

# A Gaussian Window for Interference Mitigation in Ka-band Digital Beamforming Systems

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**Abstract**—This paper proposes the use of a Gaussian window on the array factor as an interference mitigation method, aiming to avoid the computational complexity of the MVDR algorithm at the cost of a slight performance reduction. We show that by optimizing the parameters of the Gaussian window, it is possible to effectively mitigate the interfering signal if it is received within a certain angular range from the desired signal, while being still effective beyond that range. Finally, we show that the effectiveness of this approach is maintained across the full frequency reception range of the Ka-band, and confirm its validity using  $8 \times 8$  and  $16 \times 16$  array sizes.

**Index Terms**—Antenna arrays, array pattern, digital beamforming, phased arrays, uniform rectangular arrays, windowing

## I. INTRODUCTION

The past few decades have been marked by a continuous development of wireless communications, offering tremendous possibilities for integrating technologies that were once offline. Moreover, the economic, social, and political impact of human connectivity technologies worldwide is undeniable today.

Therefore, global broadband coverage using satellite constellations has seen several initiatives and investments, many of them working at the Ka-band, which have a frequency range in the downlink of 17.7 GHz to 21.2 GHz and in the uplink of 27.5 GHz to 31.0 GHz [1]. However, due to the directive nature of antennas, solutions are needed to allow for efficient communications with moving satellites. Besides, the use of millimeter-wave Ka-band for these applications makes it very susceptible to interference and channel-fading.

In this context, electronic beam scanning using phased array antennas has been pointed as a good solution for wireless communications [2]. Phased array antennas consist of multiple radiating elements that collectively form a beam in the far field of the antenna, hence, the designation of beamforming. The use of several elements allows the beam to be steered or shaped electronically by providing the appropriate amplitude and phase excitation at each of the radiating elements [3]. Digital beam forming (DBF) consists in performing digital amplitude and phase control directly at baseband, which requires an RF front-end and data converter per antenna element [4]. Despite this apparent complexity, which is overcome by the recent advances

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in silicon processes, DBF can be used to generate or receive multiple beams at the same time, to handle wideband signals without beam squint effects as well as to rapidly reconfigure beam shapes and pointing directions, possibly in real time [4].

Several tasks are involved in the beamforming process. In the receiver side, the main ones are: (i) the angle-of-arrival (AOA) estimation, that filters the power spectrum to identify the angles where the received signals better correlate with the expected ones; and, after knowing the AOA, (ii) the computation of weights for each antenna array element, which defines the steering angle of the array and also sets minima on the direction of interferers to mitigate their effects.

Considering the previous knowledge of the AOA, there are several potential solutions to calculate the weights of the array in the beamforming process. The most basic way is to use the array factor to define the direction of the beam to be generated or received by the antenna array [3]. The array factor is a function of the geometry of the array, the position of the elements and the phase of each element. While this creates relevant sidelobes on the array radiation pattern, windowing methods can be used to mitigate their effect by applying different amplitudes to the elements of the array [3]. By reducing the sidelobe level of the radiation pattern, the window also facilitates the spatial filtering of the interferers, a feature which will be further explored in this paper.

The Capon method, commonly called Minimum Variance Distortionless Response (MVDR), is a correlation based beamforming process. It takes in consideration information about the incoming signal and interferers to generate a correlation matrix, which can be used to maximize the signal at the desired direction, while placing nulls in the direction of known interferers. This allows for a higher resolution but also comes with a much higher computational cost [5]. The MVDR algorithm depends on the inverse of the correlation matrix, which takes much longer to be processed, both in software and hardware implementations.

Several solutions have been proposed in the literature to reduce the computational burden of the beamforming process while preserving the achieved resolution and minimizing the interferers. Some of them use hardware optimizations to accelerate the MVDR processing efficiency through pipelining and mathematical strategies to avoid the matrix inversion [6], [7],

while others propose the use of simpler methods considering the trade-off between the quality of results and computational cost [8].

