Optimization of Cascaded Hydro Units Modeled as Price Makers Using the *linprog* Function of MATLAB® and Considering the Tailwater Effect

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***Abstract*— This paper describes an enhanced model for the Short Term Hydro Scheduling Problem, HSP, that includes the impact of operation decisions on the market prices and the possibility of adjusting the tailwater level. Additionally, the efficiency of hydraulic turbines is treated as a variable dependent on the discharged flows. The developed solution algorithm uses an iterative approach that solves in each iteration a linearized HSP problem using the *linprog* function of the MATLAB® Optimization Toolbox. In each iteration, the value of the head to use in the next iteration is updated. The paper reports results from a realistic Case Study based on the cascade of 9 hydro stations (4 of them with pumping) installed in the Portuguese section of the Douro River.**

***Index Terms*-- Electricity Markets, hydro units, optimization, short-term hydro scheduling, tailwater effect.**

I. Introduction

In 2016, almost 30% of the Portuguese electricity demand was supplied by energy generated by hydro plants. In Portugal, hydropower capacity represents a significate portion of the national energy mix, 5738 MW in a total of 18563 MW installed capacity which means that the development of models and algorithms to adequately plan their operation is of crucial importance namely in view of the participation of these units in the common electricity market that includes Portugal and Spain.

Several works addressing the Hydro Scheduling Problem, HSP, treat hydro plants as price takers, that is, considering that the energy produced or consumed by the units to be optimized do not influence market prices [1] [2]. These approaches consider prices as exogenous variables of the optimization problem that remain unchanged during the entire optimization process.

However, in power systems such as the Portuguese one, using HSP optimization models that treat hydro units as passive agents regarding their relation with market prices is an inaccurate approach and the results obtained by these models can be quite unrealistic. On Iberian Electricity Market (MIBEL) there is a very significant correlation between hydro generation and market prices, since the hydro units play an important role in supplying the demand.

The introduction of competition in activities as generation and retailing justifies the development of new models and tools to ensure that the most adequate decisions are adopted as reported in [3] [4]. In this scope, to gain advantage over other agents, generation companies develop and adopt optimization models to adequately plan the use of their assets. On the other hand, the Portuguese trade balance is heavily penalized by the import of hydrocarbon fuels. Therefore, the construction of new hydro plants and the repowering of several existing units is a key issue to ensure a more sustainable future and to increase the Portuguese energetic independence.

Planning problems, particularly in energy systems, often have non-linear objective functions. Existing limitations established due to legal or technical requirements have non-linear characteristics in most cases. Some published papers mention the successful application of non-linear techniques to solve the Short-Term Hydro Scheduling Problem, STHSP [5], [6]. However, the effect of dimensionality occurs here, because as the size and complexity of the problem augments, these techniques become very computational heavy.

Regarding the operation of hydroelectric power plants, several publications use evolutionary and genetic techniques that are reported to provide acceptable solution times as in [7]-[10]. One of biggest drawbacks associated to the application of meta-heuristics to the HSP is the difficulty in introducing new operational constraints as mentioned in [11].

Previous attempts to implement stochastic dynamic programming algorithms to the multi-reservoir HSP problem indicated that there is an unprecedented growth in the number of states, rendering this approach impractical [12] [13] because prices and inflows are also considered as states.

The complexity of the HSP problem is increased when including the hydraulic interconnections between units in the same basin as well as the pumping capacity of some units. The non-linear relationship between the head, the flow and the power output also requires using a large computational effort, specially for real sized systems [14]. Altogether, this means that the computational requirements associated with this optimization problem rapidly increases as the planning horizon gets larger and the number of plants is increased.

In the computer application reported in this paper we adopted an iterative approach to solve the original non-linear HSP problem in which the value of the net head is assumed fixed in each iteration. When an iteration ends, the net head is

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recomputed and an updated value is then used in the next iteration. The developed model includes the possibility of pumping to increase the realism and applicability of the developed application as well as the impact of the operation decisions on the markets prices. In this sense, the procedure starts with an initial set of prices that are used to obtain an initial set of generation or pumping decisions for the units. Using these decisions for each hour of the horizon, the market curves provided as input are updated so that a new set of prices are obtained. As a result, the benefit of the operation of the hydro units is estimated in a more realistic way, because market prices tend to increase when pumping exceeds generation and tend to reduce if generation exceeds pumping. Additionally, the developed solution algorithm also includes the effect of the variability of tailwater levels in the value of gross head and the efficiency of hydraulic turbines is treated as a variable dependent on the discharged flows. All these aspects contribute to increase the realism of the solution approach and the accuracy of the results.

