Optimal Power Flow Solution for Distribution Networks using Quadratically Constrained Programming and McCormick Relaxation Technique

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***Abstract*— This paper presents a quadratically constrained programming (QCP) model to tackle the optimal power flow (OPF) problem in distribution networks. The proposed model is fast, reliable, and precise enough to be embedded into the multi-temporal power system analysis. The proposed model benefits from a standard QCP to solve the branch active and reactive power flows. The second-order conic programming (SOCP) approach has been applied to address the quadratic constraints. The nonconvex feature of the OPF problem has been relaxed utilizing the McCormick envelopes. To find the minimum current of each branch, the lossless power flow model has been first solved and the obtained results have been considered for solving the OPF problem. The IEEE 33-bus test system has been selected as the benchmark to verify the efficient performance of the proposed OPF model. The simulation study confirms that the McCormick envelopes used in the QCP approach lead to precise results with a very fast convergence time. Overall, the presented model for the OPF can be extended for both planning and operation purposes in distribution system studies.**

Keywords—Optimal Power Flow, Distribution Network, Convex Optimization Model, Second-Order Conic Programming.

NOMENCLATURE

Sets

$i, j / \Omega^b$	Index/Set of buses
l / Ω^l	Index/Set of distribution feeders

Parameters

A_{il}	Incidence matrix of the network (bus to feeder)
R_l, G_l	Resistance/conductance of feeder *l*
X_l, B_l	Reactance/susceptance of feeder *l*
Z_l	Impedance (absolute value) of feeder *l*
Y_l	Admittance matrix of network
PD_i	Active power demand at bus *i*
QD_i	Reactive power demand at bus *i*
$S_l^{\max}$	Maximum apparent power of feeder *l*

Variables

$f(PG_i)$	Cost function
PG_i	Active power generation at bus *i*
QG_i	Reactive power generation at bus *i*
P_l'	Net active power transmitted by feeder *l*
P_l	Lossless active power transmitted by feeder *l*
P_l^{Loss}	Active power loss of feeder *l*
Q_l'	Net reactive transmitted power by feeder *l*
Q_l	Lossless reactive power transmitted by feeder *l*
Q_l^{Loss}	Reactive power loss of feeder *l*
V_i	Node voltage at bus *i*
I_l	Feeder current of line *l*
ω	McCormick decision variable

Symbols and Abbreviations

min	Minimum (lower bound)
max	Maximum (upper bound)

I. INTRODUCTION

Power flow (PF) and optimal power flow (OPF) problems one of the most significant problems in power system studies. The applications of PF and OPF in the operation and planning of power systems have been extensively studied in the corresponding literature [1]. Indeed, OPF is known as one of the indispensable tools in power system studies [2]–[4]. Several practical solutions and methods have been presented in the literature in terms of solving the OPF problem. However, this is still known as a challenging and computationally intensive problem especially in large-scale and complex networks with a considerable number of branches and power generating units. In this regard, the efforts on developing proper models to address the nonconvex and inherent non-linear constraints of this problem are still producing new formulations [5].

There are some simplified models of the OPF available in the transmission networks for large-scale and long-term studies. In the simplified models, the resistance of the transmission lines can be neglected due to the X/R ratio. Besides, the node voltages are very close to 1 p.u.; therefore, it can be assumed that the voltage profile is levelled. The simplified OPF problem is the so-called "DCOPF''. In large-scale power systems, the application of DCOPF has been thoroughly examined and the functionality of such tools in both operation and planning studies has been extensively approved by researchers. However, at the distribution level, the aforementioned assumptions, i.e. the X/R ratio and therefore, the lossless models are not valid. The voltage drops

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and active power losses are the main challenging issues to this end.

The classic approaches for solving the PF problem, like the Newton-Raphson technique, are unable to find acceptable results due to the low X/R ratio of the distribution feeders [6]. The shortcomings of the Newton-Raphson method in ill-conditioned distribution networks have been addressed in [7].

