Multiple vehicle synchronisation in a full truck-load pickup and delivery problem: a case-study in the biomass supply chain

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Abstract

The search for higher efficiency in transportation planning processes in real life applications is challenging. The synchronisation of different vehicles performing interrelated operations can enforce a better use of vehicle fleets and decrease travelled distances and non-productive times, leading to a reduction of logistics costs. In this work, the full truck-load pickup and delivery problem with multiple vehicle synchronisation (FT-PDP-mVS) is presented. This problem is motivated by a real-life application in the biomass supply chain "hot-system", where it is necessary to simultaneously perform chipping and transportation operations at the forest roadside. The FT-PDP-mVS consists in determining the integrated routes for three distinct types of vehicles, which need to perform interrelated operations with minimum logistics costs. We extend existing studies in synchronisation of multiple routes by acknowledging several synchronisation aspects, such as operations and movement synchronisation. A novel mixed integer programming model (MIP) is presented, along with valid inequalities to tighten the formulation. A solution method approach is developed based on the fix-and-optimise principles under a variable neighbourhood decomposition search. Results of its application to 19 instances based on a real-world case-study demonstrate its performance. For a baseline instance, the synchronisation aspects tackled in this problem allowed for significant gains when compared to the company's current planning approach. Furthermore, the proposed approach can enhance planning and decision making processes by providing valuable insights about the impact of key parameters of biomass logistics over the routing results.

Keywords: transportation, routing, pickup and delivery, synchronisation, OR in natural resources

Highlights:

- A routing problem synchronising multiple types of vehicles is presented;
- A mathematical formulation is presented for the problem at hand;
- A fix-and-optimise matheuristic is proposed for solving large scale instances;
- A case study is solved with real-world instances of a biomass supplier from Finland;
- Managerial insights are drawn from a set of computational experiments.

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1 Introduction

Increasing efficiency of transportation planning processes in real life applications is a challenging task. In recent years, there have been many examples of new generation transportation management systems that propose cost-efficient vehicle routes complying with specific business requirements, and without compromising quality of service and reliability. Synchronisation of operations and vehicles' routes can play a major role to increase cost efficiency. Moreover, when the vehicle service time is affected by the existence of other vehicles with similar schedules in the same location, there may be delays or unnecessary idle times that impact the plan flexibility and lead to an increase of the transportation costs.

Different situations can occur in respect to transportation planning under synchronisation constraints. There is often a need for two or more vehicles (or crews) to meet at the same location and time to perform an interrelated operation in order to avoid unnecessary travel efforts, such as additional travel costs, idle times or vehicles. Contrariwise, when there are several vehicles performing similar transportation services, synchronisation at origin and/or destination should prevent queuing and congestion, in order to minimize the idle time. Furthermore, some applications also need to acknowledge the existence of active and passive types of vehicles, where passive vehicles need to be transported by an active vehicle in order to change locations. This paper focuses on performing interrelated operations between vehicles at a given location, as well as ensuring the transportation of passive vehicles through the use of active means of transport. The goal is to find minimum cost routes for distinct types of vehicles engaged in interrelated operations, therefore synchronisation is desirable for minimising the total transportation costs.

This problem is motivated by a real-life application in the biomass supply chain "hot-system", but it has similarities to problems in other sectors. The "hot-system" consists of performing simultaneously chipping and transportation operations at the forest roadside (or pickup location p), as illustrated in Figure 1. For this purpose, the vehicle fleet is composed by several trucks (Figure 2a), chipping machines (loaders in a general case, Figure 2b) and lorries (Figure 2c). The chipping machine fragments forest residues into wood chips and directly feeds them into a chargeable container of a truck. Once the container is full, the truck carries the wood chips from the pickup location up to a delivery location d for consumption or storage. Once the pile of forest residues is fully chipped, the chipper is transported to another pickup location p'. Because chippers are often trailer-mounted machines incapable of autonomous movement, they require the service of a lorry. In the end of the planning horizon (usually one day) all vehicles must return to their depot 0. In this framework, synchronisation of the vehicles can reduce chippers and trucks idle times and the costs related with the use of the lorry, therefore decreasing the logistics costs. Since logistics can represent up to 50% of the total cost of the biomass business (Allen et al., 1998), the optimisation of biomass supply operations can in fact improve the cost-efficiency of biomass value chains, ultimately making woody biomass more competitive than alternative fossil fuels.

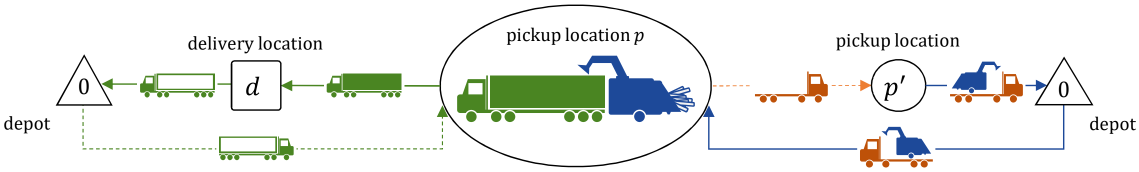


Figure 1: Chipping, transportation and hauling network

This real-world application includes a combination of conditions that are not found in the literature. The cost structure of such logistic operations comprises a fixed cost for each vehicle

(a) truck (b) loader (c) lorry

Figure 2: Types of vehicles

that is used, regardless of its type, as well as two variable cost components, namely the distance travelled by each vehicle and its expended time since the start of the working schedule.

Firstly, the optimal route and schedule for each truck should account for the synchronisation of operations between the loader and the truck at the pickup location. As stated before, both types of vehicles should be present at the same location and at the same time so that the interrelated operations can occur. The loader serves one truck at a time until its container is full, so other trucks that arrive at that pickup location need to queue until the loader becomes again available. The loader remains idle if no truck is present. The literature about the Log-Truck Scheduling Problem (LTSP) addresses many business specificities of wood trucks scheduling, but do not consider these synchronisation aspects (e.g., Hachemi et al. (2009)): typically the loader location is known beforehand and it is unchangeable during the planning horizon, therefore disregarding the chippers transportation cost. Other studies in Vehicle Routing Problems with Multiple Synchronisation Constraints (VRPMSs) do address operations synchronisation to some extent, namely in the prevention of queueing at locations (e.g., Lehuédé et al. (2015); Grimault et al. (2017)), but need to be extended to practical applications in the biomass supply chain, such as ensuring that a loader has been transported and is present at pickup locations.

Secondly, this study addresses the movements of the loader between pickup locations. The route of the loader starts at the depot and encompasses a sequence of pickup locations where there is material to be processed/loaded. The route ends in the depot at the end of the planning horizon, the latest. The loader remains in the same location until all the material there has been loaded in trucks, and only then can be moved. In the end of the horizon, all the pickup locations were visited once by one loader. Moving the loader is a non-productive time that should be avoided. Furthermore, the loader movement also represents extra transportation costs related with the usage of the dedicated lorry that carries the trailer-mounted machine between pickup locations, as well as the variable travel costs, proportional to the total distance travelled by the lorry. The lorry route starts at the depot and encompasses a sequence of visits to pickup locations for pickup and drop-off of the loaders. Only one loader is transported at a time. After each delivery, the lorry may either engage a new pickup service, come back to the depot empty, or remain in that location. The latter may be convenient in a case of a later pickup service nearby, thus avoiding unnecessary trips with the empty lorry. The studies related with the truck and trailer routing problem (TTRP) already cover some of these business requirements (e.g. Meisel and Kopfer (2012)), but further extensions are needed, as discussed in the next section.

This research builds on a literature review on VRP variants with similarities to our problem. The first contribution of this paper is a new mathematical formulation for integrated route planning for the three types of vehicles (lorries, loaders and trucks), which models the synchronisation constraints based on the approach of Kim et al. (2010). The model can be solved to optimality for small-scale instances, and a set of valid inequalities is presented to improve its performance. The second contribution of this paper is a matheuristic approach developed to obtain better quality solutions in a reasonable computational time for larger problem instances. The third contribution is to demonstrate the effectiveness of this approach, exemplified in a case-study in the biomass-for-bioenergy supply chain in Finland. For this purpose, the obtained routes and schedules are compared with the outcomes of the current planning approach. Furthermore, valuable managerial insights are provided for planners of the biomass logistics, thus enhancing decision-making processes.

The remainder of this paper is as follows. In Section 2, a literature review of similar problems is presented. The problem description is detailed in Section 3. In Section 4 the novel mathematical formulation and valid inequalities are presented. The proposed solution method for real-world applicable instances is described in Section 5. Section 6 is devoted to the computational results and managerial insights supported by a case-study in the biomass-for-bioenergy supply chain in Finland. Finally, this paper concludes by summarising the main achievements and future work.

2 Literature review

Vehicle Routing Problems (VRPs), firstly introduced by Dantzig and Ramser (1959), aim to find the minimum cost routes for vehicles subjected to a series of constraints. Over the years, several VRP variants were proposed to deal with more realistic applications. The problem in hand covers several aspects of known VRP variants, including the VRP with multiple synchronisation constraints (VRPMS), the pickup and delivery problem (PDP), the log-truck scheduling problem (LTSP) and the truck and trailer routing problem (TTRP). Table 1 summarises the literature review of each of the routing problem variants according to each of its features. The following subsections will frame this problem into each one of the enunciated VRP variants. Finally, a brief overview of rich VRPs that mingle some of these VRPs variants will be performed, focusing on problems that show similarities to the one we are tackling.

2.1 Vehicle routing problem with multiple synchronisation constraints

The Vehicle Routing Problem with Multiple Synchronisation Constraints (VRPMS), introduced in Drexl (2012), aims to find the minimum cost routes for several vehicles, which need to be synchronised in some nodes to fulfil common tasks. The major distinctive aspect of these problems when compared to traditional VRPs, concerns to the fact that vehicles' routes are dependent of each other. This interdependence leads to complexity when developing solution methods for these types of problems, as a change in one vehicle's route may render all other routes infeasible. In this work, the author distinguishes five types of synchronisation aspects in VRPs, namely load, task, resource, operations (OS) and movement (MS) synchronisation. While load, task and resource synchronisation are intrinsically present in any standard VRP, OS and MS are often dealt separately and in the context of real-life applications. The problem under study can be viewed as a VRP with OS and MS, not only because different vehicles will need to be synchronised in time when arriving at certain locations to perform certain operations, but also because certain vehicles (namely lorries and loaders) will need to move between locations simultaneously.

One of the first works explicitly addressing OS in vehicle routing was Bredström and Rönnqvist (2008). This work focus on applications in homecare staff scheduling, although it enumerates several alternative applications, such as planning of security guards and forest management. In this problem, each staff member has certain locations to visit, and occasionally customers must be visited by two staff members simultaneously or within a given precedence (e.g., visiting elderly or disabled people at the same time for lifting purposes or with a fixed time offset to apply medical treatments after a meal). From a modelling point of view, the staff members can be viewed as vehicles, or generically resources needed for performing tasks. Besides introducing a Mixed Integer Programming (MIP) model for the problem using a vehicle flow formulation, a solution method is

Table 1: Summary table of the literature review

Framework	Problem denomination	Examples of references	Depot: Multi	Depot: Single	Fleet – Capacity: Homogeneous	Fleet – Capacity: Heterogeneous	Fleet – Type of vehicle: Single	Fleet – Type of vehicle: Multiple	Active / passive vehicles: Fixed coupling	Active / passive vehicles: Flexible coupling	Pickup/delivery requests: From/to depot	Pickup/delivery requests: Between customers	Pickup/delivery requests: Paired requests	Pickup/delivery requests: Split deliveries	Pickup/delivery requests: Full truck-loads	Synchronisation: Operation simultaneity	Synchronisation: Operation precendence	Synchronisation: Simultaneous movement	Solution approaches
	OUR PROBLEM		•		•			•		•		•	•	•	•	•	•	•	Matheuristic
VRPMS	HHCRSP: Home Health Care Routing and Scheduling Problem	Bredström and Rönnqvist (2008)		•	•		•									•	•		Matheuristic
VRPMS		Mankowska et al. (2014)		•				•								•	•		Meta-heuristic
VRPMS	CMVRP: Combined man-power vehicle routing problem	Kim et al. (2010)		•			•			•							•	•	Heuristic
GPDP	PDP: Pickup and Delivery Problem	Gendreau et al. (1999)	•		•		•					•	•						Meta-heuristic
GPDP		Korsvik et al. (2011)	•				•					•	•	•	•				Meta-heuristic
GPDP		Ropke and Pisinger (2006) Petersen and Ropke (2011) Ghilas et al. (2016) Gschwind et al. (2018) Ropke et. al (2007) Veenstra et al. (2017)		•		•	•					•	•						Meta-heuristic Column generation Branch-and-price Branch-and-cut
GPDP	LTSP: Log-truck Scheduling Problem	Gronalt and Hirsch (2007) Palmgren et. al. (2004) Flisberg et al. (2009)	•		•		•					•	•		•				Meta-heuristic Column generation Branch-and-price
TTRP	GTTRP: Generalized Truck and Trailer Routing Problem	Drexl (2007) Drexl (2011)		•		•		•		•								•	Branch-and price Branch-and-cut
TTRP	TTRP: Truck and Trailer Routing Problem	Chao (2002) Scheuerer (2006) Lin et al. (2009) Derigs et al. (2013) Parragh and Cordeau (2017)		•	•			•	•								•	•	Meta-heuristic Branch-and-price Branch-and-cut
VRPMS / GPDP	Full truckload pickup and delivery problem with resource synchronisation	Grimault et al. (2017)	•			•	•					•	•	•	•	•	•		Meta-heuristic
VRPMS / GPDP / TTRP	Long-haul freight transportation problem	Neves-Moreira et al. (2016)	•		•			•		•		•	•		•	•		•	Matheuristic
VRPMS / GPDP / TTRP	Active-passive VRP	Meisel and Kopfer (2014) Tilk et al. (2018)		•	•			•		•		•	•		•			•	Meta-heuristic Branch-and-price Branch-and-cut

presented, consisting in a heuristic approach where restricted MIP problems are solved iteratively. Kim et al. (2010) present a VRP with OS with the purpose of obtaining a schedule of vehicles that visit a set of customers, where a certain number of tasks has to be performed in a fixed sequence. This problem also intends to assign vehicles to staff, allowing for staff "transshipments" at relay stations in order to respect working time regulations. This paper presents a mathematical formulation based on four-index decision variables x_{ij}^{km}, allowing to account for simultaneous movement of crew m through vehicle k across an arc (i, j). More recently, Mankowska et al. (2014) present a Home Health Care Routing and Scheduling Problem (HHCRSP), which consists in a generalisation of the problem introduced by Bredström and Rönnqvist (2008) where a set of customers needs to be serviced by a set of home carers. However, the specific requirements for the home health care sector are approached differently, as customers may need simultaneous services by more than one home carer at the same time (or with temporal precedence) and not all carers are able to perform certain services. The mathematical model that is presented explicitly handles staff qualifications and simultaneity/precedence requirements. A solution method is also presented, where several neighbourhood structures are devised and used under an Adaptive Variable Neighbourhood Search (AVNS) framework.