This paper proposes the use of a Gaussian window on the array factor as an interference mitigation method, that avoids the computational complexity of the MVDR method at the cost of a slight performance reduction. In particular, we study the reception of two signals in the Ka-band, a desired signal at boresight and an interferer at an arbitrary direction (from boresight up to 90°). We show that by optimizing the parameters of the gaussian window, it is possible to effectively mitigate the interfering signal if it is received within a certain angular range. We analyze the Q<sup>2</sup>-factor performance metric as a function of the angle between the signal and the interferer, and confirm the validity of this approach using 8 × 8 and 16 × 16 array sizes. Furthermore, we show that the effectiveness of this approach is maintained across the full frequency reception range of the Ka-band.

The rest of this paper is organized as follows. Section II presents some background information about the beamforming theory and Section III presents the proposed approach. In Section IV we discuss the obtained results, and finally, Section V presents our conclusions about the work.

## II. BEAMFORMING ARRAY THEORY

Consider that two signals are arriving from two directions  $(\theta_0, \phi_0)$  and are received by a uniform rectangular array (URA) composed of  $M \times N$  antenna elements. The geometry of an URA is depicted in Fig. 1. The URA is in the  $xy$ -plane such that the boresight direction is the positive  $z$ -axis. The array elements are equally spaced and have equal amplitudes. The directions are composed of azimuth angle  $\phi \in [0, 2\pi]$  and elevation angle  $\theta \in [0, \pi/2]$ .

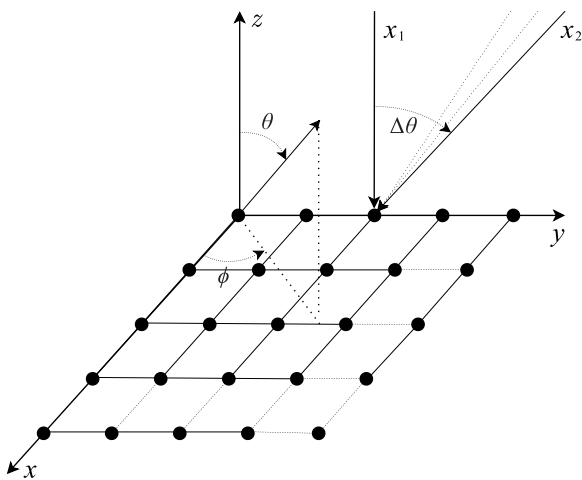


Fig. 1. Geometry of an  $M \times N$  rectangular planar array, where  $x_1$  and  $x_2$  represent the received signals,  $\phi$  and  $\theta$  are the azimuth and elevation angles, respectively, and  $\Delta\theta$  is the elevation angle between two received signals.

The received signals are given by

$$\mathbf{x}(k) = \mathbf{A}s(k) + \mathbf{n}(k), \quad (1)$$

where  $\mathbf{A}$  is the matrix of steering vectors,  $s(k)$  is the vector of incident signals,  $\mathbf{n}(k)$  is the noise vector at each element with zero mean and variance  $\sigma_n^2$ , and  $k$  represents the time sample.

The steering vectors containing the response of the array elements are calculated as follows

$$\mathbf{a}(\theta_0, \phi_0) = e^{j(kd_x m \sin(\theta_0) \cos(\phi_0) + kd_y n \sin(\theta_0) \sin(\phi_0))}, \quad (2)$$

where  $k = 2\pi/\lambda$  is the wavenumber;  $d_x$  and  $d_y$  represent the element spacing in the  $x$ - and  $y$ -directions, respectively, given in multiples of the center wavelength  $\lambda_c$ ;  $m = [0, \dots, M - 1]$  and  $n = [0, \dots, N - 1]$  represent the elements in the  $x$ - and  $y$ -directions, respectively [3].

Thus, the matrix of steering vectors is obtained as

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1) \mathbf{a}(\theta_2, \phi_2)]. \quad (3)$$

The array correlation matrix [3] is given by the approximate time-averaged correlation

$$\hat{\mathbf{R}} \approx \frac{1}{K} \sum_{k=1}^K \mathbf{x}(k)\mathbf{x}^H(k). \quad (4)$$

The minimum variance optimum weights [3] can be calculated as follows

$$\mathbf{w} = \frac{\mathbf{R}_u^{-1} \mathbf{a}(\theta_0, \phi_0)}{\mathbf{a}(\theta_0, \phi_0) \mathbf{R}_u^{-1} \mathbf{a}(\theta_0, \phi_0)} \quad (5)$$

where  $\mathbf{R}_u$  is the undesired correlation matrix accounting for the correlation matrices for interferers and noise and  $\mathbf{a}_0$  is the array steering vector containing the directions of the received signals. For simplification purposes, it is assumed that the angles of arrival of the two signals are known.