This work was developed in the scope of the MSc thesis submitted by the first author to the Faculty of Engineering of Univ. of Porto, Portugal, in June 2016 and it was prepared with the collaboration of EDP Gestão da Produção SA.

Apart from this Introduction, Section II details the mathematical formulation of the Hydro Scheduling Problem, HSP, and Section III describes the developed solution approach. Section IV presents the results that were obtained using the mentioned Case Study and Section V includes the most relevant conclusions.

II. Mathematical Formulation of the HSP Problem

Given a time horizon and a set of hydro units, the Hydro Scheduling Problem (HSP) is an optimization problem to schedule these units along this horizon. In the developed work we addressed short terms scheduling problems of 1 day to 1 week discretized in periods of 1 hour aiming at getting the best operation strategy for each unit in terms of having an operation decision as a generator or as a pump or eventually having the unit stopped. Portugal and Spain share a common electricity market, MIBEL, to which generation companies have to submit their selling bids (if operating as generators) or buying bids (if pumping is available) for the each hour of the next day. In order to maximize the revenues, generation companies should submit the most adequate bids and to get more insight in order to formulate the most adequate bids, generation companies can run simulation models as the HSP to identify the most adequate operation strategies which justifies the relevance gained by this problem in recent years.

For an hydro unit i in hour k, the generated power is given by (1) incorporating the losses in the hydro circuit. The density of the water is approximately equal to 1000 kg/m^3 and the gravitational acceleration is given by g = 9.8 m/s^2. The power required to pump a flow q_{ik} is given by (2).

$$P_{Gik} = 9.8 \times q_{ik} \times (h_{ik} - \beta \times q_{ik}^{2}) \times \eta_G \quad (1)$$

$$P_{Pik} = 9.8 \times q_{ik} \times (h_{ik} + \beta \times q_{ik}^{2}) \times \mu_P \quad (2)$$

In these equations:

q_{ik} – discharged/pumped water flow in m^3/s;

h_{ik} - gross head in meters;

η_G and μ_P - generation and pumping efficiencies;

β - head loss coefficient.

The head loss coefficient is given by (3) and it depends on the nominal head loss Δhn and on the nominal discharge flow qn. Once the head loss coefficient is calculated, expression (1) indicates that there is a non linear dependency between the power, the discharge flow and the head because of the $\beta \times q_{ik}^2$ term as illustrated by the blue curve in Fig. 1. This term causes a reduction of the head and it depends on the flow.

$$\beta = \frac{\Delta hn}{qn^2} \quad (3)$$

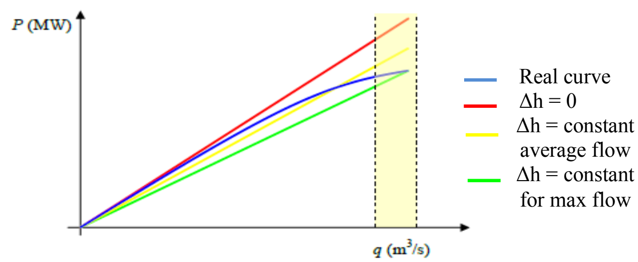

Figure 1. Non linear relation between the power, the flow and the head.

Several approximations have been used to address this non linear aspect and so to introduce some approximations in the original problem as illustrated in Fig. 1. For instance, if the head loss term is neglected, then the blue curve is replaced by the red one. The linear red curve is above the real one leading to a poor approximation for larger flows. This is inappropriate because operation decisions tend to use as much water as possible to increase the revenues. On the other hand, we could set the head at a fixed value, namely the maximum or the average leading to the green and yellow curves in Fig. 1. This could be unrealistic for instance for small or run of river units in which the head can vary a lot after some hours of operation. If the head loss term is computed for the maximum flow, the error is smaller for large flows and this can be interesting given the trend to operate with as much flow as possible.