2.2 General pickup and delivery problem

The General Pickup and Delivery Problem (GPDP) consists in a vehicle routing problem where customers pickup and delivery locations are acknowledged and its purpose is to perform the pickup of load at pickup nodes and deliver it to delivery nodes. Parragh et al. (2008a,b) presented a framework for classifying GPDPs, where these problems are split into two main variants, depending on whether the pickup and delivery requests occur from and/or to the depot, or whether the pickups and deliveries occur between customers. The problem under study frames itself in this latter variant and an overview of its characteristics is now provided.

2.2.1 Pickup and delivery problem

The Pickup and Delivery Problem (PDP), as it is named by Parragh et al. (2008a,b), is a particularisation of the GPDP. It is the sub-class of GPDPs in which our problem is best framed because it assumes that merchandise from pickup locations must be delivered to a predetermined delivery location.

The PDP is especially relevant in applications where commodities are differentiated and only can be delivered to a specific customer, such as courier transportation (Gendreau et al., 1999) or maritime shipping (Korsvik et al., 2011).

Typical solution approaches for PDPs consist in heuristic methods. One of the most renowned solution approaches is the Adaptive Large Neighbourhood Search (ALNS), introduced by Ropke and Pisinger (2006). The main advantage of the ALNS consists in its flexibility, providing a good balance between diversification and intensification in the search process (Pisinger and Ropke, 2010). The ALNS is employed for several PDPs (Ropke and Pisinger, 2006; Petersen and Ropke, 2011; Ghilas et al., 2016). Alternative solution approaches for PDPs include Column-Generation (CG) algorithms (Gschwind et al., 2018) and Branch-and-Price (B&P) and Branch-and-Cut (B&C) algorithms (Ropke et al., 2007; Veenstra et al., 2017).

2.2.2 Log-truck scheduling problem

GPDPs can be also be classified as less-than-truck-load problems and full-truck-load problems (Parragh et al., 2008b). The former need to consider capacities of vehicles and the quantities of each node, so that vehicles capacity constraints are met, while the latter does not need to consider quantities, as the delivery unit consists in a single truck-load. When considering the PDP with paired pickups and deliveries as a full-truck-load problem, it is possible to infer that the pickup performed by a truck will necessarily be followed by the delivery of the same load to its corresponding customer. This is a valid assumption in several contexts, namely in the forestry sector, as well as in our case. The Log-truck Scheduling Problem (LTSP) is a particular case of the PDP considering full truck-loads, where previously paired pickup and delivery tasks must be sequenced among the vehicles in order to obtain minimum costs (Gronalt and Hirsch, 2007). Given the high quantities of raw materials involved in log-truck transportation, the quantities available at one location usually exceed a full truck-load, which also turns this problem into a split delivery PDP. In the context of our problem, the LTSP resembles the routes done by trucks transporting the merchandise.

For solving the LTSP, Palmgren et al. (2004) presented a B&P algorithm, although Tabu Search (TS) strategies are also common in the literature: Gronalt and Hirsch (2007) uses the unified tabu search (UTS) algorithm (Cordeau et al., 2001) to solve the LTSP and modifies it by allowing neighbourhood size oscillations in the search process. Flisberg et al. (2009) uses a two-phase iterative procedure, where a linear programming problem is solved in the first phase to determine the origin and destination nodes of each full truck-load and, in the second phase, the UTS algorithm is applied for determining the vehicles routes.

2.3 Truck and trailer routing problem

Firstly introduced in Chao (2002), the Truck and Trailer Routing Problem (TTRP) encompasses a set of trucks (active vehicles) with coupled trailers (passive vehicles) to perform the necessary deliveries to customers. However, trailers cannot visit all locations, as certain customers have site-dependent vehicle restrictions, so they must first visit a transshipment node where trailers are uncoupled from trucks and loads are transferred from trailers to trucks and/or vice-versa. Afterwards the truck will perform sub-tours to trailer-incompatible customers while leaving the trailer uncoupled at the transshipment node. It is possible to state that the TTRP is a particular case of the VRPMS, due to the occurrence of movement synchronisation between two different types of vehicles.

A typical application of the TTRP is the raw milk collection problem with trucks and trailers, where a dairy company collects raw milk from farmers, and trailers need to be collected by trucks when they are full at milk collection locations. Drexl (2007) performs some extensions to the TTRP, one of them allowing the truck-trailer combinations to be changed within the time horizon, also presenting mathematical formulations using different decision variables: turn variables, designating if a node j is visited immediately after i and immediately before k, arc variables, where routing is set by the arcs that are traversed, and the traditional path variables, where routing is given by whether a node j is visited immediately after node i. In Drexl (2011), the Generalized TTRP (GTTRP) is presented, where time windows and a heterogeneous fleet are introduced.

The TTRP has similarities to our problem when we consider the routes performed by the lorry that moves the loaders between pickup locations. The solution methods for the TTRP present in the literature usually consist in meta-heuristic algorithms, such as TS (Chao, 2002; Scheuerer, 2006), Simulated Annealing (SA) (Lin et al., 2009) and Large Neighbourhood Search (LNS) (Derigs et al., 2013). B&P and B&C algorithms are also rather common (Drexl, 2007, 2011; Parragh and Cordeau, 2017).

2.4 Rich vehicle routing problems

Several rich VRPs may be found in the literature that relate to the work in this article, namely in the application of real-world problems. We highlight some of them, due to their intrinsic similarities to our problem.

One of the rich VRPs that exhibits several similarities to our problem is the active-passive VRP, introduced by Meisel and Kopfer (2012), which consists in a PDP where the fulfilment of the pickup and delivery requests depend on the simultaneous movement of both an active and passive vehicle, such as in a TTRP. Several applications can be found for this problem, such as the transportation of containerised goods. Although the underlying similarities to the TTRP are clear, in this problem there is no pre-defined assignment of active vehicles to passive vehicles, meaning that active vehicles may couple/uncouple any passive vehicle as many times as necessary. Meisel and Kopfer (2012) propose an ALNS approach as a solution method, and since then, exact approaches have also been envisaged in Tilk et al. (2018). The main distinction between this problem and ours consists in the fact that passive vehicles only have synchronisation requirements with active vehicles. In our case the passive vehicle (loader) not only needs to be synchronised with an active vehicle (lorry), but it is also necessary to synchronise its operations with a third vehicle (truck).

Grimault et al. (2017) presents an ALNS algorithm for a PDP applied to construction, in which construction sites need materials to be delivered from quarries or asphalt concrete plants. Different pickup/delivery requests need to be synchronised, as loading operations require a resource that only loads one vehicle at a time. The main distinction between this problem and ours consists in the assumption that the loading resource is available at each location throughout the entire planning horizon, which does not occur in our case. The problem is solved using instances from a case-study and from the LTSP literature instances in Hirsch (2011). This problem can therefore be framed not only as a PDP, but also as a VRPMS with resource synchronisation.

Neves-Moreira et al. (2016) presents a long-haul freight transportation problem where tractors are allowed to visit transshipment points in the course of their routes in order to perform exchanges of semi-trailers. The problem is solved using a fix-and-optimise matheuristic for real-world instances. This problem can be framed into any of the problem variants addressed before, as synchronisation is present by acknowledging trucks and trailers, as well as the pickup and delivery nature of the routing problem.

3 Problem statement

According to the findings of the literature review, our problem can be considered a full truck-load pickup and delivery problem with multiple vehicle synchronisation (FT-PDP-mVS). In this problem, distinct types of vehicles – trucks and lorries and trailers (henceforth called loaders) – are considered over a transportation network of pickup (supply) and delivery (demand) locations, considering that loaders and lorries are subject to movement synchronisation (MS), and loaders and trucks are subject to operations synchronisation (OS). The considered planning horizon consists in one work day, and all the previously established pickup/delivery requests must be performed in this time frame. The problem objective is to minimize the overall transportation costs, which consist in a trade-off between the number of vehicles used (fixed costs), its travelled distance (distance costs) and the total expended time each vehicle needed to perform its operations (time costs).

3.1 Problem entities and features

3.1.1 Requests

A request is characterised as a truck-load that needs to be transported from a pickup location to a delivery location. Each truck-load will be delivered to a single delivery location. It is assumed that the commodity assignments from pickup to delivery locations are previously set at a higher planning level, where different objectives from the ones in this paper are considered. Therefore, these assignments are not a decision to be obtained in this problem. Examples of tactical planning problems in the biomass-for-bioenergy supply chain can be found in Marques et al. (2018); Boukherroub et al. (2017); Meyer et al. (2015).

All requests must be performed within the established planning horizon. When the amount required by a delivery location exceeds a full truck-load, requests are split into two or more requests, considering the minimum number of truck-loads needed to satisfy the demand.

3.1.2 Vehicles – loaders, lorries and trucks

Loaders are entities that perform an operation on raw materials at pickup locations, prior to its transportation to delivery locations. They are assumed to be homogeneous in terms of its productivity, i.e. the amount of material produced per unit of time. Loaders consist in trailer-mounted machines that are pulled by a lorry whenever they need to be moved (to and from the depot, and between pickup locations).

Lorries are support vehicles and only carry out operations related with the loaders pickup and drop-off. A lorry can only transport one loader at a time but it is able to transport multiple loaders throughout the course of their routes, i.e. different loaders can be coupled to (or decoupled from) lorries as many times as necessary. There may exist some particular cases where this situation does not apply (i.e. lorries and loaders are unique vehicles and cannot be separated, precluding the possibility of a lorry performing loader drop-offs and/or pickups during its route).

Trucks are the vehicles responsible for delivering the truck-loads from the pickup locations to the delivery locations. The trucks fleet is considered homogeneous, i.e. all vehicles exhibit equal transportation capacity.

3.1.3 Transportation network

The transportation network encompasses a set of depots, a set of pickup locations and a set of delivery locations.

Depots are starting and ending locations for vehicle routes. All vehicles are associated with a starting depot location, which is previously known, and it is not mandatory that they end their route at that same depot. A depot also serves as a location for picking up and/or dropping off a loader in the course of a lorry route.

Pickup locations are characterised for being geographically dispersed, each having a known amount of material that is available to be transported to delivery locations. The amount of material present at a pickup location typically exceeds the unit capacity of a truck, thus yielding multiple pickup requests per location. Furthermore, the location requires a loader on site so that trucks' containers can be loaded. As for lorries, they will visit the same pickup location at most twice, one for machine drop-off and another for machine pickup. Due to space availability restrictions at pickup locations, only one loader can be present at the location.

Delivery locations receive cargo that is transported directly from pickup locations by trucks. Trucks may visit the same delivery location more than once in order to deliver multiple truck-loads of material.

3.2 Operations and movement synchronisation

Synchronisation is essential in pickup locations, since loaders need to be transported to the location by lorries (MS), as well as coordinate its temporal availability with both lorries and trucks (OS) so that operations can be performed with the smallest idle time possible. Loaders only visit pickup locations once, and therefore can only leave after all material is loaded into trucks.

Figure 3a presents an example of feasible routes for two lorries, $R = \{r_1, r_2\}$, three loaders, $L = \{l_1, l_2, l_3\}$, and three trucks, $K = \{k_1, k_2, k_3\}$, over a transportation network composed by two depots, $O = \{o_1, o_2\}$, four pickup locations, $P = \{p_1, p_2, p_3, p_4\}$, and three delivery locations, $D = \{d_1, d_2, d_3\}$. An example of a feasible route for truck k_1 is $\{o_2, p_2, d_1, p_1, d_2, o_2\}$. A feasible route for loader l_1 is $\{o_1, p_2, o_1\}$. The lorry r_1 starts the route carrying loader l_1 from the depot to p_2. Then, it returns empty to depot o_1, where it engages with loader l_2 and transports it to p_4. Lorry r_1 waits there until all operations of l_2 are over, and then transports it to p_3. After dropping off loader l_2 in p_3, the lorry will finish its service by collecting loader l_3 at p_1 (left there previously by lorry r_2) and returning it to the depot. In sum, a feasible route of lorry r_1 is $\{o_1, p_2, o_1, p_4, p_3, p_1, o_1\}$. Regarding lorry r_2, a feasible route is $\{o_1, p_1, p_2, o_1, p_3, o_1\}$.

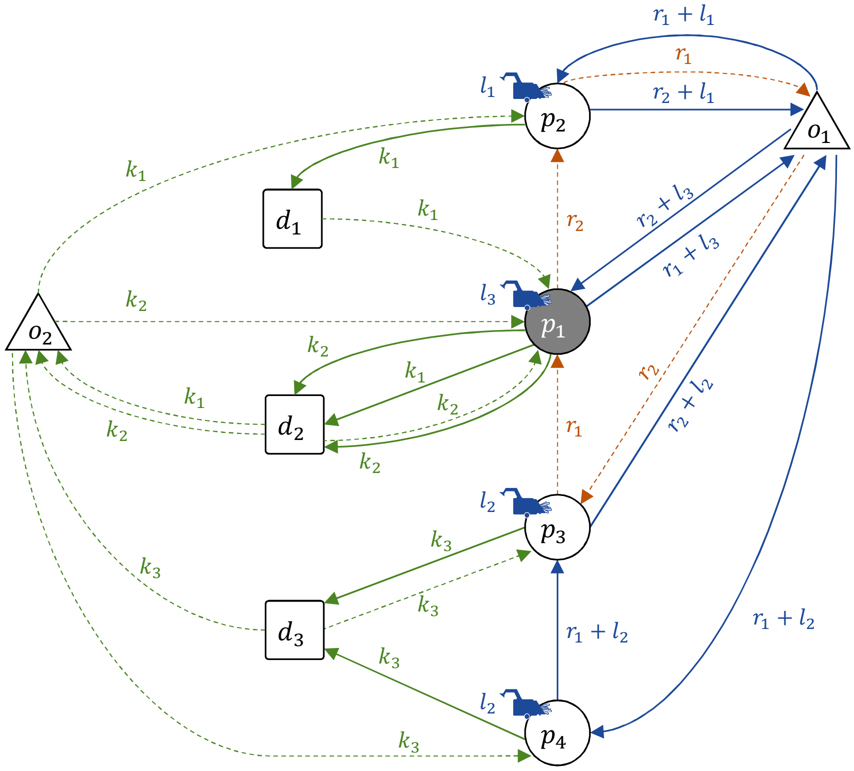
Movement synchronisation is shown in the similarities between the routes of the loader l_1 and lorries r_1 and r_2. The first part of the route of the loader l_1 is coincident with the beginning of the route of lorry r_1, while the second part of the route is coincident with the ending of the route of lorry r_2. The same logic applies to loaders l_2 and l_3.

