A Hybrid Approach to Investigating the Distributional Aspects Associated with Reliability System Indices

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***Abstract*—The power system analysis area has recently been challenged by the merge between the automotive and power industries and their combined objective to work towards a low-carbon electrified transport economy. In order to study the hourly effects on the reliability of power systems, this paper first proposes a discussion of the risk methodologies and then a novel way of examining the probability distributions of the reliability indices based on a hybrid approach. The effects of the substantial use of renewable energy combined with the integration of electric vehicles will be discussed in relation to the IEEE test systems and a planning configuration of the Portuguese Generation System.**

Keywords—Power system reliability, Wind power generation, Monte Carlo methods.

I. INTRODUCTION

Power industries have faced several new challenges over the last decades. The rapid growth of renewable energy shows that the environmental problems are serious. The integration of wind and solar power by different countries around the world has become a reality and the needs for significantly more sustainable solutions are, undoubtedly, a priority for the energy sector. In addition to the new renewable electricity producing mechanisms, the challenge seems to be for automotive and power industries to both work towards a low-carbon electrified transport economy [1].

Such a combination has consequences for the analysis of power systems, since increased amounts of renewable power sources are already causing an effective increase in the number of random variables and the operational complexities in the system. From a power system tools perspective, three different approaches have appeared in the literature to evaluate power systems in an attempt to find more comprehensive solutions: simulation approaches, analytical tools and/or hybrid methodologies. Undoubtedly, sequential Monte Carlo simulation (SMCS) is the most suitable tool for simulating chronological aspects. Differently from the analytical approaches, the SMCS is able not only to calculate the common reliability indices but also to investigate the distributional aspects associated with them. However, the computational burden of SMCS is often more substantial than other approaches [2]. Simulation methods are usually divided into SMCS, non-SMCS, pseudo-SMCS [3], [4] and Population-Based methods [5]. A suitable approach based on the combination of SMCS and Cross-Entropy method was recently proposed [6]. The only feature that this approach lacks in comparison to the normal SMCS is the ability to assess the probability distributions of the reliability indices [6]. Analytical calculations are usually based on recursive approaches [7] involving frequency and duration methods [8], and/or cumulant procedures [9]. Hybrid approaches use both simulation and analytical characteristics [10].

In order to study the hourly effects on the reliability of power systems, this paper presents a discussion of the risk methodologies and then a novel way of examining the probability distributions of the reliability indices based on a hybrid approach.

II. STOCHASTIC AND TIME-DEPENDENT CAPACITIES

The SMCS is often chosen to assess reliability indices due to its ability to preserve the relations between all variables and it also manages time-dependent characteristics. These indices are generally based on the following power balance equation

$$R = G - L \tag{1}$$

where G represents the amount of power available at hour t, L is the total system load at hour t and R is the static reserve at hour t. The random variable G depends on the availability of the equipment, which is usually modeled using a two state and/or a multi-state Markov model [3]-[7]. Also, when taking the effects of the capacity fluctuation into account it is necessary to include time-dependent characteristics in the system representation. Thus, prior to composing the system's capacity, the random variable G can be decoupled into two capacity slices, such as a stochastic capacity slice, which is only linked to the stochastic behavior of each component, and a time-dependent capacity slice, which captures the time-dependent variations linked to the resources. Furthermore, the random variable L consists of hourly observations of capacities. In fact, a large slice of this capacity can be classified as time-dependent, whereas the stochastic capacity slice of L, may depend on, for instance, the short- and long-term uncertainties of load representations, which can also be included in any hourly load model.

Hence, it is possible to decouple G and L representations on several slices of capacity in accordance with each type of generation technology to represent the behavior of the power system's components. In general, hybrid methodologies are directly or indirectly based on these principles.

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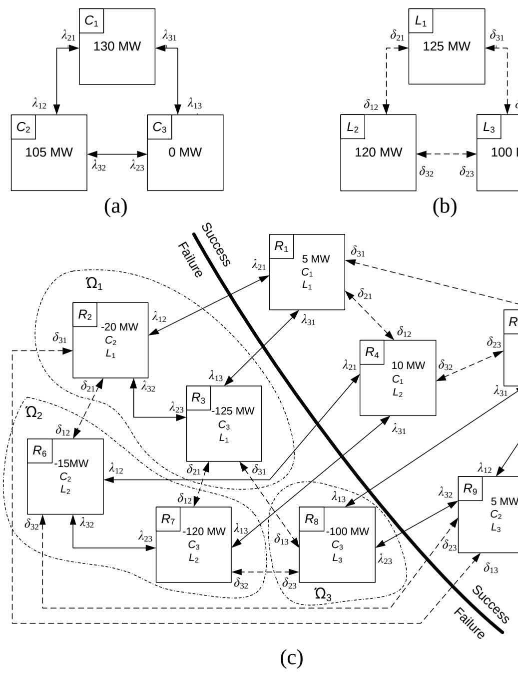

Figure. 1. State space representation.

