Intrinsic symmetry of Ampère's circuital law and other educational issues

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Abstract: This paper explores Ampère's circuital law (ACL) from an educational perspective. The interchangeability of the ampèrian loop with the current loop, an intrinsic symmetry of ACL that is seldom addressed in the literature or textbooks, is illustrated here. It is verified that the symmetry axis of a circular current is an ampèrian loop. The attempt to apply ACL to a finite wire, a common source of student misunderstanding, is used to highlight the limitations of ACL. The generalisation of ACL is illustrated using an instructive example where the displacement current is unconfined and not spatially uniform. This work is primarily intended for teachers and more advanced undergraduate students, who may benefit from the ideas that are presented here.

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Résumé : Nous réétudions ici le théorème d'Ampère, dans une perspective pédagogique. Nous illustrons l'interchangeabilité de la boucle d'Ampère et la boucle de courant, une symétrie intrinsèque du théorème qui est rarement discutée dans la littérature ou les manuels. Nous vérifions que l'axe de symétrie d'un courant circulaire est une boucle d'Ampère. Afin d'illustrer les limitations du théorème, nous utilisons l'application du théorème à un fil de longueur finie, une source fréquente de malentendus chez les étudiants. Nous illustrons la généralisation du théorème d'Ampère à l'aide d'un exemple pédagogique où le courant de déplacement n'est pas confiné ni spatialement uniforme. Ce travail vise surtout les enseignants et les étudiants des 2^{e} et 3^{e} cycles qui peuvent profiter des idées présentées ici.

[Traduit par la Rédaction]

1. Introduction

Ampère's circuital law (ACL) is an important topic in electromagnetism (e.g., see refs. 1–4) partly because of its use in situations, such as the infinite current wire and the infinite current plate, where the choice of a suitable symmetrical ampèrian loop enables determination of the magnetic field with greater simplicity than the Biot–Savart law.

While such geometrical symmetries, which are specific to each situation, have been extensively treated in textbooks, an intrinsic symmetry of ACL that has received much less attention is the interchangeability of the ampèrian and current loops. As far as is known to us, although this topic is presented in the context of topology and physics [5], it is absent from electromagnetism textbooks and literature, in spite of its conceptual and educational relevance.

Moreover, students experience some learning difficulties with ACL [6–8] and an awareness of the interchangeability of the ampèrian and current loops may prevent the incorrect use of ACL in physical problems, such as the finite current wire, which is a common student misunderstanding.

In order to overcome the limitations of ACL it is necessary to use the generalized ACL where the key concept is the displacement current, a subtle issue that is normally introduced to students using the discharging capacitor example. This study presents an instructive and interesting example that is not treated in the literature and whose symmetry enables the determination of the magnetic field while illustrating the concept of displacement current.

The foregoing analysis touches on issues that, as far as is known to us, are rarely found in the literature and aims to provide valuable help on this topic to both teachers and undergraduate students at a more advanced level.

2. Ampèrian and current loops' interchangeability

ACL [1] relates the circulation of the magnetic field, $\boldsymbol{B}$, around a closed curve, C, called hereinafter an ampèrian loop, to the stationary electric current loop, I, intersecting any open surface, S, bounded by C. In a vacuum ACL takes the form

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$$\oint_C \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I \quad (1)$$

where μ_0 is the magnetic permeability and $\mathrm{d}\boldsymbol{\ell}$ is a vector tangent to curve C with magnitude equal to the length of the infinitesimal line element. If the current loop does not intersect S, the right-hand side of (1) is zero.

Let us consider two arbitrary closed curves, C and C′, as illustrated in Fig. 1.

If a current I exists in the curve C, each elementary current $I\mathrm{d}\boldsymbol{\ell}$ produces an infinitesimal magnetic field $\mathrm{d}\boldsymbol{B}$ at each element $\mathrm{d}\boldsymbol{\ell}'$ of the curve C′. Using the Biot–Savart law [1] one can write $\mathrm{d}\boldsymbol{B}$ as

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{\ell}\times \boldsymbol{r}}{r^3} \quad (2)$$

where $\boldsymbol{r}$ is the position of $\mathrm{d}\boldsymbol{\ell}'$ in relation to $\mathrm{d}\boldsymbol{\ell}$.