In the traditional distribution networks, according to the radial design and operation of the network, the power flow can be easily determined by the current injection from the reference bus. The backward, forward, and hybrid backward/forward sweep power flow models can achieve reasonable results in well-behaved distribution networks. The main advantages of the mentioned sweep techniques are the independencies of the models from the Jacobian matrix of the network and from the local power generation by the installed distributed generators (DGs). Thus, these approaches are useful for online and real-time operation studies. The main drawbacks of such techniques are unsuccessful convergence in highly loaded networks and low efficiency for large-scale networks [8]. Such formulations are not suit for recent networks with high penetrated renewable resources and multiple DGs.

In the literature, the implementation of OPF for reducing the power losses [9], feasibility study and optimal operation [10], and planning of the network [11] has been performed by using two-stage optimization models. Indeed, one problem is defined as a master problem while the feasibility and optimality of the first stage is assessed throughout the lower level by utilizing the OPF solvers. The main advantage of such approaches is to alleviate the computational load and complexity of the model [12]. However, such techniques may not achieve the global optimum. Embedding the OPF as a set of constraints and modifying the objective function of the OPF problem will result in a more complex and large-scale optimization model, and this feature will threaten the convergence of the entire optimization problem [13].

The optimal operation of the power system in the presence of volatile renewable energy resources will increase the complexity of the short term operation of the microgrids [14]. In this regard, new optimization models for short-term operational planning have been introduced for different operational planning problems [15]–[17].

In the literature, there are several models for simplifying the OPF problem, addressing the convex and non-convex features of the optimization problem. The linearization of the quadratic cost function [18], voltage deviation [19], and active power losses [20] has also been presented. There are some novel models for solving the OPF problem using semidefinite programming or second-order conic programming (SOCP) in the distribution networks. The main idea of such optimization models is to introduce the second-order terms of branch currents and node voltages as decision variables. After solving the optimization problem, the absolute values of branch currents and node voltages can be obtained, accordingly.

Optimal distribution network operation tools have become even more relevant with the increased integration of DG, distributed storage and flexible loads. However, the non-convex and non-linear feature of the OPF problem is still a challenging issue, requiring high computational capacity, particularly in (near) real-time operation planning [21]. Hence, the distribution system operator needs the simplified OPF models for improved computational efficiency while preserving the accuracy and convergence speed for highly demanding utilization in practical applications [22].

In the literature, there are some research papers in the field of simplification of the ACOPF utilizing McCormick envelopes to strengthen the well-known second-order cone (SOC) relaxation of the ACOPF problem. In [23] , the authors applied the McCormick envelopes to tightening the bounds of voltage at the slack bus. The effectiveness of such tightening approach has been demonstrated on a real distribution network. It was concluded that the stronger relaxations enable more efficient global solution of the ACOPF problem and can improve computational efficiency of mixed-integer non-linear programming (MINLP) problems with AC power flow constraints. In [24], the convex model of the OPF problem utilizing trilinear monomials techniques have been presented. To relax the polar voltage coordinates, different approached have been studied. The functionality of the recursive McCormick approach has been compared with other techniques to show the effectiveness of them for OPF problems.

In this paper, a new Quadratic Constrained Programming (QCP) model for solving the OPF problem is developed using McCormick envelopes to convert the non-convex feature of the original OPF model into a convex QCP problem. The proposed model improves the solution efficiency while preserving the accuracy of the model. Therefore, it is appropriate for both planning and operation problems embedding the OPF studies.

The rest of this paper is presented as follows: the generalized OPF problem formulation is given in Section II. The principles of the McCormick relaxation technique for nonlinear and non-convex constraints are introduced in Section III. Then, the OPF problem is reformulated by using the SOCP concept in Section IV with the corresponding quadratic constraints. The McCormick envelopes extracted for the branch flow are provided in this section as well as the minimum and maximum boundary estimation for the decision variables. Section V provides the simulation results and the main conclusions of this paper are addressed in Section VI.

II. DISTRIBUTION OPTIMAL POWER FLOW

The generalized mathematical formulation of the OPF problem is provided in this section. The objective function can be defined as the minimization of total power generation cost, while in networks supplied from one end, which is normally the case of traditional distribution networks, the main objective function has been defined as the minimization of the total power losses. Note both objectives will result in identical solutions for networks supplied from one end.

