value for these parameters, we compute upper bounds rather than their true value. By defining

$$T^{on+off}_{min} = \min_j \{T^{on}_{min,j} + T^{off}_{min,j}\}$$

we obtain a limit for the maximum number of switches as

$$S = 1 + 2 * 24 \text{ DIV } \left(T^{on+off}_{min}\right)$$

and for the maximum magnitude of the stretch of an interval as

$$s_{max} = 24/\min_j\{T^{on}_{min,j}, T^{off}_{min,j}\}.$$

For convenience, let us also define the index sets:

$$\mathbb{I} := \{0,1,\ldots,S\} \text{ - switching times indexes,}$$
$$\mathbb{J} := \{1,2,\ldots,N\} \text{ - generation unit indexes,}$$

and the time horizon

$$\mathbb{T} := [0,24] \text{ - time horizon interval.}$$

Decision/Control Variables:
The model has two types of control variables, since two types of decisions are taken. On the one hand, one has to decide for how much time each unit is in each status, that is the magnitude of stretch applied to each time interval for each unit, $\alpha_{i,j}$. On the other hand, one also must decide on the amount of power production for each unit at each time instant. In our case, we do this by deciding on the variation of the production at each time instant $\delta_j(t)$.

$\alpha_{i,j}$: stretch magnitude applied to the time interval $[t_i, t_{i+1})$ for unit j. These are real-valued variables in the range $[1, s_{max}]$.

$\delta_j(t)$: rate of change (increase or decrease) for the production of unit j at instant t. These variables are also real-valued an must be within $[-\Delta^{dn}_j, \Delta^{up}_j]$.

State Variables:
The state variables characterize the system and are as follows.

$t_{i,j}$: i-th switching time of unit j;
$u_{i,j}$: Status of unit j in the interval $[t_i, t_{i+1})$, (1 if the unit is on; 0 otherwise);

$u_j(t)$: Status of unit j at instant t, (1 if the unit is on; 0 otherwise);
$y_j(t)$: Power generation of unit j at instant t, in $[MW]$;

Objective Function:
The objective of the UC problem is the minimization of the total costs for the whole planning period, in which the total costs are expressed as the sum of fuel costs and start-up and shut-down costs of the generating units. Therefore, the objective function is as follows:

Minimize

$$\sum_{j\in\mathbb{J}} \int_0^T \Big(F_j\left(y_j(t)\right) u_j(t) + S_j(t)\left(1-u_j(t-1)\right) u_j(t) + S_{dj}\cdot\left(1-u_j(t)\right)\cdot u_j(t-1)\Big)\, dt.$$

Dynamic Constraints:
We must define the unit status during each time interval. Unit j must have it status

switched at the beginning of each interval $[t_i,t_{i+1})$. Thus if in the interval $[t_i,t_{i+1})$ the unit was 1 (on), then in the interval $[t_{i+1},t_{i+2})$ it becomes 0 (off) and vice-versa.

$$u_{i+1,j} = |u_{i,j} - 1|, \quad j \in \mathbb{J}, i \in \mathbb{I}.$$

The ending time instant of a time interval, which is the beginning of the next one, is obtained by adding up the starting time instant with the length of the interval.

$$t_{i+1,j} = t_{i,j} + \alpha_{i,j}[T^{on}_{min,j}u_{i,j} + T^{off}_{min,j}(1-u_{i,j})], \quad j \in \mathbb{J}, i \in \mathbb{I}.$$

In addition, we also must define the power production at each time instant and, for convenience, also the unit status at each time instant.

$$u_j(t) = u_{i,j}, \quad j \in \mathbb{J}, i \in \mathbb{I}, t \in [t_i,t_{i+1}),$$

$$y_j(t) = \begin{cases} 0 & \text{if } u_j(t) = 0, \\ y_j(t_i) + \int_{t_i}^{t} \delta_j(s)ds, \text{with } i = \max\{i : t_i \leq t\}, & \text{if } u_j(t) = 1, \end{cases} \quad t \in \mathbb{T}, j \in \mathbb{J}.$$