Operations synchronisation at pickup location p_1 along the time is illustrated in Figure 3b. Operations start when lorry r_2 arrives at the location and decouples loader l_3. The first loading operation of loader l_3 starts when truck k_2 arrives. Since truck k_1 arrives while k_2 is still being served, k_1 waits on site and is served right after. After serving truck k_1 there is no other truck available, so loader l_1 remains idle until truck k_2 returns to location p_1. Next, loader l_3 will be moved to another pickup location because the requests (i.e. amount of material available) are equivalent to three full truck-loads, now exhausted. Operations at p_1 end when lorry r_1 engages loader l_3 and departures from p_1. It is noteworthy that operations synchronisation with precedences can also be verified in this example. In fact, decoupling the loader from the lorry must be performed before any loading task with trucks. The same applies to the coupling operation, as it must be the final operation to be performed at the pickup location.

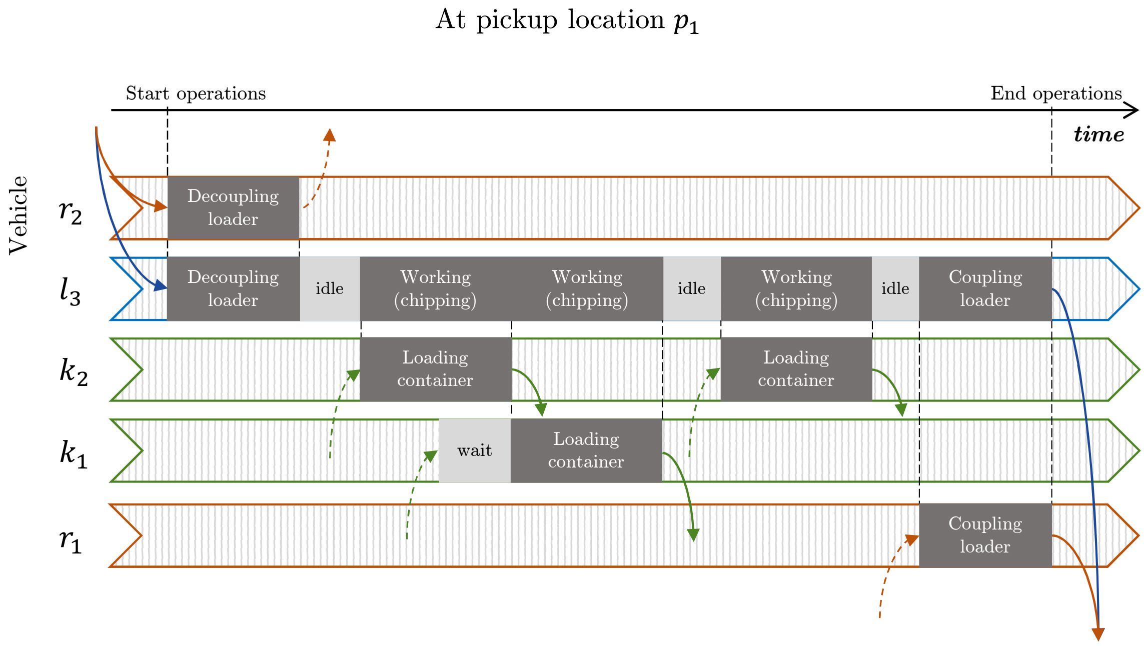
4 Problem modelling

The modelling approach encompasses three routing sub-problems, one for each type of vehicle – lorries, loaders and trucks –, all of them intertwined with synchronisation constraints in order to ensure the problem requirements. For each one of the routing sub-problems, a vehicle flow formulation is used, where the generic decision variables state if a given vehicle traverses arc (i, j) or not.

Given that multiple operations may be performed at each location, it is necessary to consider artificial nodes in the transportation network. For the sake of clarity, we use the term "location" to designate a real-world geographical site and the term "node" is used for referring to the nodes of the transportation network, used for modelling purposes. Therefore, a location may have one or more nodes associated with it.



(a) Feasible routes for the three types of vehicles, where $O = \{o_1, o_2\}$ are depots, $P = \{p_1, p_2, p_3, p_4\}$ are pickup locations, $D = \{d_1, d_2, d_3\}$ are delivery locations, $R = \{r_1, r_2\}$ are lorries, $L = \{l_1, l_2, l_3\}$ are loaders and $K = \{k_1, k_2, k_3\}$ are trucks; dotted lines are trips with an empty truck (in green) or lorries without the trailer-mounted loader (in orange)



(b) Operations synchronisation at pickup location p_1

Figure 3: Example of routing plan considering multiple vehicle synchronisation

4.1 Transportation network preprocessing

A data preprocessing phase is crucial for reducing the model size and complexity, and consequently reducing the solution time when using a commercial solver. This phase consists in the creation of the nodes and arcs in a way that some constraints related with the solution consistency are dealt *a priori* and do not need to be included in the mathematical model.

The network preprocessing phase consists in the creation of artificial nodes and the generation of arcs, explained in the following subsections. Each vehicle type will have its own transportation network, generated according to the specific tasks it is able to perform. To illustrate the rationale behind the network preprocessing, Figure 4 revisits the example given in the problem statement section.

4.1.1 Depot nodes

Start of vehicles routes: each depot location $o \in O$ contains a start node O_o^+, where vehicle routes will necessarily start, regardless of the type of vehicle. Each vehicle is associated with an initial depot location.
End of vehicles routes: each depot location $o \in O$ contains a sink node O_o^-, where a vehicle ends its route. The sink node that a vehicle visits may not necessarily correspond to the same location it started from. There are no restrictions to the number of vehicles each depot sink node can receive.
Pickups and drop-offs of loaders: each depot location $o \in O$ will have a fixed number n of intermediate nodes $\left\{\hat{O}_o^1, ..., \hat{O}_o^n\right\}$. These nodes are not mandatory to be visited, and their purpose is to allow for a lorry to visit the depot in the middle of its route, either to drop off or to pick up a loader present at that location. The number of intermediate nodes that are created corresponds to the maximum number of pickups and drop-offs the depot will be able to handle.

In Figure 4a it is possible to verify that lorries $r_1, r_2 \in R$ and loaders $l_1, l_2, l_3 \in L$ all leave node O_1^+, which is the start node of depot o_1. The same applies for trucks $k_1, k_2, k_3 \in K$ in the start node of depot o_2. Also, lorry r_1 visits intermediate node $\hat{O}_1^1$, at where loader l_2 also arrives from O_1^+ and engages with r_1. The inverse situation can also be verified, as lorry r_2 performs the drop-off of loader l_1 at intermediate node $\hat{O}_1^3$, after which loader l_1 finishes its service by visiting the depot sink node and lorry r_2 proceeds with the remainder of its route. All these vehicles end their routes at the sink node of depot o_1.

4.1.2 Delivery location nodes

Unloading raw materials: each delivery location $d \in D$ has a fixed number n of unloading nodes, $\left\{D_d^{1\star}, ..., D_d^{n\star}\right\}$, each one of them corresponding to the reception of a truck-load from its corresponding origin, thus acknowledging the split delivery nature of this problem.

In Figure 4b this situation is easily observable, as trucks $k_1, k_2 \in K$ visit delivery location d_2 three times, once for each unloading node $D_2^{1\star}, D_2^{2\star}, D_2^{3\star}$ to deliver the truck-loads originating from pickup location p_1.

4.1.3 Pickup location nodes

Decoupling loader from lorry: each pickup location $p \in P$ contains a decoupling node P_p^+. These nodes are used for considering movement synchronisation between lorries and loaders when these vehicles arrive at the pickup location simultaneously. After the loader drop-off is complete, the lorry will be able to leave the location and perform other services.

Coupling loader to lorry: each pickup location $p \in P$ contains a coupling node P_p^-. These nodes are used for considering movement synchronisation between lorries and loaders when these vehicles leave the pickup location simultaneously. A lorry will arrive at this node, as well as the loader that was assigned to that location and has finished its service. Arcs to this node are generated taking into account that this node can only be visited when all other nodes of the location have already been visited.

Loading raw materials: each pickup location $p \in P$ has a fixed number n of loading nodes, $\left\{\hat{P}_p^1, ..., \hat{P}_p^n\right\}$, which correspond to the number of truck-loads trucks will need to transport from that location (corresponding to the requests defined beforehand). In each of these nodes, a loader and a truck will arrive in order to start loading operations for the truck-load associated with that node.

In Figure 4c, pickup location p_1 comprises three loading nodes, corresponding to three truck-loads. Initially lorry r_2 and a loader l_3 arrive simultaneously at decoupling node P_1^+. From this point onwards, the lorry's and loader's routes will diverge, as lorry r_2 will perform the pickup of another loader at pickup location p_2, and loader l_3 will remain at location p_1 until all work is complete. The loader will "travel" to one of the possible loading nodes $\left\{\hat{P}_1^1, \hat{P}_1^2, \hat{P}_1^3\right\}$ to begin loading operations. Trucks will also arrive at the same node where loader l_3 has gone to, ensuring that loading operations can start. Afterwards the truck leaves the location to deliver the merchandise, as loader l_3 continues to travel to the remaining loading nodes. Finally, when all loading nodes are visited (and therefore all work is complete), the loader will visit the coupling node P_1^-, where lorry r_1 arrives to pick it up and finish their routes by visiting the sink node of depot o_1.

4.1.4 Arc generation

The arc generation rules, synthesised in Table 2, aim to remove arc possibilities from the transportation network that would lead to inconsistencies. For example, arcs where a lorry and a loader leave a decoupling node may be eliminated, since the lorry must have already left the loader in the location.

It is worth noting that arc generation for lorries and loaders present in Table 2 is valid for the general case, where loaders may be decoupled from lorries. For cases where a lorry and a loader act as an unique vehicle and cannot be split, movement synchronisation must be met at all times, and decision variables for these vehicles only need to be generated this vehicle combination. In these cases, the network can be significantly simplified. For example, arcs heading and originating from depot intermediate nodes would be removed, and lorries would necessarily go from a decoupling node to the coupling node of the same location, i.e. cannot leave the pickup location as long as the loader does not finish all of its loading tasks.

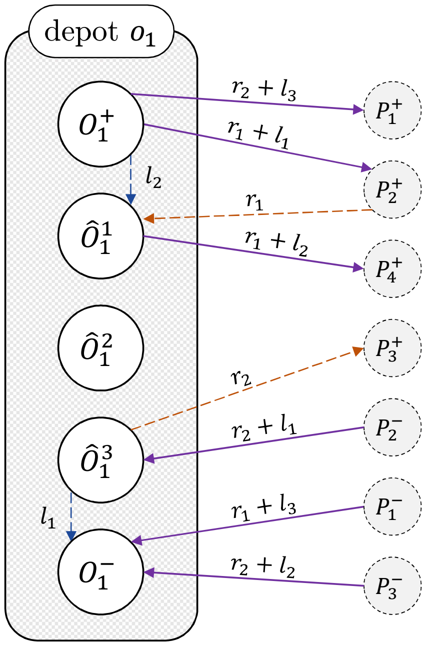
In the event that all lorries and loaders act as unique vehicles, depot intermediate nodes would have no use in the model, and therefore they would be eliminated.

4.2 Mathematical formulation

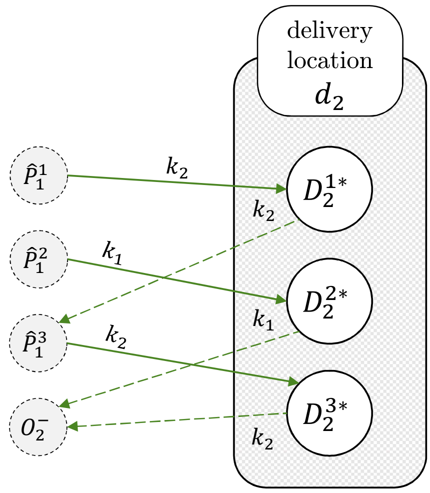
The mathematical formulation of the mixed integer programming model for the FT-PDP-mVS is now presented. The sets and parameters will firstly be presented, followed by the decision variables, the objective function and the model constraints.

4.2.1 Sets and parameters

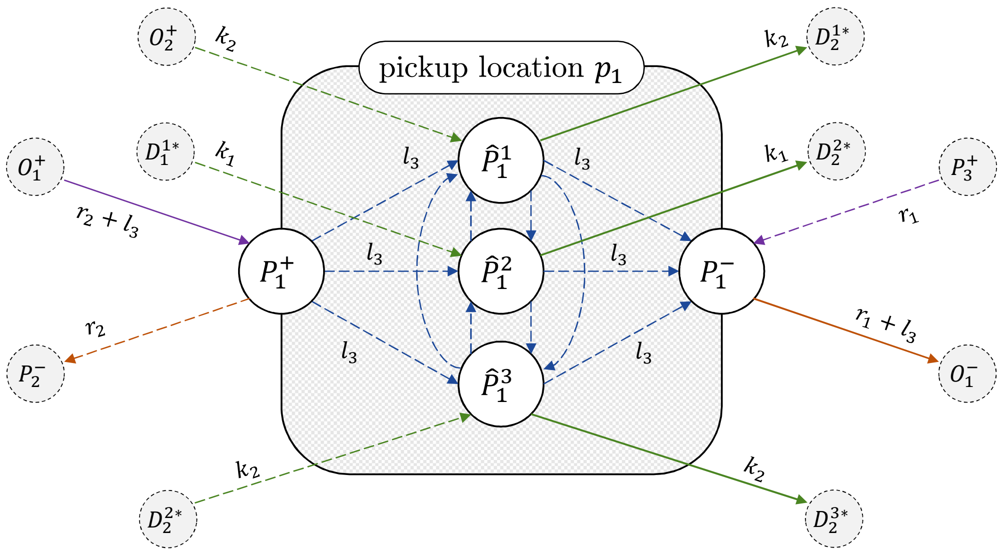
Most of the model sets have already been presented, but they are presented again for the sake of clarification.



(a) For depot location $o_1 \in O$



(b) For delivery location $d_2 \in D$



(c) For pickup location $p_1 \in P$

Figure 4: Node and arc generation

Table 2: Arc generation for lorries, r, loaders, l, and trucks, k

Origin \ Destination		Depots: Start node, O^+	Depots: Intermediate nodes, $\hat{O}$	Depots: Sink nodes, O^-	Pickup locations: Coupling nodes, P^+	Pickup locations: Loading nodes, $\hat{P}$	Pickup locations: Decoupling nodes, P^-	**Delivery locations** Unloading nodes, D^*
Depots	Start node, O^+		$l^\#$		$r+l$	k	r	
	Intermediate nodes, $\hat{O}$			$l^\#$	$r+l$		r	
	Sink nodes, O^-							
Pickup locations	Coupling nodes, P^+		r	r		$l^\#$	r	
	Loading nodes, $\hat{P}$					$l^\#$	$l^\#$	k
	Decoupling nodes, P^-		$r+l$	$r+l$	$r+l$			
Delivery locations Unloading nodes, D^*				k		k		

- within the same location

Locations and nodes: the sets containing locations and nodes are the following. In order to simplify notation throughout the formulation, sets of nodes may be added a subscript relating to a specific location, in which case we only refer to nodes of that specific location.

