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A HIERARCHICAL APPROACH TO THE CIRCLE COVERING PROBLEM

Outline

- Motivation
- Problem Description
- Approach
- Preliminary Results
- Conclusions
- Future Work

Motivation

- Find an improved collision detection (CD) structure for piece placement in nesting problems, using circles to represent polygons;
- CD is easy and fast to compute between pairs of circles;
- Build a platform that can be used to develop and test nesting algorithms, using circle CD;
- Test behaviour of the approach;

Problem Description

- In nesting problems “complexity” increases with the number of pieces;
- CD tests are used to adjust and validate piece positions;
- These CD tests compare pairs of pieces and its “complexity” increases with the “detail” of each piece;
- CD performance depends heavily on the chosen representation for the polygons;

Problem Description

- How to “accurately” represent a polygon through circles?
  - Impossible to get an exact representation of the polygon;
  - Circles can overlap and have different sizes;
  - Cannot have holes inside the outline of the circle coverage region;
  - Polygon outline can be approximated by outside (is contained by the outline of the circle cover);
  - Polygon outline can be approximated by inside (contains the outline of circle cover);
  - Polygon outline can be approximated by both inside and outside (circle cover outline is outside the outline of the polygon in some regions, and inside it on other regions);

Problem Description

- Since the results returned from the circle coverage region will be used as input on other problems, the quality of the solution will greatly influence their performance;
- Hard to solve two conflicting objectives:
  - Lower number of circles;
  - Most accurate outline representation possible;

Approach

- Difference of the polygon area to the area of the circle coverage region must be bellow a given threshold (α);
- Minimum number of circles that is compatible with the previous limit is computed;
- Circle coverage region will completely contain the polygon;

Approach

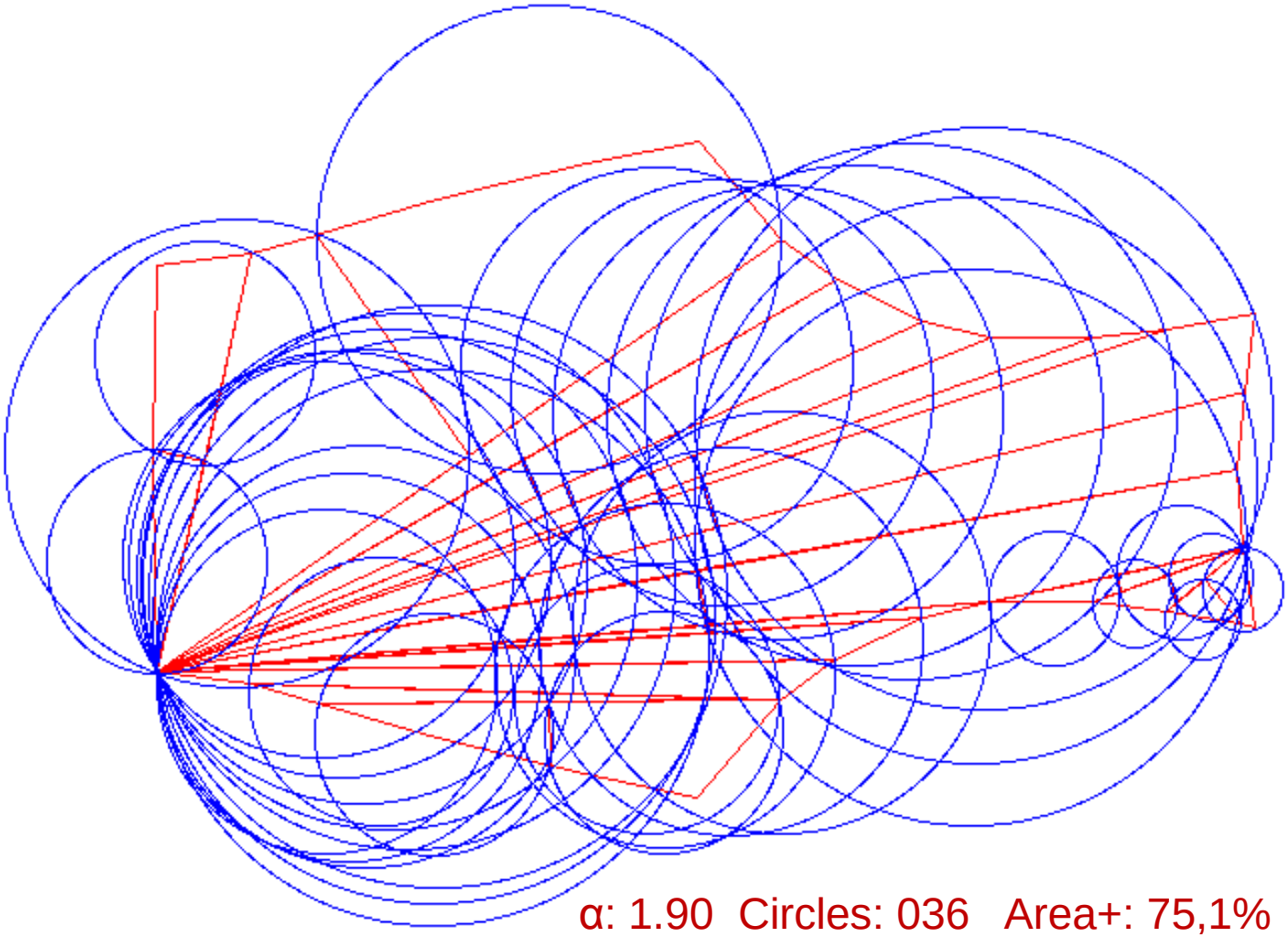
- Convex decomposition of the polygon (partition), and further division;
- Each convex sub-polygon has a minimum enclosing circle (MEC) associated;
- Advantages:
  - Since all convex polygons are adjacent, no holes are generated inside the circle coverage region;
- Disadvantages:
  - Creates high area of overlapping between circles (hard to compute exactly);