Control Constraints:
Due to the mechanical characteristics and thermal stress limitations, the instantaneous output variation level of each online unit is restricted by ramp rate constraints, both up and down.

$$\delta_j(t) \in [-\Delta^{dn}_j, \Delta^{up}_j], \quad j \in \mathbb{J}, t \in \mathbb{T}.$$

The magnitude of the stretch is limited both from below and from above, since one must assure that the $T^{on/off}_{min,j}$ are satisfied and that the scheduling does not go beyond the scheduling horizon.

$$\alpha_{i,j} \in [1,A], \text{ for } i \in \mathbb{I}, j \in \mathbb{J}.$$
$$\text{with } A = \begin{cases} \frac{24-t_i}{T^{on}_{min,j}u_{i,j} + T^{off}_{min,j}(1-u_{i,j})} & \text{if } t_i \leq 24 - T^{on}_{min,j}u_{i,j} + T^{off}_{min,j}(1-u_{i,j}), \\ 1 & \text{otherwise,} \end{cases}$$

Pathwise State Constraints:
Each unit has maximum and minimum output capacity limits.

$$y_j(t) \in [Ymin_j u_j(t), Ymax_j u_j(t)] \quad j \in \mathbb{J},\ t \in \mathbb{T},$$

The power generated at each time instant must meet the respective load demand.

$$\sum_{j\in\mathbb{J}} y_j(t) \geq D(t) \quad t \in \mathbb{T},$$

The spinning reserve is the amount of real power available from on-line units net of their current production level and it must satisfy a pre-specified value, at each time instance.

$$\sum_{j\in\mathbb{J}} Ymax_j(t)u_j(t) \geq R(t)+D(t) \quad t\in\mathbb{T},$$

Initial State Constraints:
The initial status of each unit is given.
$u_{0,j} = InitialStatus_j, \quad j\in\mathbb{J}.$
Also
$u_j(0) = u_{0,j}, \quad j\in\mathbb{J}.$
The first switching interval starts at the beginning of the scheduling horizon and thus
$t_{0,j} = 0, \quad j\in\mathbb{J}.$
Finally, the power production of each on-line unit has to be within its capacity limits.

$$y_j(0)\in[Ymin_j u_{j,0}, Ymax_j u_{j,0}], \quad j\in\mathbb{J}.$$

The numerical solution of continuous-time optimal control problems has been a well-studied subject for many decades [6] and also has been having recent developments and available solvers such as ICLOCS [12], BOCOP [5], ACADO [16]. Although the use of one of these solvers is recommended, an alternative is always to discretize the problem, transcribe it into a nonlinear programming problem and use directly an NLP solver.

The use of a continuous-time formulation for the UC problem has some advantages: (i) the possibility of accommodating any changes in the data or parameters that occur not an hourly basis, but at any time in between; (ii) in particular, the formulation proposed can deal with continuous-time varying demand (which is more realistic), resulting in an output strategy that responds with continuous-time variations; (iii) however, in case the demand and all remaining data varies only on an hourly basis, the resulting output strategy will follow very closely the one obtained with a discrete-time model; (iv) the complexity of the optimization problem obtained is not increased, possibly being easier to find an optimal solution, since the decision variables involved are all real-valued. It is well-known that real-valued nonlinear programming problems are, in general, less difficult to solve than mixed-integer nonlinear programming problems.

5 Conclusions

We have addressed the UC problem, a well-researched problem in the literature, which is usually formulated using a mixed-integer nonlinear programming model. Here we have explored the formulation of this problem using optimal control models. Previous works on an optimal control approach to the UC problem, as far as we are aware of, are limited to the work in [13], that uses a discrete-time optimal control model.

We have proposed here a formulation of the UC problem using a continuous-time optimal control model. An interesting feature of the continuous-time formulation is the fact that, contrary to the usual mixed-integer programming models in the literature, all decision variables are real-valued, which enables the use of more efficient optimization methods for its solution.

Additional advantages of the continuous-time optimal control formulation are the possibility of dealing more accurately with data that is provided with irregular or fast sampled time intervals, or even continuous-time varying. In particular, this formulation can deal appropriately with continuous-time varying demand data.

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