O	Set of depot locations
P	Set of pickup locations
D	Set of delivery locations
$O^+, \hat{O}, O^-$	Set of depot start, intermediate and sink nodes, respectively
$P^+, \hat{P}, P^-$	Set of pickup decoupling, loading and coupling nodes, respectively
$D^\star$	Set of delivery unloading nodes

Auxiliary sets are also defined for depot locations, $O^\star = O^+ \cup \hat{O} \cup O^-$, and pickup locations, $P^\star = P^+ \cup \hat{P} \cup P^-$, for designating the sets of all depot and pickup nodes, respectively.

Vehicles: let R, L and K be the sets of lorries, loaders and trucks, respectively. Also, let σ and χ be a fictitious lorry and loader, respectively, and with that in mind, $R^\star = R \cup \{\sigma\}$ and $L^\star = L \cup \{\chi\}$. These fictitious vehicles are used for modelling arcs where one travels without the other. Sets $R^\star$ and $L^\star$ are used for defining the model's decision variables, which are described in this subsection. For designating all real (i.e. non-fictitious) vehicles, set V is also defined, $V = R \cup L \cup K$.

Directed graphs: the problem as a whole is defined through two directed incomplete graphs. For lorries and loaders, set $G = (N, A)$, with $N = O^\star \cup P^\star$ is used. Let δ^+_{irl} and δ^-_{irl} be the sets of nodes that follow and precede node $i \in N$ when using vehicles $r \in R^\star$ and $l \in L^\star$, respectively. The directed graph for trucks is $G' = (N', A')$, with $N' = O^+ \cup O^- \cup \hat{P} \cup D^\star$, and φ^+_{ik} and φ^-_{ik} define the sets of nodes that follow and precede $i \in N'$ when using vehicle $k \in K$, respectively. As expected, if a given node i is not possible to be visited by a vehicle (or pair of vehicles) due to the network preprocessing, sets $\varphi^+_{ik}(\delta^+_{irl})$ and $\varphi^-_{ik}(\delta^-_{irl})$ will result in an empty set.

Parameters: the problem parameters are defined as follows.

C_v	Fixed usage cost of vehicle $v \in V$
c'_v	Usage cost per distance unit of vehicle $v \in V$
d'_{ij}	Travel distance between node $i \in (N \cup N')$ and node $j \in (N \cup N')$
c_v	Usage cost per time unit of vehicle $v \in V$
d_{ij}	Travel time between node $i \in (N \cup N')$ and node $j \in (N \cup N')$
s_i	Service time of operation being performed at node $i \in (N \cup N')$
T	Duration of the planning horizon

Note that parameter s_i relates to the necessary time for each operation to be performed by a vehicle in a specific node, e.g., the time a loader takes to load a specific truck-load, or the time a truck needs to perform the unloading operation at unloading nodes.

4.2.2 Decision variables

For each one of the routing sub-problems, a vehicle flow formulation is used. For the sub-problem concerning the trucks routes, a standard set of binary three-index path decision variables was considered. In respect to lorries and loaders, a new set of decision variables was needed to address movement synchronisation aspects. To that effect, a similar modelling approach to the one in Kim et al. (2010) is adopted. Consider a unique set of binary four-index path decision variables x_{ij}^{rl}, taking the value 1 if arc (i, j) is traversed by lorry r and loader l simultaneously, 0 otherwise. For situations where the lorry moves without the loader, for example, a fictitious entity replaces the index of the missing entity. Therefore, $x_{ij}^{r\chi}$ would mean that lorry r traverses arc (i, j) carrying the fictitious loader χ. By using these decision variables, movement synchronisation is easily introduced (or removed) in the model, thus avoiding the addition of unnecessary synchronisation constraints.

Additionally, due to the need to ensure synchronisation requirements, it is necessary to define additional decision variables which will account for the arrival time of vehicles to locations.

Therefore, the models decision variables are the ones that follow.

$$x_{ij}^{rl} \quad \begin{cases} 1 & \text{if arc } (i, j) \in A \text{ is traversed by lorry } r \in R^\star \text{ and loader } l \in L^\star \text{ simultaneously;} \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij}^{k} \quad \begin{cases} 1 & \text{if arc } (i, j) \in A' \text{ is traversed by truck } k \in K; \\ 0 & \text{otherwise.} \end{cases}$$

t_{iv} Arrival time of vehicle $v \in V$ at location $i \in (N \cup N')$

4.2.3 Objective function

The objective function (equation (1)) consists in minimising the overall costs. It contains three main components: equation (1a) corresponds to the sum of fixed costs for each one of the types of vehicles (lorries, loaders and trucks), and equation (1b) corresponds to the costs associated with the travelled distance for each type of vehicle. Equation (1c) corresponds to vehicle time costs: it accounts for the total time each vehicle spent in performing its routes and therefore is computed by using the arrival time of each vehicle at depot sink nodes. This summand is the one that guides the MIP model towards the minimisation of the makespan of each vehicle, which corresponds to the elapsed time since the start of the work day up to the time instant it finishes its route. All vehicles start the work day at time 0.

$$\min \sum_{r\in R}\sum_{l\in L^\star}\sum_{j\in O^-}\sum_{i\in\delta^-_{jrl}} C_r\cdot x^{rl}_{ij} + \sum_{l\in L}\sum_{r\in R^\star}\sum_{j\in O^-}\sum_{i\in\delta^-_{jrl}} C_l\cdot x^{rl}_{ij} + \sum_{k\in K}\sum_{j\in O^-}\sum_{i\in\varphi^-_{jk}} C_k\cdot y^{k}_{ij} \tag{1a}$$

$$+\sum_{r\in R}\sum_{l\in L^\star}\sum_{(i,j)\in A} d'_{ij}\cdot c'_r\cdot x^{rl}_{ij} + \sum_{l\in L}\sum_{r\in R^\star}\sum_{(i,j)\in A} d'_{ij}\cdot c'_l\cdot x^{rl}_{ij} + \sum_{k\in K}\sum_{(i,j)\in A'} d'_{ij}\cdot c'_k\cdot y^{k}_{ij} \tag{1b}$$

$$+\sum_{v\in V}\sum_{j\in O^-} c_v\cdot t_{jv} \tag{1c}$$

4.2.4 Constraints

The constraints directly related to lorries correspond to equations (2)-(8). Constraints (2)-(3) guarantee the depot is the start and end of all lorry routes. Constraints (4) ensure the intermediate depot nodes are only visited by lorries if they have already left the depot start node. Constraints (5) establish the route inflow-outflow conditions for lorries. Constraints (6)-(7) set the lorries arrival times at nodes. Constraints (8) are linking constraints between the arrival times at nodes and the assigned routes to lorries.

$$\sum_{l\in L^\star}\sum_{i\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} = \sum_{l\in L^\star}\sum_{j\in O^-}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} \qquad \forall r\in R \tag{2}$$

$$\sum_{l\in L^\star}\sum_{i\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} \le 1 \qquad \forall r\in R \tag{3}$$

$$\sum_{l\in L^\star}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} \le \sum_{l\in L^\star}\sum_{o\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{oj} \qquad \forall r\in R, \forall i\in\hat{O} \tag{4}$$

$$\sum_{l\in L^\star}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} = \sum_{l\in L^\star}\sum_{i\in\delta^+_{jrl}} x^{rl}_{ji} \qquad \forall r\in R, \forall j\in N\setminus(O^+\cup O^-) \tag{5}$$

$$s_i + d_{ij} \le t_{jr} + M\cdot\left(1-\sum_{l\in L^\star} x^{rl}_{ij}\right) \qquad \forall r\in R, \forall i\in O^+, \forall j\in N\setminus O^+ \tag{6}$$

$$t_{ir} + s_i + d_{ij} \le t_{jr} + M\cdot\left(1-\sum_{l\in L^\star} x^{rl}_{ij}\right) \qquad \forall r\in R, \forall i\notin O^+, \forall j\in N\setminus O^+ \tag{7}$$

$$t_{jr} \le M\cdot\sum_{l\in L^\star}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} \qquad \forall r\in R, \forall j\in N\setminus O^+ \tag{8}$$

The constraints related to loaders correspond to equations (9)-(15) and are now introduced. Constraints (9)-(10) guarantee the depot is the start and end of all loader routes. Constraints (11) ensure the intermediate depot nodes are only visited by loaders if they have already left the depot start node. Constraints (12) establish the route inflow-outflow conditions for loaders. Constraints (13)-(14) set the loaders arrival times at nodes. Constraints (15) are linking constraints between the arrival times at nodes and the assigned routes to loaders.

$$\sum_{r\in R^\star}\sum_{i\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} = \sum_{r\in R^\star}\sum_{j\in O^-}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} \qquad \forall l\in L \tag{9}$$

$$\sum_{r\in R^\star}\sum_{i\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} \le 1 \qquad \forall l\in L \tag{10}$$

$$\sum_{r\in R^\star}\sum_{j\in\delta^+_{irl}} x^{rl}_{ij} \le \sum_{r\in R^\star}\sum_{o\in O^+}\sum_{j\in\delta^+_{irl}} x^{rl}_{oj} \qquad \forall l\in L, \forall i\in\hat{O} \tag{11}$$

$$\sum_{r\in R^\star}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} = \sum_{r\in R^\star}\sum_{i\in\delta^+_{jrl}} x^{rl}_{ji} \qquad \forall l\in L, \forall j\in N\setminus(O^+\cup O^-) \tag{12}$$

$$s_i + d_{ij} \le t_{jl} + M\cdot\left(1-\sum_{r\in R^\star} x^{rl}_{ij}\right) \qquad \forall l\in L, \forall i\in O^+, \forall j\in N\setminus O^+ \tag{13}$$

$$t_{il} + s_i + d_{ij} \le t_{jl} + M\cdot\left(1-\sum_{r\in R^\star} x^{rl}_{ij}\right) \qquad \forall l\in L, \forall i\notin O^+, \forall j\in N\setminus O^+ \tag{14}$$

$$t_{jl} \le M\cdot\sum_{r\in R^\star}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} \qquad \forall l\in L, \forall j\in N\setminus O^+ \tag{15}$$

The constraints related to trucks correspond to equations (16)-(21) and are the ones that follow. Constraints (16)-(17) guarantee the depot is the start and end of all truck routes. Constraints (18) establish the route inflow-outflow conditions for trucks. Constraints (19)-(20) set the trucks arrival times at nodes. Constraints (21) are linking constraints between the arrival times at nodes and the assigned routes to trucks.

$$\sum_{i\in O^+}\sum_{j\in\varphi^+_{ik}} y^k_{ij} = \sum_{j\in O^-}\sum_{i\in\varphi^-_{jk}} y^k_{ij} \qquad \forall k\in K \tag{16}$$

$$\sum_{i\in O^+}\sum_{j\in\varphi^+_{ik}} y^k_{ij} \le 1 \qquad \forall k\in K \tag{17}$$

$$\sum_{i\in\varphi^-_{jk}} y^k_{ij} = \sum_{i\in\varphi^+_{jk}} y^k_{ji} \qquad \forall k\in K, \forall j\in N'\setminus(O^+\cup O^-) \tag{18}$$

$$s_i + d_{ij} \le t_{jk} + M\cdot\left(1-y^k_{ij}\right) \qquad \forall k\in K, \forall i\in O^+, \forall j\in N'\setminus O^+ \tag{19}$$

$$t_{ik} + s_i + d_{ij} \le t_{jk} + M\cdot\left(1-y^k_{ij}\right) \qquad \forall k\in K, \forall i\notin O^+, \forall j\in N'\setminus O^+ \tag{20}$$

$$t_{jk} \le M\cdot\sum_{i\in\varphi^-_{jk}} y^k_{ij} \qquad \forall k\in K, \forall j\in N'\setminus O^+ \tag{21}$$

The following equations (22)-(27) establish that all pickup and delivery nodes are visited by its appropriate vehicles exactly once. Constraints (22) ensure that exactly one lorry and one loader arrive at decoupling nodes at pickup locations. Constraints (23) and (24) guarantee that exactly one loader and one truck can visit loading nodes, respectively. Constraints (25) and (26) ensure that exactly one lorry and one loader can visit coupling nodes, respectively. Finally, constraints (27) guarantee that all loading and unloading nodes are visited exactly once by a truck.

$$\sum_{r\in R}\sum_{l\in L}\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} = 1 \qquad \forall j\in P^+ \tag{22}$$

$$\sum_{l\in L}\sum_{i\in\delta^-_{j\sigma l}} x^{\sigma l}_{ij} = 1 \qquad \forall j\in\hat{P} \tag{23}$$

$$\sum_{k\in K}\sum_{i\in\varphi^-_{jk}} y^{k}_{ij} = 1 \qquad \forall j\in\hat{P} \tag{24}$$

$$\sum_{r\in R}\sum_{i\in\delta^-_{jr\chi}} x^{r\chi}_{ij} = 1 \qquad \forall j\in P^- \tag{25}$$

$$\sum_{l\in L}\sum_{i\in\delta^-_{j\sigma l}} x^{\sigma l}_{ij} = 1 \qquad \forall j\in P^- \tag{26}$$

$$\sum_{k\in K}\sum_{i\in\varphi^-_{jk}} y^{k}_{ij} = 1 \qquad \forall j\in(P^\star\cup D^\star) \tag{27}$$

Constraints (28) are sub-tour elimination constraints. They avoid lorries and loaders from performing sub-tours through depot intermediate nodes and travelling a lower distance than from visiting a location directly. Depending on how the distance matrix of the problem is conceived, these constraints may or not be needed. If euclidean distances (both for time and distance) are considered, these constraints are not necessary, as vehicles will always prefer to travel a lower distance. If real distance matrices are considered where the objective is to minimize travel time or distance, these constraints should be introduced into the model, as in these situations the triangle inequality assumption does not hold. With these additional constraints, vehicles will not be able to visit an intermediate depot node without either performing a loader drop-off or a pickup, i.e. vehicles cannot leave these nodes in the same form as they arrived.

$$\sum_{i\in\delta^-_{jrl}} x^{rl}_{ij} + \sum_{i\in\delta^+_{jrl}} x^{rl}_{ji} \le 1 \qquad \forall j\in\hat{O}, \forall r\in R^\star, \forall l\in L^\star \tag{28}$$

The synchronisation constraints that link the different routing sub-problems correspond to equations (29)-(30).

$$\sum_{r\in R} t_{ir} = \sum_{l\in L} t_{il} \qquad \forall i\in(P^+\cup P^-) \tag{29}$$

$$\sum_{l\in L} t_{il} = \sum_{k\in K} t_{ik} \qquad \forall i\in\hat{P} \tag{30}$$

Constraints (29) establish that lorries and loaders must arrive to the pickup decoupling and coupling nodes at the same time and therefore must wait in their previous nodes until both are available to arrive at the same time. Constraints (30) are analogous to constraints (29) and ensure that trucks arrive at pile loading nodes at the same time as loaders.