Analogously, if current I exists in curve C′, each elementary current $I\mathrm{d}\boldsymbol{\ell}'$ creates a magnetic field $\mathrm{d}\boldsymbol{B}'$ at each element $\mathrm{d}\boldsymbol{\ell}$ of the curve C, given by

$$\mathrm{d}\boldsymbol{B}' = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{\ell}'\times (-\boldsymbol{r})}{r^3} \quad (3)$$

Considering (2) and (3), the inner products $\mathrm{d}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{\ell}'$ and $\mathrm{d}\boldsymbol{B}'\cdot\mathrm{d}\boldsymbol{\ell}$ are, respectively, given by

$$\mathrm{d}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{\ell}' = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{\ell}\times \boldsymbol{r}\cdot\mathrm{d}\boldsymbol{\ell}'}{r^3} \quad (4)$$

$$\mathrm{d}\boldsymbol{B}'\cdot\mathrm{d}\boldsymbol{\ell} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{\ell}'\times (-\boldsymbol{r})\cdot\mathrm{d}\boldsymbol{\ell}}{r^3} \quad (5)$$

Because the right-hand sides of (4) and (5) are equal, one can thus write

$$\iint \mathrm{d}\boldsymbol{B}'\cdot\mathrm{d}\boldsymbol{\ell} = \iint \mathrm{d}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{\ell}' = \frac{\mu_0 I}{4\pi}\iint\frac{\mathrm{d}\boldsymbol{\ell}'\times\mathrm{d}\boldsymbol{\ell}\cdot\boldsymbol{r}}{r^3} \quad (6)$$

The last integral in (6) depends *only* on the geometry of curves C and C′ and has the value [5]

$$\iint\frac{\mathrm{d}\boldsymbol{\ell}'\times\mathrm{d}\boldsymbol{\ell}\cdot\boldsymbol{r}}{r^3} = \begin{cases} 4\pi & \text{if C and C}' \text{ are interlocked} \\ 0 & \text{if C and C}' \text{ are not interlocked}\end{cases} \quad (7)$$

Therefore, in ACL the roles of current and ampèrian loops are interchangeable, an interesting symmetry that brings about further insights into ACL and is illustrated in the next section with some examples.

3. Instructive examples

Consider two interlocked circumferences of equal radius, R, as illustrated in Fig. 2 (R is also the distance between their centres, P and Q). The planes containing the circumferences are orthogonal and their intersection contains the segment PQ.

Regardless of knowing relation (6), the present symmetry allows us to take any one of the circumferences as the current I and the other one as the ampèrian loop C. However, though the symmetry guarantees the interchangeability of I and C, it is not sufficient to determine the magnetic field $\boldsymbol{B}$ along C, because its magnitude is not constant.

Fig. 1. Two arbitrary closed curves C and C′. Any of the curves can be taken as the electrical current while the other takes the role of the ampèrian loop.

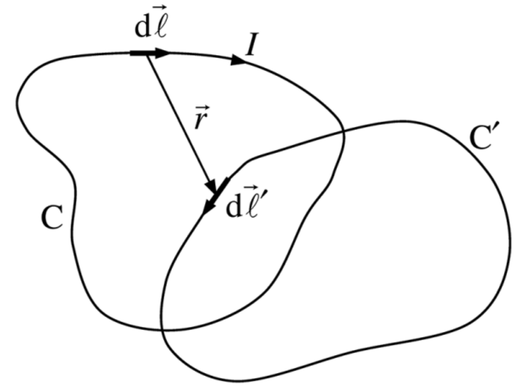
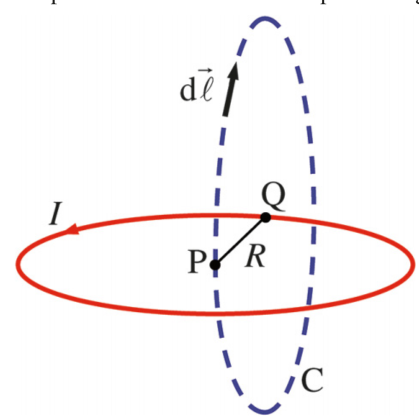


Fig. 2. Two interlocked circumferences of the same radius, R. Because of symmetry the roles of the current loop I and the ampèrian loop C are interchangeable. The choice of current loop as full line and ampèrian loop as dashed line is also adopted in Figs. 3–5.



