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Based on different mathematical modelling approaches to the production in the first stage, we develop a sequence-oriented formulation and a product-oriented formulation, and propose decomposition-based heuristics to solve this problem efficiently.
By considering these dependencies arising in practical production processes, our model can be applied to various industrial cases, such as the beverage industry or the steel industry. Computation tests on instances from an industrial application are provided at the end of the paper.

# Tactical production and distribution planning with dependency issues on the production process

Wenchao Wei*, Luis Guimarães*, Pedro Amorim*, Bernardo Almada-Lobo*‡

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**Keywords:** tactical integrated production-distribution planning, two-stage production process, sequence dependent setup times and costs, MILP based heuristics.

## 1. Introduction

In many industrial supply chains, raw materials are gradually transformed into end products through a series of production stages, and then delivered to scattered clients to meet their demands. With market globalisation and international trade expansions, many firms have been trying to optimise their production and distribution systems simultaneously in the most efficient and economical way possible, such that the overall costs are minimised and all client requirements are met. For example, the production sites are usually geographically close to clients and are dedicated to few types of products in order to reduce the distribution and production costs, respectively. In an environment where demands are seasonal, the production rates and the inventory levels of some products are kept in a systematic way to balance inventory costs and customer satisfaction. Moreover, some groupings of products of different types and amounts over a large time scale for a one-off delivery may increase the usage of the transportation capacities, and thus decrease the total delivery costs. The integrated production-distribution models consider all the factors above throughout the decision-making process, thus providing the tools to investigate the supply chains in a more macroscopic way than in a dissolved model (Chandra & Fisher, 1994).

Following the hierarchical supply chain management, company practitioners perform strategic, tactical and operational oversight to monitor and improve the integrated production-distribution process. The strategic level decisions are usually the first step in developing such a process, for instance, by choosing the site and functionality of factories or creating a reliable transportation network. The process itself is substantially defined at tactical level, where issues about demand satisfaction, cost control and risk management are addressed. Common concerns in the tactical level include production schedules, transportation and warehousing solutions, or inventory logistics. Operational level refers to day-to-day processes, such as detailed management of good-in-process, or managing incoming and outgoing products. In this study, we investigate the production and distribution planning in a tactical level by considering the dependency constraints arising at operational level. The motivation is to provide the company practitioners with a global vision of the entire production-distribution process, and to find key bottlenecks for further improvements.

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Over the last decades, research has been conducted on this integration, as well as on its industrial applications. For a review, the reader should go to Mula *et al.* (2010). However, most papers focus mainly on the interdependencies between production and distribution, and thus fail to capture the dependency attributes within the production process itself, which tends to be rather simplified. For example, in a multi-product production site, the type of raw material entering the production system determines the characteristics of the products being produced within a certain period. There is also the need for cleansing the production entities in order to guarantee the purity of the products. The cost and time for cleansing often depend on the sequence of product lots and the quality of the products produced. Clearly, the above-mentioned dependencies substantially determine the detailed production management in the operational level, and will eventually influence the final integrated production and distribution planning in the tactical level as well.

In this study we consider production processes that are tightened with these dependencies. These processes can be modeled using a two-stage production structure which consists of resource preparation (where raw materials are transformed into continuous resources) in the first stage, and product forming in the second. One industrial motivation comes from the mass production of glass containers (Almada-Lobo *et al.*, 2008), which are manufactured at a rate of thousands of products per minute on each production line. In the first stage, raw materials such as sand, soda ash, limestone and cullet are mixed and melted into glass paste (which we call continuous resources) around $1500°$C. In the second stage, the paste is drawn into production lines that shape them into final products, which are glass containers. This kind of structure captures the essential parts of the production process in many other industrial cases. A similar process transferring alloy into various vessels can be found in the steel industry (Araujo *et al.*, 2007; Santos-Meza *et al.*, 2002). There is also a two-stage production process with soft drinks (Ferreira *et al.*, 2009), where the syrup is prepared in tanks in the first stage, and then distributed to parallel bottling machines in the second stage. The production system of the spinning industry processes fibres (of different fibre blends) in the first stage, which are then used to produce different types of yarn on parallel machines in the second stage (Camargo *et al.*, 2014). In these examples, the two-stage manufacturing structure must be addressed within the production-distribution planning procedure, which may eventually alter the quantities of the production lot, as well as the timing and magnitude of the distribution. The production sequences, lot sizes, inventory and distribution plans should therefore be balanced in a comprehensive way such that the trade-offs between manufacturing and transportation costs, demand satisfaction, and backlog and sale losses are optimised simultaneously.

In this paper, we study a model that integrates production and distribution planning in the tactical level for a yearly planning horizon. By considering a two-stage manufacturing structure, this model is a variation of the production-distribution models suggested by Fahimnia *et al.* (2013). The major modelling concern is synchronising the production and distribution plans, while respecting the dependency constraints arising in the production stages. Particularly, we consider that the setup times and costs consumed in the first production stage are sequence-dependent, while the setup times and costs in the second stage are just product-dependent. The justification for this modelling choice are presented afterwards in Section 3. The unfulfilled demands are either shifted to subsequent production periods as backlog, or penalised as lost sale if they cannot be supplied within a certain time lag. As a response to seasonally varied demand, moderate amounts of products are normally produced in advance and stored in warehouses incurring inventory costs. The transportation costs are determined by the distance between the plants and clients, and the quantities of products that will be delivered. The quantities of backlog, lost sale, inventory and delivery are determined explicitly in the model.

We focus on big-bucket formulations rather than on small-bucket formulations, since the former provide much better lower bounds (Wolsey, 2002). It is clear that in the two-stage production process, second-stage decisions are dominated by the decisions made in the first production stage. Different mathematical formulations are derived based on the modelling choices adopted in the first production stage. For instance, one can define the changeovers (occurring in the first production stage) between products explicitly in the model, or alternatively, use a collection of pre-defined sequences of products, such that the changeovers between products in the first production stage follow a given sequence. Accordingly, we develop two mixed integer linear programming (MILP) formulations: a product-oriented and a sequence-oriented formulation, which are then compared.

The difficulty of solving our problem is synchronising the production activities in the two production stages across all the machines in order to maintain a certain output rate. Additionally, the model is complicated by the existence of sequence-dependent setup times and costs. In light of the existing literature, such as Pochet & Wolsey (2006) and Helber & Sahling (2010), MIP-based decomposition algorithms are capable of solving our problem efficiently. Different decomposition statistics are adopted based on the problem characteristics, such as, for example, time periods, products, machines, or demands. Specifically, we develop a relax-and-fix heuristic based on time-decomposition to find an initial solution, and a fix-and-optimise improvement approach embedding a variable neighbourhood search (decompositions based on time periods and entities in the first production stage). Our computation experiments on an industrial application show that our solution approach is efficient in finding good solutions.

This paper contributes to this research area in three ways: firstly, we shed light on tactical production-distribution planning with integrated dependency issues on the production process; secondly, we examine two mixed integer linear programming formulations for the integrated model; thirdly, we develop heuristic algorithms to solve the problem efficiently, and then provide managerial analysis.

The paper is organised as follows: Section 2 lists relevant research contributions in the integration of production and distribution planning. Section 3 introduces the problem features and the basic assumptions. Section 4 describes two different approaches for modelling the tactical production and distribution planning. Section 6 elaborates on a relax-and-fix heuristic and on a fix-and-optimise heuristic procedure for both formulations. Section 7 presents our computational experiments conducted on industrial instances and summarise the results. Finally, Section 8 concludes this paper and outlines future research topics.

## 2. Literature review

Among the first attempts to combine production and distribution, Burns *et al.* (1985) study the delivery of products in a simplified production system in order to minimise inventory, production and transportation costs.

Chandra & Fisher (1994) address a single manufacture site, with a multi-product problem. The authors study the coordination of production planning and vehicle routing to minimise setups, inventories and transportation costs. Their computational studies show that considering production and distribution simultaneously decreases total operating costs. Pyke & Cohen (1993) restrict the supply chain to a single plant and try to determine optimal batch sizes. A Markov chain model is proposed for the case of single product and an approximation scheme for a multiple product environment in Pyke & Cohen (1994).

Dhaenens-Flipo & Finke (2001) discuss a multi-facility, multi-product, multi-period production and distribution problem in the form of a network flow problem, applied to a real industrial problem. Park (2005) considers a production and distribution planning problem to maximise the total net profit in a multi-plant, multi-retailer, multi-item, and multi-period logistic environment, and propose optimisation models and a heuristic solution for both integrated and decoupled planning.

Chen & Vairaktarakis (2005) study an integrated scheduling model of production and distribution operations where products are processed and then delivered directly to the customer without intermediate inventory. Their objective takes into account both customer service level and total distribution cost. Lee *et al.* (2006) suggest an integrated mathematical model for the semiconductor industry supply chain consisting of production and distribution chains, where production re-entry, binning and substitution flows are considered. Kopanos *et al.* (2012) address a production process with a semi-continuous mixed integer programming model for simultaneous production and logistics operations planning, based on the definition of families. A review of recent research on the integrated planning of production and distribution is well described in Mula *et al.* (2010) and Fahimnia *et al.* (2013).

We list another stream of literature concerning the modelling choices of detailed lot sizing and scheduling problems. The problem is often formulated on a discrete time scale using either a big-bucket time scale where multiple setups can be scheduled in each period (see Lasserre, 1992 and Clark & Clark, 2000), or a small-bucket time scale where only one setup is scheduled in each period (see Lasdon & Terjung, 1971 and Fleischmann, 1994). Hybrid models combining big-bucket and small-bucket time scales are studied by Fleischmann & Meyr (1997) and Transchel *et al.* (2011). The disadvantage of discrete time scales formulation is its untraceability for large problems due to the binary variables assigned to each discrete time period. Using continuous time scale formulations could avoid this problem, but it renders a more complicated model (Floudas & Lin, 2004). For a recent research on two-stage lot-sizing and scheduling formulations, go to Camargo *et al.* (2012). A continuous common resource is considered in the first stage, and lot-sizing and scheduling with sequence-dependent setup times and costs are considered in the second stage. This work provides three discrete and continuous time-based scale formulations, as well as comparisons between them.