Finally, decision variables domain is presented in equations (31):

$$x^{rl}_{ij}\in\{0,1\}, \qquad y^{k}_{ij}\in\{0,1\}, \qquad 0\le t_{iv}\le T. \tag{31}$$

4.2.5 Valid inequalities

We now present valid inequalities that can tighten the mathematical formulation and induce improvements in the branch-and-bound procedure. All of the presented inequalities can be added statically with the model constraints.

One of the three sets of valid inequalities that follow can be used to reduce the occurrence of symmetric solutions on the vehicle allocation. They are VRP-specific valid inequalities adapted from the literature (e.g. Sherali and Smith (2001); Adulyasak et al. (2014)). Assuming that $R_o = \{r_1, r_2, ..., r_{|R_o|}\}$, $L_o = \{l_1, l_2, ..., l_{|L_o|}\}$, $K_o = \{k_1, k_2, ..., k_{|K_o|}\}$ define the sets of lorries, loaders and trucks that depart from depot $o \in O$, respectively, and that the cost structures (i.e. fixed and variable costs) of each vehicle are equal to the ones of its peers, the symmetry-breaking constraints SB1–SB3 that follow can be applied between ordered pairs of vehicles, according to the desired vehicle allocation order.

The first set of symmetry-breaking constraints (SB1) is presented in equations (32)-(34) and it specifies that a certain vehicle can only be used if another identical vehicle is already being used.

$$\sum_{l \in L^\star} \sum_{i \in O^+} \sum_{j \in \delta^+_{ir_{(n+1)}l}} x_{ij}^{r_{(n+1)}l} \leq \sum_{l \in L^\star} \sum_{i \in O^+} \sum_{j \in \delta^+_{ir_nl}} x_{ij}^{r_nl} \qquad \forall o \in O, 1 \leq n \leq |R_o| - 1 \tag{32}$$

$$\sum_{r \in R^\star} \sum_{i \in O^+} \sum_{j \in \delta^+_{irl_{(n+1)}}} x_{ij}^{rl_{(n+1)}} \leq \sum_{r \in R^\star} \sum_{i \in O^+} \sum_{j \in \delta^+_{irl_n}} x_{ij}^{rl_n} \qquad \forall o \in O, 1 \leq n \leq |L_o| - 1 \tag{33}$$

$$\sum_{i \in O^+} \sum_{j \in \varphi^+_{ik_{(n+1)}}} y_{ij}^{k_{(n+1)}} \leq \sum_{i \in O^+} \sum_{j \in \varphi^+_{ik_n}} y_{ij}^{k_n} \qquad \forall o \in O, 1 \leq n \leq |K_o| - 1 \tag{34}$$

The second set of symmetry-breaking constraints (SB2) is presented in equations (35)-(37), which consist in alternative tighter constraints that order vehicle allocation by the number of traversed arcs by each vehicle. These constraints further reduce the number of symmetric solutions, compared with SB1.

$$\sum_{l \in L^\star} \sum_{(i,j) \in A} x_{ij}^{r_{(n+1)}l} \leq \sum_{l \in L^\star} \sum_{(i,j) \in A} x_{ij}^{r_nl} \qquad \forall o \in O, 1 \leq n \leq |R_o| - 1 \tag{35}$$

$$\sum_{r \in R^\star} \sum_{(i,j) \in A} x_{ij}^{rl_{(n+1)}} \leq \sum_{r \in R^\star} \sum_{(i,j) \in A} x_{ij}^{rl_n} \qquad \forall o \in O, 1 \leq n \leq |L_o| - 1 \tag{36}$$

$$\sum_{(i,j) \in A'} y_{ij}^{k_{(n+1)}} \leq \sum_{(i,j) \in A'} y_{ij}^{k_n} \qquad \forall o \in O, 1 \leq n \leq |K_o| - 1 \tag{37}$$

The third set of symmetry-breaking constraints (SB3), presented in equations (38)-(40), also reduces the number of symmetric solutions by ordering allocated vehicles by their travelled distance in the solution.

$$\sum_{l \in L^\star} \sum_{(i,j) \in A} d'_{ij} \cdot x_{ij}^{r_{(n+1)}l} \leq \sum_{l \in L^\star} \sum_{(i,j) \in A} d'_{ij} \cdot x_{ij}^{r_nl} \qquad \forall o \in O, 1 \leq n \leq |R_o| - 1 \tag{38}$$

$$\sum_{r \in R^\star} \sum_{(i,j) \in A} d'_{ij} \cdot x_{ij}^{rl_{(n+1)}} \leq \sum_{r \in R^\star} \sum_{(i,j) \in A} d'_{ij} \cdot x_{ij}^{rl_n} \qquad \forall o \in O, 1 \leq n \leq |L_o| - 1 \tag{39}$$

$$\sum_{(i,j) \in A'} d'_{ij} \cdot y_{ij}^{k_{(n+1)}} \leq \sum_{(i,j) \in A'} d'_{ij} \cdot y_{ij}^{k_n} \qquad \forall o \in O, 1 \leq n \leq |K_o| - 1 \tag{40}$$

Besides breaking solution symmetry by ordering vehicle allocation, it is also possible to order identical pickup and delivery requests. In fact, if the total quantity demanded by a delivery location coming from a certain pickup location exceeds a full truck-load, the arrival times of vehicles of loading and unloading nodes can be established in a given order. Note that this only applies to truck-load requests whose associated quantities are equal, as well as its origin and destination locations. This fourth set of symmetry-breaking constraints (SB4) is presented in equations (41)-(42).

$$\sum_{v\in(L\cup K)} t_{iv} \leq \sum_{v\in(L\cup K)} t_{i'v} \qquad \forall p \in P, i, i' \in \hat{P}_p \tag{41}$$

$$\sum_{k\in K} t_{jk} \leq \sum_{k\in K} t_{j'k} \qquad \forall d \in D, j, j' \in D^{\star}_d \tag{42}$$

Finally, we present a set of valid inequalities whose purpose is to set lower bounds (LB1) to vehicles arrival times at depot sink nodes. The principles behind these valid inequalities are presented in Meisel and Kopfer (2012) and were adapted to our specific case. These constraints are present in equations (43)-(45), taking into account the minimum required travel and service times for each vehicle type.

For lorries, constraints (43) take into account the minimum required travel times for arriving and leaving a pickup location, plus the service times for performing the decoupling and coupling operations of a loader. These lower bounds become active if lorries visit these pickup locations.

For loaders, constraints (44) take into account the minimum required travel times for arriving and leaving a pickup location, plus the service times at loading, decoupling and coupling nodes. These lower bounds become active if loaders visit these pickup locations.

For trucks, constraints (45) take into account the minimum required travel times for arriving at a pickup location of a specific request, the service time for the request's loading node, the travel time from the pickup and the delivery location, the service time for the request's unloading node and the minimum required travel time when departing from the delivery location. These lower bounds become active if trucks perform a specific request.

$$\begin{aligned}\sum_{j\in O^-} t_{jr} \geq & \sum_{l\in L}\sum_{p\in P}\sum_{i\in\delta^-_{P^+_p rl}} \min_{i\in N}\left\{d_{ip} + s_{P^+_p}\right\} \cdot x^{rl}_{iP^+_p} + \\ & + \sum_{l\in L}\sum_{p\in P}\sum_{j\in\delta^+_{P^-_p rl}} \min_{i\in N}\left\{d_{ip} + s_{P^-_p}\right\} \cdot x^{rl}_{P^-_p j} \qquad \forall r \in R\end{aligned} \tag{43}$$

$$\sum_{j\in O^-} t_{jl} \geq \sum_{r\in R}\sum_{p\in P}\sum_{i\in\delta^-_{P^+_p rl}} \left(\min_{j,j'\in N}\{d_{jp} + d_{pj'}\} + \sum_{p'\in P^{\star}_p} s_{p'}\right) \cdot x^{rl}_{iP^+_p} \qquad \forall l \in L \tag{44}$$

$$\sum_{j\in O^-} t_{jk} \geq \sum_{p\in\hat{P}}\sum_{d\in D^{\star}} \min_{i,j\in N}\{d_{ip} + s_p + d_{pd} + s_d + d_{dj}\} \cdot y^k_{pd} \qquad \forall k \in K \tag{45}$$

It should be noted that symmetry-breaking constraints SB1–SB3 cannot be combined in the same model, as they would render it infeasible. On the other hand, constraints SB4 and LB1 may be combined with any of the other presented inequalities.

5 Matheuristic solution method

Routing problems with inter-vehicle synchronisation have an additional difficulty when developing solution methods, as changing a vehicle's route may have negative effects in the feasibility of other routes.

The solution approach proposed comprises two main phases. The first phase consists in obtaining an initial solution, where slight adaptations to the MIP model need to be considered. The second phase consists in the implementation of a fix-and-optimise matheuristic approach grounded on the principles of a variable neighbourhood decomposition search (VNDS). The following subsections will outline these two phases.

5.1 Initial solution

Due to all the intricacies present in the problem at hand and in VRPs with synchronisation in general, the main bottleneck when developing solution methods for large instances of these problems consists in solution feasibility.

The initial tests performed with the largest case-study instances showed that the commercial solver had difficulties obtaining an initial solution. For some cases the solver was not able to find any feasible solution within a two hour time limit. In order to overcome this problem and to allow a fast convergence into the feasible domain, high penalty costs for unaccomplished pickup/delivery requests were introduced into the objective function, thus allowing infeasible solutions to be considered in early stages of the algorithm.

The addition of penalties for unaccomplished requests has some repercussions in the model described in the previous section. Constraints (22)-(27), which ensure that all nodes are visited, were adapted with an additional decision variable together with the routing decision variables, so that a node visit can be skipped. To that effect, new decision variables are defined as follows.

$$z_{ij} \quad \begin{cases} 1 & \text{if request from loading node } i \in \hat{P} \text{ to unloading node } j \in D^{\star} \text{ is not being satisfied;} \\ 0 & \text{otherwise.} \end{cases}$$

$$z'_p \quad \begin{cases} 1 & \text{if no requests at pickup location } p \in P \text{ are being satisfied;} \\ 0 & \text{otherwise.} \end{cases}$$

Decision variables z_{ij} will be applied to constraints relating to the corresponding nodes of the request they refer to. Decision variables z'_p are applied to the constraints relating to the pickup coupling and decoupling nodes and their purpose is to ensure that these nodes are not visited when no requests are being satisfied in that location. These two new sets of variables are linked by constraints (46).

$$z'_p \geq \sum_{i \in \hat{P}_p} \sum_{j \in D^{\star}} z_{ij} - \left|\hat{P}_p\right| + 1 \qquad \forall p \in P \tag{46}$$

The interpretation of these constraints is as follows: for a given pickup location $p \in P$, if $\sum z_{ij} = \left|\hat{P}_p\right|$, i.e. the number of unaccomplished requests in p is equal to the number of loading nodes, these two summands cancel each other and z'_p will necessarily be equal to 1. Note that it is not necessary to include additional constraints ensuring the contrapositive situation, i.e. that z'_p can only take value 0 if $\sum z_{ij} < \left|\hat{P}_p\right|$, as the model's underlying transportation network is already able to ensure this situation.

The addition of these penalty variables allows the solver to immediately obtain an initial (trivial) solution, where all requests are unaccomplished. Afterwards, the solver is given an initial time limit

TL_0 to search for better solutions and start satisfying some requests, so that the matheuristic algorithm that follows does not start so far from the feasible domain.

5.2 Solution approach: fix-and-optimise

The proposed solution method consists in an improvement matheuristic under a fix-and-optimise (FO) framework, similar to the one proposed by Helber and Sahling (2010). The concept behind FO resides in solving smaller MIP sub-problems in an iterative manner so that the computational burden from the high number of integer variables is reduced.

In each iteration, FO uses the best found solution (incumbent solution) as an initial solution for the mathematical solver. From the incumbent solution, a sub-problem is defined by fixing a portion of the binary variables which value is equal to 1, leaving the remaining decision variables (not used in the solution) to be optimised by the mathematical solver.

The outline of the sub-problems is a critical aspect of the FO procedure. Sub-problems need to take into account possible variable interdependencies in order to avoid entrapment in local optima. In our case, variable interdependencies can be verified through vehicles routes dependency. In fact, if two vehicle routes need to be synchronised at a given location and only one of them is fixed, the other route will typically remain the same, as synchronisation constraints will need to be met. Therefore, it is necessary to reach a compromise so that variables dependencies are attenuated and sub-problem size is not significantly large.

The outline of the matheuristic is present in Algorithm 1. FO sub-problems construction is outlined in lines 7-12. It consists in randomly selecting a given number of pickup locations for which all associated routing decision variables will be released, regardless of the type of vehicle, while fixing all remaining variables in the solution whose value is equal to 1. In the early stages of the algorithm, it is likely that the incumbent solution will be "infeasible" (i.e. having unaccomplished requests), and in these situations, the selection of the first location to be included in the sub-problem will only consider locations whose requests are not yet completely satisfied, thus allowing to reach problem-feasible solutions more quickly.

The optimisation process of each sub-problem is outlined in line 14. Each optimisation iteration is given an initial time limit TL_{it}, which is increased by TL_{inc} if a new incumbent solution is found during that initial time.

After obtaining an initial solution and objective function values (x_0, f_0) in lines 1-2, the matheuristic is executed under a VNDS framework where the number of locations in a sub-problem is increased until new incumbent solutions are found. After each FO iteration the resulting solution (x_{solve}, f_{solve}) from the optimisation process is evaluated against the incumbent solution (x_{cur}, f_{cur}) in lines 15-17. In case of an improvement, sub-problem construction is restarted with two random locations $(n = 2)$. Otherwise, an additional location is added into the current sub-problem up to the maximum sub-problem size, given by parameter N.

6 Computational experiments

The proposed model was applied to a case-study in the biomass supply chain, inspired in a wood chips supplier company operating in Southern Finland. The mathematical model was implemented in the Gurobi 7.5 commercial solver. The solution method was implemented in Python 3.6. The performance of the proposed solution method was tested in problem instances of increasing size in terms of number of pickup/delivery requests and number of vehicles. The results for a baseline

Algorithm 1: Matheuristic outline

input: MIPmodel (mixed integer programming model),
$\mathsf{TL_0}$ (time limit for obtaining an initial solution),
$\mathsf{TL_{it}}$ (initial time limit for a solver iteration),
$\mathsf{TL_{inc}}$ (time limit increment in solver iteration if new incumbent solution is found),
N (maximum neighbourhood size)

x_0, f_0 = `MIPsolve`(MIPmodel, $\mathsf{TL_0}$);
$x_{cur} = x_0$; $f_{cur} = f_0$;
while *termination criteria not met* **do**
  n = 1; *selected_locs* = $\varnothing$;
  while n < N **do**
    n = n + 1;
    release binary variables of MIPmodel
    while |*selected_locs*| < n **do**
      append random location **to** *selected_locs*
    foreach var **in** x_{cur} **do**
      if var = 1 **and** var **not** *associated with any location* **in** *selected_locs* **then**
        fix var
    feed MIPmodel with initial solution x_{cur}
    x_{solve}, f_{solve} = `MIPsolve`(MIPmodel, $\mathsf{TL_{it}}$, $\mathsf{TL_{inc}}$);
    if $f_{solve} < f_{cur}$ **then**
      n = 1; *selected_locs* = $\varnothing$; $x_{cur} = x_{solve}$; $f_{cur} = f_{solve}$;
    else n = n + 1;
print x_{cur}, f_{cur}

instance are described in detail, with emphasis on fleet sizing, the vehicle routes and schedules and the accomplishment of the synchronisation aspects.