The OPF problem with the aim of minimization of the power loss is defined as follows:

$$Min \sum_{i\in\Omega^b} \left[PG_i - PD_i \right] \tag{1}$$

subject to:

$$PG_i - \sum_{l\in\Omega^l} P'_l - PD_i = 0 \quad \forall i \in \Omega^b \tag{2}$$

$$QG_i - \sum_{l\in\Omega^l} Q'_l - QD_i = 0 \quad \forall i \in \Omega^b \tag{3}$$

$$P'_l + jQ'_l = V_i \sum_{j\in\Omega^b} Y_l^* V_j^* \quad \forall l \in \Omega^l, i \neq j \tag{4}$$

$$P'_l = G_l \left[V_i^2 - V_i V_j \cos(\delta_i - \delta_j) \right] + B_l V_i V_j \sin(\delta_i - \delta_j) \tag{5}$$

$$Q'_l = B_l \left[V_i^2 - V_i V_j \cos(\delta_i - \delta_j) \right] - G_l V_i V_j \sin(\delta_i - \delta_j) \tag{6}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{7}$$

$$PG_i^{\min} \le PG_i \le PG_i^{\max} \tag{8}$$

$$QG_i^{\min} \le QG_i \le QG_i^{\max} \tag{9}$$

$$\sqrt{(P_l')^2 + (Q_l')^2} \le S_l^{\max} \tag{10}$$

where $G_l=\frac{R_l}{Z_l^2}$, $B_l=\frac{X_l}{Z_l^2}$, $Z_l^2=R_l^2+X_l^2$ and the net injected active and reactive powers are defined as $P_l'=P_l+P_{loss}$ and $Q_l'=Q_l+Q_{loss}$, respectively. The set of distribution lines and buses are assumed as Ω^l and Ω^b, respectively. The active and reactive power flows of line l, are modeled as P'_l and Q'_l respectively, while R_l and X_l are the resistance and the reactance of line l, respectively. PG_i, QG_i, PD_i, and QD_i are active and reactive power of generating units and load demand at bus i, respectively.

The objective function is defined in (1) to represent the total network loss, while the only generating unit is the upstream grid in traditional OPF, however, it can be extended in the presence of the other generating units installed with the same concept and for the sake of simplicity. The net active and reactive power injection at each bus are presented in Eq. (2) and (3), respectively. Eq. (4) is the generalized power flow equation while Eqs. (5) and (6) are the active and reactive power flows of the branches of the network. The node voltage bounds are modeled in (7) and the minimum and maximum acceptable power generations are expressed in (8) and (9) for active and reactive power generation. Finally, the apparent power capacity of the feeders is modeled in (10). It should be noticed that in the provided model, the power flow of the feeders includes the power losses in the branches as well. It should be noted that the current model considers the generalized representation of the distribution networks. In the presence of shunt compensators such as capacitor banks and static var compensators, the new reactive power injected by such devices can be applied at each bus. For the sake of simplicity, in the current model, the shunt injections have not been addressed in this paper. In addition, the application of online tap changing is not modeled in the current problem formulation.

III. The McCormick Envelopes for Relaxing Non-convex Constraints

One of the efficient methods to tackle bilinear non-linear programming models is McCormick envelopes. Usually, such envelopes would be utilized to handle MINLP problems by relaxing the original MINLP problem into a convex NLP. Tackling the resulting convex NLP would lead to a lower bound on the original MINLP problem's optimal solution.

In general, handling non-convexities in optimization problems would be very sophisticated. In this respect, the non-convex term should be first converted into a convex one through parameter relaxation on the problem. By applying some relaxations on the bounds using convex relaxation, the problem would be more computationally efficient.

Note that the solution obtained from the relaxed problem may not necessarily relate to the original problem any longer. However, the solution derived from the convex relaxed problem specifies a lower bound, pertaining to the optimal solution. In this regard, any convex tighter relaxation will lead to a lower bound approaching the solution. Hence, relaxing the problem by a convex technique associated with the tightest bounds would be of great significance.