In this paper, production and distribution planning are combined to tackle a multi-site tactical decision-making problem. Production sites are selected for conducting certain types of production activities, and then paired with clients for delivering satisfactory quantities of products. To the best of our knowledge, the dependence constraints arising in the production stage have not yet been studied in this integrated context. Detailed production planning needs to be underlined when examining the entire planning system. Specifically, we are interested in a two-stage production process where the outputs on each stage are related. Moreover, we consider multiple types of resources in the first stage. Looking at both literature streams, our problem is described as a multi-plant, multi-client, multi-period and multi-product production and distribution problem with a two-stage production process.

# 3. Problem description

Let us consider a company which has several plants and clients scattered in a wide international region (see Figure 1 for an illustration). The products are produced in the plants and then delivered to the clients to meet their demands. A tactical integrated production and distribution plan for this company consists of production decisions (such as machine utilisation, lot sizes, setup sequences of products), inventory solutions and delivery management (such as quantity of products transported from plants to clients) over a large time horizon with regard to all environmental constraints. Our objective is to find an integrated plan that minimises the total setup, inventory, backlog, lost sale and transportation costs.

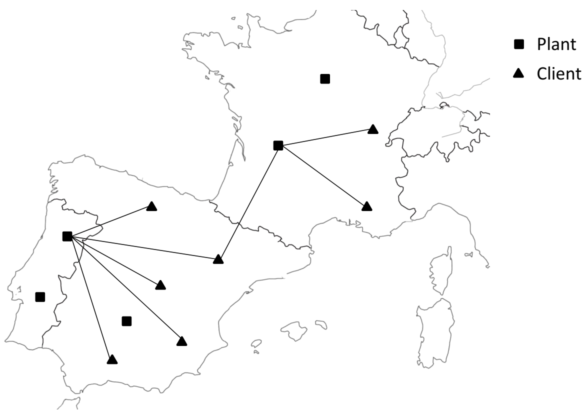


Figure 1: An illustrative distribution network

The model considered in this paper is based on a number of assumptions. Firstly, we present assumptions which also hold for the integrated production-distribution models, as described in Mula *et al.* (2010):

- The products are delivered to the clients directly from the plants without passing by a distribution centre (as shown in Figure 1).
- The transfer cost per product unit only depends on the product's characteristics and distance between the plants and clients.
- Warehouses are only located at each plant. Therefore, demands for each period cannot be supplied in advance. The storage capacity of warehouses is assumed to be sufficiently large.
- Unsatisfied demands are either classified as backlog or penalised as lost sale (if they cannot be met within a certain period).

The assumption of transportation cost may be seen as strict. However, with the advent of supply chain management in various industrial sectors, many enterprises started to dismiss their own fleet of vehicles and to transport final goods in cooperation with external partners, which are the called third-party logistics service providers. In the long run, the unitary transportation cost is considered constant. Thus, classical optimisation models which coordinate production and distribution volumes at an aggregate level are appropriate for a long-term production planning.

Backlog makes it possible to further group demand and improves machine capacity utilisation; however, backlog is penalised because it leads to a deterioration of the customer service. In production environments with high levels of capacity utilisation, as well as in other applications, the consideration of backlogs, also known as back orders, is imperative. Otherwise, no feasible solution would exist (Quadt & Kuhn, 2008).

Particularly, we consider a two-stage production process (see Figure 2 for an illustration) in the integrated production-distribution planning. In the first-stage production, the raw materials are processed into continuous *resources*, and then transformed into end products in the second stage. *Feeding machines* are the entities that carry over the continuous batch of resources and feed them into *manufacturing machines*, which are the entities that conduct the discrete production of end products. The outcome of the first stage production is thereafter called a *resource*, which essentially determines the texture of the end products. The end product is the output of the second-stage production and there are different manufacturing (forming) *processes* to produce them. Next, a new production lot in the first stage is called a *resource campaign*, and a production lot in the second stage is a (manufacturing) *process campaign*.

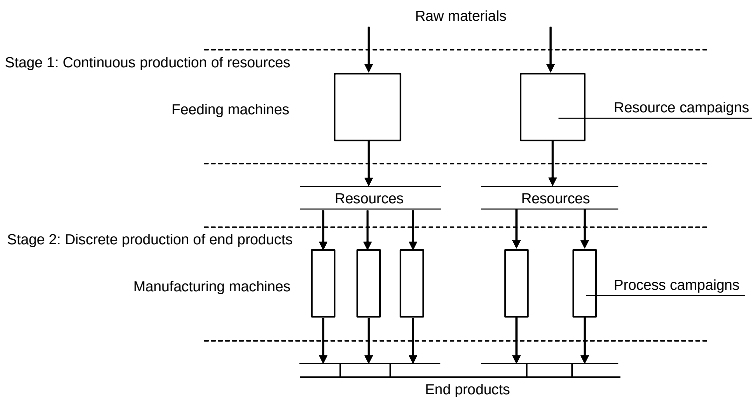


Figure 2: An illustrative two-stage production process

To address the two-stage production process in the model, further assumptions are needed:

- The procurement of raw materials is assumed to be reliable and is not considered.
- Each manufacturing machine is assigned unequivocally to a feeding machine.
- Simultaneous production of resources and end products is possible.
- At each point in time, one resource at most is provided by a feeding machine, thus all the manufacturing machines connected to this feeding machine should manufacture products that require the same resource.
- The processing of resources is considered a semi-continuous task at different throughput rates. The manufacturing machines also have different throughput rates for each product and are allowed to be idle only when the feeding machine is being set up to another resource campaign.
- The setup state of a machine is conserved from one period to the next.
- The setup times and costs of changing from one resource campaign to another are sequence-dependent due to technical and/or quality issues. In addition, the triangle inequality holds, which means that it is never faster to change from one resource to another by means of a third resource campaign. The setup times and costs of changing from one process campaign to another are considered constant.
- Setup *carryover* holds: a feeding machine starts a period by processing the same resource as the one from the end of the previous period; a manufacturing machine starts a period with a process campaign carried over from the previous period.
- No changeover may overlap the boundaries of periods (i.e., setup crossover is not allowed in the model).
- Products are divided into *families*, each of which is described as a set of technologically similar products sharing some resources or consuming the same resource. A changeover from one product family to another consists of changing the (first-stage) resource, or the forming (second-stage) process, or both. The changeovers between products of the same family only require simple operations, and thus are neglected in the model. Therefore, family is the scheduling unit and only the quantities of each product produced are determined.
- A product cannot be produced in a given period unless the desired resource campaign and process campaign are already set up during that period.

The assumption that setup state can be conserved over time periods is a real-world feature (specifically in a 24-7 production system) which may promote savings in setup costs and time, and decrease inventory levels since setups consume a large amount of the machine's capacity.

Changeovers between different process campaigns normally incur in setup times and costs that are much lower than those of the resource campaigns. Therefore, without loss of generality, both setup times and costs are considered constant. This assumption is practical on a tactical planning level for many two-stage production processes (such as in the glass container or steel industry).

The assumptions of setup carryover and non-overlapping render a much more compact formulation. A natural consequence is that if a resource campaign is set up in the beginning of both the current and the successive period, then that resource campaign is the only one in the current period. This conservation applies *mutatis mutandis* to process campaigns, thus restricting the solution space to a reasonable size. The potential advantage of setup crossovers has a relatively small effect on the total costs on this planning level, as time buckets are large. Due to the triangle inequality of setup costs and times, at most one setup is allowed for each resource on each feeding machine, during each period.

Table 1: Relations between family, resource and process for the illustrative example

| *Family* $f$ | $\{Product\}$ $\{i\}$ | *Resource* $u$ | *Manufacturing process* $o$ |
|---|---|---|---|
| 1 | $\{1, 2\}$ | 1 | 1 |
| 2 | $\{3\}$ | 1 | 2 |
| 3 | $\{4, 5\}$ | 2 | 2 |

We consider the consumption of resources, as well as the production quantities, machine throughput rates and product demands in the same unit of measurement (for instance, in tonnes). All the time related parameters and variables, such as processing time or setup time, are measured in the same time unit (for instance, hours).