Approach

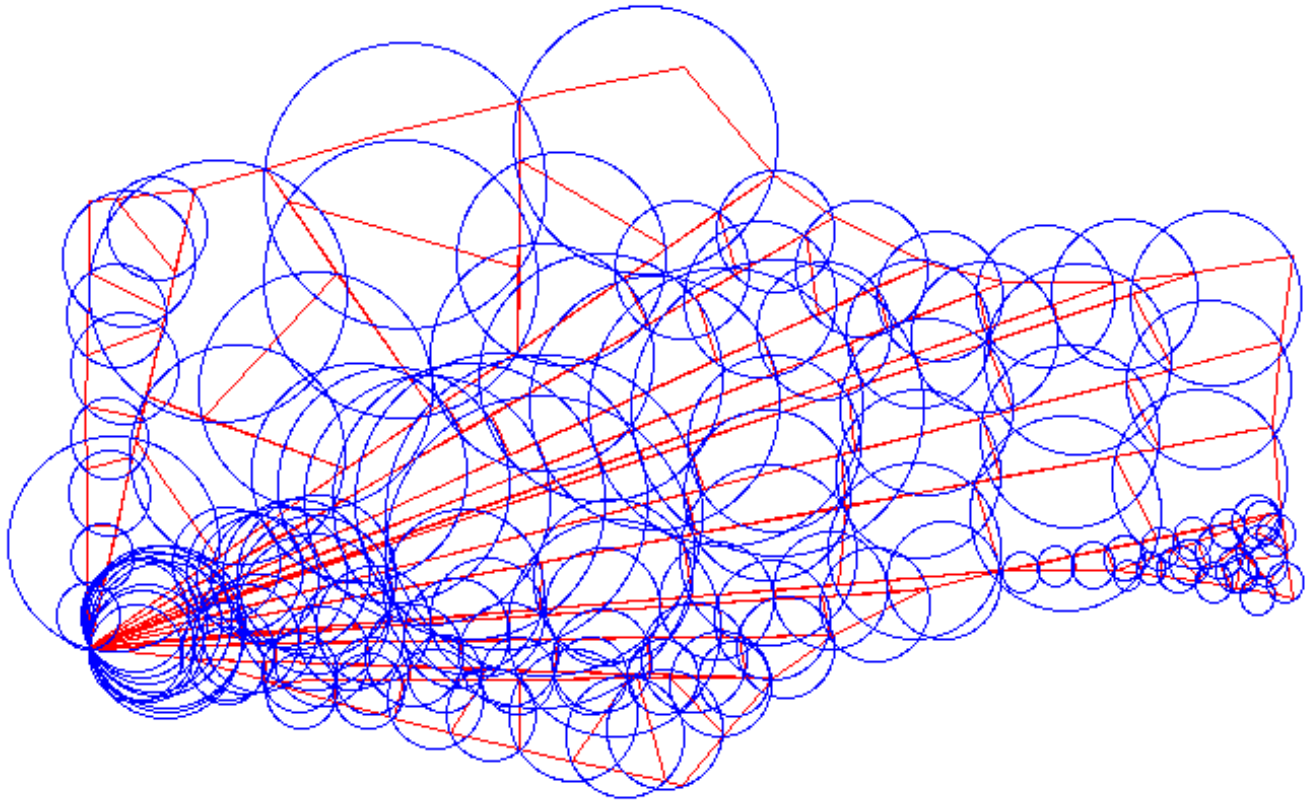
Simplified Algorithm:

- Step 0:
  - Convex decomposition (partition);
- Step 1:
  - Create MEC for each polygon;
  - Compare initial polygon and circle coverage areas;
  - If area difference is above threshold (α):
    - Divide polygons and go to Step 1;
  - Else
    - Solution found!