Then, the weighted array output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \quad (6)$$

where  $(\cdot)^H$  denotes the Hermitian transpose.

For the sake of simplicity, throughout this work, the element spacing is  $d_x = d_y = \lambda_c/2$  and the antenna arrays consist of isotropic antenna elements, which allowed us to focus on the proposed solution.

Fig. 2 shows the performance when no MVDR beamforming algorithm is employed, i.e., when the algorithm is not trying to cancel the interferer, for the two different array sizes.

Also, the performance was obtained at the center frequency  $f_c = 18.9$  GHz, as well as two other frequencies corresponding to the limits of the downlink Ka-band,  $f = 17.7$  GHz and  $f = 20.2$  GHz. The performance oscillation is due to the maxima and minima of the sidelobes of the array factor, which also depends on the frequency of the signal. The performance difference at the limits of the band is due a small change in the wavelength in the wavenumber  $k$ , since  $\lambda \neq \lambda_c$ .

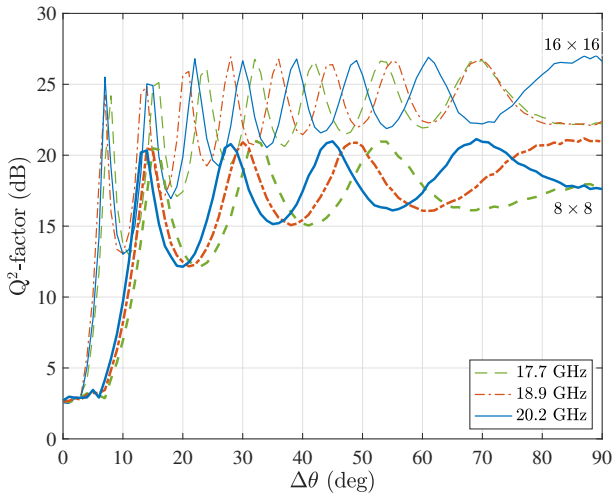


Fig. 2. Performance of the 8 × 8 and 16 × 16 arrays when no MVDR is employed, at different frequencies.

Moreover, the impact of noise was also assessed. Fig. 3 shows the performance obtained at  $f_c = 18.9$  GHz for the 8 × 8 URA, showing the effect of different noise levels, when MVDR is employed and when no MVDR is employed. This result shows that when no MVDR is employed the performance varies with  $\Delta\theta$ . Besides, it can be observed that the maxima are limited by the same performance obtained using the MVDR algorithm. This occurs at specific angles that cannot be controlled and depend on the signal frequency as seen above. It is clear that increasing the noise level leads to a reduction of the performance oscillation when no MVDR is employed, but at the cost of a performance decrease, since the performance is limited by the noise. In the analysis carried out in this work, it was assumed a noise variance  $\sigma_n^2 = 1$ .

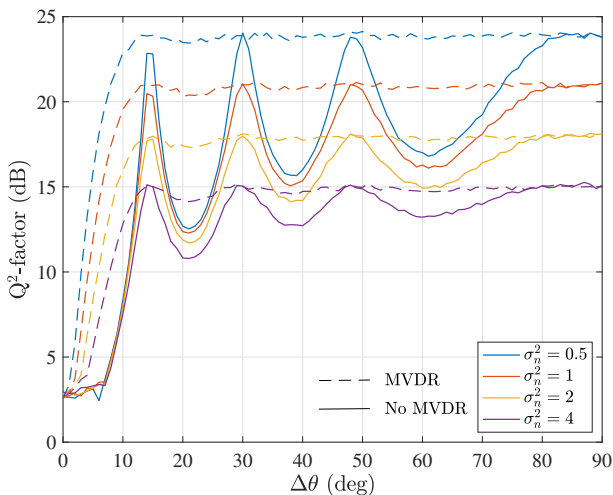


Fig. 3. Performance for different noise levels ( $\sigma_n^2 = 0.5, 1, 2, 4$ ), with and without MVDR, at  $f_c = 18.9$  GHz, for the 8 × 8 URA.