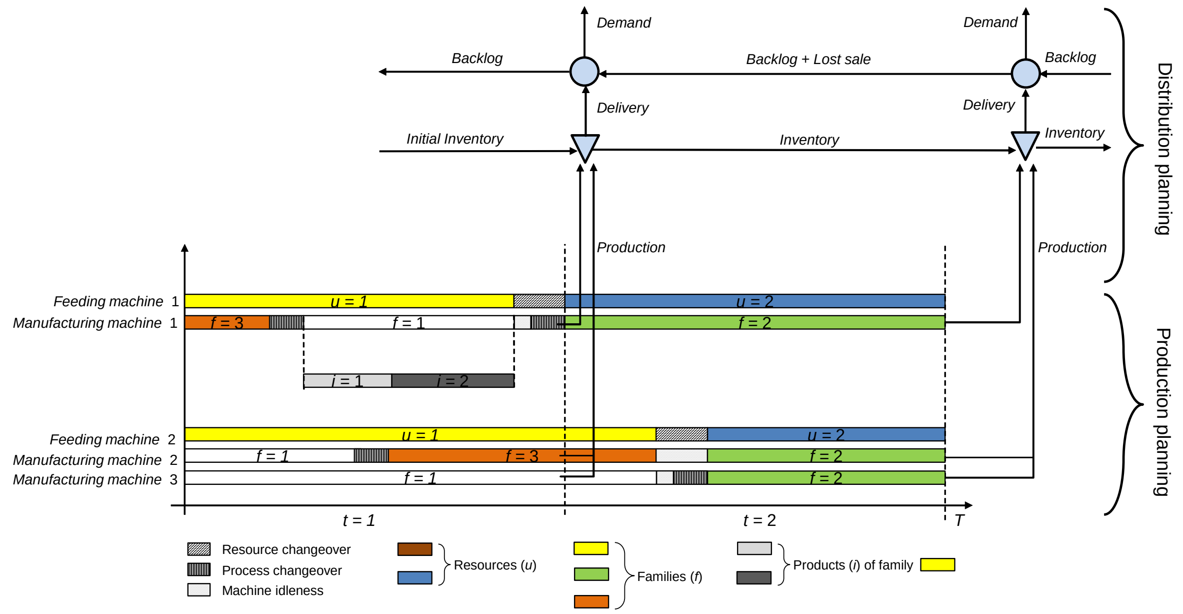


Figure 3: An illustrative production and distribution planning

Without loss of generality, we illustrate the integrated production-distribution planning with a two-stage production process on a single plant. This plant consists of two feeding machines and three manufacturing machines. Manufacturing machine 1 belongs to feeding machine 1; manufacturing machines 2 and 3 belong to feeding machine 2. The indices of other components of the problem and the relations between the components are given in Table 1. These two feeding machines can process the two resources and the three manufacturing machines can perform two manufacturing processes. A possible production-distribution plan over periods 1 and 2 is presented in Figure 3. During period 1, the changeover from family 3 to family 1 on feeding machine 1 requires only a process changeover, while the changeover from family 1 to family 2 needs both a resource changeover on the feeding machine 1 and a process changeover on the manufacturing machine 1. These two changeovers are performed simultaneously. The process changeover occupies a small part of the time slot of the resource changeover, with the machine remaining idle for the rest of the slot. During period 2, the changeover from family 3 to family 2 requires only one resource changeover on feeding machine 2. Note that changeovers between products, for example product 1 and product 2 in period 1, require very low capacity (few minutes) and incur low costs, and are therefore neglected in the model.

At the end of time period 1, the products 1 produced are either delivered to clients to meet their demands in period 1, or stored in the plant and then used to fulfil the demands in period 2. The quantities used for each purpose are determined explicitly in the model. Since product 3 is not produced in period 1, its demand in period 1 is either backlogged to period 2 or classified as lost sale if it cannot be satisfied at the end of period 2 (if the demands can be backlogged for 1 time period at most).

## 4. Mathematical formulations

The main challenges of the mathematical formulations are synchronising the resource campaigns in the first production stage with the process campaigns in the second production stage, and also incorporate scheduling and lot-sizing decisions of resource campaigns. Since the major sequencing and scheduling decisions relate to the first stage of production, we developed two mathematical formulations that differ in the modelling choices adopted in the resource campaigns. The first one is a sequence-oriented formulation, where a collection of pre-defined sequences of resource campaigns is used; the second one is a product-oriented formulation, where the changeover between resource campaigns is explicitly defined. The second stage production schedules, inventory and transportation solutions are either directly or implicitly determined thereafter. In presenting these formulations, the following notation is used.

*Indices*

| | |
|---|---|
| $i \in \mathcal{N}$ | Products |
| $f \in \mathcal{F}$ | Product families |
| $j \in \mathcal{P}$ | Plants |
| $k \in \mathcal{K}$ | Feeding machines |
| $m \in \mathcal{M}$ | Manufacturing machines |
| $k(m) \in \mathcal{K}$ | Feeding machine to which manufacturing machine $m$ belongs to |
| $t \in \mathcal{T}$ | Planning time periods |
| $c \in \mathcal{C}$ | Clients |
| $u, l \in \mathcal{U}$ | Resources |
| $o \in \mathcal{O}$ | Processes |

*Sets*

| | |
|---|---|
| $\mathcal{M}_k^K$ | Set of manufacturing machines connected to feeding machine $k$ |
| $\mathcal{M}_j^P$ | Set of manufacturing machines belonging to plant $j$ |
| $\mathcal{F}_m^M$ | Set of product families that can be produced on manufacturing machine $m$ |
| $\mathcal{F}_u^U$ | Set of product families whose production requires resource $u$ |
| $\mathcal{N}_m^M$ | Set of products that can be produced on manufacturing machine $m$ |
| $\mathcal{N}_f^F$ | Set of products belonging to family $f$ |
| $\mathcal{N}_u^U$ | Set of products whole production requires resource $u$ |

*Parameters*

| | |
|---|---|
| $d_{ict}$ | Demand of client $c$ for product $i$ in period $t$ |
| $p_{im}$ | Throughput rate of product $i$ on manufacturing machine $m$ (tons/hour) |
| $h_{ij}$ | Unitary holding cost of product $i$ at plant $j$ |
| $b_{ic}$ | Unitary backlog cost of product $i$ at client $c$ |
| $l_{ic}$ | Unitary lost sale cost of product $i$ at client $c$ |
| $r_{ijc}$ | Unitary transfer cost of product $i$ from plant $j$ to client $c$ |
| $q_{kt}$ | Working time of feeding machine $k$ in period $t$ |
| $cap_{kt}$ | Producing rate of feeding machine $k$ in period $t$ (tons/hour) |
| $stp_m$ | Setup time incurred for starting a new process campaign on manufacturing machine $m$ |
| $scp_m$ | Setup cost incurred for starting a new process campaign on manufacturing machine $m$ |
| $\epsilon$ | Maximum number of time periods a demand can be backlogged |
| $\theta$ | A large value |

*Continuous variables*

| | |
|---|---|
| $D_{ukt}$ | Production of resource $u$ on feeding machine $k$ in period $t$ (hours) |
| $X_{imt}$ | Production of product $i$ on manufacturing machine $m$ in period $t$ (hours) |
| $I_{ijt}$ | Stock of product $i$ at plant $j$ at the end of period $t$ |
| $S_{ijct}$ | Supply quantity of product $i$ from plant $j$ to client $c$ in period $t$ |
| $\hat{S}_{ijctt'}$ | Quantity of product $i$ produced in plant $j$ in period $t$ that is used to satisfy demand of client $c$ for product $i$ in period $t'$ |
| $B_{ict}$ | Backlog quantity of product $i$ in client $c$ in period $t$ |
| $L_{ict}$ | Lost sale quantity of product $i$ in client $c$ in period $t$ |

*Binary variables*

| | |
|---|---|
| $Y_{fmt}$ | (=1) if a setup occurs to family $f$ on manufacturing machine $m$ in period $t$ |
| $Z_{fmt}$ | (=1) if family $f$ on manufacturing machine $m$ is the first to be produced in period $t$ |
| $Q_{mt}$ | (=1) if no family starts up on manufacturing machine $m$ in period $t$ |

## 4.1 Sequence-oriented formulation

Inspired by the sequence-oriented formulation of single stage, the single machine lot-sizing problem proposed by Haase & Kimms (2000), this formulation is based on the concept of using a collection of pre-defined sequences which describe the items to be produced and their order. Similar choices have been made in Dhaenens-Flipo & Finke (2001) for a network flow model of multi-products, multi-facilities and for a multi-period production-distribution planning problem. Here we denote a sequence $s \in \mathcal{S}$ as a permutation of resource campaigns scheduled in the first production stage. To present the sequence-oriented formulation, we further define the following parameters:

| | |
|---|---|
| $\widehat{sc}_s$ | Setup cost incurred if sequence $s$ is selected |
| $\widehat{st}_s$ | Setup time incurred if sequence $s$ is selected |
| $g_{us}$ | (=1) if resource $u$ is present in sequence $s$ |
| $f_{us}$ | (=1) if resource $u$ is first in sequence $s$ |
| $l_{us}$ | (=1) if resource $u$ is last in sequence $s$ |

and a binary decision variable:

| | |
|---|---|
| $W^S_{skt}$ | (=1) if sequence $s$ is selected for feeding machine $k$ in period $t$ |

The information of resource changeovers is already indicated in each sequence. For the instance mentioned before (in Section 3), the set of possible sequences for each feeding machine is $\mathcal{S} = \{1, 2, 12, 21\}$. Then we illustrate our modelling considerations and choices in the follow order: first stage planning, second stage planning, a combination of the first and second stages, and inventory management.

## First stage planning

The setups of resource campaigns on feeding machines are modelled in constraints (1) - (3). $\mathcal{S}_k$ is the set of sequences that can occur on feeding machine $k$. Constraint (1) selects exactly one sequence $s \in \mathcal{S}_k$ for each feeding machine $k$, in each period $t$. In the problem description, we assume that in each period a feeding machine must start with the same resource campaign as the one it had processed at the end of the previous period. Therefore, the sequence chosen for a given period must be compatible with the sequences chosen for the same feeding machine during the preceding and the subsequent periods. This compatibility is expressed in constraint (2). Constraint (3) prevents a resource campaign from starting until it has been set up.

$$\sum_{s \in \mathcal{S}_k} W^S_{skt} = 1, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{1}$$

$$\sum_{s \in \mathcal{S}_k | f_{us}=1} W^S_{skt} = \sum_{s \in \mathcal{S}_k | l_{us}=1} W_{sk,t-1}, \quad \forall\, u \in \mathcal{U},\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{2}$$

$$D_{ukt} \leq q_{kt} \cdot \sum_{s \in \mathcal{S}_k |\, g_{us}=1} W^S_{skt}, \quad \forall\, u \in \mathcal{U},\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{3}$$

As mentioned before, switching from one resource campaign to another on a feeding machine depends on the sequence they are processed. The feeding machine utilization constraint reads:

$$\sum_{u \in \mathcal{U}} D_{ukt} + \sum_{s \in \mathcal{S}_k} \widehat{st}_s \cdot W^S_{skt} = q_{kt}, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{4}$$

and the feeding machine capacity constraint:

$$\sum_{m \in \mathcal{M}^K_k} \sum_{i \in \mathcal{N}^M_m} p_{im} \cdot X_{imt} + cap_{kt} \cdot \sum_{s \in \mathcal{S}_k} \widehat{st}_s \cdot W^S_{skt} \leq q_{kt} \cdot cap_{kt}, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{5}$$

where the quantity of each product produced on each manufacturing machine in a period is function of the manufacturing machine processing rate. Replacing $q_{kt}$ by $\sum_{u\in\mathcal{U}} D_{ukt} + \sum_{s\in\mathcal{S}_k} \widehat{st}_s \cdot W^S_{skt}$ in constraint (5) leads to

$$\sum_{m\in\mathcal{M}^K_k} \sum_{i\in\mathcal{N}^M_m} p_{im} \cdot X_{imt} \leq D_{ukt} \cdot cap_{kt}, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T}$$

which is more compact and used to substitute constraint (5).