How does it work?

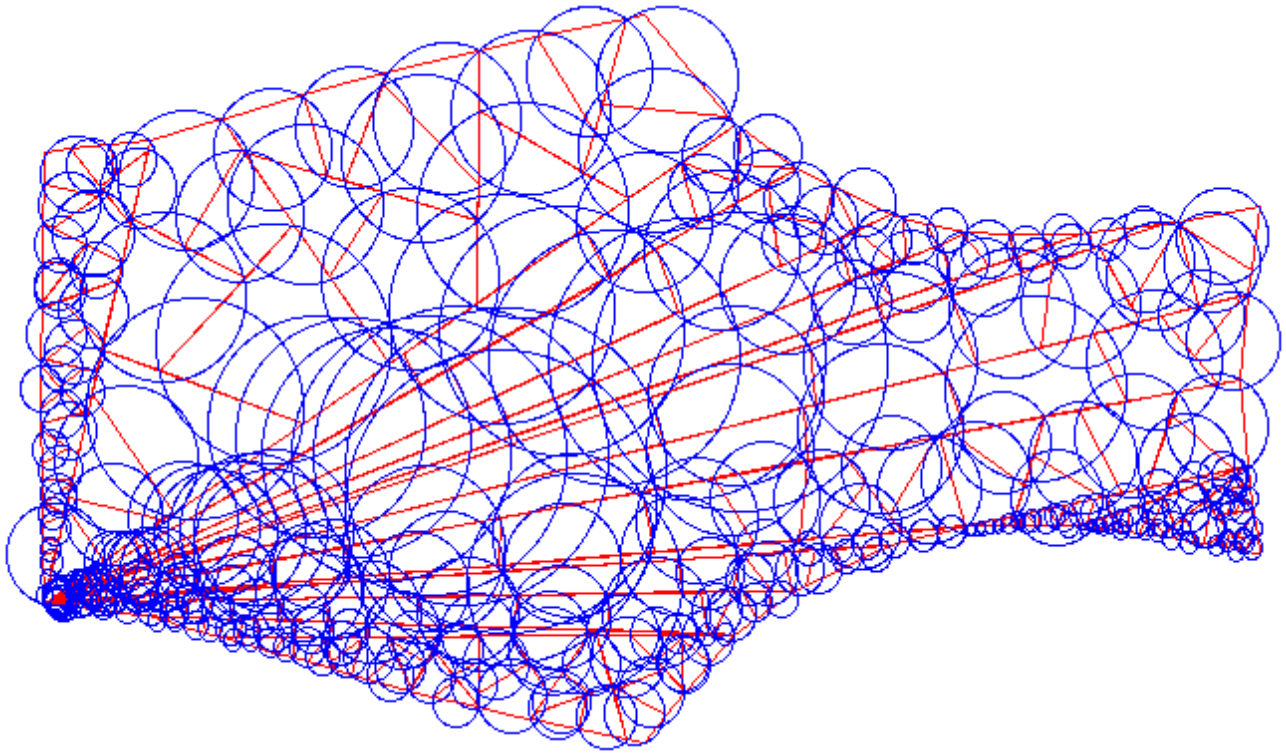


How does it work?