III. A Discussion of the Analytical and Simulation Approaches

Analytical and simulation techniques are frequently used to adequacy evaluation of generation systems. Analytical approaches generally adopt the state space representation, while simulation can either adopt the state space representation or the chronological representation. In general, an analytical approach must be based on two major assumptions: a) the capacity and load states are independent and b) the probability of occurrence of two or more states during the same time step is negligible. These two conditions ensure the evaluation process takes place in a certain order. As an example, Fig. 1 illustrates a hypothetical state space representation, where a capacity model shown in Fig. 1a is combined with a load model shown in Fig. 1b, resulting in the reserve model shown in Fig. 1c. In the latest figure, the reserve state R_1 is a combination of the capacity state C_1 and the load state L_1, and it does not transit directly to the reserve state R_9, because two transitions would be needed during the same time step (e.g. from capacity state C_1 to C_2 and from load state L_1 to L_3).

From the simulation point of view, the sequential Monte Carlo representation assumes that two consecutive sampled system states differ from one single state component. This assumption follows the ordering principle previously described for the analytical approach. However, it must be highlighted that the probability of occurrence of two or more states during the same discrete time step could be taken into account in the simulation process. This feature is also the major difference between the non-sequential and sequential representations.

The analytical and non-sequential based models also have another major constraint: system state residence times are not considered. In order to attain the reliability indices, such as the frequency and duration indices, the success/failure border (see Fig. 1c), which separates the positive reserve states (success) from the negative reserve states (failure), must be monitored and information on the probability and frequency of negative reserve states must be collected [7]. During this procedure, any chronological sense is broken and it is not possible to track the existing dependences between states. On the other hand, in the chronological representation, the history of a system is simulated in discrete time steps. More specifically, the sequential approach simulates the system operation by sampling the component state durations, which in turn are dependent on each component's mean-time-to-failure (MTTF) and mean-time-to-repair (MTTR). Fig. 2 illustrates a composition of six different system states using the same capacities presented in Fig. 1.

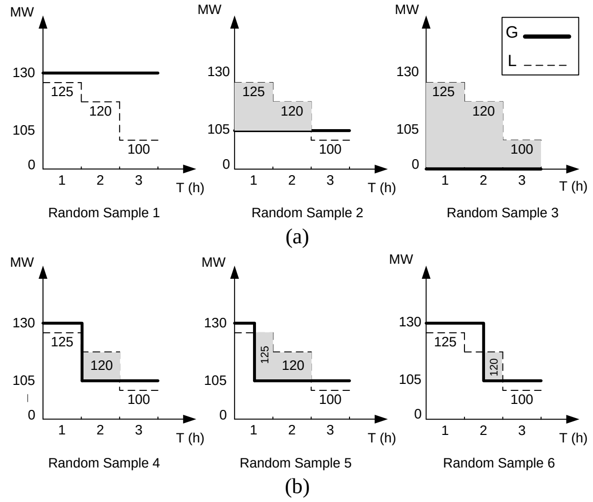

Figure. 2. Chronological representation – SMCS sampled example.

A. Meeting Point between Analytical and Statistical Methods

The random sample 1, in Fig. 2a, can be compared with the set of reserve states R_1, R_4, and R_5 in Fig. 1c, performing a completely positive reserve state in both representations. The main difference between these representations is that the corresponding reserve states R_1, R_4, and R_5, have no sampled residence time. The sampled system state in the simulation representation (see Fig. 2a) is composed of the three reserve states visited in the analytical approach.

The random sample 2, in Fig. 2a, starts in a failure state during the first two hours. This is followed by a success state in the third hour which is due to the load transition from 120 to 100 MW. The random sample 2, in Fig. 2, can be compared with the reserve states R_2, R_6, and R_9, in Fig. 1c. First, two failure states are visited and then a transition to a success state is assigned. Similarly, the random sample 3, in Fig. 2a, can be compared with the reserve states R_3, R_7, and R_8, in Fig. 1c. It shows a totally negative reserve state in both representations. It is interesting to verify how the sequence of states observed in the chronological representation is composed of different subsets in the analytical approach. In fact, when the capacity is fixed during a certain period of time, the load variation always leads to a transition between reserve states. As stated previously, although the probability of occurrence of two or more states during the same time step may be considered small, it would not be negligible in the simulation approach.