In order to avoid these problems, in this research we adopted the under-relaxed iterative technique described in [14] and [15]. This approach solves the non-linear problem in an iterative way along which the value of the net head is updated. Using the values of the flows obtained at the end of one iteration, we update the net head to be used in the next iteration. As a result of this approach, the HSP problem to solve in each iteration is a linear problem that can be solved using the *MATLAB linprog* function.

In order to update the value of the head, after solving the HSP in iteration *it*, the value of the head for unit i in hour k in iteration *it+1* is updated using a function f in which the head depends on the volume in the reservoir at the end of iteration *it* (4). The function relating the head and the volume is known as volume-level curve and the experience shows that just using the volume obtained in iteration *it* can originate a large change in the head that may cause the divergence of the process. In order to avoid this we used (4) to update the value of the head h_{ik} from iteration *it* to *it+1.* In this expression the argument of function f includes not only the volume v_{ik}^{it} obtained at the end of iteration *it* but also the volume v_{ik} at iteration *it-1*.This expression also uses a parameter $\alpha>0$ that has to be adjusted for the hydro system under analysis [14].

$$h_{ik}^{it+1} = f(v_{ik}^{it}) = f(v_{ik}^{it} + \alpha(v_{ik}^{it-1} - v_{ik}^{it})) \quad (4)$$

The linearized HSP problem to solve in each iteration of this process is formulated by (5) to (13). The objective function is given by (5) for an hydro system with I units and for a planning period with K sub periods usually of 1 hour each. This problem maximizes the revenue obtained by the generation company along the K subperiods as a result of selling the generated electricity P_{Gik} in period k at the price π_k subtracting the amount paid to buy electricity to pump, P_{Pik}. The third term in (5) penalizes spilling given by s_{ik} using a coefficient ps set at a value much larger that the electricity prices π_k.

$$\max\ Z = \sum_{i=1}^{I}\sum_{k=1}^{K} (\pi_k \times P_{Gik}) - (\pi_k \times P_{Pik}) - (ps \times s_{ik}) \quad (5)$$

Subject to:

$$v_{ik} = v_{i(k-1)} + a_{ik} - q_{ik} - s_{ik} + qp_{ik} + \sum_{m \subset M_i}^{L} q_{m(k-\phi_m)} + s_{m(k-\lambda_m)} - qp_{m(k-w_m)} \quad (6)$$

$$v_{ik}^{\min} \le v_{ik} \le v_{ik}^{\max} \quad (7)$$

$$q_i^{\min} \le q_{ik} \le \min(q_i^{\max}, qn_i \times \sqrt{\frac{h_{ik}}{hn_i}}) \quad (8)$$

$$qp_i^{\min} \le qp_{ik} \le \min(qp_i^{\max}, qpn_i - \delta_i \times (h_{ik} - hn_{Pi})) \quad (9)$$

$$0 \le s_{ik} \le \infty \quad (10)$$

$$v_{iK} = v_{iK}^{final} \quad (11)$$

$$vol_i^{\min} \le q_{ik} + s_{ik} - qp_{ik} \le vol_i^{\max} \quad (12)$$

In this formulation:

I - total number of hydro plants;
K - total number of periods;
π_k - market price in hour k;
P_{Gik} - generation power of unit i, in hour k, in MW;
P_{Pik} - pumping power of unit i, in hour k, in MW;
p_s - penalty factor for spills;
s_{ik} - spill of reservoir i, in hour k;
L - number of upstream reservoirs;
v_{ik} - volume of reservoir i, in hour k;
a_{ik} - inflow of reservoir i, in hour k;
q_{ik} - discharge volume of reservoir i, in hour k;
q_{Pik} - pumping volume of reservoir i, in hour k;
M_i - set of upstream reservoirs of reservoir i;
φ_m - delay of turbine discharge volumes;
λ_m - delay of spill volumes;
ω_m - delay of pumping volumes;
vol_i^{min} / vol_i^{max} – min/max launch volume of reservoir i;
v_i^{min} / v_i^{max} - min/max volume limit of reservoir i;
q_i^{min} - turbine minimum discharge limit of reservoir i;
q_i^{max} - turbine maximum discharge limit of reservoir i;
qp_i^{min} - turbine minimum pumping limit of reservoir i;
qp_i^{max} - turbine maximum pumping limit of reservoir i;
qn_i - nominal turbine discharge volume of reservoir i;
qpn_i - nominal turbine pumping volume of reservoir i;
hn_i - nominal turbine head of reservoir i;
hn_{Pi} - nominal pumping head of reservoir i;
h_{ik} - head of reservoir i, in hour k;
δ_i - pumping coefficient of reservoir i;
v_{iK}^{final} - volume specified for reservoir i in the last scheduling period, K.