### III. PROPOSED APPROACH

To avoid the computational complexity imposed by the calculation of the inverse of the correlation matrix, we propose and evaluate an alternative method to the MVDR beamforming algorithm. In this method, the beamforming weights are calculated simply resorting to the steering vectors of  $x_1$ , i.e.,  $w = \mathbf{a}(\theta_s, \phi_s)$ , and then use a Gaussian window to mitigate the effect of the interferer.

The main advantage of using a window is that the weights can be pre-calculated and used directly on the signal, depending only on the window configuration. The implementation of the window would mean one multiplication per antenna element. Considering that each multiplication has  $O(1)$  computational complexity, for all elements it would have complexity  $O(n)$ . On the other hand, the MVDR method is data-dependent, therefore, it needs techniques to calculate and invert the correlation matrix, as seen in eq. (5). Using the same metric, the inversion of the matrix would have complexity  $O(n^3)$ , considering the Gauss–Jordan elimination method.

The coefficients of a Gaussian window are calculated as follows

$$w(n) = e^{-n^2/2\sigma^2} \tag{7}$$

where  $\sigma = (L-1)/(2\alpha)$  is the standard deviation of a Gaussian random variable,  $L$  is the window length, and  $\alpha$  is the width factor, which is inversely proportional to the width of the window [9].

Thus, the window width can be varied according to  $\alpha$ , which in turn can be used to control the angle of the first null. The first null as a function of  $\alpha$  is shown in Fig. 4 and can be approximated by the polynomial fittings shown in the same figure, for the two array sizes.

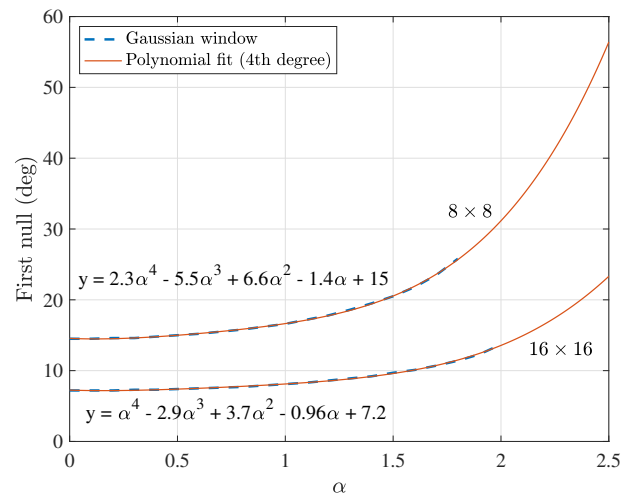


Fig. 4. First null in degrees versus  $\alpha$ , for the 8 × 8 and 16 × 16 URA, and respective polynomial approximations.

If the direction of the interferer is known and lies in the range of  $\theta_{\text{null}} = [14 - 25]^\circ$  and  $\theta_{\text{null}} = [7 - 13]^\circ$ , regarding the

$8 \times 8$  and  $16 \times 16$  URAs, respectively, it can be mitigated by obtaining the weights of a Gaussian window calculated for the optimal  $\alpha$  that provides a null in the direction of the interferer.

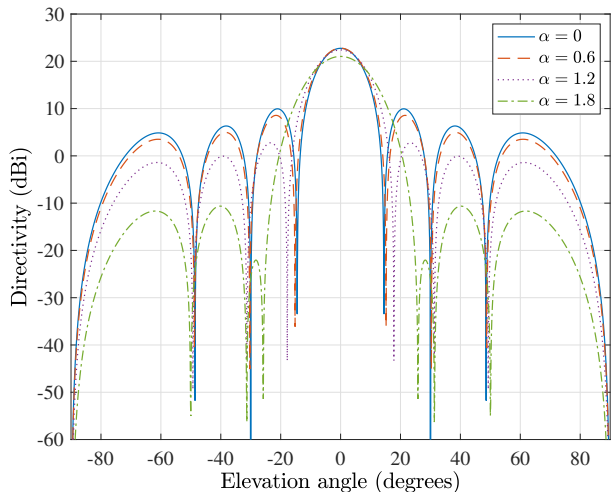


Fig. 5. Directivity (elevation cut, azimuth angle =  $0.0^\circ$ ) for the  $8 \times 8$  URA.