## Second stage planning

Each manufacturing machine $m$ is set up for exactly one family $f \in \mathcal{F}^M_m$ at the beginning of each time period:

$$\sum_{f\in\mathcal{F}^M_m} Z_{fmt} = 1, \quad \forall\, m \in \mathcal{M},\, t \in \mathcal{T} \tag{6}$$

In line with constraint (2) from the first stage, no changeover occurs in the boundaries of periods; constraint (7) expresses that during each period a manufacturing machine starts by producing the family that was being produced in the previous period. Constraint (8) reinforces this by imposing that if such a family is produced right at the beginning of the previous period, then it is the only family that produced in the previous period. Constraint (9) defines the auxiliary variable $Q_{mt}$.

$$Z_{fm,t+1} \leq Z_{fmt} + Y_{fmt}, \quad \forall\, m \in \mathcal{M},\, f \in \mathcal{F}^M_m,\, t \in \mathcal{T} \tag{7}$$

$$Z_{fm,t+1} + Z_{fmt} \leq 1 + Q_{mt}, \quad \forall\, m \in \mathcal{M},\, f \in \mathcal{F}^M_m,\, t \in \mathcal{T} \tag{8}$$

$$Y_{fmt} + Q_{mt} \leq 1, \quad \forall\, m \in \mathcal{M},\, f \in \mathcal{F}^M_m,\, t \in \mathcal{T} \tag{9}$$

The production time of each product $i$ is limited by the machine utilization:

$$X_{imt} \leq q_{k(m),t} \cdot (Z_{fmt} + Y_{fmt}), \quad \forall\, m \in \mathcal{M},\, t \in \mathcal{T},\, f \in \mathcal{F}^M_m,\, i \in \mathcal{N}^F_f \tag{10}$$

## Combining first and second stages

At the beginning of each period $t$, the resource $u$ feeding each manufacturing machine $m$ for family $f \in \mathcal{F}^M_m \cap \mathcal{F}^U_u$ must be available on the corresponding feeding machine:

$$\sum_{f\in\mathcal{F}^M_m\cap\mathcal{F}^U_u} Z_{fmt} = \sum_{s\in\mathcal{S}_k |\, f_{us}=1} W^S_{skt}, \quad \forall\, m \in \mathcal{M},\, u \in \mathcal{U},\, t \in \mathcal{T} \tag{11}$$

and family $f \in \mathcal{F}^M_m \cap \mathcal{F}^U_u$ can only be produced when resource $u$ is available:

$$\sum_{f\in\mathcal{F}^M_m\cap\mathcal{F}^U_u} Y_{fmt} \leq \theta \cdot \sum_{s\in\mathcal{S}_k |\, g_{us}=1} W^S_{skt}, \quad \forall\, m \in \mathcal{M}, u \in \mathcal{U},\, t \in \mathcal{T} \tag{12}$$

Constraint (13) links the family production time to the duration of the resource campaign, as well as the duration of the process campaign. Every manufacturing machine has the same working time as the feeding machine it belongs to. This happens for economic reasons related to the feeding machine's energy consumption. More specifically, the utilisation of a machine is divided into three parts: producing products (connected to feeding machines), being set up for another process campaign and remaining idle (when the corresponding feeding machine is being set up for another resource campaign). In each time period and on each manufacturing machine, the sum of the production time of each product and the total processing setup time is equal to the duration of the resource campaign when the corresponding feeding machine is active. Such condition is expressed on a resource campaign basis, according to constraint (13). The number of process setups is subtracted by 1 if there is a resource changeover. This happens because the process and the resource changeover are performed simultaneously.

$$\sum_{i \in \mathcal{N}_u^U} X_{imt} + (\sum_{f \in \mathcal{F}_u^U} Y_{fmt} - \sum_{\substack{s \in \mathcal{S}_k | f_{us}=0, \\ l_{us}=g_{us}=1}} W_{skt}^S) \cdot stp_m = D_{u,k(m),t} \quad \forall\, u \in \mathcal{U},\, m \in \mathcal{M},\, t \in \mathcal{T} \quad (13)$$

Take the production planning depicted in Figure 3 for example, where there is no resource campaign changeover on feeding machine 2 in period 1. Therefore, on manufacturing machine 2, the sum of the total production time of all products (either from family 1 or 3) and the process setup time (from family 1 to family 3) is equal to the duration of the resource campaign ($u = 1$). On manufacturing machine 2 in period 2, the production time of products from family 2 fully occupies the duration of the resource campaign ($u = 2$) since the process setup is conducted simultaneously with resource setup on feeding machine 2. In the first case, we have $\sum_{\substack{s \in \mathcal{S}_k | f_{1s}=0, \\ l_{1s}=g_{1s}=1}} W_{s21} = \sum_{\substack{s \in \mathcal{S}_k | f_{2s}=0, \\ l_{2s}=g_{2s}=1}} W_{s2t} = 1$, and in the second case we have $\sum_{\substack{s \in \mathcal{S}_k | f_{1s}=0, \\ l_{1s}=g_{1s}=1}} W_{s22} = 0, \quad \sum_{\substack{s \in \mathcal{S}_k | f_{2s}=0, \\ l_{2s}=g_{2s}=1}} W_{s22} = 1$

## Inventory management

The inventory constraint (14) states that at the end of each period $t$, for any product $i$, the quantity delivered to the set of customers that order $i$ to be delivered by plant $j$ is equal to the quantity of this product produced in period $t$ minus the inventory balance of product $i$ at plant $j$.

$$I_{ij,t-1} + \sum_{m \in \mathcal{M}_j^P} p_{im} \cdot X_{imt} = I_{ijt} + \sum_{c \in \mathcal{C}} S_{ijct}, \quad \forall\, i \in \mathcal{N},\, j \in \mathcal{P},\, t \in \mathcal{T} \quad (14)$$

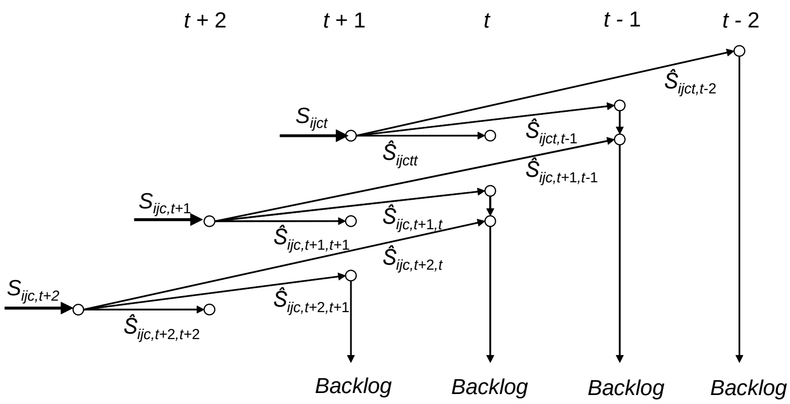


Figure 4: Illustration of disaggregated supply, backlog

In light of the facility location reformulation of Krarup & Bilde (1977), constraint (15) disaggregates $S_{ijct}$ (see Figure 4) to satisfy the demand of client $c$ for product $i$ at current time period $t$ and the backlog of product $i$ from period interval $[t - \epsilon, t - 1]$.

$$S_{ijct} = \sum_{t'=t-\epsilon}^{t} \hat{S}_{ijctt'}, \quad \forall\, i \in \mathcal{N},\, j \in \mathcal{P},\, c \in \mathcal{C},\, t \in \mathcal{T} \quad (15)$$

The demand can only be backlogged for $\epsilon$ time periods at most and there is only inventory at the plants (in modelling assumption), which implies that $\hat{S}_{sjctt'} = 0$ for all $t' < t - \epsilon$ and $t' > t$.

Accordingly, the demand of client $c$ for product $i$ at period $t$ is met by supplying product $i$ during the time interval $[t, t + \epsilon]$, while the unmet demand is considered lost sale at period $t$ (see constraint (16)). Note that $\epsilon$ refers to the number of time periods that a certain demand can be met after its due date without leading to lost sales. It is also worth noting that, in a deterministic model, lost sales of demands in a period only occur at the end of that period.

$$d_{ict} = \sum_{t'=t}^{t+\epsilon} \sum_{j \in \mathcal{P}} \hat{S}_{ijct't} + L_{ict}, \quad \forall\, i \in \mathcal{N},\, c \in \mathcal{C},\, t \in \mathcal{T} \tag{16}$$

As displayed in Figure 4, the backlog of product $i$ for client $c$, at period $t$, is the sum of the backlog (occurred before period $t$) of met demand after period $t$ (but still within maximum backlog period $\epsilon$).

$$B_{ict} = \sum_{t'=t+1-\epsilon}^{t} \sum_{r=t+1}^{t'+\epsilon} \sum_{j \in \mathcal{P}} \hat{S}_{ijcrt'}, \quad \forall\, i \in \mathcal{N},\, c \in \mathcal{C},\, t \in \mathcal{T} \tag{17}$$

### Problem statement

The objective is to minimise the sum of total costs related to inventory, transportation, backlog, lost sale, and setup. The sequence-oriented formulation $\Pi_{SO}$ reads:

$$\begin{aligned} \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left( h_{ij} \cdot I_{ijt} + \sum_{c \in \mathcal{C}} r_{ijc} \cdot S_{ijct} \right) + \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}} \left( b_{ic} \cdot B_{ict} + l_{ic} \cdot L_{ict} \right) + \\ \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}_k} \sum_{t \in \mathcal{T}} \widehat{sc}_s \cdot W^S_{skt} + \sum_{m \in \mathcal{M}} \sum_{f \in \mathcal{F}^M_m} \sum_{t \in \mathcal{T}} Y_{fmt} \cdot scp_m \end{aligned} \tag{18}$$

subject to (1) - (17) and

$$D_{ukt}, X_{imt}, I_{ijt}, S_{ijct}, \hat{S}_{ijctt'}, B_{ict}, L_{ict} \in \mathbb{R}^+_0; Y_{fmt}, Z_{fmt}, Q_{mt}, W^S_{skt} \in \{0, 1\}. \tag{19}$$