The random sample 4, in Fig. 2b, shows two transitions during the same time step, where the system capacity changes from 130 to 105 MW and the system load changes from 125 to 120 MW. In this case, the transition between reserve states identified in the simulation process cannot be compared to the state space representation in Fig. 1c. This is mainly because there is no direct transition between the reserve states R_1 and R_6. Intuitively, it can be concluded that the number of reserve states presented in Fig. 1c does not change when the assumption b) is not taken into account in the analytical approach. Nevertheless, when two or more state transitions during the same time step are included in the evaluation, the state probability and frequency information change. In fact, the Markovian process behind the analytical and simulation approaches can be characterized by the relationship between the states. From the analytical point of view, this organization is based on the assumption b) where the communication between states can be divided into equivalent classes where the states in an equivalent class communicate with each other, but they do not communicate with any state outside of that class [11]. An important observation is that both approaches are able to deal with frequency unbalance [12]. However, the frequency unbalance is intrinsic to the simulation, whereas a correction factor should be applied to the analytical approach [13].

The random sample 5 includes the ordinary behavior of the simulation, where the residence time of the sampled states differ from the time step commonly used for load models (usually 1 h). The random sample 6 shows another way of attaining the reserve state R_6, where the simulation approach can be compared with R_1, R_4, R_6 and R_9.

B. *Remarks on Analytical Methodologies*

Analytical methodologies are well known for their efficiency. Hence, before presenting the proposed methodology, the analytical methods will be briefly discussed. The aim is to identify the best analytical approach in which some time-dependent characteristics can be included without losing efficiency. Three different analytical methods were implemented in order to compare their efficiency and essentially their flexibility in coping with chronology. These methods were implemented using JAVA language and were based on three different techniques: the Recursive Algorithm [7], a fast convolution technique based on the Fast Fourier Transform (FFT) algorithm [14], and the Fourier Transform method based on the Gram-Charlier expansion [7].

In general, these methods use a generalized frequency and duration methodology and their accuracy can be controlled by a capacity rounding increment (CRI) in MW, and also by a truncation probability (TProb) [14]. Clearly, the full potential of these three algorithms will not be completely explored with the IEEE-RTS 79: 32 generating units and 3405 MW of installed capacity with a load model covering 8736 h. However, this system is a benchmark and the results can be easily replicated. For the simulations, only one index was chosen to carry out the comparisons: LOLP (Loss of Load Probability). In addition, the following parameters are used: CRI = 1, 2, and 5 MW, for TProb = 0. The Recursive algorithm was chosen as a reference. In the next sections, a more complete range of indices such as LOLE (Loss of Load Expectation), EENS (Expected Energy Not Supplied), LOLF (Loss of Load Frequency) and LOLD (Loss of Load Duration) will be explored. Table I shows the results in terms of accuracy and time elapsed (τ).

The recursive algorithm [7] is based on a simple combination of the probabilities and frequencies using the failure and repair rates of each generating unit. Assuming that repair and failure rates are constant, this process an exact result, which is achieved using a recursive conditioned probability approach. In spite of the results, the consideration of all states may affect the efficiency of the method (see Table I), mainly because the number of states increases with the number of generating units, if no derated states are included. Thus, the application of this algorithm in large systems can be very time consuming. Alternatively, truncation or rounding techniques can be used.

In the FFT algorithm [12] [14] the up and down states are represented as positive impulses. This process is similar to the recursive method. However, in order to convolve the generators through the FFT method, the distance between the impulses should be maintained by a constant step. The use of the FFT algorithm accelerates the convolution in comparison to a simple combination of states, as shown in Table I. Nevertheless, to keep a constant distance in a train of impulses, a simple weighted-averaging rounding technique must be applied [14]. During this process an error is introduced and its magnitude strictly depends on the step used, as also shown in Table I.

TABLE I. RESULTS FOR THE IEEE-RTS 79 SIMULATION – PEAK LOAD

	LOLP	τ (s)	Accuracy
FFT (1 MW)	0.084578	0.468	0
FFT (2 MW)	0.084550	0.234	2.81E-05
FFT (5 MW)	0.084413	0.093	1.65E-04
GCE (1 MW)	0.092276	1.093	7.70E-03
GCE (2 MW)	0.092276	0.406	7.70E-03
GCE (5 MW)	0.089888	0.187	5.31E-03
Recursive	0.084578	0.952	-

The Fourier Transform method, based on Gram-Charlier expansion [7], is a cumulant method that approximates the train of impulses of the probabilities using a normal distribution curve. Following this approximation, the probability of having an outage capacity higher than a defined value can be directly obtained. The computational effort of this cumulant method does not depend on the number of generating units, nor does it depend on the magnitude of the system, as shown in Table I. However, the accuracy of the algorithm is dependent on a fitting curve. Thus, as shown in [9], the error is acceptable when the system is composed of identical units with relatively large forced outage rates and the accuracy decrease when units with low forced outage rates are added. Therefore, the FFT algorithm has been chosen to be explored further.