From this initial configuration three singular cases can be built: the radius of the current loop, R, is made arbitrarily large, keeping Q at the centre of C (Fig. 3); the ampèrian loop radius, R, is made arbitrarily large, keeping P at the centre of I (Fig. 4); and the current and ampèrian loops' radii, R, are both made arbitrarily large, keeping P and Q in their respective positions (Fig. 5).

Case 1. Starting from the two circumferences of Fig. 2, keeping Q at the centre of C, as current radius tends to infinity the infinite current wire is obtained, as shown in Fig. 3.

This problem is widely discussed in undergraduate textbooks. The magnetic field at any point, P, of C can be obtained either by the Biot–Savart law or by ACL, although the latter is simpler to use owing to symmetry. We have then

$$\oint_C \boldsymbol{B}\cdot\mathrm{d}\boldsymbol{\ell} = 2\pi R B = \mu_0 I \quad (8)$$

from which one obtains the well-known result for an infinite current wire

$$B = \frac{\mu_0 I}{2\pi R} \quad (9)$$

Fig. 3. Starting from configuration of Fig. 2, the radius R of current I is made arbitrarily large, keeping Q at the centre of C, obtaining in the limit the infinite current wire.

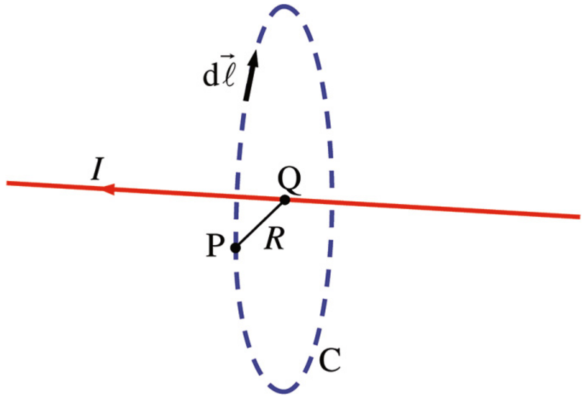
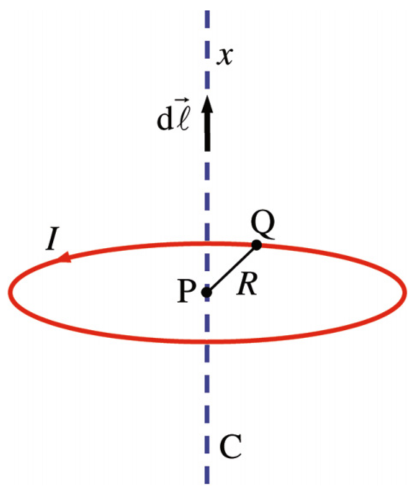


Fig. 4. Keeping the point P at centre of I, the radius of the circular ampèrian loop C in Fig. 2 is made arbitrarily large, leading in the limit to the axis of the circular current I.



Case 2. If, in Fig. 2, instead of being the radius of I that tends to infinity, it is the radius of C tends to infinity, keeping P at the centre of I, one gets the situation illustrated in Fig. 4. The current loop is a circular wire with its axis being the ampèrian loop, which is the previous situation with current and ampèrian loops interchanged.

The magnetic field due to the circular current I at a point on the axis at a distance x from the centre of the current is given by [1]

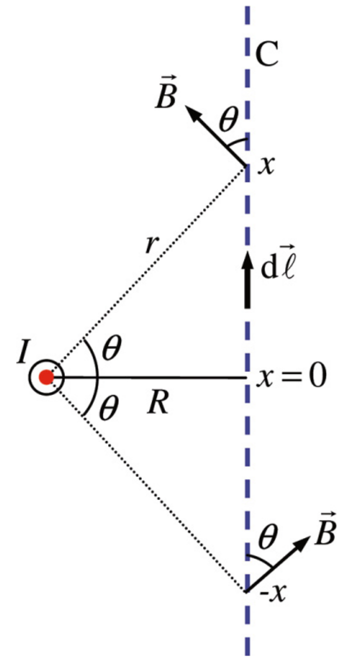
$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \tag{10}$$