The objective function (5) is subjected to a set of constraints that includes the water balance equation (6) and operational and technical constraints for each unit i and for each period k, (7) to (13). The balance equations are established considering the hydro connections between the units as well as the time lag to allow the discharged water from one unit to arrive to next one, inflows and spills. The problem also includes maximum and minimum flow constrains and maximum and minimum generated and pumping limits. For each unit it is also established a constraint imposing the volume at the end of the final scheduling period (11). This value can be obtained from medium or long term planning problems to define longer term strategies for the use of the water. This means that in this paper the HSP problem has a short-term nature but it profits from information from medium to long term water management problems.

III. Developed Solution Approach

To consider the impact of the schedule of a set of hydro units on market prices we admitted that the generation company has estimates of the buying and selling curves of all other market agents for each hour k of the planning horizon and we also used the MATLAB *linprog* to implement an ancillary function that maximizes the Social Welfare Function associated with this set of bids for each hour under analysis. The market clearing price for hour k corresponds to the dual variable of the generation/demand balance equation of this market clearing problem.

This procedure gives a set of initial prices that are used to solve the HSP problem described in Section II. As a result, we obtain the operation decisions for each hydro and for each hour (operation as generator or as pump or not operating). Using this information, we obtain the total generation and pumping powers for each hour and these values are used to change the selling and buying curves for that hour. The selling curve is shifted to the right as much as the power generated by the units under analysis. The buying curve is shifted to the right as much as the pumping power of the units under analysis in each hour. This means including a zero price selling bid with an amount equal to the power generated by the hydros and to include a buying bid at the maximum market price with an amount equal to the pumping power in that hour. Once these bids are considered for each hour, the Social Welfare Function is maximized providing the new market price in that hour that will be used to solve the HSP in the next iteration.

The above process corresponds to an outer cycle illustrated in the flowchart in Fig. 2 that in each iteration w calls the HSP detailed in Section II. On its turn, the HSP is itself solved using an iterative process during which the value of the head of each unit is updated as also described in Section II. In order to smooth the impact of the operation decisions coming from the HSP problem in the market prices, we used an expression similar to (4) to update the prices. In this case, the market prices were updated based on the calculation of the weighted moving average, using a memory term regarding the prices of the previous iterations. This technique aims at improving the convergence of the iterative process, especially in operation periods in which the hydro production has a strong impact on market prices. Therefore, the price in hour k to use in the iteration $w+1$ of the outer cycle is given by (13).

$$\pi_k^{w+1} = \pi_k^{aux} + \beta(\pi_k^{w} - \pi_k^{aux}) \quad (13)$$

In this expression and for a given hour k:

π_k^{w+1}- market price in iteration $w+1$;

π_k^{w} - market price used in iteration w of the HSP;

π_k^{aux} - market price from the maximization of the Social Welfare Function, using the new selling and buying bids;

β - smoothing parameter.