C. *Hybrid Evaluation of Reliability*

One of the first proposals using hybrid reliability evaluations was to represent the model depletion in the output capacities of hydroelectric units [10]. This cited work proposes a simulation/analytical approach to reliability evaluation in large hydroelectric systems, where the random variable G is decoupled between reservoir depletion and equipment outages. In this case, simulation is responsible for handling the time-dependent characteristics of reservoir depletion through complex reservoir distribution functions and an analytical approach is used to handle equipment outages without time-dependent characteristic. One of the main conclusions is that the equipment outages have a very small effect on the energy state of large hydroelectric systems.

Another analytical model that considers time-dependent characteristics is proposed in [15] and it allows unconventional sources, such as wind power and photovoltaic to be included in the assessment of the reliability of power systems. This methodology is based on the division of the generation system in order to incorporate the effects of the primary energy fluctuations as well as the failure and repair characteristics of the energy generating units. In this case, the overall electric system is decoupled into subsystems containing the conventional and unconventional units. The simulation process controls the calculation of the power output of the unconventional subsystems for each hour under study, so as to include the effect of fluctuating energy.

As verified, both hybrid methodologies cited apply a principle that is commonly used in SMCS, where the random variable G is decoupled into subsystems in order to model the stochastic behavior of power system components and their time-dependent characteristics. Intuitively, a chronologic sense is introduced through the impact of hydro inflow, or, for example, wind and solar power characteristics.

IV. Proposed Methodology

The proposed methodology follows the intuitive way that is based on decoupling G and L in different subsystems. This is mainly to convolve all stochastic capacities G and L at an appropriate moment during the evaluation, where the time-dependent effects of each capacity slice or subsystem can be appropriately represented through the natural load chronology.

The representation of thermal power plants is based on the two-state Markov model [7] where the failure/repair cycles for all thermal power technologies, such as nuclear, coal, oil, gas are included. The power available is assumed to be dependent on the unit unavailability. Other models that consider more than two states (derated states) can be used if the necessary parameters are available. Similarly, the hydro power available is assumed to be dependent on the failure/repair cycle using the two-state Markov model [7] and on the water storage of each reservoir. The power available for each unit is assumed to be proportional to the level of water storage in its respective reservoir as proposed in [3].

Wind power technology is usually modeled using several generating units (turbines) that are grouped into an equivalent multi-state Markov model [3]. The energy production of the wind generating units is usually defined for each hour according to the hourly wind series of each geographic region. This paper applies another approach, where the production of wind power is considered using the historical wind production (wind series), which captures the wind speed, the power conversion characteristics and also the stochastic behavior (up/down cycles) of each generating turbine. This latter assumption can be viewed as a simplified way of representing the stochastic behavior of the wind. This is true mainly when the historical yearly series only represents a single scenario and does not have statistical representation. However, this assumption can be enough to assess wind scenarios. Therefore, historical yearly wind power series fluctuations with an hourly resolution will be used, which represents some specific wind scenarios.

A. Representation of a Conventional Generation System

As stated previously, the conventional units, such as hydro and thermal units, are modeled using their capacity states, which may include derated levels. The probabilities and frequencies are given using the transition rates between the states. The method used to combine them is based on the FFT algorithm [14]. The first step of the proposed methodology is a convolution of the thermal units' outage capacities, which does not depend on the time representation (hour, week, or month resolution). The result of this convolution is stored and combined with the monthly availability of hydro power. Although, there is a range of different thermal capacities, the period considered in the convolution process should be the same for all units, in order to keep the coherence with the number of impulses. This is a requirement of the FFT method.

The next step is the convolution of the monthly cycles of the hydro units. The proposed model aims to capture the hydro units' behavior during the dry and wet months of the year. Therefore, in each step of the proposed approach, their affected capacities are convolved with the stored results of the thermal units' convolution. The main idea of the capacity affectation is also presented in [15]; however, the authors have affected the capacities after the combination of the generators. In this paper, the opposite order is used, which allows different levels of affectation for each unit. At the end of the hydro units' convolution, it is possible to write the static generation subsystem model, incorporating hydro fluctuations as $G = \{c_G; p_G; f_G^k\}$, where each state is modeled using its capacity vector (c_G), probability vector (p_G) and incremental frequency vector (f_G^k).