The calculation of the line integral of $\boldsymbol{B}$ along C is straightforward

$$\int_{-\infty}^{+\infty} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \int_{-\infty}^{+\infty} \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}\mathrm{d}x = \int_{0}^{+\infty} \frac{\mu_0 I R^2}{\left(R^2 + x^2\right)^{3/2}}\mathrm{d}x \tag{11}$$

which leads to

Fig. 5. From Fig. 2, when the current and ampèrian loops' radii R are both made arbitrarily large, keeping P and Q in their respective positions, two orthogonal infinite lines separated by R are obtained. One of them is the current I and the other one is the ampèrian loop C.



$$\int_{-\infty}^{+\infty} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \frac{\mu_0 I}{R}\left[\frac{x}{\sqrt{1 + x^2/R^2}}\right]_{x=0}^{x=+\infty} = \mu_0 I \tag{12}$$

The preceding result together with (8) is consistent with the interchangeability of the current and ampèrian loops, given by (6), showing that the axis of a circular current can be taken as an ampèrian loop, an example that as far as is known to us is ignored in the literature.

Case 3. Finally, from Fig. 2, when the current and ampèrian loops' radii, R, are both made arbitrarily large, keeping P and Q in their respective positions, the situation is the one shown in Fig. 5. Two orthogonal infinite lines separated by R are obtained, one of them being the current, I, and the other one the ampèrian loop C.

Using (9) and the geometry of Fig. 5, the magnetic field component, B_x, in the direction of $\mathrm{d}\boldsymbol{\ell}$ is

$$B_x = \frac{\mu_0 I}{2\pi r}\cos\theta = \frac{\mu_0 I R}{2\pi}\frac{1}{R^2 + x^2} \tag{13}$$

The line integral of $\boldsymbol{B}$ along C is, therefore,

$$\int_{-\infty}^{+\infty} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \frac{\mu_0 I R}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{R^2 + x^2}\mathrm{d}x = \frac{\mu_0 I R}{\pi}\int_{0}^{+\infty} \frac{1}{R^2 + x^2}\mathrm{d}x \tag{14}$$

which leads to

$$\int_{-\infty}^{+\infty} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \frac{\mu_0 I R}{\pi}\left[\frac{1}{R}\tan^{-1}\left(\frac{x}{R}\right)\right]_{x=0}^{x=+\infty} = \frac{1}{2}\mu_0 I \tag{15}$$

The factor 1/2 in (15) is unexpected, as it leads us to conclude that the infinite line does not behave as one ampèrian loop, but just as half of one. If this is correct, it immediately raises a question: in the previous example (Fig. 4), an identical infinite line was taken as an ampèrian loop and in that case the circulation of $\boldsymbol{B}$ is $\mu_0 I$. What explains this apparent inconsistency?

Considering an infinitely long line as an ampèrian loop, such a line closes itself at infinity to form the loop. In situations like that of Fig. 4, there is no need to concern ourselves with what happens at infinity, because at infinity the contribution of $\boldsymbol{B}$ to the circulation is zero, as B decreases with x at $1/x^3$. Figure 6*a* gives a different perspective from that of Fig. 4 and shows that the axis of a circular current when closed at infinity always encloses the current, I, regardless of whether this happens through the left or through the right.

However, in the present situation (Fig. 5), how line C closes at infinity determines whether the ampèrian loop encloses the current or not. Moreover, the circulation of $\boldsymbol{B}$ along C is independent of the distance from the current I. This is illustrated in Fig. 6*b*, which shows that when C closes at infinity the circulation of $\boldsymbol{B}$ is either $\mu_0 I$ or zero depending on whether C encloses the current, I, or not. It is instructive to note that, as the result given by (15) is independent of R, although $\boldsymbol{B}$ goes to zero as $R \to \infty$, its circulation is nonzero for $R = \infty$ (Fig. 6*b*).

Interestingly, two orthogonal infinite lines can be seen as two *semi*-interlocked curves, because in this case the integral in (7) gives 2π, a result that may be interpreted topologically as meaning that the translation of one of the orthogonal lines relative to the other is restricted to half of space.