Fig. 5 shows the directivity of the  $8 \times 8$  URA when applying a Gaussian window, for different values of  $\alpha$ . It can be seen that applying a Gaussian window greatly reduces the sidelobes, but at the cost of a larger beamwidth and a gain reduction, in particular for  $\alpha = 1.8$ . In this work, we assume a scenario where two signals,  $x_1$  and  $x_2$ , are being received by an URA, where  $x_1$  is the desired signal and  $x_2$  is an interferer, i.e., undesired signal. In this scenario, the direction of  $x_1$  is fixed at  $(\theta_1, \phi_1) = (0^\circ, 90^\circ)$ , while the interferer is at  $(\theta_2, \phi_2) = (\Delta\theta, 90^\circ)$ . This allowed to evaluate the impact of the proximity of the two signals by varying the direction of the interferer from  $\Delta\theta = 0^\circ$  up to  $\Delta\theta = 90^\circ$ .

In these assessments, 4-QAM signals were transmitted, and the quality of the received signals was evaluated for two different array sizes:  $8 \times 8$  and  $16 \times 16$ . The performance metric used was the  $Q^2$ -factor following a definition for quadrature amplitude modulation (QAM) signals [10].

#### IV. RESULTS

To assess the proposed method, we analyzed the performance of the optimal  $\alpha$  calculated as described in Section III, as well as the effect of the Gaussian window for different values of the  $\alpha$  factor. For comparison, the performance of the MVDR algorithm is also provided.

Fig. 6 shows the performance of the Gaussian window for  $\alpha = 0.6, 1.2$  and  $1.8$ , as well as  $\alpha = 0$  (no window) for the  $8 \times 8$  URA, at  $f_c = 18.9$  GHz. The results show that, if the interferer is within the  $\Delta\theta$  range discussed in the previous section, the optimal  $\alpha$  can null the interferer, providing a performance close to that of the MVDR algorithm.

Besides, the performance provided by the optimal  $\alpha$  is above or the same of the performance provided for the other  $\alpha$  values. For  $\Delta\theta$  values above that range, the optimal  $\alpha$  is the maximum

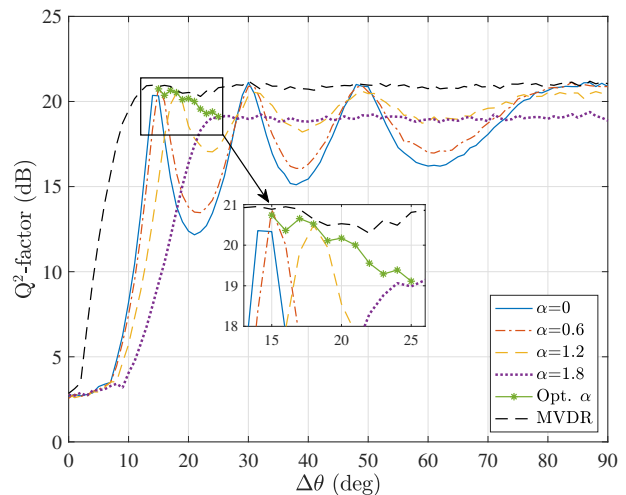


Fig. 6. Performance of the Gaussian window for  $\alpha = 0, 0.6, 1.2$  and  $1.8$ , optimal  $\alpha$ , and MVDR, at  $f_c = 18.9$  GHz, for the  $8 \times 8$  URA. The inset highlights the performance of using the optimal  $\alpha$ , which lies between the MVDR and the other  $\alpha$  values.