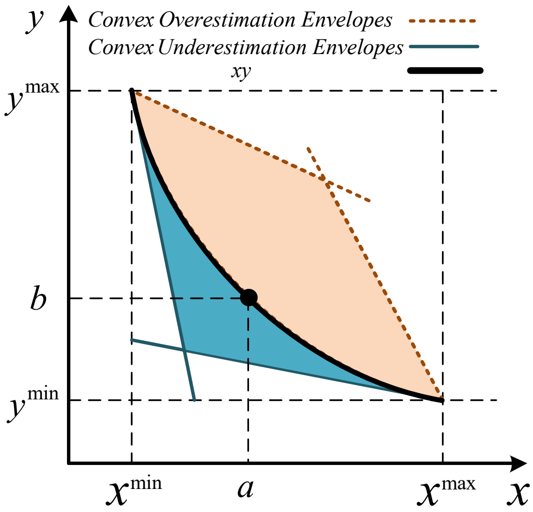

Fig. 1. Convex relaxation of *xy* using McCormick envelopes

A very well-known and effective relaxation method is McCormick envelopes, ensuring that the resulting relaxed problem is convex with abundantly tight bounds. By converting the original non-convex optimization problem into a convex one by using the McCormick envelopes, the shortfall due to the existence of multiple local minima, mistakenly chosen by the solver as the global solution, would be overcome. Accordingly, the minimum value of the objective function of the relaxed model is the global minimum. On the other hand, tackling the primary non-convex optimization problem by applying the solution of the relaxed model and evaluating the feasibility gives an upper bound. By using the McCormick envelopes, an envelope will be obtained preserving the convexity, besides mitigating the dimension of the resulting feasible space. Consequently, the derived solution for the lower bound by applying the envelopes will approach the optimal solution. It is noteworthy that utilizing tighter bounds gives the solution in a significantly lower time. In case the lower and upper bounds are not available, estimation should be employed to obtain their values. Elimination of the objective function and maximization/minimization of the variable while considering the constraints of the non-convex optimization problem would be an efficient method to determine the lower and upper bounds of the variable. It is worth mentioning that applying such values leads to relaxing the problem in a tighter manner while including the optimal solution.

In the next section, the OPF problem in distribution networks is represented as a QCP where one of the constraints can be relaxed using McCormick envelopes.

$$\omega = xy \tag{11}$$

$$x^{\min} \le x \le x^{\max}, \quad y^{\min} \le y \le y^{\max} \tag{12}$$

The underestimators of the function are represented by:

$$a=(x-x^{\min}), \qquad b=(y-y^{\min}) \tag{13a}$$

$$ab \ge 0 \rightarrow (x-x^{\min})(y-y^{\min}) \ge 0 \tag{13b}$$

$$\omega \ge x^{\min}y + xy^{\min} - x^{\min}y^{\min} \tag{13c}$$

$$a=(x^{\max}-x), \qquad b=(y^{\max}-y) \tag{14a}$$

$$ab \ge 0 \rightarrow (x^{\max}-x)(y^{\max}-y) \ge 0 \tag{14b}$$

$$\omega \ge x^{\max}y + xy^{\max} - x^{\max}y^{\max} \tag{14c}$$

The overestimators of the function are represented by:

$$a=(x^{\max}-x), \qquad b=(y-y^{\min}) \tag{15a}$$

$$ab\geq 0 \rightarrow (x^{\max}-x)(y-y^{\min})\geq 0 \tag{15b}$$

$$\omega \leq x^{\max}y+xy^{\min}-x^{\max}y^{\min} \tag{15c}$$

$$a=(x-x^{\min}), \qquad b=(y^{\max}-y) \tag{16a}$$

$$ab\geq 0 \rightarrow (x-x^{\min})(y^{\max}-y)\geq 0 \tag{16b}$$

$$\omega \leq xy^{\max}+x^{\min}y-x^{\min}y^{\max} \tag{16c}$$

IV. QUADRATIC CONSTRAINED OPTIMIZATION MODEL OF OPF

The OPF problem is represented as a QCP model in subsection *A*. This problem has a nonlinear and nonconvex constraint that needs to be relaxed by using proper approaches. The McCormick relaxation and its corresponding constraints are provided in subsection *B*. The boundary limits determinations for the McCormick envelopes are addressed in subsection *C*.