The model was also used to compare the obtained plan with the currently adopted planning approach by the supplier, in which synchronisation between lorries and loaders is not taken into account. Finally, some additional managerial insights are provided about the biomass logistics that are relevant to improve planning and decision making in this sector.

6.1 Case-study

The company under study is responsible for sizing, assigning and scheduling the vehicles needed for performing the daily chipping and transportation operations that correspond to the contracted wood chips requests. A request is a certain amount of spruce wood chips to be transported between each pickup location and each destination, established beforehand as a result of the biomass distribution plan for that week, as discussed in Marques et al. (2018). The amounts of material to be transported are expressed in bulk cubic meters (b-m^3), which translate the effective amount of wood chips obtained from raw materials in its "unchipped" state. Empirically, it is commonly accepted that 1 m^{3}of roundwood corresponds to 3 b-m^{3}of wood chips and 1 tonne of wood chips corresponds to 4 b-m^{3}of spruce wood chips (Francescato et al., 2008).

The baseline case for this research is the biomass operations scheduling plan for a certain region "A", encompassing 15 requests of full truck-loads for a given planning day. The capacity of each truck is 76 b-m^{3}and the total amount of wood chips to be delivered is 940.3 b-m^3. There are 5 wood piles of spruce forest residues geographically dispersed and ready to be chipped, also called pickup locations.

The number of available vehicles are 3 lorries, 3 loaders and 7 trucks. Vehicles fixed costs correspond to € 200 for lorries and € 4500 for loaders. The costs related to vehicles travelled distance is 1.00, 1.50 and 1.20 euros per travelled km for lorries, loaders and trucks, respectively. Finally, vehicle costs per expended minute correspond to 0.67, 5.00 and 1.42 euros for lorries, loaders and trucks, respectively.

The considered productivity of loaders is 42 b-m^3/h, meaning that a total of 42 b-m^{3}of wood chips can be produced per hour from the original raw materials (Yoshida et al., 2016). The service time of each truck-load request is calculated by the quotient of its amount by the loader productivity. A full truck-load will have a loading operation time of 109 min. Given the total available amount of 940.3 b-m^3, the loaders total working time will amount to 22.4h.

Trucks transportation capacity is assumed to be equal to 19 t, which corresponds to a maximum capacity of 76 b-m^3, a value coherent with the estimates given by Francescato et al. (2008). Each unloading operation at delivery nodes will be proportional to the amount of each transported truck-load so that unloading a truck at its full capacity will amount to 20 min of expended time. Additionally, it is assumed that loader decoupling and coupling operations at depots and pickup locations take 10 min.

The considered duration of the planning horizon is 12 hours, from 8:00 to 20:00. The average travel time between locations is equal to 17.5 min, varying between 3 and 34 min. Travel distances between locations, on the other hand, vary between 2 and 27 km, with an average distance of 13.8 km. The distances between locations were computed by resorting to the national road database of Finland (`http://www.liikennevirasto.fi/avoindata/digiroad`).

During the data preprocessing phase, the pickup locations were split into truck-load equivalents, as explained in the node creation procedure, which originated the 15 pickup loading nodes corresponding to each request. The average number of requests fulfilled by a pickup location is 3. There are 3 different delivery locations for the sum of the daily requests, meaning that each delivery location, in average, will receive 5 truck-loads. For the baseline instance, a single depot is considered.

Additional instances were considered for analysing the performance of the solution method, as presented in Table 3, where instance A5 corresponds to the baseline case described earlier. They consist in variants of the baseline case instance of region "A" through the addition or removal of pickup locations. Specifically, instance A4 corresponds to removing the pickup location that is farther from the depot, instance A3 corresponds to removing the two farthest pickup locations from the depot, while instance A6 corresponds to adding a new pickup location that is closest to the depot, and so on. For the purpose of completeness, a second region "B" is also considered, with different locations and a different fleet. The rationale behind these instances is analogous to the one used for generating instances for region A. For each instance the number of vehicles of each type was progressively reduced or enlarged according to the number of requests demanded in each case. Finally, instances "C" are multi-depot instances, used for the sole purpose of evaluating the adequateness of the solution approach in these situations.

The total quantities to be delivered vary between 535.0 and 1712.3 b-m^3. Due to instances being based on real data, adding or removing one location may result in a big or small increase or decrease in the total quantity to be transported, depending on the actual quantity the new location has available.

6.2 Performance of the solution approaches

The contents of this subsection are threefold: first, the baseline results for the MIP solver approach are introduced and analysed. Afterwards, we evaluate the impact of introducing different combinations of the valid inequalities presented earlier in this paper. Finally, we compare the results obtained by the matheuristic approach with the baseline results.

All computational experiments were performed in an Intel Xeon E5-2680v2 @ 2.80 GHz CPU, with capacity for 20 simultaneous processing threads, with a maximum time limit of two hours for each instance.

6.2.1 Exact approach: mixed integer programming solver

Table 3 summarizes the obtained results for the MIP solver in all 19 problem instances. In it, the main features of each problem instance are shown, as well as the main indicators of the solution approach, namely the final value of the objective function, the percentual MIP gap computed by Gurobi, the runtime after which no solution improvement was found within the established time limit, as well as the initial and final linear relaxation of the branch-and-bound procedure (root relaxation and best bound, respectively).

Table 3: Results for the MIP solver approach

Instance	**Instance Size**								**MIP Solver**							
	Locations			Vehicles available			Requests		Objective Function	MIP Gap	Runtime (s)	Root Relaxation	Best Bound	Model size		Vehicles used (R / L / K)
	$\lvert O\rvert$	$\lvert P\rvert$	$\lvert D\rvert$	$\lvert R\rvert$	$\lvert L\rvert$	$\lvert K\rvert$	No.	Total quantity (b-m3)						Decision variables (bin. / cont.)	Constraints	
A2	1	2	1	2	2	7	9	535.0	**14,386**	12.5%	4,409	9,203	12,584	960 / 179	1,591	1 / 2 / 3
A3	1	3	1	3	3	7	11	682.3	**19,363**	15.5%	4,093	14,321	16,359	1567 / 248	2,368	2 / 2 / 4
A4	1	4	2	3	3	7	13	815.3	**25,549**	31.7%	4,997	16,607	17,449	2110 / 294	3,026	3 / 3 / 4
A5	1	5	3	3	3	7	15	940.3	**27,845**	16.1%	5,144	19,294	23,365	2733 / 340	3,758	3 / 3 / 4
A6	1	6	3	3	3	7	16	974.3	**28,097**	16.0%	4,026	19,910	23,611	3172 / 369	4,245	2 / 3 / 5
A7	1	7	4	4	4	8	18	1,123.3	**41,878**	33.0%	6,982	27,194	28,069	5068 / 504	6,113	3 / 4 / 7
A8	1	8	4	4	4	8	19	1,183.3	—	—	—	28,479	29,409	5800 / 540	6,802	—
A9	1	9	4	5	5	9	21	1,279.3	**49,914**	26.7%	6,791	35,290	36,590	8821 / 702	9,296	3 / 4 / 6
B2	1	2	1	2	2	7	8	500.0	**13,131**	4.4%	1,814	8,035	12,557	792 / 163	7,452	2 / 2 / 4
B3	1	3	1	3	3	7	11	720.0	**19,268**	13.0%	420	14,269	16,772	1543 / 248	12,422	2 / 2 / 4
B4	1	4	2	3	3	7	14	900.0	**26,431**	30.6%	2,418	17,954	18,349	2297 / 311	17,210	3 / 3 / 5
B5	1	5	2	3	3	7	15	961.7	**27,256**	13.8%	6,445	19,149	23,496	2711 / 340	1,371	3 / 3 / 5
B6	1	6	2	3	3	7	16	986.7	**28,269**	15.4%	1,822	20,234	23,914	3154 / 369	2,344	3 / 3 / 4
B7	1	7	3	4	4	8	18	1,116.7	**34,840**	21.4%	4,237	26,880	27,371	5044 / 504	3,285	3 / 3 / 5
B8	1	8	3	5	5	9	22	1,386.7	**50,890**	24.7%	5,645	37,186	38,335	8600 / 705	3,740	4 / 4 / 7
B9	1	9	3	6	6	10	26	1,636.7	—	—	—	50,576	51,299	13594 / 938	4,227	—
C4	2	4	2	4	4	12	17	1,035.0	**34,243**	20.9%	6,908	27,098	27,098	5788 / 612	6,089	3 / 3 / 6
C6	2	6	2	6	6	12	22	1,402.3	**61,048**	28.1%	7,141	43,899	43,899	11532 / 900	9,413	5 / 4 / 10
C8	2	8	4	6	6	12	27	1,715.3	—	—	—	52,834	52,834	16812 / 1098	13,699	—

The results show that Gurobi is unable to prove optimality for any of the solutions if finds within the established time limit, even for smaller instances. For the smaller instance A2 and B2, the estimated MIP gap is 12.5% and 4.4%, respectively, obtained after 4,409 and 2,270 seconds. For the baseline instance A5 the final objective function value with the MIP solver is obtained after 5144 seconds with a gap of 16.1%. Larger instances (e.g. A8, A9, B8, B9, C6, C8) yield a gap between 20% and 32%. Additional tests for for instances A2, A5 and C6 show that even when the stopping criterion is set to 48 hours the MIP solver still fails to find optimum solution with little improvements in the MIP gap for instances A5 and C6, and the obtained solution for instance A2 is proven to be optimal in approximately 40 hours.

Within the established time limit, Gurobi is able to increase the initial root relaxation in 16 out of the 19 instances, exceptions being instances C4, C6 and C8. In general, the results show that, as instance size increases, the increase in the root relaxation value is less expressive. For example, in instance A2 the root relaxation increases by a order of 3,000, while instance A9 only yields an increase of around 1,000. This analysis suggests that, as instance size increases, Gurobi is not able to keep up with the increase of the number of nodes in the search tree, due to the combinatorial nature of the problem at hand. This conclusion motivates the tests that follow, where several valid inequalities will be tested with the purpose of reducing the size of the branch-and-bound tree in need to be explored by the MIP solver.

6.2.2 Exact approach: valid inequalities

In order to validate the adequacy of the valid inequalities in facilitating solution convergence, a new set of computational experiments was run in Gurobi. In these tests, all feasible combinations of valid inequalities were tested for each problem instance, resulting in a total of 15 combinations of valid inequalities and a total of 285 solver runs.

The main indicators used for the assessment of the valid inequalities were the percentual improvements of the root relaxation, best bound and final solution values against the baseline MIP solver approach, without valid inequalities, and within the established time limit. Table 4 summarises these indicators.

Table 4: Main indicators of the use of valid inequalities

Valid inequalities			Root Relaxation Improvement				Best Bound Improvement				Solution Improvement			
SB1 / SB2 / SB3	**SB4**	**LB1**	Minimum	Median	Average	Maximum	Minimum	Median	Average	Maximum	Minimum	Median	Average	Maximum
—	—	LB1	10.3%	14.5%	14.1%	17.4%	2.5%	11.5%	14.0%	39.9%	-2.1%	0.0%	1.2%	18.4%
	SB4	—	7.9%	11.5%	11.0%	15.2%	4.6%	10.2%	12.7%	39.8%	-1.5%	0.2%	0.7%	7.0%
		LB1	10.3%	14.5%	14.1%	17.4%	4.6%	13.1%	15.8%	42.3%	-3.9%	0.0%	1.2%	17.2%
SB1	—	—	7.9%	11.5%	11.2%	17.5%	2.5%	10.5%	11.5%	21.7%	-13.1%	0.0%	-1.3%	1.2%
		LB1	10.3%	14.5%	14.3%	19.2%	2.0%	12.2%	16.1%	39.9%	-1.7%	0.0%	0.7%	12.2%
	SB4	—	7.9%	11.5%	11.3%	17.5%	4.6%	11.5%	14.0%	39.5%	-3.2%	0.0%	1.5%	19.7%
		LB1	10.3%	14.5%	14.3%	19.2%	4.6%	14.6%	17.4%	40.8%	-13.8%	-0.1%	0.5%	17.3%
SB2	—	—	7.9%	11.5%	11.2%	17.5%	1.5%	9.0%	8.5%	12.2%	-16.0%	-0.3%	-1.6%	1.0%
		LB1	10.3%	14.5%	14.1%	18.3%	1.4%	11.9%	14.4%	40.0%	-19.8%	-0.9%	-2.2%	1.0%
	SB4	—	7.9%	11.5%	11.3%	17.5%	4.6%	9.7%	13.0%	39.2%	-7.9%	-0.1%	-0.8%	1.1%
		LB1	10.3%	14.5%	14.3%	19.2%	4.6%	12.4%	15.1%	42.7%	-18.1%	0.0%	-2.5%	1.3%
SB3	—	—	7.9%	11.5%	11.2%	17.5%	2.1%	9.1%	8.6%	12.2%	-19.2%	-0.9%	-2.7%	1.0%
		LB1	10.3%	14.5%	14.1%	18.3%	3.1%	11.5%	14.2%	39.8%	-20.0%	-1.0%	-2.4%	1.0%
	SB4	—	7.9%	11.5%	11.3%	17.5%	3.9%	9.7%	12.7%	41.3%	-17.4%	-0.4%	-1.8%	1.0%
		LB1	10.3%	14.5%	14.3%	19.2%	4.6%	12.5%	15.0%	41.5%	-17.6%	-0.5%	-2.2%	1.0%

In all combinations of valid inequalities it was possible to obtain an increase of the root relaxation value. The minimum root relaxation percentual increase is 7.9%, and its maximum value is 19.2%. From the different combinations that were tested, the introduction of valid inequality LB1 into the mathematical model has a significant effect in the root relaxation improvement: in fact, these valid inequalities yield a minimum of 10.3% increase in this indicator. Compared with only using constraints LB1, there is no significant change in the root relaxation improvement when symmetry-breaking constraints are introduced into the model together with constraints LB1, thus suggesting this latter set of constraints absorbs most of the root relaxation improvement.