B. Representation of Load, Unconventional Sources and Electric Vehicles

The time-dependent power sources, like wind power, are rarely included in conventional energy generation models. Although, wind turbines behave in a similar way to hydro or thermal units from a stochastic point of view, the key factor linked to the wind capacity representation is the wind speed and direction. The annual variations in wind speed are often represented through a sequence of percentage values (wind series), similar to the hydro depletion of the reservoirs. However, the wind power resolution of these sequences is usually modeled in the same way as the load, using 8760 capacity points. Hence, a simple method to combine the probabilities and frequencies of the wind power with the probabilities and frequencies of the conventional generation consists of converting the wind capacity sequence of a wind farm into impulses to perform a convolution. Thus, there are at least three possible ways of considering the wind power's effect on the proposed evaluation: a) the wind capacities of each wind farm are convolved *one by one* with the conventional generation, b) the capacities of each wind farm are added together hour by hour creating a *huge wind farm*, which is convolved with the conventional generation and c) the hourly wind power of each wind farm is added as a *negative capacity* on the load points, maintaining the chronological characteristic of both models. In the latter case, the ordinary procedure for the load model is followed, that is described in [12].

In order to assess the chronological representation accuracy of these three approaches, a simple experiment is proposed, where each case is compared with a SMCS. For this task, the IEEE-RTS-96 HW [16], with an installed capacity of 11,391 MW and a peak load of 8,550 MW is used. The SMCS is set with β = 1%. Three different wind scenarios are also assessed in order to compare the LOLE index in different conditions. Taking the SMCS as a reference, Fig. 3 shows the results for the comparisons.

As expected, the *one by one* and *huge wind farm* cause a pessimistic LOLE index for each of the three scenarios studied. The differences to the SMCS results come from considering a set of unrealistic failures states, composed of load states and wind power states inadequately assumed to have occurred at same time. Naturally, the *negative capacity* provides similar results to those obtained using SMCS, since chronology is explicitly represented. In fact, on the one hand the representation used in SMCS considers the hourly output of each wind farm as a capacity addition on G. On the other hand, the analytical negative capacity considers the hourly output of each wind farm as a capacity reduction on L. The effect in (1) can be considered the same. However, it is important to highlight that SMCS is able to represent the stochastic behavior of each turbine, whereas the proposed analytical approach captures the turbine's up and down cycles using only one wind power series. This may cause some differences depending on the purpose of the study.

So far, only the effect of wind power has been represented preserving their time-dependent characteristics. However, other

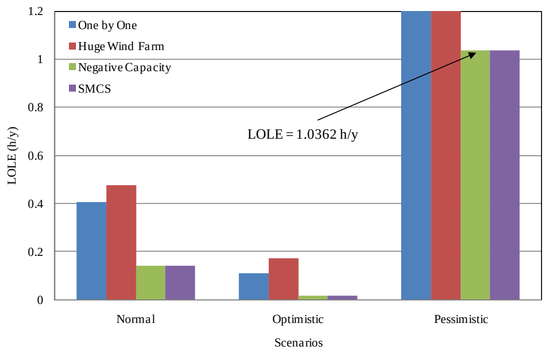

Figure. 3. Convolution comparisons with SMCS result.

effects that are time–dependent, such as EV, should also be included. A simple way of including EV could be by applying the same concept used for wind power.

In fact, this approach models EV as positive capacities when added onto the system load. In order to monitor the chronological sense of the load, wind power and EV subsystems, they can be modeled in a similar way as $CS = \{c_{CS}; p_{CS}; f_{CS}^{k}\}$, where CS is defined as the chronological subsystem, which contains the capacity vector (c_{CS}), the probability vector (p_{CS}), and the incremental frequency vector (f_{CS}^{k}). This model can clearly be extended to include other unconventional sources.

C. Reserve Model and Indices Calculation

The conventional generating capacity model represented by the random variable G can be combined with the chronological subsystem capacity model, represented by the random variable CS, to define the capacity reserve model $R = G - CS$ in which $R = \{c_R; p_R; f_R\}$. Hence, probabilities and frequencies of the reserve model as well as the traditional reliability indices are obtained through the combination of the two conventional and chronological subsystems, according to the same equations used in [12].

As shown in Fig. 1c the failure state space of the reserve model can be organized as subsets of failure states Ω_j, where each subset contains the combination of cumulative states of G against individual states of CS. At this point, it is important to highlight that the model alters the failure state space of Fig. 1, mainly due to the presence of the unconventional sources and EV. However, for didactic purposes the failure state space is considered to be the same as the one shown in Fig 1. From this perspective, one of the first remarks might be the possibility of assessing system reliability indices for each chronological capacity observation. This means an hourly reliability assessment, since the load, EV and unconventional sources are represented using hour resolution, although any other resolution could be used.