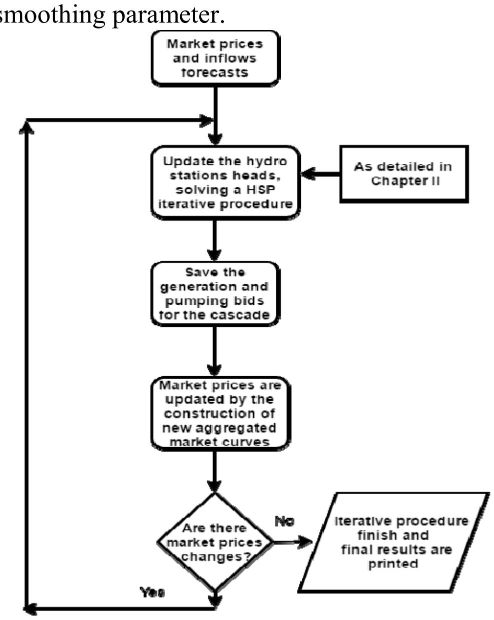

Figure 2. Market prices updating outer cycle.

The outer cycle continues updating the prices and calling the HSP problem to update the operation decisions until a convergence criterium is reached. In this case, the convergence of the external cycle is checked based on the Mean Absolute Percentage Error (MAPE) for the prices in two consecutive iterations, $w+1$ and w (14). If the computed MAPE is smaller than a specified limit, then the external iterative cycle stops and the results are displayed.

$$MAPE = \frac{1}{K} \frac{\sum_{k=1}^{K} \left| \pi_k^{w+1} - \pi_k^{w} \right|}{\sum_{k=1}^{K} \left| \pi_k^{w+1} \right|} \quad (14)$$

In the case of some smaller or run of river units, solving the HSP problem taking the downstream levels as constant can cause larger errors due to the possibility of tailwater levels elevations. Comparing to other hydroelectric scheduling optimization works, the developed approach incorporates an important improvement because it updates the downstream level along the mentioned outer cycle.

To consider this effect, the tailwater of unit i in hour k to use in iteration $w+1$ of the outer cycle ($tailwater_{ik}^{w+1}$) is updated using (15) based on the nominal tailwater of the unit. At the end of iteration w we compute the variation of the tailwater using (16) that results from the analysis made by EDP Gestão da Produção using measurements for the flows and tailwater levels for the units in the Douro river cascade. This expression indicates that the tailwater level can be elevated at the maximum of 15% regarding the nominal value, namely when the flow reaches its maximum. The computed tailwater level is then used to reduce the net head thus implying a reduction of the generated power. For a different hydro system or if particular hydro conditions exist for a specific unit, measured values for the flows and tailwater levels should be used to build the most adequate expression.

$$tailwater_{ik}^{w+1} = tailwater_{i}^{nom} + \Delta tailwater_{ik}^{w+1} \quad (15)$$

$$\Delta tailwater_{ik}^{w+1} = 0.15 \times (q_{Gik}^{w} / q\max_{i}) \times tailwater_{i}^{nom} \quad (16)$$

In these expressions:

$tailwater_i^{nom}$ - nominal tailwater level of reservoir i;

$qmax_i$ - maximum flow of reservoir i;

q_{Gik}^{w} - discharge flow of reservoir i in hour k in iteration w of the outer cycle.

Finally, along the iterative process of the outer cycle it is also possible to update the generation and pumping efficiencies used in (1) and (2) since they depend on the flows. Regarding the discharge efficiency, (17) and (18) are polynomial expressions based on regression studies on the efficiency curves provided by manufactures of Kaplan and Francis turbines respectively. An analog process can also be used to update the pumping efficiencies. The order of the polynomials was defined having into account the zones of greater efficiency.

$$\eta_{Gik}^{new} = \eta_{Gi}^{0}(-9.67q_{ik}^{4} + 24.05q_{ik}^{3} - 21.17q_{ik}^{2} + 7.70q_{ik} + 0.02) \quad (17)$$

$$\eta_{Gik}^{new} = \eta_{Gi}^{0}(-1.16qq_{ik}^{2} + 2.12q_{ik} + 0.03) \quad (18)$$

In these expressions:

η_{Gi}^{0} – nominal discharge efficiency of reservoir i;

η_{Gik}^{new} – discharge efficiency of reservoir i, in hour k.

IV. Results For the Douro River Cascade

The cascade in the Portuguese section of the Douro River has 9 plants according to the scheme in Fig. 3. It has a total installed capacity of 1504.8 MW and the units B, C, E and H have pumping capacity. Table I includes the most relevant technical data of this hydro system. The market prices used in the simulations were downloaded from the Iberian Market Operator and the inflows were based on historical data provided by EDP – Gestão da Produção, S.A. The two cases reported below correspond to wet weeks during which the inflows were admitted to be approximately constant along the period of 168 h that was studied.