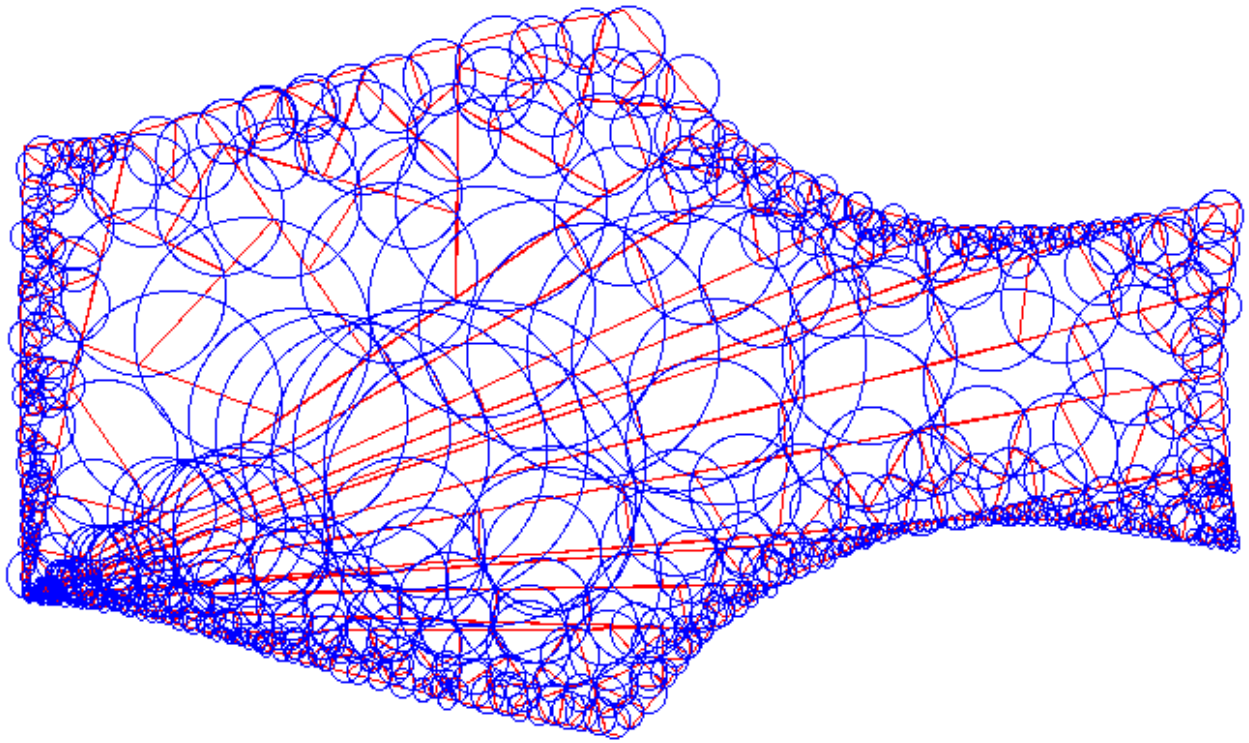
α : 1.20 Circles: 129 Area+: 18,9%

How does it work?



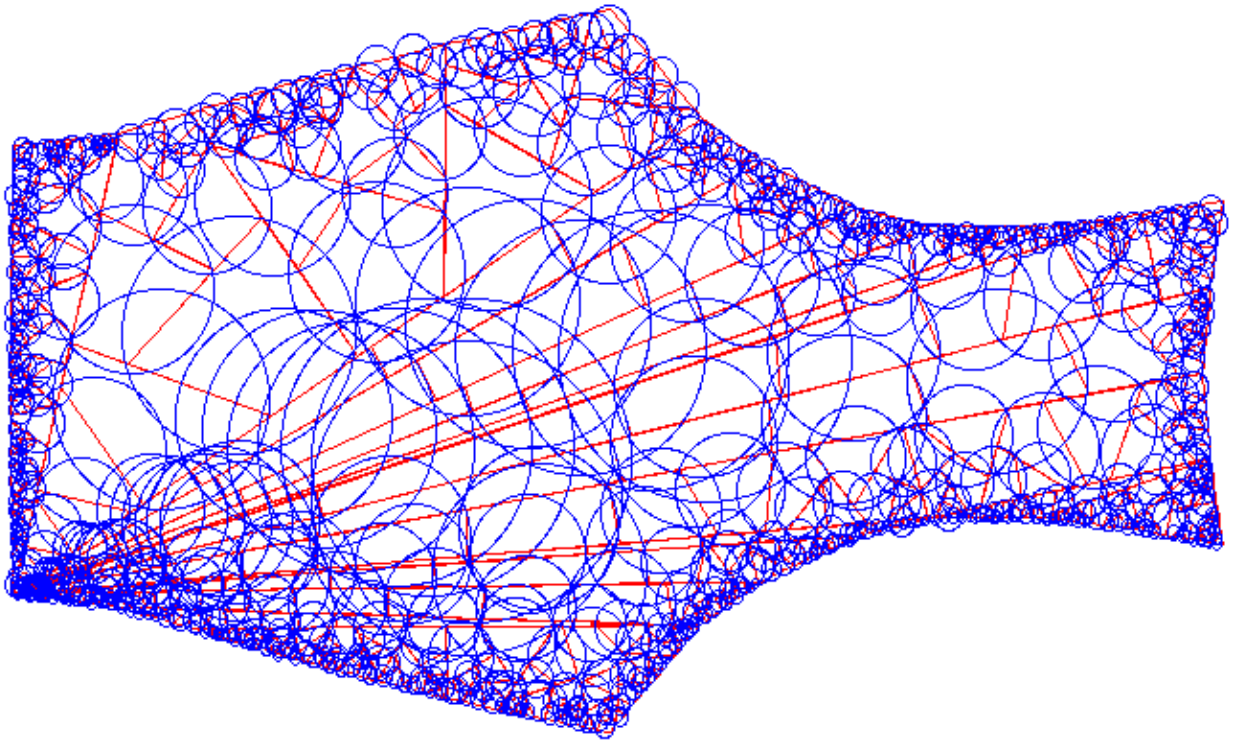
α : 1.10 Circles: 302 Area+: 6,9%

How does it work?