## 4.2 Product-oriented formulation

Instead of giving a set of pre-defined sequences of resource campaigns, we can alternatively incorporate the resource changeover decisions into the model, and implicitly optimise them during the solution procedure. In light of the single machine capacitated lot-sizing problem with sequence dependent setup costs (CLSD), we developed a product-oriented formulation presented below. Instead of sequence-oriented variable $W^S_{skt}$, we propose using two binary decision variables, $W^P_{ulkt}$ and $F_{ukt}$, to schedule first stage production activities. We introduce variable:

| | |
|---|---|
| $W^P_{ulkt}$ | (=1) if a changeover from resource $u$ to resource $l$ happens on feeding machine $k$ in period $t$ |
| $Fc_{ukt}$ | (=1) if the feeding machine $k$ is set up for resource $u$ at the beginning of period $t$ |
| $V_{ukt}$ | An auxiliary variable |

and parameters:

$st_{ulk}$ Setup time incurred when performing a changeover from resource $u$ to resource $l$ on feeding machine $k$

$sc_{ulk}$ Setup cost incurred when performing a changeover from resource $u$ to resource $l$ on feeding machine $k$

Only constraints related to the first stage and its links to the second stage need to be modified, namely constraints (1)-(5) and (11)-(13). The constraints regarding the second stage and inventory management remain the same.

### First stage planning

Constraint (20) imposes each feeding machine $k$ to start by processing exactly one resource campaign in each period $t$.

$$\sum_{u \in \mathcal{U}} Fc_{ukt} = 1, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{20}$$

Constraint (21) is the flow balance requirement which handles the setup state carryover on each feeding machine in each period. This constraint is also proposed by Haase (1996).

$$Fc_{ukt} + \sum_{l \in \mathcal{U}} W^P_{lukt} = \sum_{l \in \mathcal{U}} W^P_{ulkt} + Fc_{uk,t+1}, \quad \forall\, u \in \mathcal{U},\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{21}$$

As previously mentioned, a feeding machine starts a period with the same resource campaign as the one processed at the end of the previous period. Therefore, processing sequences of resources for adjacent periods must be connected. Constraint (22) is the disconnected sub-tour elimination constraint motivated by the travelling salesman problem, and suggested by Almada-Lobo *et al.* (2007) to cut off disconnected sub-tours in the lot-sizing problem. Sub-tours are sequences that start and end at the same setup state. Constraint (22) links the resource campaign sequences in adjacent time periods. Without it, one may find infeasible solutions. Other possible disconnected sub-tour elimination constraints can be found in Smith-Daniels & Ritzman (1988) and Haase (1996).

$$V_{ukt} \geq V_{lkt} + 1 - N \cdot (1 - W^P_{lukt}) - N \cdot Fc_{lkt}, \quad \forall\, u, l, l \neq u, \in \mathcal{U},\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{22}$$

Constraints (23) states that a resource campaign can only be conducted after it has been set up.

$$D_{ukt} \leq q_{kt} \cdot \sum_{l \in \mathcal{U}} (Fc_{ukt} + W^P_{lukt}), \quad \forall\, u \in \mathcal{U},\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{23}$$

The feeding machine utilization is guaranteed by constraint (24).

$$\sum_{u \in \mathcal{U}} D_{ukt} + \sum_{u,l \in \mathcal{U}} st_{ul} \cdot W^P_{ulkt} = q_{kt}, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{24}$$

and the respective capacity constraint by constraint (25):

$$\sum_{m \in \mathcal{M}^K_k} \sum_{i \in \mathcal{N}^M_m} X_{imt} \cdot p_{im} \leq D_{ukt} \cdot cap_{kt}, \quad \forall\, k \in \mathcal{K},\, t \in \mathcal{T} \tag{25}$$

### Linking first and second stages

Constraints (26) - (28) perform the same function as constraints (11) - (13), respectively.

$$\sum_{f \in \mathcal{F}_m^M \cap \mathcal{F}_u^U} Z_{fmt} = F_{u,k(m),t}, \quad \forall\, m \in \mathcal{M}, u \in \mathcal{U},\, t \in \mathcal{T} \tag{26}$$

$$\sum_{f \in \mathcal{F}_m^M \cap \mathcal{F}_u^U} Y_{fmt} \leq \theta \cdot \sum_{l \in \mathcal{U}} W_{lukt}^P, \quad \forall\, m \in \mathcal{M}, u \in \mathcal{U},\, t \in \mathcal{T} \tag{27}$$

$$\sum_{i \in \mathcal{N}_m^M \bigcap \mathcal{N}_u^U} X_{imt} + (\sum_{f \in \mathcal{F}_u^U} Y_{fmt} - \sum_{l \in \mathcal{U}: l \neq u} W_{lukt}^P) \cdot st_m = D_{uk(m),t}, \quad \forall m \in \mathcal{M},\, u \in \mathcal{U},\, t \in \mathcal{T} \tag{28}$$

### Problem statement

The product-oriented formulation $\Pi_{PO}$ reads:

$$\begin{aligned} \min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left( h_{ij} \cdot I_{ijt} + \sum_{c \in \mathcal{C}} r_{ijc} \cdot S_{ijct} \right) + \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}} \left( b_{ic} \cdot B_{ict} + l_{ic} \cdot L_{ict} \right) + \\ \sum_{u,l \in \mathcal{U}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} sc_{ulk} \cdot W_{ulkt} + \sum_{m \in \mathcal{M}} \sum_{f \in \mathcal{F}_m^M} \sum_{t \in \mathcal{T}} Y_{fmt} \cdot cp_m \end{aligned} \tag{29}$$

subject to (6)-(10), (14)-(17), (20)-(25), (26)-(28), and

$$D_{ukt}, X_{imt}, I_{ijt}, S_{ijct}, \hat{S}_{ijctt'}, B_{ict}, L_{ict}, V_{ukt} \in \mathbb{R}_0^+;\, Y_{fmt}, Z_{fmt}, Q_{mt}, W_{ulkt}^P, Fc_{ukt} \in \{0,1\}. \tag{30}$$

## 4.3 Remarks

In the product-oriented formulation, sequences of resource campaigns are defined by the model, while in the sequence-oriented formulation the model has a pre-determined set of sequences. A sequence-oriented formulation corresponds to the selection of a connected sequence to be applied in each time period. Therefore, it does not require additional constraints to ensure the connectivity (for example, constraint (22)). A production-oriented formulation selects the setups to be performed in each time period, hence the so-called disconnected sub-tour elimination constraints, which can have an exponential size, and are often required to ensure the connection between the sub-sequences induced by setup decisions.

This is the major difference between these two formulation approaches and explains why sequence-based formulations are easier to model. However, this potential advantage has a drawback of the number of possible sequences (decision variables) growing exponentially with the number of products present in the problem instance. The sequence-oriented formulation has more decision variables than the product-oriented formulation but less constraints. Haase & Kimms (2000) suggest that one may only consider the so-called effective sequences and still find the optimal solution. An effective sequence is defined as a permutation of a subset of items, which has the minimum total setup cost (time) within all the permutations of the items in this subset. We argue that, in a continuous production environment, especially when the problem is tight capacitated, this might not hold, for example when the feeding machines are not allowed to be idle. A counter example is given in Appendix A. Therefore, the computation effort of our industrial example involves all possible sequences.

The practical restrictions on the number of resources processed on each feeding machine can significantly reduce the number of available sequences for one feeding machine during a period. This makes it possible to outline all sequences that correspond to the possible states of a feeding machine during a period.

## 5. An illustrative instance

In this section, we illustrate an integrated production-distribution plan on a concrete instance. The instance consists of 5 products grouped into 2 product families and 3 manufacturing machines allocated to 2 feeding machines, each on a different plant. The relations between families, resources and processes are given in Table 1. The relations between the entities that carryover the production activities are provided in Table 2. Feeding machine 1 can only process resource 1, and feeding machine 2 can only process resource 2; therefore, products 1 and 2 can only be produced on machine 1 and products 3, 4 and 5 can only be produced on machines 2 and 3. These three machines are identical. Let $scp_m = 1, sct_m = 0.1, b_{ic} = 4, l_{ic} = 4, h_{ij} = 2, p_{im} = 5$ for $i = 1, \cdots, 5, c = 1, 2, j = 1, 2, m = 1, 2, 3$, $q_{kt} = 10$ for $k = 1, 2, t = 1, 2, 3$, $\epsilon = 1$ and

$$(d_{ict}) = \left( \begin{pmatrix} 0 & 25 & 20 \\ 0 & 20 & 20 \end{pmatrix}, \begin{pmatrix} 0 & 20 & 15 \\ 0 & 30 & 20 \end{pmatrix}, \begin{pmatrix} 51 & 13 & 25 \\ 55 & 22 & 40 \end{pmatrix}, \begin{pmatrix} 12 & 10 & 0 \\ 10 & 20 & 0 \end{pmatrix}, \begin{pmatrix} 10 & 16 & 0 \\ 6 & 5 & 0 \end{pmatrix} \right),$$

$$(cap_{kt}) = \begin{pmatrix} 5 & 5 & 5 \\ 10 & 10 & 10 \end{pmatrix}, (st_{ul}) = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, (r_{pc}) = \begin{pmatrix} 0.4 & 0.8 \\ 0.8 & 0.4 \end{pmatrix}.$$

Furthermore, let $sc_{ul} = 25 st_{ul}$ for $u, l = 1, 2$.

Table 2: Parameter structure of the illustrative example

| Plant | {Feeding machine} | {Manufacturing machine} |
|---|---|---|
| 1 | $\{1\}$ | $\{1\}$ |
| 2 | $\{2\}$ | $\{2, 3\}$ |

We sum up the demand corresponding to each resource type and present the aggregated demand, as well as the capacity profile, in Figure 5, on the right. The proportion of each product is immediately below the demand profile, on the left of Figure 5. Thereafter, we intuitively observe that there are insufficient capacities to meet both demands.