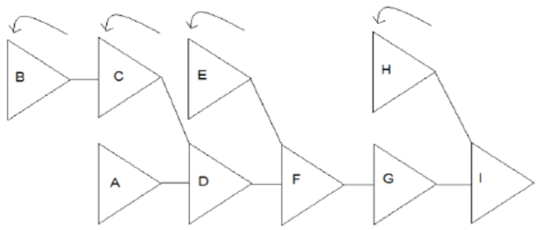

Figure 3. Scheme of the cascade in the Portuguese section of the Douro.

TABLE I
TECHNICAL DATA OF THE HYDRO STATIONS

unit	h_n m	q_G m^3/s	q_P m^3/s	P_G MW	ηG	μP	type
A	20.5	1077	--	186	0.90	--	Kaplan
B	94.0	170	135	140	0.89	0.91	Francis
C	30.0	120	85	30.8	0.89	0.91	Francis
D	30.5	900	--	240	0.92	--	Kaplan
E	93.6	310	238	270	0.89	0.92	Francis
F	27.0	744	--	180	0.93	--	Kaplan
G	33.3	705	--	201	0.91	--	Kaplan
H	51.5	320	279	140	0.89	0.92	Francis
I	10.6	1350	--	117	0.86	--	Kaplan

Fig. 4 presents the market prices profile used for the simulated 168 hours of the period of analysis. The used price scenario presents an average value of 46.65 €/MWh, with a standard deviation of 7.38 €/MWh. The initial market prices considered in this paper were based on real data downloaded from the OMIE, the Operator for the Iberian Electricity Market. To access more accurately the impact of the operation decision in the Iberian electricity market prices, the market prices taken from the OMIE webpage were pre-processed in order to exclude from the real aggregated market curves the units in the Portuguese section of Douro River.

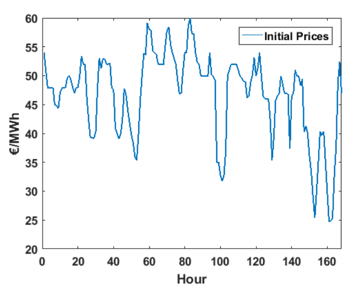

Figure 4. Set of initial market prices.

<u>CASE I</u> – As indicated in Section II, in each iteration of the developed approach we solve a linearized HSP problem enabling solving large size problems with a reduced computational effort. In this case, the optimization problem involved a total of 2016+1680 = 3696 decision variables (4 hydros × 168 hours × 3 for hydros with pumping plus 5 × 168 × 2 for hydros without pumping) and 10593 constraints (9 hydros × 168 hours × 7 technical constraints + 9 mandatory final volume constraints). In the first place, the HSP was run using data for a wet week and not considering the impact of the operation decisions on the prices, that is, admitting that the hydro units are price takers. This means running the HSP just once, or in an equivalent way, to run just the first iteration of the outer cycle detailed in Section III. The results indicate that generation was correctly positioned in the periods of larger prices and pumping was located in the lower price periods. The total generation was 33803.33 MWh, the energy used to pump was 10022.12 MWh and the total revenue was 1506.33 k€.

<u>CASE II</u> - Secondly, we ran the HSP for the same wet week as in Case I now considering the impact on prices, that is, the hydro units are assumed as price makers, as well as the possibility of adjusting the tailwater levels. Fig. 5 illustrates the results along the 168 hours of week.

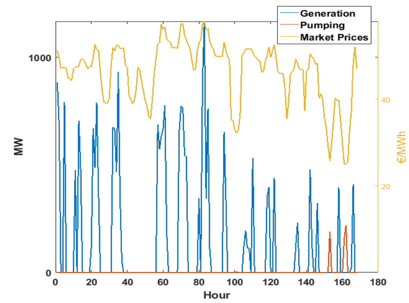

Figure 5. CASE II - Results for the total generated (blue curve) and pumping power (red curve). Final prices in €/MWh in the yellow curve, right axis.