A. Nonlinear Quadratic Model of OPF Problem

The mathematical problem formulation using the QCP approach is as follows:

$$Min \sum_{i\in\Omega^b} f\left(PG_i\right) \tag{17}$$

subject to:

$$PG_i-\sum_{l\in\Omega^l} A_{il}P_l-\sum_{l\in\Omega^l} A_{il}R_l\left|I_l\right|^2=PD_i \qquad \forall i\in\Omega^b \tag{18}$$

$$QG_i-\sum_{l\in\Omega^l} A_{il}Q_l-\sum_{l\in\Omega^l} A_{il}X_l\left|I_l\right|^2=QD_i \qquad \forall i\in\Omega^b \tag{19}$$

$$\left|V_i\right|^2-\left|V_j\right|^2=2\left[R_lP_l+X_lQ_l\right]+Z_l^2\left|I_l\right|^2 \qquad \forall i,j\in\Omega^b \tag{20}$$

$$P_l^{Loss}=R_l\left|I_l\right|^2 \tag{21}$$

$$Q_l^{Loss}=X_l\left|I_l\right|^2 \tag{22}$$

$$\left|V_i^{\min}\right|^2\leq\left|V_i\right|^2\leq\left|V_i^{\max}\right|^2 \qquad \forall i\in\Omega^b \tag{23}$$

$$\left|I_l^{\min}\right|^2\leq\left|I_l\right|^2\leq\left|I_l^{\max}\right|^2 \qquad \forall l\in\Omega^l \tag{24}$$

$$\left|V_j\right|^2\left|I_l\right|^2=P_l^2+Q_l^2 \qquad \forall j\in\Omega^b, l\in\Omega^l, l:i\rightarrow j \tag{25}$$

where $|I_l|$ is the current magnitude of line *l* and $|V_j|$ is the voltage magnitude of the end bus *j* in this model. It should be noted that A_{il} is the node to branch incidence matrix of the distribution network. The generalized operation cost of generating units is considered as the cost function in this study (17). The cost function $f(PG_i)$ can be represented as a linear or piecewise linear term. In quadratic programming, the cost function can be represented as a quadratic function of power generation. Equations (18) and (19) express the nodal active and reactive power balances according to Kirchhoff's current law, KCL. Equation (20) describes the net summation of voltage drops of all lines in a planar loop, which has to be equal to zero according to Kirchhoff's voltage law, KVL. The active and reactive power losses are expressed in (21) and (22), respectively. In the traditional distribution networks supplied at one end, i.e. from the upstream network, the total power losses can be considered the main objective function. However, the current problem formulation is the generalized representation of the OPF in the distribution network operation. Equations (23) and (24) deal with the limits of quadratic node voltages and branch currents, respectively.

With such representation and considering the $|V_i^2|$ and $|I_l^2|$ as the decision variables of the OPF problem, the model can be represented as the standard QCP model, while the only quadratic constraint is the active and reactive power transmitted through the feeders, i.e. Eq (25). The left-hand side (LHS) of the mentioned equation is a non-convex and non-linear function. Therefore, it is needed to be relaxed by using a proper technique. In this paper, the McCormick envelopes method has been applied to this specific equation.

B. McCormick Relaxation of Non-linear Constraint

As mentioned in the previous subsection, the active and reactive power transmitted through the distribution feeders would be expressed as a nonlinear constraint. Therefore, the McCormick relaxation can be applied to convert this term by a set of linear and convex equations. With the selection of the associated decision variable representing the behavior of the LHS of Eq. (25), the McCormick envelopes can be expressed as:

$$\omega_{jl}\geq P_l^2+Q_l^2 \qquad \forall j\in\Omega^b, l\in\Omega^l \tag{26}$$

where $\omega_{jl}\triangleq\left|V_j\right|^2|I_l|^2$, and as the decision variables of the QCP are supposed to be $x=\left|V_j\right|^2$ and $y=|I_l|^2$, the McCormick envelopes are as follows:

$$\omega_{jl}\geq\left|V_j^{\min}\right|^2\left|I_l\right|^2+\left|V_j\right|^2\left|I_l^{\min}\right|^2-\left|V_j^{\min}\right|^2\left|I_l^{\min}\right|^2 \tag{27}$$