considered  $\alpha$  in Fig. 4 ( $\alpha = 1.8$  and  $\alpha = 2.0$  for the  $8 \times 8$  and  $16 \times 16$  arrays, respectively), since it provides an approximately constant performance for higher  $\Delta\theta$  angles, with a penalty of  $\sim 2$  dB compared to that of the MVDR. This results from the previously observed reduction of the directivity of the main beam with the increase of  $\alpha$  and the system becoming noise limited rather than interference limited. We also observe a small increase on lowest acceptable  $\Delta\theta$  of the interferer which is a fundamental limitation of the pattern resulting from the array factor (without windowing). In fact, the MVDR algorithm can null the interferer for smaller  $\Delta\theta$ , however at the cost of increased computational complexity. Fig. 7 shows a similar outcome for the  $16 \times 16$  URA.

Fig. 8 and Fig. 9 show the performance obtained for the frequencies at the limits of the Ka-band, for the  $8 \times 8$  and  $16 \times 16$  URAs, respectively. These results show that the proposed approach is effective within the operating frequency range since it mitigates the variability and degradation of performance resulting from the different frequencies of the signal.

#### V. CONCLUSIONS

We have proposed and evaluated a Gaussian window as an alternative to the MVDR method for the mitigation of interfering signals in Ka-band digital beamforming systems. We have concluded that an optimal value of  $\alpha$  can be chosen to cancel the interfering signal when it exists in the range  $\theta_{\text{null}} = [14 - 25]^\circ$  and  $\theta_{\text{null}} = [7 - 13]^\circ$  for the  $8 \times 8$  and  $16 \times 16$  URAs, respectively, so that the performance practically matches that of the MVDR. However, there is an inevitable small increase on lowest acceptable  $\Delta\theta$  of the interferer which is a fundamental limitation of the pattern resulting from the array factor (without windowing). Additionally, we concluded that for an interferer at a  $\Delta\theta$  above that, the proposed method is still effective in mitigating this signal at the cost of slight

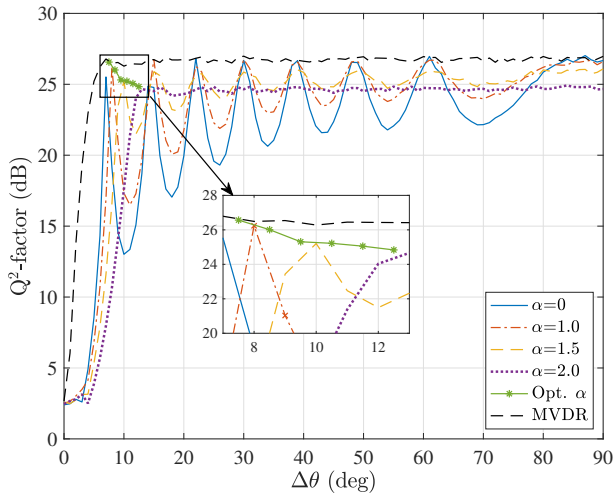


Fig. 7. Performance of the Gaussian window for  $\alpha = 0, 1.0, 1.5$  and  $2.0$ , optimal  $\alpha$ , and MVDR, at  $f_c = 18.9$  GHz, for the  $16 \times 16$  URA. The inset highlights the performance of using the optimal  $\alpha$ , which lies between the MVDR and the other  $\alpha$  values.

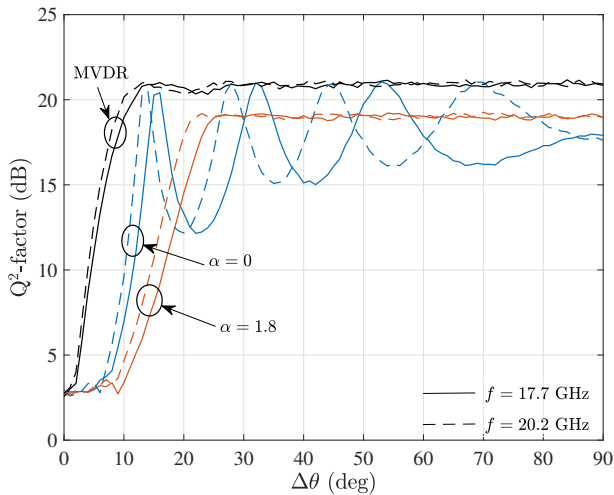


Fig. 8. Performance of MVDR and Gaussian window at the limits of the band, for the  $8 \times 8$  URA.

reduction in the  $Q^2$ -factor performance of approximately 2 dB which results from the reduction of the directivity of the main beam and system becoming noise limited. We have further concluded that this approach can also mitigate the variability and degradation of performance resulting from the different frequencies of the signal. Future work includes evaluating this approach under arbitrary reception angles for the desired signal, as well as the integration of angle-of-arrival algorithms in the analysis.