The total generated and consumed energies are 25155.34 and 272.84 MWh and the revenue gets reduced to 1316.73 k€. The reduction of the generated energy is due to the dependency of the generation efficiency and of the tailwater level on the flows. Additionally, when hydro generation is larger than pumping then the prices tend to reduce due to the new zero price selling bids, as indicated by the graphs in Fig. 6. Conversely, when pumping is larger, then prices tend to increase due to the demand increase. As a result of these impacts on the prices, the revenues tend to reduce because when selling the prices are lower and when buying the prices are a bit larger. The solution also shows a reduction of pumping regarding Case I as a way to reduce the amount paid to buy electricity (second term in (5)) and to elevate as much as possible the total revenue given by (5).

Fig. 7 shows the average deviations for all the 9 units of the tailwater levels along the 168 hours regarding the initial values. According to this graph, the periods with the largest tailwater deviations coincide with the periods with more generation. However, it is important to notice that the variation of the global mean tailwater level is not directly proportional to the generated power because each hydro station has a different historical mean tailwater level.

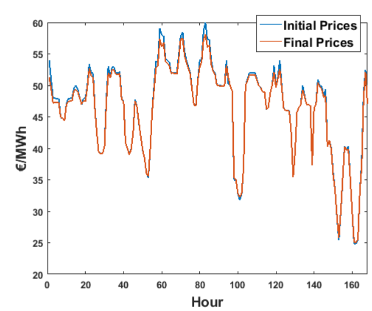

Figure 6. CASE II – Market prices before and after the iterative procedure.

Figure 8 displays the minimum and the maximum water volumes as well as the evolution of the stored water in the set of 9 hydro units. It is clear that in the periods of larger generation there is a larger reduction of the stored volume. On the other hand, when the period under analysis ends at hour 168 the final volume constraints (11) are respected.

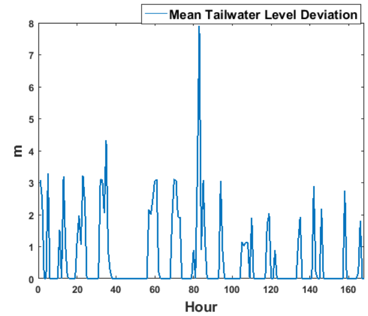

Figure 7. CASE II – Cascade mean tailwater level deviation.

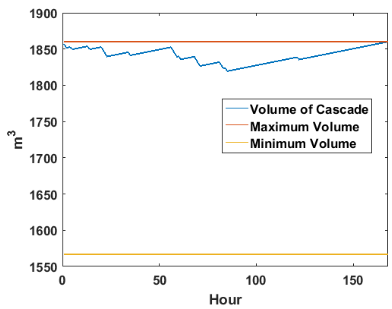

Figure 8. CASE II – Maximum and minimum volume limits (yellow and red curves) and total volume for each hour of the analysed period (blue curve).

V. Conclusions

Hydro units are playing an increasingly important role in power systems namely in view of their flexibility of operation and their capacity to accommodate variations from other renewable and volatile sources as wind and solar PV. On the other hand, their correct scheduling allows generation companies to obtain large revenues given their virtually zero marginal operation cost. In view of these aspects, this paper describes a realistic formulation for the HSP problem including the impact of operation decisions on market prices, on the operation efficiencies and on the tailwater. These features allow obtaining more accurate results as illustrated by the reduction of 12.6% of the revenues when going from Case I to Case II of the reported Case Study. These aspects are very relevant in the Iberian Electricity Market where the hydro capacity is close to 25% of the total. Generation companies can therefore profit from this type of models to get more insight on the operation of their units and to take more sounded decisions namely in terms of preparing their selling and buying bids to the market operator.

Acknowledgments

The first author thanks EDP Gestão da Produção SA, for their help, support and useful discussions held during the development of this research work. This work is financed by the ERDF – European Regional Development Fund through the Operational Program for Competitiveness and Internationalisation - COMPETE 2020 within project «POCI-01-0145-FEDER-006961», and by National Funds through FCT - Portuguese Foundation for Science and Technology as part of project UID/EEA/50014/2013.

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Biographies

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