$$\omega_{jl}\geq\left|V_j^{\max}\right|^2\left|I_l\right|^2+\left|V_j\right|^2\left|I_l^{\max}\right|^2-\left|V_j^{\max}\right|^2\left|I_l^{\max}\right|^2 \tag{28}$$

$$\omega_{jl}\leq\left|V_j^{\max}\right|^2\left|I_l\right|^2+\left|V_j\right|^2\left|I_l^{\min}\right|^2-\left|V_j^{\max}\right|^2\left|I_l^{\min}\right|^2 \tag{29}$$

$$\omega_{jl}\leq\left|V_j\right|^2\left|I_l^{\max}\right|^2+\left|V_j^{\min}\right|^2\left|I_l\right|^2-\left|V_j^{\min}\right|^2\left|I_l^{\max}\right|^2 \tag{30}$$

where the superscripts *min* and *max* are corresponding to the minimum and maximum levels of the decision variables. Eq (27) and (28) are the function's underestimators while Eq. (29) and (30) are the function's overestimators. The functionality of the McCormick envelopes is dependent upon the proper selection of the minimum and the maximum acceptable ranges of each variable.

C. Boundary Determination of the Non-linear Constraint

The proper selection of the minimum and maximum bounds of the decision variables of McCormick relaxation is the key point to guarantee the functionality of the model and then, the optimality of the solution. Regarding the voltage, the minimum and maximum acceptable bounds are determined by the system operator as a hard constraint. A rough estimation method to determine the minimum current of each branch is using the lossless power flow results in which the active and reactive power losses are ignored. By applying such an assumption, the problem should be solved by using the lossless power flow, while considering the voltage drops in Eq. (20). To have a tighter relaxation, the updated maximum voltage level at each node from the lossless power flow model can be used in the second phase, i.e. the QCP with McCormick envelopes.

V. SIMULATION RESULTS

In this section, the functionality of the proposed QCP with McCormick relaxation approach in solving the OPF problem for distribution networks is studied. The standard IEEE 33-bus test system is selected and the obtained results are compared with the optimal solutions achieved by MATPOWER. The single-line diagram of the test system is depicted in Fig. 2. This network has four laterals and it is served by the upstream network connected to bus 1. Table I presents the nodal active and reactive powers of this network. The total active and reactive power demands are 3.715 MW and 2.300 MVAr, respectively. The minimum voltage occurs at bus 18 since it is located at the end of the feeder. In this case, the voltage of bus 1 is set to 1 p.u. for all case studies, and the minimum and maximum acceptable ranges for node voltages are supposed to be 0.9 and 1.1 p.u., respectively.

By considering the MATPOWER results as the reference, it can be seen that the obtained results for the OPF utilizing the proposed method have an error of less than 0.05%, which means that the results are accurate enough. On the other hand, the simulation time is less than the other non-linear approaches with reasonable accuracy. It should be noted that the simulation time is 0.11 seconds for the QCP including 0.02 seconds for the first stage, i.e. the lossless OPF, and 0.09 seconds for the McCormick relaxation used in the second stage. Fig. 3 illustrates the voltage profile obtained by utilizing the QCP method developed in this study.