4. Limitations of ACL

When introducing ACL to students, issues often arise regarding its application to specific situations [6–9]. For instance, let us consider the problem of determining the magnetic field, $\boldsymbol{B}$, due to a current-carrying finite segment XY at point P a distance r from it (Fig. 7), where the circular surface S is bounded by the circular ampèrian loop C.

By symmetry, at every point in C, $\boldsymbol{B}$ would have the same magnitude and be tangent to C, so that

$$\oint_{\mathrm{C}} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = 2\pi r B \tag{16}$$

From (1), the magnitude of $\boldsymbol{B}$ would be given by

$$B = \frac{\mu_0 I}{2\pi r} \tag{17}$$

This result would then be the same as that for an infinitely long wire, as the same symmetry arguments apply. However, it does not agree with the result obtained from the Biot–Savart law [1], which depends on the length of the current segment as

$$B = \frac{\mu_0 I}{4\pi r}\left(\frac{d_1}{\sqrt{d_1^2 + r^2}} + \frac{d_2}{\sqrt{d_2^2 + r^2}}\right) \tag{18}$$

Moreover, ACL must give the same result for *any* surface bounded by C, but for a finite current segment one can always choose a surface S′, as the one shown in Fig. 7, that does not intersect the current, giving in this case $B = 0$, in contradiction with (17). In addition, it is physically impossible to have a *stationary* current in an open circuit. It follows from these considerations that ACL does not apply to a current in a finite segment, as a stationary current can only exist in a closed circuit. To overcome this difficulty, if one regards the finite current segment as a portion of a stationary current loop, then the symmetry arguments fail, as they do not take into account that $\boldsymbol{B}$ is created by the entire current. In other words, connecting the points X and Y to close the loop breaks the existing symmetry, that is, it is no longer feasible to determine $\boldsymbol{B}$ by ACL because of the difficulty of solving the integral in (1).

As illustrated in Fig. 8, the current, I, can be expressed as the flux of the current density vector, $\boldsymbol{J}$, through *any* open surface, S, bounded by the ampèrian loop, C, $I = \int \boldsymbol{J}\cdot \mathrm{d}\boldsymbol{a}$. The direction of $\boldsymbol{J}$ is given by the direction of I, and the directions of $\mathrm{d}\boldsymbol{\ell}$ and $\mathrm{d}\boldsymbol{a}$ satisfy the right-hand rule. From a didactical point of view it is thus more instructive write ACL as

$$\oint_{\mathrm{C}} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \int_{\mathrm{S}} \boldsymbol{J}\cdot \mathrm{d}\boldsymbol{a} \tag{19}$$

Though a stationary current cannot exist in an open circuit it is, however, possible to establish a transient current in such a circuit, a situation that requires the generalization of ACL through the displacement current. An interesting educational example illustrating the application of Ampère's generalized law to an open circuit with an unconfined displacement current is given in the next section.

5. Application of generalized ACL

The generalization of ACL is often illustrated in textbooks with the classic example of the discharging capacitor circuit [1, 2] shown in Fig. 9, where the decrease in time of the electric field within the capacitor results in a spatially uniform displacement current, I_{D} [10, 11], which is confined to the region between the plates and is numerically equal to the conduction current, I. Both of these currents are accounted for by Ampère's generalized law [2]

$$\oint_{\mathrm{C}} \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0\left(\int_{\mathrm{S_0}} \boldsymbol{J}\cdot \mathrm{d}\boldsymbol{a} + \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathrm{S_0}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{a}\right) = \mu_0 (I + I_{\mathrm{D}}) \tag{20}$$

where ε_0 is the electrical permittivity, $\boldsymbol{E}$ is the electric field, and $\mathrm{S_0}$ is *any* open surface bounded by the curve C.

The choice of circuit in Fig. 9 is pedagogically relevant because it intuitively shows that the displacement current, I_{D}, across S′ is numerically equal to, and can be taken as a continuation of, the conduction current, I, so that the circulation of $\boldsymbol{B}$ along C is always $\mu_0 I$. However, a knowledge of circulation does not enable the determination of $\boldsymbol{B}$ because of lack of symmetry, unless the wire connecting to the capacitor is taken to be sufficiently long for the contributions from the remaining circuit to be negligible.