Meanwhile, based on [12] and as shown in Fig. 1 and 2, some stochastic properties can be explored within the failure state space in order to attain an analytical risk model. For instance, the probability of residence in the cumulative subspace Ω_1 and its respective frequency and average duration can be obtained as follows.

$$LOLP_{\Omega_1} = p_{R_2} + p_{R_3} \quad (2)$$

$$LOLF_{\Omega_1} = p_{R_2}(\delta_{12} + \delta_{13} + \lambda_{21}) + p_{R_3}(\delta_{12} + \delta_{13} + \lambda_{31}) \quad (3)$$

$$LOLD_{\Omega_1} = LOLP_{\Omega_1}/LOLF_{\Omega_1} \quad (4)$$

where $LOLP_{\Omega_1}$ is the loss of load probability, $LOLF_{\Omega_1}$ is the loss of load frequency, and $LOLD_{\Omega_1}$ is the loss of load duration all referred to subset Ω_1. These concepts can be extended to all subsets Ω_j. Therefore, an analytical risk model Γ can be written as follows.

$$\Gamma = \{p_\Omega; f_\Omega; d_\Omega\} \quad (5)$$

where the p_Ω, f_Ω, d_Ω, are vectors with $LOLP_{\Omega j}$, $LOLF_{\Omega j}$, $LOLD_{\Omega j}$, entries respectively. These entries are computed according to the equations below.

$$LOLP_{\Omega_j} = p_{CS_j} p_{G_j} \quad (6)$$

$$LOLF_{\Omega_j} = p_{CS_j} f_{G_j} + p_{G_j} f_{CS_j} \quad (7)$$

$$LOLP_{\Omega_j} = LOLP_{\Omega_j}/LOLF_{\Omega_j} \quad (8)$$

where p_{CS_j} is the individual probability of CS, and p_{G_j} is the cumulative probability of G, both at hour j. Observe that Γ is a risk model that contains probabilities, frequencies and average durations for all subsets Ω. Therefore, through the manipulation of these vectors it is possible to achieve hourly, daily, weekly, monthly and annual reliability indices for the generation system.

D. Distributional Aspects Associated with Reliability System Indices – A Statistical Perspective

From a statistical perspective, the mechanisms used to explore the distributional aspects associated with the reliability system index mean values are based on time observations. Such time observations allow a comprehensive image of each system failure for each assessed random sample. For instance, to analyze the distributional aspects of the LOLE index for the hypothetical system shown in Fig. 1, Fig. 4 shows six different possible random samples for the system. In order to estimate the LOLE index, an estimator LLD_i must be defined, as in [2], which is calculated using the total loss of load duration observed throughout year i. In this case, a LOLE index can be estimated as follows.

$$\widehat{LOLE} = E[LLD] = \frac{1}{N}\left(\sum_{i=1}^{N} LLD_i\right) \quad (9)$$

where N is the number of samples and T is the period of study (usually 8760 h). In Fig. 4 the study period is 3 hours and the number of samples is 6. Using (9), a possible LOLE index could be 1.33 h/yr. Clearly, the small number of samples does not reveal any conclusions about the LOLE index, without the confidence interval evaluation [11]. By storing LLD_i values it is possible to estimate a probability distribution function for the LOLE index.

Another approach to the LLD estimator can be obtained by dividing LLD_i by time portions along random samples (see Fig. 4). Each portion of LLD can be linked to a year sample i, and an hour j. Hence, the LLD estimator for the random sample i can be expressed as follows.

$$LLD_i = \sum_{j}^{T} LLD_{i,j} \quad (10)$$

where, $LLD_{i,j}$ can be read as loss of load duration (in hours) observed in random sample i in the hour j. Therefore, another potential way of verifying the distributed characteristics of the LOLE index is, instead of observing LLD_i in each random sample i, one can observe $LLD_{i,j}$ in overall random samples N as shown in Fig. 4. As stated in previous sections, from an analytical perspective, an analytical risk model Γ was proposed for all subsets Ω. Analogously, it is possible to write a statistical

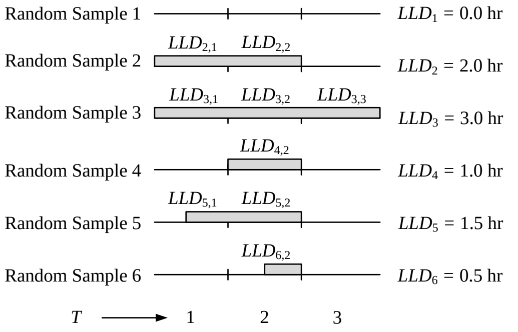

Figure. 4. Distributional aspect of LOLE index – LLD estimator.