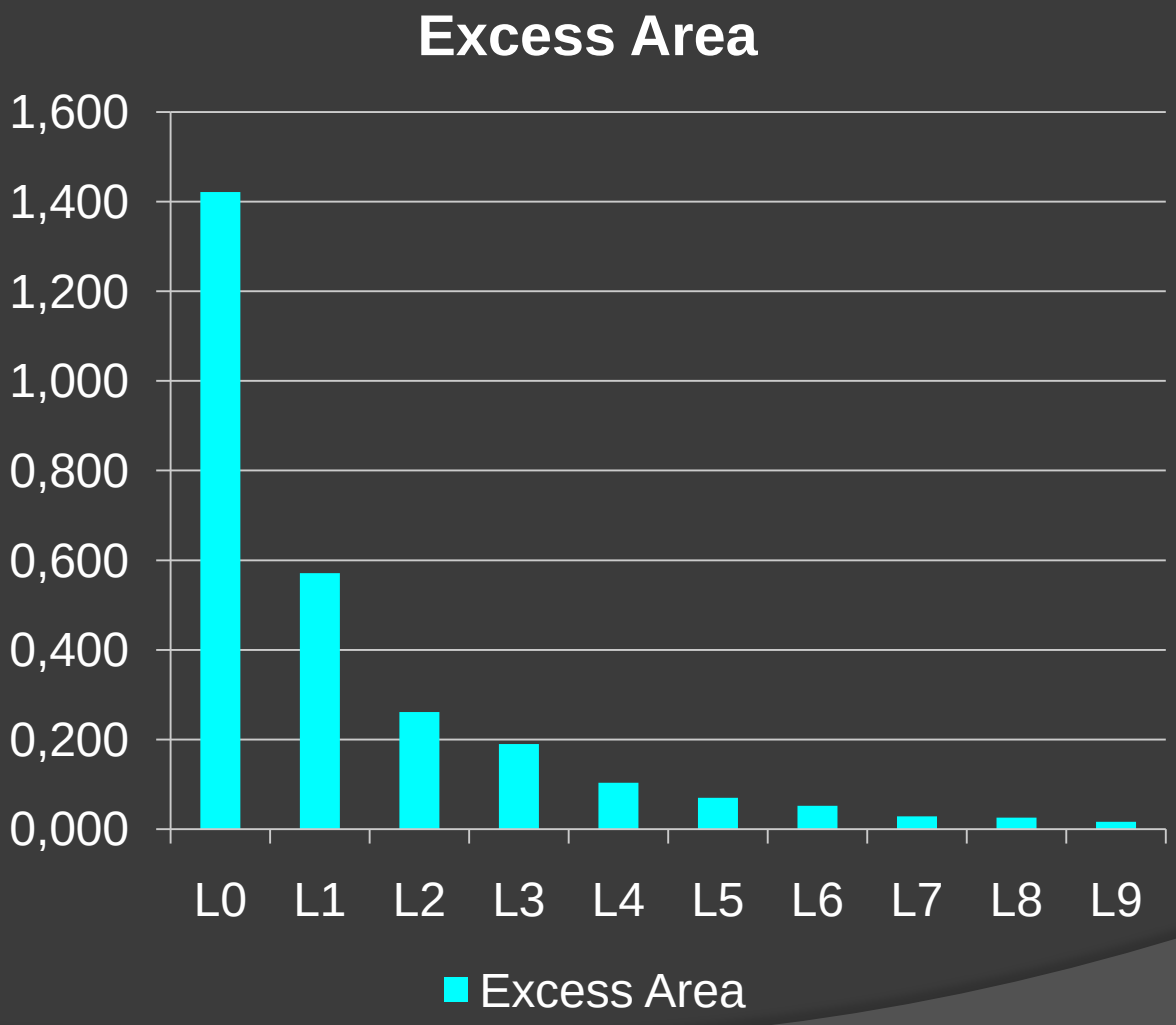
α : 1.05 Circles: 558 Area+: 2,9%

How does it work?