Unlike the root relaxation improvement, the distribution of the best bound improvement values is far more asymmetric. The percentual improvements range from 1.4% up to 42.7%. The variability of this indicator is lowest when using only one of the SB1, SB2 or SB3 types of valid inequalities, although its low variability also determines a significant lower average improvement in the best bound values.

Despite the improvements in the linear relaxation values, this does not necessarily translate into improvements in the final solution: the average improvements in the final solution are mostly negative or around zero percent for each combination of valid inequalities. The results show that the most promising valid inequalities for the problem are the combination of SB1 and SB4 constraints, leading to an average solution improvement of 1.5%. Anyhow, this improvement is observed due to an increase in variability.

In conclusion, these results suggest that the impact of the valid inequalities in the performance of the MIP solver is largely instance-dependent. Therefore, the branch-and-bound procedure reveals itself to be inadequate for these problems, even after strengthening the model with valid inequalities.

6.2.3 Matheuristic approach: fix-and-optimise

The used parameters for the matheuristic approach are exhibited in Table 5. These parameters were obtained after evaluating the convergence process of the method with different values of these parameters in a preliminary stage. The Gurobi MIP solver was also fed hint values of zero to the penalty variables z_{ij} and z'_p. This feature of Gurobi allows us to inform the solver that these hint values should yield good quality solutions, therefore changing the branching decisions it makes to explore the search tree. All other Gurobi parameters were set to its default values.

Table 5: Used parameters for the matheuristic approach

Parameters		**Value**
Termination criteria	Global time limit (TL)	7200s
Obtaining an initial solution	Unit penalty values for z_{pd} variables	10000
	Time limit (TL_0)	300s
Sub-problem construction	Sub-problem sizes	2, 3, 4 pickup locations
	MIP solver iteration time limit (TL_{it})	15s
	Time limit increment in MIP solver iteration if new incumbent is found (TL_{inc})	30s

Table 6 summarizes the matheuristic results for the 19 problem instances, including the average value of the best objective function value (bOF) after 10 repetitions for each instance, its standard deviation, the average runtime after which no solution improvement was found and the number of vehicles used. The percentual difference between the bOF obtained with both methods (% diff.) is also presented.

Table 6: Comparison of the MIP solver approach with the matheuristic approach

Instance	**Instance Size**								**MIP Solver**				**Fix-and-Optimise**				**% diff.**
	Locations			Vehicles available			Requests		Objective Function	MIP Gap	Runtime (s)	Vehicles used (R / L / K)	Objective Function		Runtime (s)	Vehicles used (R / L / K)	
	$\|O\|$	$\|P\|$	$\|D\|$	$\|R\|$	$\|L\|$	$\|K\|$	No.	Total quantity (b-m^3)					Average	Std. Deviation			
A2	1	2	1	2	2	7	9	535.0	**14,386**	12.5%	4,409	1 / 2 / 3	**14,386**	**0**	69	1 / 2 / 3	0.0%
A3	1	3	1	3	3	7	11	682.3	**19,363**	15.5%	4,093	2 / 2 / 4	**19,197**	**47**	294	1 / 2 / 3	-0.9%
A4	1	4	2	3	3	7	13	815.3	**25,549**	31.7%	4,997	3 / 3 / 4	**25,461**	**65**	964	2 / 3 / 4	-0.3%
A5	1	5	3	3	3	7	15	940.3	**27,845**	16.1%	5,144	3 / 3 / 4	**27,294**	**89**	1,290	2 / 3 / 5	-2.0%
A6	1	6	3	3	3	7	16	974.3	**28,097**	16.0%	4,026	2 / 3 / 5	**27,754**	**108**	2,071	3 / 3 / 5	-1.2%
A7	1	7	4	4	4	8	18	1,123.3	**41,878**	33.0%	6,982	3 / 4 / 7	**34,850**	**138**	3,474	3 / 3 / 5	-16.8%
A8	1	8	4	4	4	8	19	1,183.3	——	——	——	——	**41,761**	**179**	4,081	4 / 4 / 6	——
A9	1	9	4	5	5	9	21	1,279.3	**49,914**	26.7%	6,791	3 / 4 / 6	**49,294**	**97**	4,412	4 / 4 / 6	-1.2%
B2	1	2	1	2	2	7	8	500.0	**13,131**	4.4%	1,814	2 / 2 / 4	**13,131**	**0**	47	2 / 2 / 4	0.0%
B3	1	3	1	3	3	7	11	720.0	**19,268**	13.0%	420	2 / 2 / 4	**19,295**	**50**	248	2 / 2 / 4	0.1%
B4	1	4	2	3	3	7	14	900.0	**26,431**	30.6%	2,418	3 / 3 / 5	**26,243**	**53**	775	3 / 3 / 5	-0.7%
B5	1	5	2	3	3	7	15	961.7	**27,256**	13.8%	6,445	3 / 3 / 5	**27,237**	**97**	1,437	3 / 3 / 5	-0.1%
B6	1	6	2	3	3	7	16	986.7	**28,269**	15.4%	1,822	3 / 3 / 4	**27,734**	**91**	1,770	3 / 3 / 5	-1.9%
B7	1	7	3	4	4	8	18	1,116.7	**34,840**	21.4%	4,237	3 / 3 / 5	**34,410**	**104**	2,910	2 / 3 / 5	-1.2%
B8	1	8	3	5	5	9	22	1,386.7	**50,890**	24.7%	5,645	4 / 4 / 7	**49,970**	**119**	5,054	3 / 4 / 7	-1.8%
B9	1	9	3	6	6	10	26	1,636.7	——	——	——	——	**69,323**	**266**	4,860	5 / 5 / 8	——
C4	2	4	2	4	4	12	17	1,035.0	**34,243**	20.9%	6,908	3 / 3 / 6	**33,904**	**150**	1,452	3 / 3 / 5	-1.0%
C6	2	6	2	6	6	12	22	1,402.3	**61,048**	28.1%	7,141	5 / 4 / 10	**56,522**	**79**	2,392	3 / 4 / 7	-7.4%
C8	2	8	4	6	6	12	27	1,715.3	——	——	——	——	**70,273**	**197**	5,350	4 / 5 / 8	——
																Average	**-2.3%**

For the matheuristic approach, small values of standard deviation were found for all problem instances, thus suggesting that 10 repetitions per instance are sufficient for obtaining representative results. As the number of pickup locations increases, as well as the number of truck-loads to be transported, the runtime is higher, as expected. For small instances (e.g. A2, B2), it takes around one minute to find good admissible solutions. For instance A5 the bOF is reached after 22min and for larger instances it takes close to 1h30min.

Within the two hours time limit, the average bOF is within the same range of the MIP solver for practically all problem instances and it is obtained in much lower computational times. However, in the majority of the cases (12 in 16) the difference of the bOF in both methods is below 2%. The major differences are found in instances A7 (yielding a bOF 16.9% better in FO than in the MIP solver) and in C6. In these cases, there is a significant difference in the total number of vehicles used. For example, for instance A7, FO proposes 3 lorries, 3 loaders and 5 trucks while the MIP solver estimates one additional loader and two additional trucks. In the case of instance C6, FO proposes 4 lorries, 4 loaders and 7 trucks, while the MIP solver allocates one more lorry and three more trucks. In the other problem instances, both solution methods point to equal conclusions regarding the number of vehicles used, therefore the difference in terms of bOF are due to distinct scheduling and synchronisation solutions.

In conclusion, the FO solution approach seems to be adequate to solve this problem. As the number of pickup locations increases, it provides good quality solutions in shorter computational time when compared with the MIP solver. For instances above 8 pickup locations the MIP solver is typically unable to even obtain an initial solution, while the matheuristic is able to obtain admissible solutions within the two hours time limit.

6.3 Baseline instance solution analysis

The results for the baseline instance A5, obtained with the FO approach, show that all daily requests can be fulfilled with a fleet composed by 2 lorries, 3 loaders and 5 trucks. The total transportation costs amount to € 27,301, from which 51% (€ 13,900) correspond to daily usage costs (i.e. fixed costs), 3% (€ 799) are travel costs and 46% to vehicle time costs. Almost all fixed costs are related to the daily use of the loaders (€ 13,500). Lorries correspond to approximately 3% of the daily usage costs. In respect to travel costs, € 493 are related with the routes of the trucks, while € 306 relate with the routes of loaders and lorries.

As expected, the loaders are the bottleneck equipment of biomass logistics. In total, the loaders remain idle only 61 min out of the used 1708 min of the shift, yielding an average of 20 min per loader. The loader remains at a pickup location 5 h in average. Consequently, lorries are sub-used, as its average idle time is 3h12min per lorry.

It is noteworthy that the model indeed captures all the business aspects that lead to the definition of feasible routes in this complex real-world problem. The routes of lorries r_1 and r_2 are showed in Figure 5a, based on the real-life coordinates of all locations (depots, pickup locations and delivery locations). As an example, r_1 departures from depot o_1 carrying l_1 to unload it in pickup location p_3. Then, it travels empty back to depot o_1, from where it departs with loader l_3 to pickup location p_4. Next, it travels back empty to depot o_1 and finishes the route. Note that all pickup locations are visited by lorries twice, both for loader drop-off and pickup, which suggests that it is more advantageous to have lorries perform multiple loader pickups and drop-offs instead of pairing with a single loader and wait in the pickup location until loading operations are done. This is a consequence of the value of the lorry daily usage cost and the ratio of the latter and the variable transportation cost, as discussed in section 6.5.

Figure 5a also presents the pickup assignments for loaders l_1, l_2 and l_3. In this case, l_2 is assigned to the farthest pickup location, p_5, with 5 truck-loads, so the schedule of l_2 consists in departing from the depot in the beginning of the day heading to pickup location p_5 (arrival at 8:18), remaining there all day and returning to the depot in the end of the day at 19:07. Conversely, l_1

and l_3 are assigned to pickup locations closer to depot o_1 and with less truck-loads, being able to visit more than one location during the day. In fact, loader l_1 is the mostly used resource during the day.

The routes and schedules for the trucks are also presented in Figure 5a. Pickup location p_5 receives 4 out of the 5 available trucks and this can be explained due to the fact that operations at p_5 have a very little time slack, and therefore the location requires trucks to arrive quickly so that the minimum amount of time is spent. After delivering truck-loads coming from p_5 to delivery location d_1, trucks usually visit other pickup locations with less truck-loads, with preference for the ones that do not lead to queueing at the location.

The timeline with the sequence of operations in pickup location p_2 is showed in Figure 5b, evidencing the synchronisation aspects that were correctly dealt by the model. Loader l_1 is dropped at location p_2 by lorry r_2 at 11:45. Loader drop-off takes 10 min and then stays idle for 21 min until the first truck (k_7) arrives at 12:11. The synchronised operations are wood chipping (by l_1) and truck container loading (by k_7), and takes about 66 min. The second truck (k_2) arrives just 2 min after the departure of k_7 and synchronised operations are immediately started. After the second truck-load is served, the availability in p_2 is exhausted and the loader is now ready for transport. Lorry r_2 returns to location p_2 at 13:41 and waits until 15:08 for the loader to be available for pickup. Movement synchronisation corresponds to both the arrival of r_2 with l_1 in the morning of the day and the departure of r_2 with l_1 in the end of the day.

6.4 Comparison with current planning process

The biomass operational planning process of our case-study consisted in elaborating the operational plans manually. This was due to the fact that no planning tool for integrated fleet sizing and vehicle routing that acknowledged inter-vehicle synchronisation was available. To that effect, the elaboration of plans consisted in decomposing the decisions into a fleet sizing phase, followed by a route planning phase.

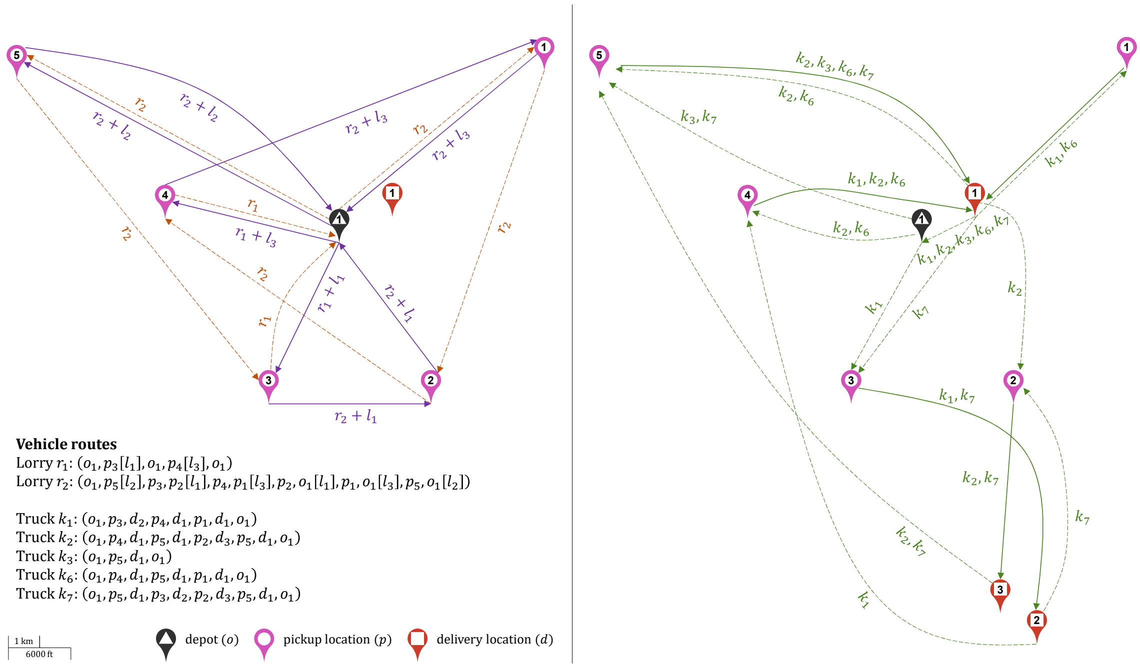
One of the main limitations of this planning process consisted in the fact that not all synchronisation aspects were considered because it would turn route construction more difficult to address manually. In fact, lorries and loaders were assumed as being unique vehicles, and therefore eventual gains from a lorry performing multiple loader pickups and drop-offs were not considered.