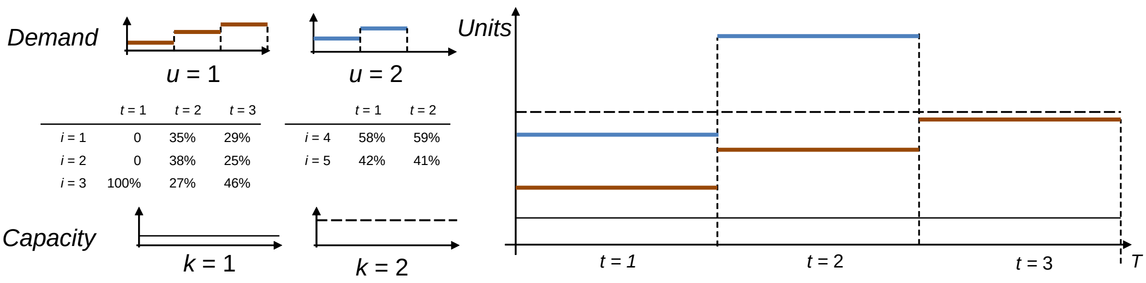


Figure 5: Illustrative profile of aggregated demand and capacity

Suppose manufacturing machine 1 is set up for family 1, and manufacturing machines 2 and 3 for family 2 at the beginning of the planning horizon. Both the product-oriented and sequence-oriented formulations can be easily solved by a MILP solver. They deliver the same optimal solution with an objective value of 633. The values for key variables are shown in Figure 6.

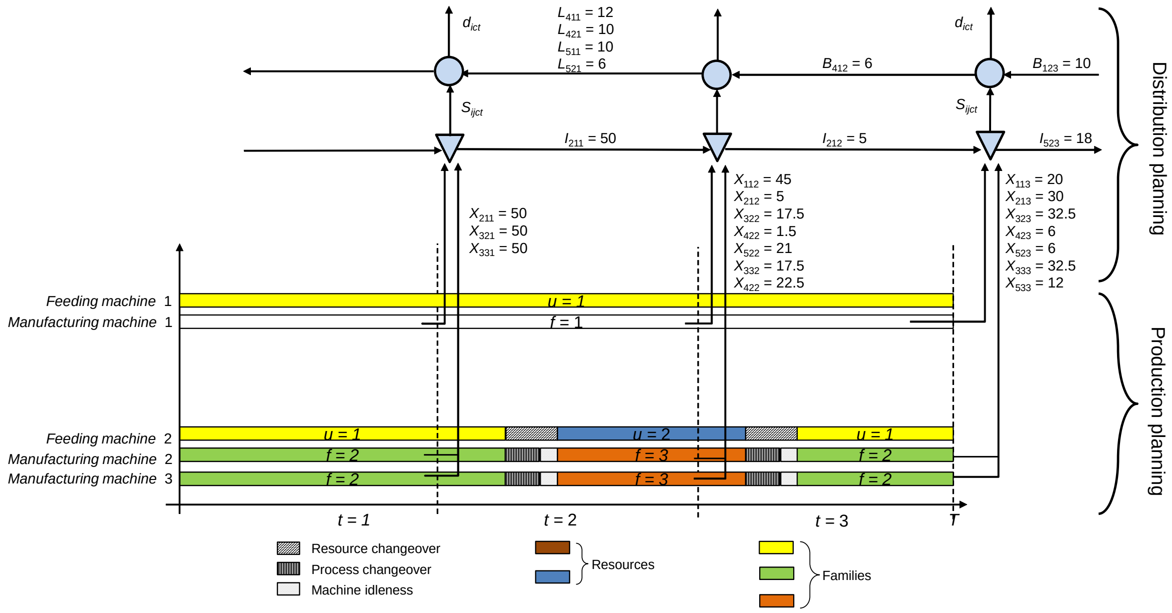


Figure 6: Optimal production and distribution planning for the illustrative instance

We observe the dependency issues in the optimal solution as follows: feeding machine 1 can only process resource 1, and therefore can only produce products 1 and 2. This leads to a large inventory (50) of product 2 at the end of period 1 (as machines cannot be idle), while the quantities of lost sale of products 4 and 5 amount to 38. Similarly, machines 2 and 3 cannot produce products 1 and 2. Therefore, there is a backlog for product 1 at the end of period 3 while 18 units of product 5 are over produced and classified as inventory. The distribution plan is affected as well. Since the capacities of the machines are tight, in period 2, the demand for products 4 and 5 cannot be fully met at the same time. Six units of product 4 are backlogged for client 1 instead of client 2, since the unitary transportation cost from plant 2 to client 1 is twice the cost from plant 2 to client 2. Therefore, backlog always occurs with clients that need higher transportation costs.

# 6. Solution approach

Solving the stand alone production planning problem is difficult. The CLSP (Capacitated lot-sizing problem) is the basic large-bucket production planning problem (Eppen & Martin, 1987), which is known to be NP-hard (Florian *et al.* 1980 and Bitran & Yanasse 1982). If positive setup times are considered, finding a feasible solution is NP-complete (Maes *et al.* 1991). Usually, solvers such as CPLEX do not satisfactorily solve real world instances of this integrated large-sized problem. Therefore, it is necessary to develop a more specific solution method. We propose two general heuristic algorithms capable of solving both formulations. The first is a relax-and-fix constructive heuristic based on a decomposition technique, and the second is a variable neighbourhood search with an adapted fix-and-optimise improvement approach to solve our problem.

## 6.1 A relax-and-fix approach

The structure of our model suggests that relax-and-fix heuristics could be a proper method to find solutions to this problem. The basic framework iteratively decomposes the original problem into a number of smaller partially relaxed sub-problems that can be solved in an easier way. By reducing the number of binary variables in each sub-problem, the computation time needed to solve each sub-problem to optimality is expected to be small. To begin with, the set of binary variables is partitioned into disjunctive subsets. At each iteration, the variables of only one of these subsets are defined as binaries, while the rest of the variables is relaxed. The resulting sub-problem is then solved to (near) optimality. A subset of binary variables of the sub-problem is then fixed at their current values and the process is repeated for all the remaining subsets. Once a binary variable is fixed, it will not be re-optimised afterwards. The decomposition of the binary variables and the criteria used to fix the binary variables mainly determine the degree of the difficulty of solving the sub-problems.

There are various strategies to divide the set of binary variables (Escudero & Salmeron, 2005). In the usual relax-and-fix strategy, the variables are grouped by periods (macro-periods) and only the binary variables are fixed at each iteration. We will use the same strategy in this paper. Another motivation for this choice is that in both formulations, all the binary variables have the same time index as one subscript (no other common subscript).

Let $\mathcal{T}^f$ be the subset of periods whose variables are fixed, $\mathcal{T}^r$ be the subset of periods whose variables are relaxed, and $\mathcal{T}^o$ be the subset of periods whose variables are optimised. These subsets are updated at each iteration. For the sequence-oriented formulation, we chose the tuple of binary variables $(W^S, Y, Z)$ as the pivot to conduct the solution procedure; other binary variables are implied thereafter. Assume that a partial or feasible solution has values $WS', Y'$ and $Z'$ (for $W^S$, $Y$ and $Z$ respectively), obtained from previous iterations. The sub-problem *subRelaxP*$(W^S, Y, Z; \mathcal{T}^f, \mathcal{T}^o, \mathcal{T}^r)$ to be solved in the next iteration reads:

$$\Pi_{SO}$$

$$\begin{cases} W^S_{skt} = WS'_{skt}, Y_{fmt} = Y'_{fmt}, Z_{fmt} = Z'_{fmt} & t \in \mathcal{T}^f \\ W^S_{skt}, Y_{fmt}, Z_{fmt} \in \mathbb{R}^+ & t \in \mathcal{T}^r \\ W^S_{skt}, Y_{fmt}, Z_{fmt} \in \{0,1\} & t \in \mathcal{T}^o \end{cases} \tag{31}$$

The first case of constraint (31) fixes the binary variables for periods that have been solved previously, the second case defines the relaxed binary variables and the last case defines the binary variable to be solved in each iteration. Similar results are obtained for product-oriented formulations by replacing $\Pi_{SO}$ with $\Pi_{PO}$, $W^S$ with $W^P$ and $WS'$ with $WP'$, where $WP'$ is the value of $W^P$ from a feasible solution.

There are multiple patterns to decompose the time intervals $\mathcal{T}^f$, $\mathcal{T}^r$ and $\mathcal{T}^o$. In light of Pochet & Wolsey (2006) and James & Almada-Lobo (2011), we adopt a partition with overlapping time intervals (overlapping between $\mathcal{T}^o$ and $\mathcal{T}^f$). The parameters used to tune the heuristic are $\alpha$ and $\beta$, which relate to the number of time periods with integrality requirements at each iteration, and the number of time periods variables are fixed at the end of each iteration, respectively. Note that $\alpha$ and $\beta$ are constant throughout the procedure. The heuristic (in sequence-oriented formulation) is described in Algorithm 1.