Fig. 6. (*a*) In Fig. 4, the infinite axis of circular loop *I* always encloses the current regardless of whether such closure is through the left or right. At infinity the contribution of ***B*** to the circulation is zero. (*b*) In contrast, for the straight current *I* in Fig. 5, how C closes at the infinity matters.

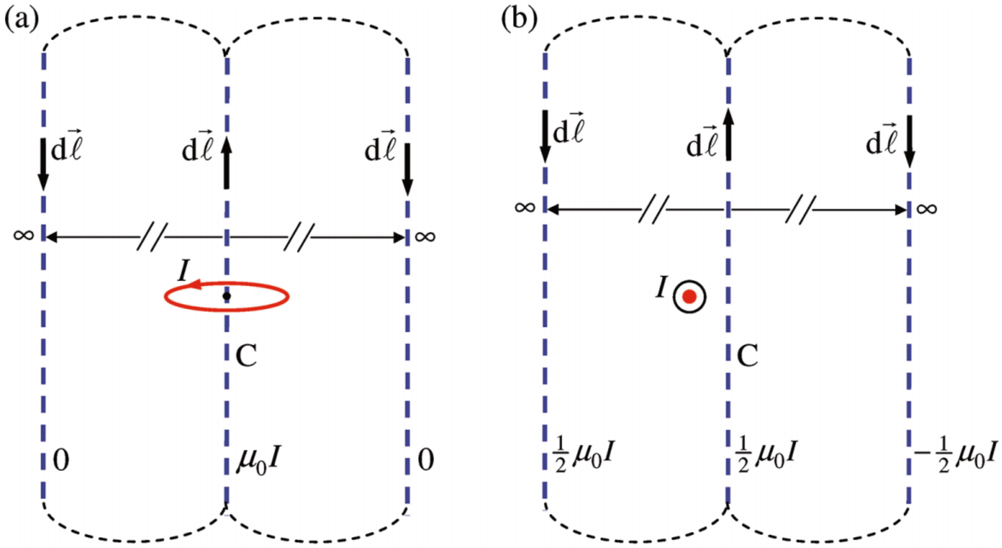


Fig. 7. Even though there would appear to be symmetry, ACL cannot be used to find the magnetic field at P owing to a finite current segment. It only applies to a stationary current, which occurs only in a closed circuit.

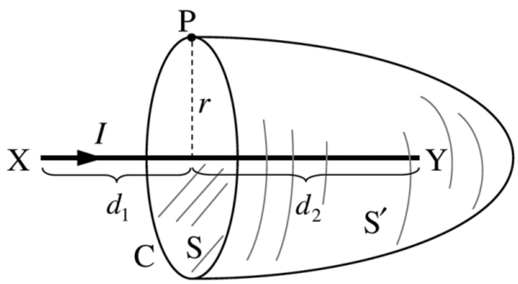
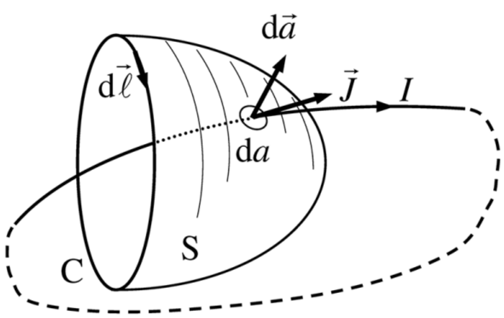


Fig. 8. Geometrical configuration and physical quantities in ACL (19).



It is instructive to consider a different example whose symmetry enables the determination of ***B*** and that can be used to illustrate the concept of a displacement current that is both unconfined and different from the conduction current. Consider two small spherical conductors with initial charges $+Q$ and $-Q$ ($Q > 0$) placed at the ends of a finite conducting segment, as shown in Fig. 10. A transient current *I* flows through the segment for a short time until the spheres become uncharged. From symmetry considerations, it is clear

Fig. 9. The discharging capacitor circuit. If the distance between the plates is small compared to their dimensions, the decreasing electric field results in a confined displacement current that is numerically equal to the conduction current, *I*.