The fleet sizing phase relies on empirically-driven rules, such as:

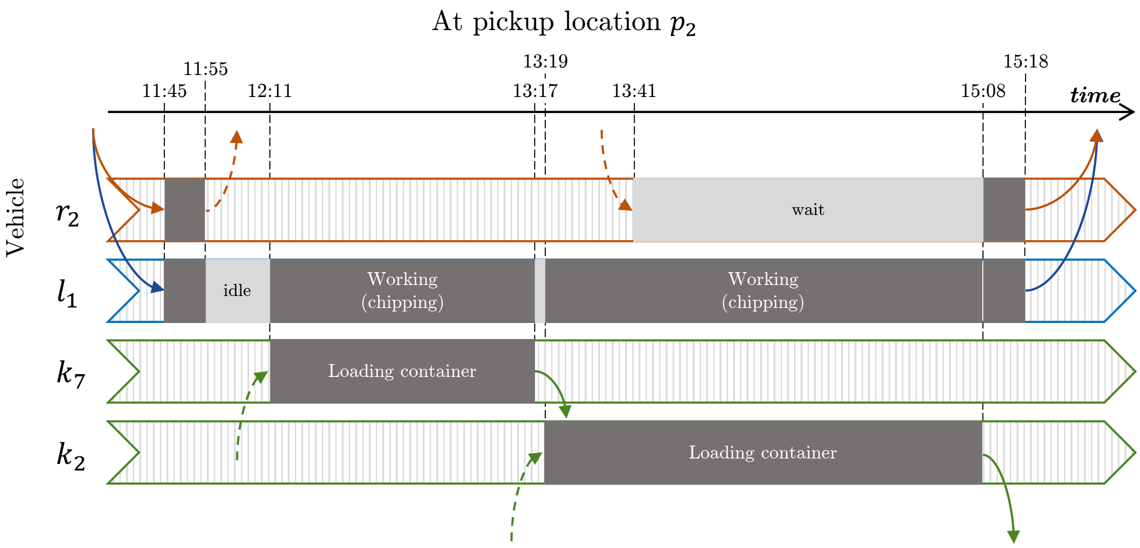
1. the number of loaders required cannot exceed the number of pickup locations, and can be estimated as a function of the total amount of material available for chipping at each pickup location, a_p, the loader average available time for chipping operations, ATL, and the loader average productivity, w (equation (47)). This function resembles the OEE (Overall Equipment Effectiveness) performance indicator that is widely used in the TPM (Total Productive Maintenance) methodology (e.g., Singh et al. (2013));

$$|L| = \frac{\sum_{p \in P} a_p}{ATL \cdot w} \tag{47}$$

2. the number of lorries is equal to the number of loaders. As already stated above, the possibility of decoupling the lorry and the loader was not considered, and therefore lorries and loaders will always move simultaneously;
3. the number of trucks required can be estimated as a function of the total amount of wood available for chipping at each pickup location, a_p, the trucks capacity, Q, the truck average available time for transportation (ATK), the average travel time between locations, TT, and



(a) Map with vehicles routes represented



(b) Operations timeline for pickup location p_2

Figure 5: Obtained solution for the baseline instance

the average loading and unloading time, TO (equation (48));

$$|K| = \frac{\sum_{p \in P} a_p \cdot 2(TT + TO)}{Q \cdot ATK} \tag{48}$$

In this study, the total amount of available material is 940.3 b-m^3. The average loader available time is estimated in 6 h based on expert's opinions, corresponding to the amount of time (in the daily shift) that the loader can be working, subtracting all stopping time (i.e. transportation to and from pickup locations, breaks, faults). The average trucks available time is estimated in 7h, based on expert's opinions, corresponding to the amount of time (in the daily shift) that the truck and driver can be working, subtracting all stopping time (i.e. transportation to and from the depot, breaks).

According to this approach the resulting number of loaders required is 3.73, rounded up to 4, therefore one more loader than in the plan obtained with the proposed solution approach. The number of lorries required is also equal to 4, which is double the number of lorries proposed by our solution approach. In respect to trucks, the number of lorries required is 7.4, rounded up to 8, therefore 2 more trucks needed than in the solution obtained in the matheuristic approach.

In order to obtain the routes corresponding to the previously sized fleet, the matheuristic approach was run again assuming the usage of this fleet and considering that lorries and loaders cannot be split. For this purpose the mathematical formulation presented in Section 4 was adapted to include constraints that enforce the use of these vehicles and the transportation network pre-processing was also changed, as explained in 4.1.4.

The obtained costs increased to € 38,142, which are 40% higher than the previously obtained plan. In fact, because the number of lorries (and loaders) is almost equal to the number of pickup locations, each lorry and loader will visit only one pickup location except for one loader and one lorry, which will visit two. Therefore, a lorry and loader visit pickup location p_1, followed by location p_2 after p_1 is completed. Due to truck-loads of these locations being smaller than the ones in the remaining locations, the plan chooses these two locations to be performed sequentially rather than simultaneously, although their relative distance is fairly large.

Table 7 shows the main indicators for this current plan. Compared with our baseline plan, we are able to acknowledge the significant increase in vehicles fixed costs, while travel costs remain stable. Vehicles idle times, however, have greatly increased for lorries and trucks. Lorries idle time has increased due to these vehicles now having to wait for loaders until they have completed all truck-loads. Trucks idle time increase can be explained by the fact that most pickup locations are being processed in an almost simultaneous manner, leading to a high demand of trucks at pickup locations practically at the same time.

From this analysis it is possible to conclude that the routing of these vehicles is simplistic when compared with the proposed plan, as such a solution will incur into unnecessary vehicle fixed costs when tasks could be concentrated in a lower number of vehicles, thus taking advantage of the length of the planning horizon. These results therefore suggest that the current planning approach did not yield satisfactory plans because it used a significantly higher number of vehicles than necessary, thus increasing fixed costs without having significant gains in vehicles idle time or travel costs that would compensate its usage in the first place.

6.5 Managerial insights for biomass logistics planning

The proposed matheuristic approach was further tested for providing additional managerial insights about the impact of key parameters of biomass logistics systems over the optimal plan. For this purpose, two main parameters were selected based on experts opinions. Alternative scenarios were built for the baseline instance by changing the parameters values within a range of possible values taken from the literature and model results were compared.

The first selected parameter is the loader's productivity, as it defines the time needed to perform the loading operations at pickup locations and therefore significantly impacts the ability of a lorry having sufficient time to perform multiple loader pickups and drop-offs during the route. The predicted impacts can range from a lower number of used lorries and/or loaders to lower vehicle idle times in general. The considered values of loaders productivity were from 36, 42 and 60 b-m^3/h, values that are coherent with the ones in Francescato et al. (2008). The second selected parameter is the lorries fixed costs, as it is believed that the ratio between this parameter and its variable costs can significantly impact the number of used lorries. The considered fixed costs were € 0, € 100 and € 200.

Consequently, 8 alternative scenarios were generated with the combination of the possible parameter values. The results are presented in Table 7.

Table 7: Main information about the different studied scenarios

Scenario	Productivity (b-m^3/h)	Lorries fixed cost (EUR)	Runtime (s)	Vehicles used (*R* / *L* / *K*)	Objective Function (EUR)	Fixed costs (EUR)	Travel costs (EUR)	Total idle time (min)			Decoupling ratio
								Lorries	Loaders	Trucks	
Baseline	**42**	**200**	**1,804**	**2 / 3 / 5**	**27,301**	**13,900**	**799**	**383**	**61**	**320**	**100%**
Current plan	**42**	**200**	**304**	**4 / 4 / 8**	**38,142**	**18,800**	**739**	**1,458**	**65**	**519**	**0%***
Scenario 1	42	100	6,502	3 / 3 / 5	26,509	13,800	803	307	5	384	100%
Scenario 2	42	0	1,546	3 / 3 / 5	26,226	13,500	804	307	8	380	100%
Scenario 3	36	200	1,707	3 / 3 / 5	28,179	14,100	812	394	15	256	100%
Scenario 4	36	100	983	3 / 3 / 5	28,020	13,800	814	408	14	348	100%
Scenario 5	36	0	526	3 / 3 / 5	27,929	13,500	798	408	14	519	100%
Scenario 6	60	200	907	2 / 2 / 4	19,296	9,400	741	363	19	346	0%
Scenario 7	60	100	1,789	2 / 2 / 4	19,183	9,200	695	548	1	450	80%
Scenario 8	60	0	765	2 / 2 / 4	19,096	9,000	719	549	5	454	80%

Decoupling ratio = No. of decoupling situations / No. total pickup locations; * - by definition.

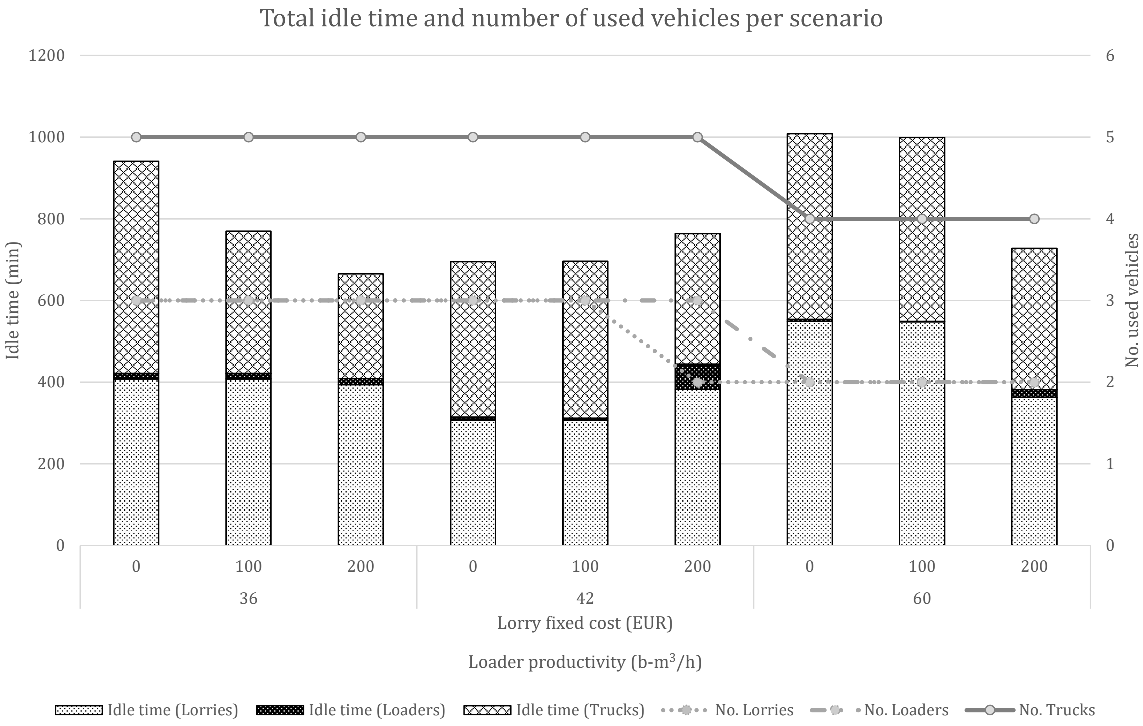


Figure 6: Main information about the different studied scenarios

The comparison of the results of the alternative scenarios shows a clear trend for using less vehicles when loaders are more productive. The decrease in the number of vehicles reflects itself not only on the number of used loaders, but also on the number of used lorries and trucks. This is due to the fact that the usage of less loaders implies the occurrence of less simultaneous loading operations, which in turn reduces the need for additional lorries and trucks.

The increase of the loader productivity from 36 b-m^3/h to 42 b-m^3/h appears to have very little impact in the obtained solution, as it does not change the number of used vehicles and the decrease of the objective function is approximately 5%. However, increasing the loader productivity from 42 b-m^3/h to 60 b-m^3/h has a very significant impact, mainly due to the decrease in vehicles fixed costs.

Results analysis also contains a decoupling ratio indicator, which tells the proportion of occurrences where a lorry was decoupled from the loader to perform different routes. In almost all the scenarios, including the baseline, the decoupling ratio is equal to 100%, meaning that for all the five pickup locations the optimal route plans encompasses decoupling the lorry and loader, i.e. the lorry departures just after the loader drop-off and the same or another lorry comes back when the loader is ready for transport. However, when the loader productivity is equal to 60 b-m^3/h and lorries fixed cost is € 200 (scenario 6), the decoupling ratio is equal to zero, meaning that no loader decouplings were made and all lorries and loaders behaved as unique vehicles.

In respect to the analysis of the vehicles idle time, represented in Figure 6, results suggest that when the loader productivity is increased to 42 b-m^3/h the total idle time also increases to approximately 1,000 min, specially due to the increase of the idle time of the lorries at the pickup locations. For the scenarios with lower productivity the total idle time is between 600 and 800 minutes, similar to the baseline.

Contrarily to our expectations, decreasing the lorry fixed cost has little impact in the increase of the total distance costs, i.e. it does not stimulate decoupling. The results also suggest that performing loader decouplings at pickup locations is an advantageous choice for low loader productivity values. For high productivity values there can be an incentive for not performing decouplings, namely the lorry fixed costs. For the studied scenarios, a lorry fixed cost of € 200 with a productivity of 60 b-m^3/h induces lorries to be used as efficiently as possible, which in this case is achieved by waiting for its loader to finish operations at the location, as loading times are now shorter.

From the obtained results it is possible to conclude that, for this instance, the main factor favouring loader decoupling is its productivity and not the variation of lorry fixed costs.

7 Conclusions and future work

In this work, a full truck-load pickup and delivery problem with multiple vehicle synchronisation is presented, motivated by a real-world case-study in the biomass supply chain in Finland. Although the literature about pickup and delivery problems is vast, there is a gap in the literature considering the synchronisation of different types of vehicles with the purpose of avoiding unproductive times while at the same time successfully modelling operations and movement synchronisation of lorries and trailers.

A novel modelling approach was developed in a systematic manner, where synchronisation aspects were dealt in the model preprocessing stage, in the definition of the decision variables and in specific time synchronisation constraints. With the purpose of tightening the mathematical formulation, a set of valid inequalities was also devised. The computational experiments performed with these valid inequalities allowed to conclude that this approach was unable to facilitate convergence into greater quality solutions.

Due to all the vehicle route dependencies and a highly pre-processed transportation network, a matheuristic approach based on the fix-and-optimise principles was developed, which not only proved to obtain better results than a traditional MIP solver but also was able to obtain feasible solutions for more constrained problem instances while the MIP solver was unable to find an initial solution.

The direct impact of this model and solution approach yields significant cost savings when compared with the current planning approach, which was a cumbersome task, performed manually. The cost savings consist in a higher utilisation ratio of each vehicle by considering that lorries can be decoupled from loaders and may perform multiple loader pickups and drop-offs, therefore decreasing their unproductive times. This increase in vehicle efficiency allows for a significant reduction of the number of necessary vehicles to perform the scheduled operations, and therefore avoids incurring into unnecessary vehicle fixed costs.

Additional tests were made with the purpose of studying the impact of two main instance parameters, namely loader productivity, which defines the average duration of each loading task at pickup locations, and the lorry fixed cost. The performed analysis suggests that a higher loader productivity can greatly impact the structure of the routing plan, as lorries and loaders will tend to behave as a single vehicle.

These last results may be interesting to consider in future work. More general models may be developed where loader productivity may be considered variable according to the location and the vehicle itself, which would allow to account for trade-offs between performing loading operations faster with more expensive loaders or performing loading operations slower with lower fixed costs. Future work may also consider alternative models for tackling multiple vehicle synchronisation under different modelling perspectives, the introduction of uncertainty factors in the problem through robust or stochastic optimisation or the development of dynamic routing approaches.

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