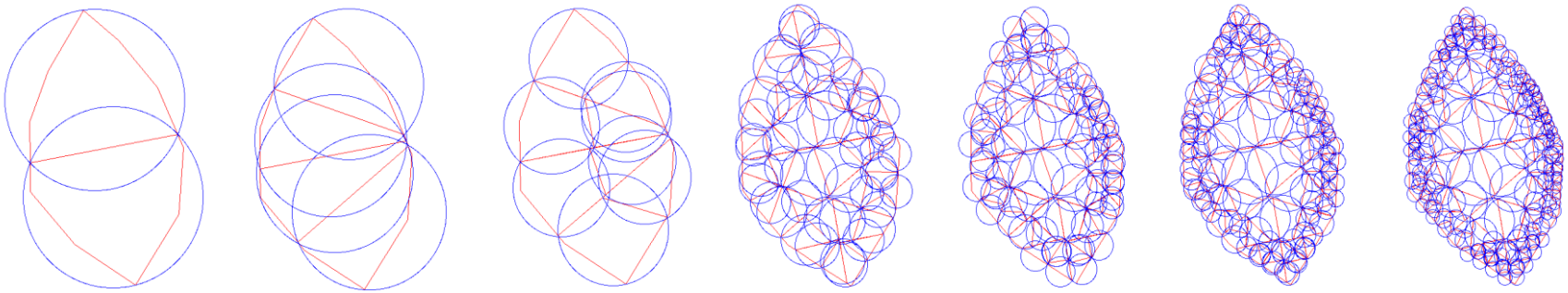
α : 1.02 Circles: 733 Area+: 1,6%

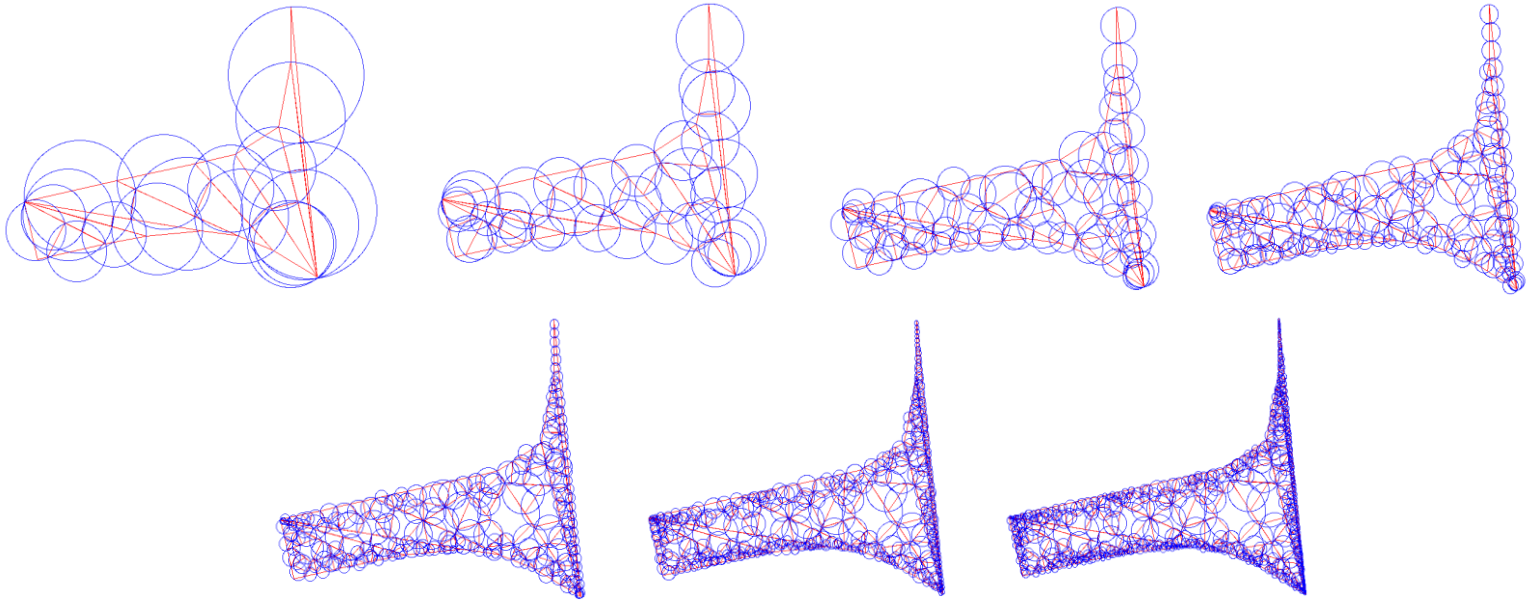
Preliminary Results



Tree Level	Circles
L0	18
L1	36
L2	72
L3	129
L4	202
L5	302
L6	427
L7	558
L8	660
L9	733

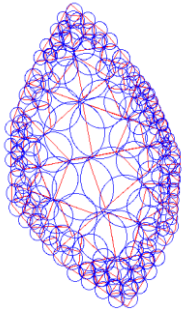
Preliminary Results





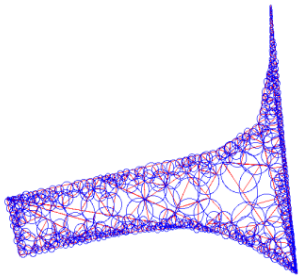
Preliminary Results

Piece 0:



Threshold	Circles	Area + (%)	Tree Level	Time (s)
20%	52	14,6	6	0,06
10%	133	7,8	8	0,17
5%	299	3,9	10	0,4
2%	594	1,9	12	0,9

Piece 1:

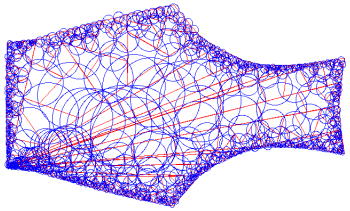


Threshold	Circles	Area + (%)	Tree Level	Time (s)
20%	124	18,9	4	0,15
10%	351	7,7	6	0,52
5%	554	4,5	7	0,85
2%	1029	1,8	10	2,65

Computer: Core2Duo 1.46Ghz, 1Gb Ram, Ubuntu 10.04

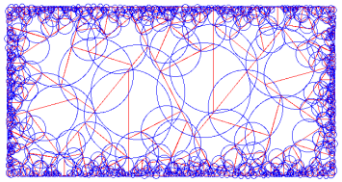
Preliminary Results

Piece 2:



Threshold	Circles	Area + (%)	Tree Level	Time (s)
20%	129	18,9	3	0,12
10%	302	6,9	5	0,35
5%	558	2,9	7	0,88
2%	733	1,6	9	1,71

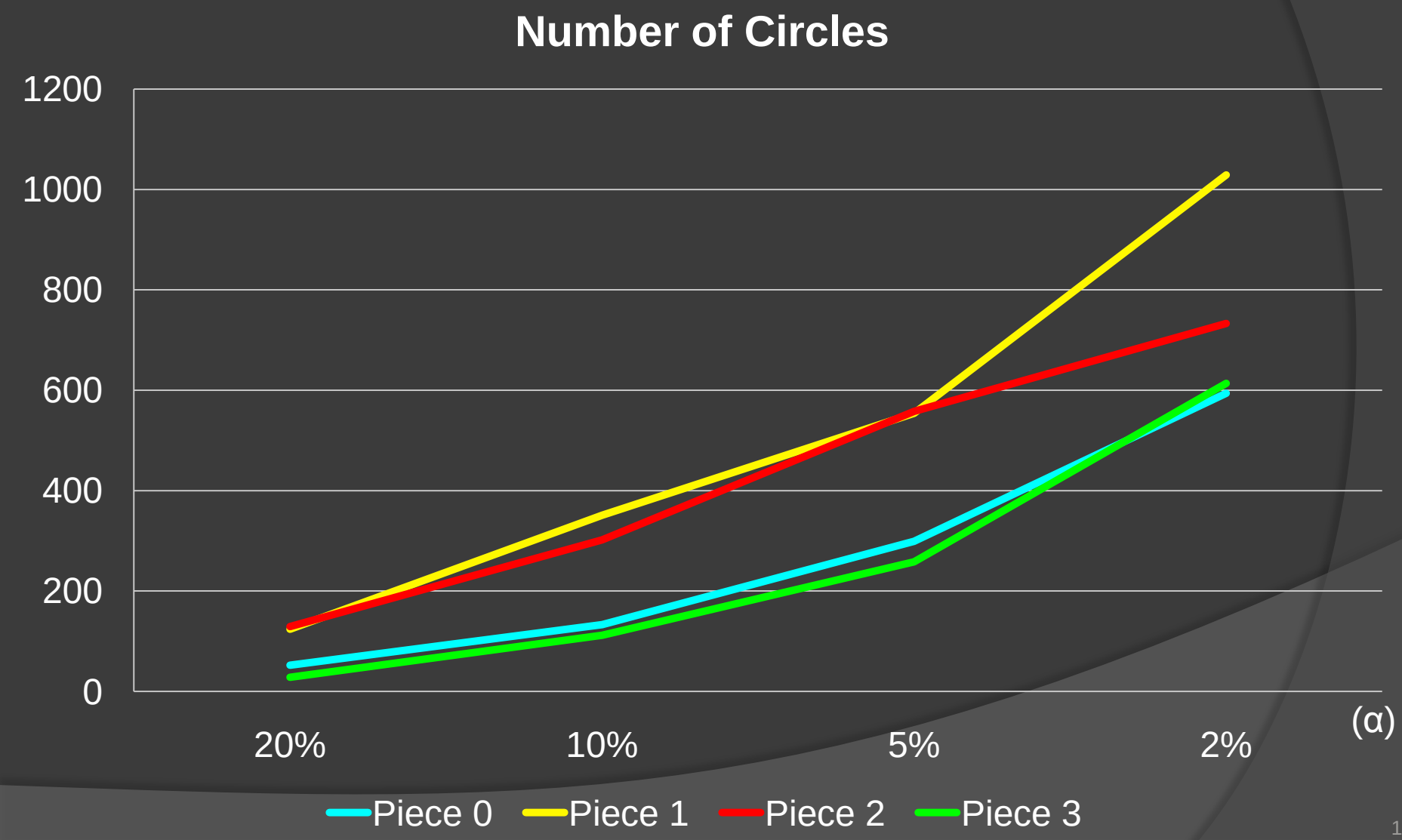
Piece 3:



Threshold	Circles	Area + (%)	Tree Level	Time (s)
20%	28	19,8	5	0,04
10%	112	8,1	8	0,13
5%	258	4,1	10	0,29
2%	614	1,3	13	0,89

Computer: Core2Duo 1.46Ghz, 1Gb Ram, Ubuntu 10.04

Preliminary Results



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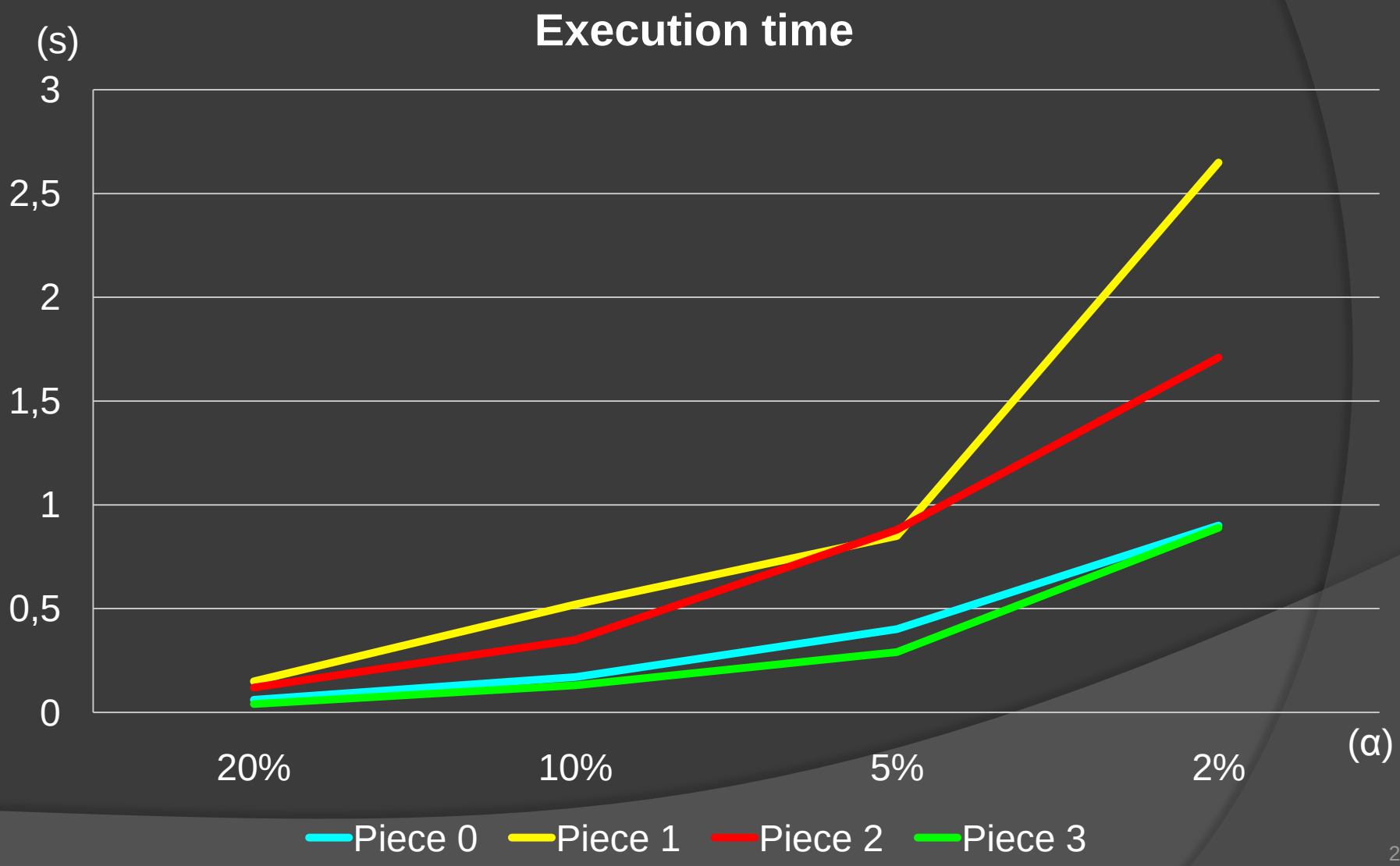

Conclusions

- The representation of polygons through circle coverage is a valid approach;
- This approach creates a high number of circles;
- Work is in early stages of development;

Future Work

- Implement other approaches and compare performance, efficiency and results;
- Enhance heuristics algorithms in order to:
  - Reduce redundant division of circles;
  - Increase the focus on the small “details” of the original form, decreasing work in regions with large segments, and focusing on the areas with small segments;

Questions?