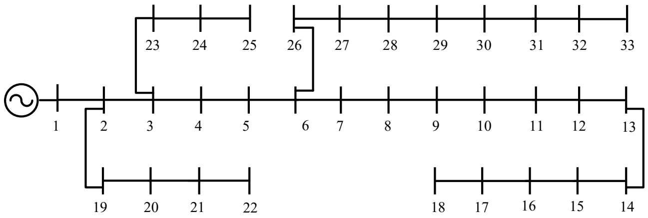

Fig. 2. Single-line diagram of IEEE 33-bus network

TABLE I. ACTIVE AND REACTIVE DEMAND OF IEEE 33-BUS

Node	P (kW)	Q (kVAr)	Node	P (kW)	Q (kVAr)
Bus 2	100	60	**Bus 18**	90	40
Bus 3	90	40	**Bus 19**	90	40
Bus 4	120	80	**Bus 20**	90	40
Bus 5	60	30	**Bus 21**	90	40
Bus 6	60	20	**Bus 22**	90	40
Bus 7	200	100	**Bus 23**	90	50
Bus 8	200	100	**Bus 24**	420	200
Bus 9	60	20	**Bus 25**	420	200
Bus 10	60	20	**Bus 26**	60	25
Bus 11	45	30	**Bus 27**	60	25
Bus 12	60	35	**Bus 28**	60	20
Bus 13	60	35	**Bus 29**	120	70
Bus 14	120	80	**Bus 30**	200	600
Bus 15	60	10	**Bus 31**	150	70
Bus 16	60	20	**Bus 32**	210	100
Bus 17	60	20	**Bus 33**	60	40

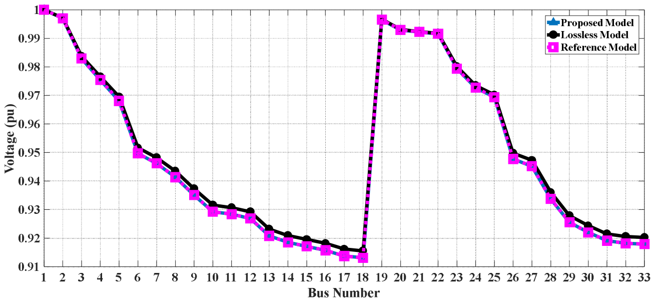

Fig. 3. Voltage profile for the first and second stage of the proposed convex QCP and the reference values obtained from MATPOWER

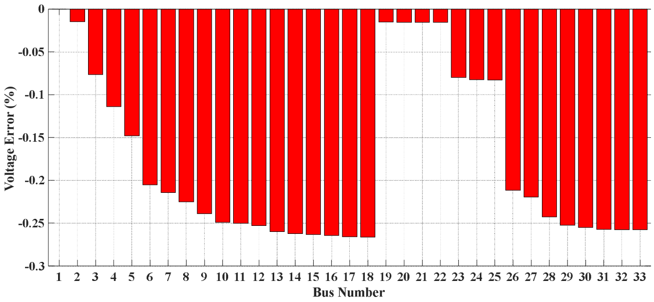

Fig. 4. Voltage error of the convex QCP results of OPF (Reference Values: MATPOWER)

TABLE II. COMPARATIVE SIMULATION RESULTS

Solver	MATPOWER (MATLAB)	CPLEX (Python)	DICOPT (GAMS)	QCP (GAMS)
Power Loss (kW)	202.0766	202.0681	202.0748	201.9851
Error (%)	0.0000	0.0042	8.9E-4	0.0453
Time (Sec.)	0.5110	0.3117	0.1204	0.110
V^{min} (pu)	0.913	0.913	0.913	0.913

As it can be seen, the proposed model and the reference model, i.e. the results obtained from the MATPOWER are almost identical. The lossless results reported in this figure are corresponding to the first stage of the proposed model for estimating the minimum and maximum bounds of the McCormick envelopes.

Fig. 4 illustrates the voltage errors of the proposed model for solving the OPF problem as a convex QCP problem. In this case, as it can be observed, the voltage of bus 1 is set to 1 p.u. for all case studies. Table II provides the simulation results reported in the literature, as well as the results obtained by applying the proposed model.

VI. CONCLUSION

In this paper, a new QCP model to tackle the OPF problem was proposed. To reduce the computational burden of the nonlinear constraints, the McCormick relaxation approach has been utilized to convert nonlinear terms to linear and convex constraints. The proposed problem formulation and the procedure for the tight boundary estimation have been introduced in the paper for a specific case study. The obtained results confirmed that the proposed model was accurate and fast enough in terms of the convergence time and precision of results. The obtained results of other standard solvers have been compared to the obtained results of the QCP with convex constraints in this paper. The future works to the current study can be extended in the following areas:

a) Developing the proposed model for multi-temporal OPF problems in the presence of volatile DGs.
b) Incorporating the proposed OPF model in the complex operational problems at the distribution level as a fast, reliable, and low computational burden sub-problem to check the feasibility and optimality studies.
c) Augmenting the model by adding electrical energy storage units for day-ahead or intraday markets.

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