**Algorithm 1** Relax-and-fix heuristic

```
Given α, β;
T^f = ∅, T^o = {t^o_1, ··· , t^o_α}, T^r = {t^r_i | t^r_i ∈ T \ T^o}
while T^o ≠ ∅ do
    Solve subRelaxP
    WS' = W^S, Y' = Y, Z' = Z
    T^f = T^f ∪ {t^o_1, t^o_2, ··· , t^o_β}
    T^o = {t^o_{β+1}, ··· , t^o_α, t^r_1, ··· , t^r_β}
    T^r = T^r \ {t^r_1, ··· , t^r_β}
end while
```

### 6.2 Fix-and-optimise heuristic

In this subsection, we propose a fix-and-optimise improvement heuristic with an adapted variable neighbourhood search (VNS) procedure, which is designed to solve large problems (Seeanner *et al.*, 2013). The fix-and-optimise heuristic decomposes the binary variables of an incumbent solution into only two subsets in every iteration. The variables in the first subset are fixed to the best solution found so far, and the other variables are optimised. No binary variables are relaxed. The criterion of decomposing the set of variables is important to the solution's quality. For example, Helber & Sahling (2010) use decomposition strategies either based on products, resources or processes to successfully solve large-time-bucket model of a multi-level capacitated lot-sizing problem. In this paper, we adopt a decomposition scheme based on a combination of feeding machines $\mathcal{K}$ and periods $\mathcal{T}$, therefore incorporating more information in each iteration. Similar decisions are also made by James & Almada-Lobo (2011) and Sahling *et al.* (2009). For the sequence-oriented formulation, the resulting sub-problem is denoted as *subFixP*:

$$\Pi_{SO}$$

$$\begin{cases} W^S_{skt} = WS'_{skt}, Y_{mft} = Y'_{mft}, Z_{mft} = Z'_{mft} & k(m) \in \mathcal{K}^f, t \in \mathcal{T}^f \\ W^S_{skt}, Y_{mft}, Z_{mft} \in \{0, 1\} & k(m) \in \mathcal{K}^o, t \in \mathcal{T}^o \end{cases} \tag{32}$$

where $\mathcal{K}^f$ represents the subset of $\mathcal{K}$ whose variables will be fixed, and $\mathcal{K}^o$ represents the subset of $\mathcal{K}$ whose variables will be optimised. Sub-problem in the production-oriented formulation is acquired by substituting $\Pi_{SO}$ for $\Pi_{PO}$, $W^S$ for $W^P$ and $WS'$ for $WP'$ in *subFixP*.

Compared to the relax-and-fix heuristic, at each iteration of the fix-and-optimise procedure, we actually exploit different parts of the solution space. The VNS serves as a diversification strategy which systematically examines the neighbourhood of current solutions, and directs the search to another region of the solution space. This strategy is achieved by gradually fixing the current solution's attributes from a set of pre-defined neighbourhood structures and the neighbourhood space is then explored by a MILP solver. If an improved solution has been found, VNS partially fixes this solution according to the structure to explore the neighbourhood of the new solution again; otherwise the VNS proceeds to the next neighbourhood structure to move to another part of the solution space. Since the MILP solver is used to find the best solution in the neighbourhood, moving to the next neighbour always means an improvement from the current solution. The VNS here mainly serves to drive the search out of a local optimum. VNS conducts the search based on each of the neighbourhood structures and only allows for a limited number of no improvements to the solutions. If all structures have been tested without an improvement in the solution, then a local optimum has been found.

Neighbourhood structures are defined on a subset of periods $\mathcal{T}^o$ and a subset of feeding machines $\mathcal{K}^o$, which index the variables to be re-optimised. Binary variables belonging to $\mathcal{T}^f = \mathcal{T} \setminus \mathcal{T}^0$ and $\mathcal{K}^o = \mathcal{T} \setminus \mathcal{T}^f$ are fixed. Specifically, given the cardinality $|\mathcal{T}^o| = \lambda$ and $|\mathcal{K}^o| = \gamma$, the neighbourhood contains all the combinations of possible $\mathcal{T}^o$ and $\mathcal{K}^o$. A high degree of the cardinalities of $\mathcal{T}^o$ and $\mathcal{K}^o$ makes it possible to search the solution space on a larger scale, therefore providing a good result. However, additional computation time is required. Since a full neighbourhood evaluation of our industrial instances is too time consuming, we apply a stochastic process that controls the neighbour selection to conduct a partial neighbourhood search.

Initially, the probabilities of selecting a period and a feeding machine are set to 1. As the procedure continues, the probability of selecting that period again is reduced. Two parameters are used to tail the selection of a period and feeding machine, namely the frequency and recency. Frequency represents the number of times the corresponding period or feeding machine has been selected. Recency indicates the number of iterations since the respective period or feeding machine were last selected. The probability of selection is determined by a weighted average of these two parameters. Intuitively, the probability decreases as the values of frequency and recency increase. A similar neighbour scoring method is described in James & Almada-Lobo (2011).

Given a pre-defined set of neighbourhood structures *Pattern* $= \{(\lambda, \gamma)\}$, the basic structure and implementation of this fix-and-optimise heuristic with VNS (in sequence-oriented formulation) is illustrated in Algorithm 2.

**Algorithm 2** Fix-and-optimise heuristic with VNS

Input: *Pattern* $= \{(\lambda, \gamma)\}$, *MaxIter*
Generate an initial solution using CPLEX and update *CurSol*
*BesSol* = *CurSol*
**while** $|Pattern| > 0$ **do**
  Initialize $\mathcal{T}^o$, $\mathcal{K}^o$ based on a randomly selected pattern $(\lambda, \gamma)$
  *count* $= 0$
  **while** *count* $\leq$ *MaxItr* **do**
    Solve *subFixP*
    Update *CurSol*
    **if** *CurSol* $<$ *BesSol* **then**
      $WS' = W^S, Y' = Y, Z' = Z$
      *BesSol* = *CurSol*
      *cout* $= 0$
    **end if**
    Update $\mathcal{T}^o, \mathcal{K}^o$
    *count*$++$
  **end while**
  Remove $(\lambda, \gamma)$ from *Pattern*
  **if** runtime limit is reached **then**
    Break
  **end if**
**end while**
Return *BesSol*

# 7. Computational results

The models are implemented in Visual Studio 2010 with CPLEX concert technology, and the experiments are run on a Dell Laptop Latitude E6400 with an Intel Core 2 2.66-GHz processor and 4 GB RAM, equipped with the Windows 7 Enterprise Service Pack 1.

In order to evaluate the performance of our approach, we use five instances to conduct the computational experiments that are generated based on real-world data sets from a company producing glass containers. The size of these instances is of practical relevance for this industry. Each of these five instance includes 5 plants, 12 feeding machines, 42 manufacturing machines, 16 clients (note that hundreds of clients have been clustered in 16 geographical areas), 1809 products grouped into 95 families according to 8 types of resources, and 12 types of processes. The planning horizon is 12 months. The demands at each period can be backlogged for one time period at most.

The parameters of the sequence-oriented and product-oriented formulations of the instances are described in Table 3. These instances differ in the set of resources each feeding machine can process and (or) in the quantities of demand forecast of each client for each product. In instances $Ins_1$ and $Ins_2$, each feeding machine can process up to three resources, while in instances $Ins_3$, $Ins_4$ and $Ins_5$, only few of the feeding machines can process up to two types of resources. Naturally, the sizes of the

Table 3: Parameters and objectives of sequence-oriented and product-oriented formulation

| | Sequence-oriented formulation | | | | Product-oriented formulation | | | |
|---|---|---|---|---|---|---|---|---|
| | Constraints | Variables | | LB | Constraints | Variables | | LB |
| | | Binary | Continuous | | | Binary | Continuous | |
| | [$10^3$] | [$10^3$] | [$10^3$] | [$10^7$] | [$10^3$] | [$10^3$] | [$10^3$] | [$10^7$] |
| $Ins_1$ | 67.076 | 106.704 | 43.903 | 3.5506 | $80,344$ | $26,652$ | $43,903$ | 3.5475 |
| $Ins_2$ | 66.533 | 100.092 | 43.111 | 3.5529 | $75,082$ | $25,396$ | $43,011$ | 3.5493 |
| $Ins_3$ | 34.163 | 4.394 | 29.850 | 3.3059 | $37,193$ | $4,589$ | $29,850$ | 3.3167 |
| $Ins_4$ | 33.754 | 4.350 | 29.619 | 3.3236 | $36,306$ | $4,520$ | $25,619$ | 3.3380 |
| $Ins_5$ | 33.048 | 4.432 | 28.971 | 3.4478 | $35,422$ | $4,491$ | $24,951$ | 3.4587 |

formulations of $Ins_3$, $Ins_4$ and $Ins_5$ are much smaller than $Ins_1$ and $Ins_2$, as shown in Table 3. Generally, the sequence-oriented formulations are more compact than the product-oriented formulations since they have fewer constraints. However, the number of binary variables grows exponentially as the number of possible sequences for each feeding machine increases, as indicated in the fourth column of $Ins_1$ and $Ins_2$. Compared to $Ins_3$, the sequence-oriented formulation of $Ins_1$ contains binary variables nearly twenty times more than $Ins_3$, while regarding the product-oriented formulation this number is reduced to four. The lower bound (LB) for each instance is obtained by means of the truncated brand-and-cut algorithm used by CPLEX with a maximum computation time of 7200 second (s). We use the values of the lower bounds as the thresholds to calculate the optimality gaps of the solutions returned by our heuristic approaches.

### 7.1 Algorithms parameters

As explained in Section 6.1, the value of $\alpha$ decides the size of the sub-problem to be solved in each iteration of the relax-and-fix heuristic, while $\beta$ controls the pace of the search procedure. Since only the sub-problem in the first iteration is a relaxation of the original problem, the solution quality of the heuristic is mainly determined in this step. Intuitively, the higher the value of $\alpha$, the better will be the solution quality the algorithm could reach, and the higher will be the computation effort. Through our experiments, we find that for low values of $\alpha$, we obtain good solutions quickly. However, it is not possible to guarantee their quality. Additionally, the solver could not solve the sub-problem in the first iteration to optimality within $3600s$ (even for $\alpha = 2$). To find a feasible solution fast, the parameters are set to $\alpha = 4, \beta = 4$ with 30 seconds ($s$) running time for each iteration (in total $90s$).

To devise a proper neighbourhood structure $(\lambda, \gamma)$ for the fix-and-optimise heuristic, we include search patterns which either allow for an extensive search on a large scale or an intensive examination within a small region. Experiments seem to indicate that the quality of the solution is mainly determined by $\mathcal{K}^o$. To keep the sub-problem in a reasonable size, only patterns with small $\gamma$ values are considered. The final adopted neighbourhood structure consists of each combination of $\lambda \in \{3, 6, 9, 2, 4, 8\}$ and $\gamma \in \{2, 3\}$.