## REFERENCES

- [1] J. Christensen, "ITU Regulations for Ka-band Satellite Networks," in *30th AIAA International Communications Satellite System Conference (ICSSC)*, p. 15179, 2012.

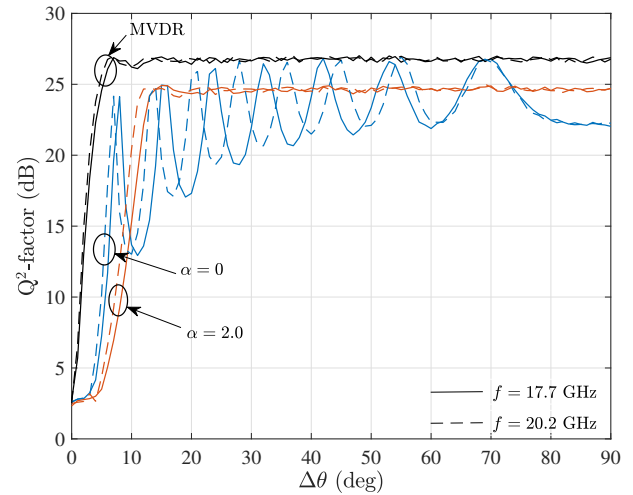


Fig. 9. Performance of MVDR and Gaussian window at the limits of the band, for the  $16 \times 16$  URA.

- [2] B. Rohrdantz, K. Kuhlmann, A. Stark, A. Geise, and A. F. Jacob, "Digital beamforming antenna array with polarisation multiplexing for mobile high-speed satellite terminals at Ka-band," *The Journal of Engineering*, vol. 2016, no. 6, pp. 180–188, 2016.
- [3] F. B. Gross, *Smart antennas with MATLAB*. McGraw-Hill Education, 2015.
- [4] D. Sikri and R. M. Jayasuriya, "Multi-beam phased array with full digital beamforming for SATCOM and 5G," *5G Phased Array Technologies*, p. 30, 2019.
- [5] S. N. Shahab, A. R. Zainun, H. A. Ali, M. Hojabri, and N. H. Noordin, "MVDR algorithm based linear antenna array performance assessment for adaptive beamforming application," *Journal of Engineering Science and Technology*, vol. 12, no. 5, pp. 1366–1385, 2017.
- [6] S. G. Sreejeesh, R. Sakthivel, and J. U. Kidav, "Superior Implementation of Accelerated QR Decomposition for Ultrasound Imaging," *IEEE Access*, vol. 8, pp. 156244–156260, 2020.
- [7] Y. Huang, M. Zhou, and S. A. Vorobyov, "MVDR Robust Adaptive Beamforming Design with Direction of Arrival and Generalized Similarity Constraints," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 4330–4334, 2019.
- [8] Q. Huang, M. Lin, J.-B. Wang, T. A. Tsiftsis, and J. Wang, "Energy Efficient Beamforming Schemes for Satellite-Aerial-Terrestrial Networks," *IEEE Transactions on Communications*, vol. 68, no. 6, pp. 3863–3875, 2020.
- [9] A. V. Oppenheim, J. R. Buck, and R. W. Schaffer, *Discrete-time signal processing. Vol. 2*. Upper Saddle River, NJ: Prentice Hall, 2001.
- [10] A. J. Lowery, L. B. Du, and J. Armstrong, "Performance of Optical OFDM in Ultralong-Haul WDM Lightwave Systems," *Journal of Lightwave Technology*, vol. 25, no. 1, pp. 131–138, 2007.