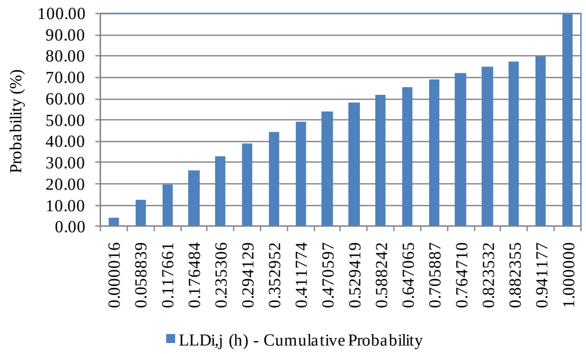

Figure. 5. Distributional aspects of $LLD_{i,j}$ – SMCS approach.

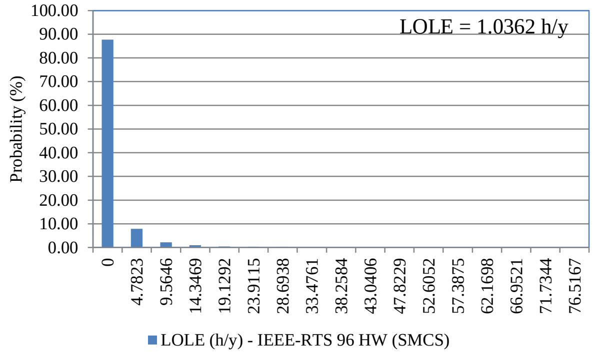

Figure. 6. Distributional aspects of LOLE – SMCS Approach.