### 7.2 Analysis of the computational results

The heuristic approaches described in Section 6 are used to solve the instance sets. Each approach consists of two phases: initial solution generation and improvement with CPU time limit $90s$ and $1800s$, respectively. In the tables presented in this subsection, B&C represents the truncated brand-and-cut algorithm used by CPLEX, R&F refers to relax-and-fix heuristic with decomposition based on time periods, and F&O means the fix-and-optimise improvement heuristic. The optimality gaps are calculated based on the corresponding lower bound values in Table 3 and summarised in Table 4 and Table 5 for the sequence-oriented and product-oriented formulations, respectively.

Table 4: Results of the heuristic approaches for the sequence-oriented formulation

| | Initial solution | | Initial solution + improvement | | | |
|---|---|---|---|---|---|---|
| | B&C | R&F | B&C+B&C | R&F+B&C | B&C+F&O | R&F+F&O |
| | [%] | [%] | [%] | [%] | [%] | [%] |
| $Ins_1$ | 1521 | 867 | 5.85 | 4.79 | 2.57 | 2.81 |
| $Ins_2$ | 1627 | 920 | 6.69 | 5.64 | 3.01 | 2.67 |
| $Ins_3$ | 1228 | 489 | 1.13 | 1.11 | 0.54 | 0.47 |
| $Ins_4$ | 1033 | 445 | 1.11 | 1.03 | 0.50 | 0.52 |
| $Ins_5$ | 1207 | 511 | 1.17 | 1.08 | 0.43 | 0.41 |
| Average | 1323 | 646 | 3.18 | 2.73 | 1.41 | 1.37 |

Table 5: Results of the heuristic approaches for the product-oriented formulation

| | Initial solution | | Initial solution + improvement | | | |
|---|---|---|---|---|---|---|
| | B&C | R&F | B&C+B&C | R&F+B&C | B&C+F&O | R&F+F&O |
| | [%] | [%] | [%] | [%] | [%] | [%] |
| $Ins_1$ | 1401 | 802 | 5.69 | 4.54 | 2.81 | 2.61 |
| $Ins_2$ | 1454 | 889 | 6.04 | 5.36 | 2.93 | 2.74 |
| $Ins_3$ | 1323 | 524 | 1.59 | 1.38 | 0.63 | 0.55 |
| $Ins_4$ | 1197 | 574 | 2.13 | 1.63 | 0.74 | 0.71 |
| $Ins_5$ | 1211 | 528 | 1.51 | 1.31 | 0.48 | 0.27 |
| Average | 1317 | 663 | 3.39 | 2.81 | 1.52 | 1.37 |

In the first phase of our heuristic approaches, initial solutions are generated by means of B&C or R&F (with the parameter setting described in Subsection 7.1). The optimality gap percentages of the initial solutions are summarised in columns "R&F" and "B&C" in Table 4 and 5. As we can see, R&F outperforms B&C when trying to quickly find a good initial solution in both formulations.

In the second phase, the initial solution is improved either by F&O or B&C. Computational results of each combination of the initial solution generation technique and improvement approach are summarised in the last four columns in Table 4 and 5. For instance, "B&C+F&O" means that the initial solution is generated by the truncated branch-and-cut algorithm, and then improved by the fix-and-optimise heuristic.

For the sequence-oriented formulation, an intuitive observation is that solutions of the last three instances converge faster than the first two, because of their smaller sizes. Additionally, R&F generates better initial solutions than B&C, and leads to final solutions with higher quality as well. F&O is much more efficient than B&C in improving the quality of the solution. The value of the optimality gap of the final solutions improved by B&C are twice higher than that improved by F&O. Overall, the combination "R&F+F&O" outperforms others. Similar observations were obtained for the product-oriented formulation.

We then compare the performance of our heuristic approaches between these two formulations. For $Ins_1$ and $Ins_2$, the initial and the final solutions in the product-oriented formulation are better than those in the sequence-oriented formulation. This is because of the large number of binary variables in the sequence-oriented formulation that slows down the solution's convergence rate. For $Ins_3$, $Ins_4$ and $Ins_5$, the sequence-oriented formulation is much smaller than the product-oriented formulation. The performance of the former is therefore better.

We take $ins_4$ and present the performance of our heuristic approaches over CPU time in these two formulations in Figure 7. The optimality gap is measured in a logarithmic scale. We observe that F&O greatly improves the solutions in the first $800s$. After $1300s$ of computation time, the improvement of

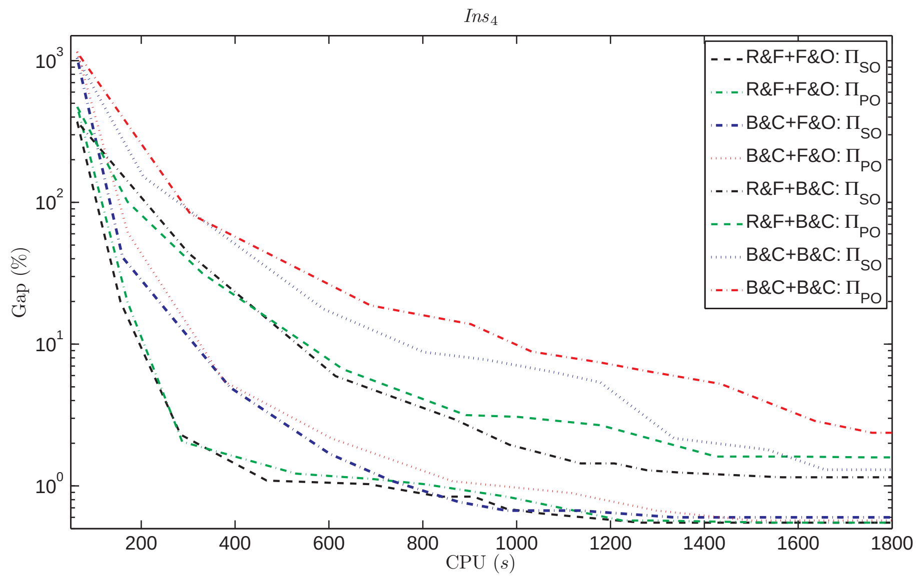


Figure 7: Optimality gap over CPU time, for each combination of heuristic approach and mathematical formulation

the solution is not so significant. B&C, however, improves the solution more slowly. The best solution is given by R&F+B&C in the sequence-oriented formulation, for which an acceptable quality is acquired only after $1500$s of computation time.

We take the results of R&F+F&O in the sequence-oriented formulation to analyse how the integrated production and distribution planning are optimised. By comparing the final result with the one acquired in the middle of the computation process (at about $200s$), we observe that the total cost decreases $13\%$, while the inventory and transportation costs remain nearly the same. There is a remarkable increase in the setup cost and a remarkable decrease in backlog and lost sales. Taking into consideration the cost term contributions, this observation reveals that by arranging the production activities wisely, it is possible to greatly reduce the backlog and lost sale, and improve customer satisfaction as well.

## 8. Conclusions

In this paper, we study the problem of integrated production and distribution planning with dependency issues on the production process. This model incorporates a two-stage production structure which is common in many industrial applications. The solution is a tactical plan of production and distribution activities over a $12$-month planning horizon. We present two formulations that differ on the first-stage modelling choices. The first is a sequence-oriented formulation based on a set of predefined sequences of resource campaigns; the second is a product-oriented formulation which models each resource changeover with a periodical variable. The predefined sequences contain all the information of resource campaigns: setup times, costs and the orders in which they will be produced. Therefore, the sequence-oriented formulation is more compact, and is generally more efficient when the number of possible sequences is moderate.

Being aware of the NP-hardness of solving integrated production, distribution problems and the large data set tackled in this study, we developed two decomposition-based approaches: relax-and-fix and fix-and-optimise. The fix-and-optimise heuristic with a VNS strategy is desirable for large problems. The algorithm is capable of performing an efficient search in the solution space based on an iterative selection of the neighbourhood structures. The fix-and-optimise systematic heuristic searches the neighbourhood space by fixing a subset of binary variables and solving the residual sub-problem to (near) optimality.

Therefore, the computation procedure is accelerated by solving a smaller sub-problem at each iteration using a commercial software. The computational studies reveal that our heuristic was able to find good solutions in less time.

We chose the glass container industry to illustrate our formulations and solution methods. However, it can be easily applied to other similar industrial cases in the future. It would also be interesting to develop stronger valid inequalities to reduce the size of the solution space and accelerate the solution approaches.

# Appendix A An illustrative example for the effective sequences losing effectiveness

Haase & Kimms (2000) define the effective sequence of products as: if the first and the last product, as well as the set of products in between are fixed, the sequence with the minimum total setup time is called effective sequence. The authors claim that if the machine is allowed to be idle, remaining in the set of effective sequences would make it possible to obtain an optimal solution. The objective is to minimise the total setup and holding cost. An example in this section shows that if the machine is not allowed to be idle (in the glass container case, the furnaces work 24 hours a day), it is not always true since a large amount of inventory might occur. We consider a single machine lot sizing problem with sequence dependent setup time and setup costs, and the value of setup time is in line with the value of setup costs. The setup times satisfy the so-called triangular inequality and given as:

$$\begin{pmatrix} 0 & 5 & 10 & 11 \\ 4 & 0 & 6 & 11 \\ 9 & 6 & 0 & 6 \\ 12 & 9 & 4 & 0 \end{pmatrix}$$

with the demand

$$\begin{pmatrix} 48 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 100 & 100 & 100 \end{pmatrix}$$

the holding cost $(100, 100, 100, 100)$, the capacity of the machine $(100, 100, 100, 100)$ and the consumption of capacity for producing a single unit of product $(1, 1, 1, 1)$.

Assume the machine is set up for product $1$ in the beginning, the product sequence of the first time period in the optimal solution is $(1, 3, 2, 4)$ with setup time $37$, while an effective sequence would be $(1, 2, 3, 4)$ with setup time $17$. We note that in this example, the holding cost per unit product is relatively high, such that the sequence with higher setup time is selected to occupy the machine in order to reduce the total holding cost.

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