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for (i from 1 to N)
| P_i^Ω ← 0;
| for (j from 1 to T)
| | if (U_{i,j} < p_{G_j}) then
| | | P_i^Ω ← P_i^Ω + p_{CS_j};
| | end if
| end for
end for
```

Figure. 7. Distributed LOLE index algorithm - Hybrid approach.

procedure to estimate Γ where one defines the expectation of a vector (matrix) as being the vector (matrix) of expectations [11].

It is therefore possible to estimate a probability distribution function of LOLE on each $'\Omega_j$. However, it would not be practical to examine 8760 probability distribution functions. Thus, in order to investigate the distributional aspects of each different load points, Fig. 5 shows the cumulative probability distribution of all load points (all hours of the year) of $LLD_{i,j}$, obtained using the SMCS on the IEEE-RTS 96 HW previously discussed. This shows that 20% of the $LLD_{i,j}$ has a risk of 1 h. This means that at least 20% of the observed failure events are reaching a value of LLD_i that is equal to or greater than 1 h/y. It

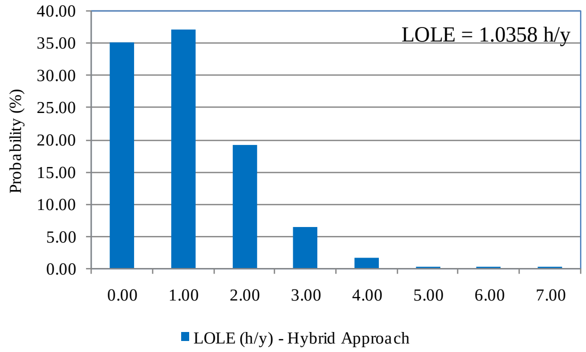

Figure. 8. Distributional aspects of LOLE – Hybrid Approach.

is also possible to see that the $LLD_{i,j}$ values range from 1.6 x 10^{-5} h to 1.0 h for all load points. In fact, the most common result obtained from the SMCS is the LOLE distribution. This index represents a probability when expressed in h/h. Fig. 6 shows the LOLE distribution for the IEEE-RTS 96 HW previously discussed. The expected value for LOLE is 1.0362 h/y.

However, it was possible to identify some rare events where the LOLE index varied from 71.7344 to 76.5167 h/y, with a very low probability.

E. Distributional Aspects Associated with Reliability System Indices – A Hybrid Perspective

After presenting a shared aspect between the analytical and statistical approaches, the hybrid methodology used to investigate the distributional aspects associated with reliability system indices will now be discussed. In summary, the proposed approach consists of building random samples, as illustrated in Fig. 4. However, instead of sampling $LLD_{i,j}$ values as in the original statistical procedure, the index distributions are obtained using the information from which Γ was derived. Following this approach one can survey the distribution of the probability P^{Ω} in which one can encounter a state $'\Omega_j$ during a year, by using the algorithm presented in Fig. 7. In this algorithm, $U_{i,j}$, denotes a uniform random sampled number at hour j and year i, and P_i^{Ω} represents the probability of encountering a state $'\Omega_j$ during the sampled year i.

This algorithm was applied to the IEEE-RTS 96 HW. It must be noted that by multiplying each P_i^{Ω} by 8760, one can obtain the number of visited hours j where a failure was assigned, for each sampled year i. Such values were organized in the probability distribution shown in Fig. 8. Clearly, the average value of this probability distribution gives an estimation for the LOLE index in h/y. Differently from the LOLE distribution obtained through the statistical procedure in which state residence times are sampled, the proposed algorithm only samples the occurrence of configurations of units which will cause a failure at hour j. Hence, Fig. 8 shows that the probability of visiting three hours with failure events is 6.54%. Note that the expected LOLE is 1.0358 h/y and it presents a similar result when compared to SMCS.

It is possible to see that the hybrid approach takes the time dependent effects into account, following the load chronology, although state residence times are not sampled.

V. APPLICATION RESULTS

The proposed methodology was applied to the planned configuration of the Portuguese Generation System (PGS). In 2015, the PGS is expected to attain an installed capacity of

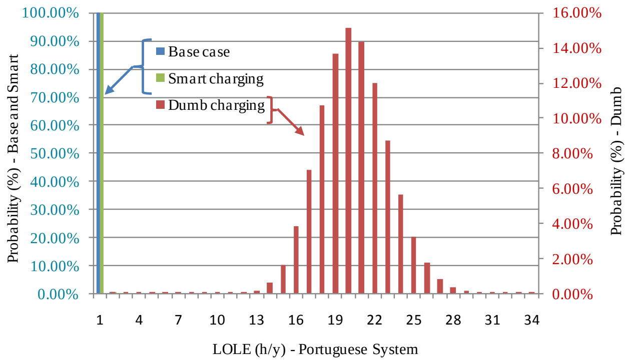

Figure. 9. Distributional aspects of LOLE – Hybrid Approach.

20,592 MW [17]. The annual peak load is predicted at 12,716 MW. The thermal installed capacity will represent 33% of the total PGS installed capacity and is divided into coal-fired and natural gas, which will mainly replace oil-fired technology and ensure flexibility to cope with wind power variation. Hydropower is expected to represent 25% of the total installed capacity. Wind power is expected to represent 24% of the installed capacity. Cogeneration shares will remain at 11% and pumping stations will increase by 2%. Both hydro and thermal power shares will decrease slightly.

Three different scenarios were studied. A base scenario considers the description previously presented. A second scenario considers an amount of EV load being added to the system load without any control. This is known in the EV literature as Dumb charging [18] which provides a load increase of 16.2%. Finally, a third scenario considers the same amount of EV load under a Smart charging strategy [18], where most of the EVs are charging in the load valley. These three scenarios are used to highlight the proposed hybrid methodology, although it also shows the effects of the integration of EV into the PGS. Table II shows the annual results for the conventional reliability indices. The Dumb charging monthly evaluation revealed the highest LOLE of 7.9063 h/y in February. Table III shows the Well-being analysis considering the largest unit criterion. Finally, Fig. 9 shows the distributed aspect of the LOLE index.

It must be noted that the number of hours with failure events increases when EV load is added to the system load using a Dumb charging scenario. The LOLE also increases from 3.61E-04 to 19.4061 h/y. However, the number of hours with failure events when EV load is added to the system, under a Smart charging strategy, remains the same.

Another important aspect related to the Hybrid approach is its computation performance. While the Hybrid approach required 226 seconds to perform the evaluation for the PGS, the SMCS took about 15 hours ($\beta = 1\%$). Evidently the representation behind the SMCS is more computationally expensive than the Hybrid approach. Clearly, the proposed approach can be viewed as a worthy alternative to investigating distributional aspects of reliability indices, due to its low computational cost.

VI. Conclusions

Different from the SMCS, which is based on the state duration sampling mechanism, the proposed hybrid approach deals with chronology using the sequence of load observations, which in turn represents a natural chronology for both approaches. While SMCS uses G as the main variable to follow the chronology, the hybrid approach uses L to deal with chronology. In fact, the hybrid approach shows to be a worth alternative to the SMCS with low computational costs, preserving time-dependent effects and allowing, for instance, Well-being analyzes. In addition, distributional aspects of reliability indices can also be investigated using the approach.

TABLE II. Conventional Indices for PGS

Index	Base	Smart	Dumb
LOLE (h/y)	3.61E-04	3.61E-04	19.4061
LOLF (oc./y)	2.98E-04	2.98E-04	16.2485
EENS (MWh/y)	0.040949	0.040949	4984.74

TABLE III. Well-being Analysis for the PGS - February

Index	Base	Smart	Dumb
E_H (h)	672.00	672.00	654.53
F_H (occ./m)	8.57E-04	8.57E-04	11.45
E_M (h)	1.08E-03	1.08E-03	9.55
F_M (occ./m)	8.23E-04	8.23E-04	4.63

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