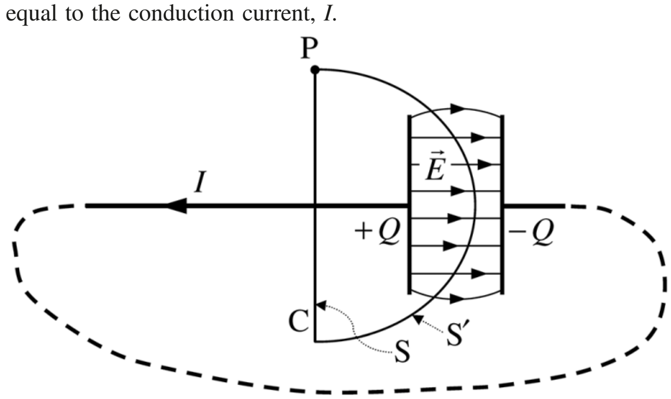
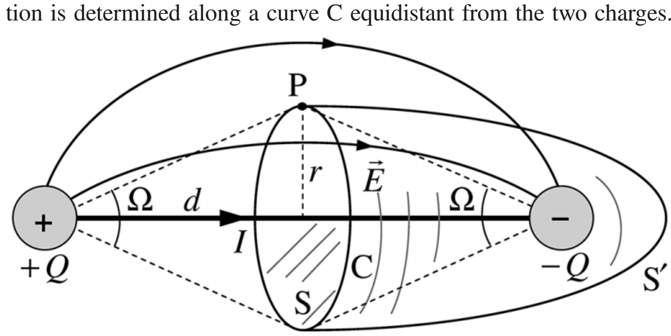


Fig. 10. Transient current $I = -dQ/dt$ is due to charges $+Q$ and $-Q$ placed at the ends of a conducting segment of length $2d$. ACL does not apply to this open circuit. For simplicity, magnetic field circulation is determined along a curve C equidistant from the two charges.



that the magnetic field, ***B***, has the same magnitude at every point P in C and is tangent to this curve.

Charges $+Q$ and $-Q$, separated by a distance $2d$, create an electric field ***E*** whose field lines begin at $+Q$ and end at $-Q$, as shown in Fig. 10. The flux, ϕ_E, of $\boldsymbol{E}$ through S is the sum of fluxes due to each charge

$$\phi_E = \phi_E^+ + \phi_E^- = \frac{Q}{2\pi\varepsilon_0}\Omega \tag{21}$$

where Ω is the solid angle through which charges see surface S.

As $I = -dQ/dt$, using (20) and (21), and taking into account that I crosses S, the circulation of $\boldsymbol{B}$ along C is given by

$$\oint_C \boldsymbol{B}\cdot d\boldsymbol{\ell} = \mu_0 I\left(1 - \frac{\Omega}{2\pi}\right) \tag{22}$$

with the displacement current across S being

$$I_D = -\frac{\Omega}{2\pi}I \tag{23}$$

If instead of S, we choose now a surface S′ that does not intersect the current, the circulation of $\boldsymbol{B}$ along C has only the contribution of the rate of change of ϕ_E across S′. In this case, while the solid angle through which the charge $+Q$ sees S′ remains Ω, for the charge $-Q$ it becomes $(4\pi - \Omega)$ (see Fig. 10). Moreover, the flux corresponding to charge $-Q$ is now negative. Thus

$$\oint_C \boldsymbol{B}\cdot d\boldsymbol{\ell} = \mu_0\varepsilon_0\frac{d}{dt}\left(\frac{Q}{4\pi\varepsilon_0}\Omega\right) - \mu_0\varepsilon_0\frac{d}{dt}\left[\frac{Q}{4\pi\varepsilon_0}(4\pi - \Omega)\right] \tag{24}$$

which, after a straightforward simplification, gives precisely (22) as one would expect. Evidently, the displacement current across S′ differs from that across S and is given by

$$I_D = \left(1 - \frac{\Omega}{2\pi}\right)I \tag{25}$$

The geometrical symmetry allows us to calculate the circulation of $\boldsymbol{B}$ along C and the solid angle Ω,

$$\oint_C \boldsymbol{B}\cdot d\boldsymbol{\ell} = 2\pi r B \tag{26}$$

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2 + r^2}}\right) \tag{27}$$

which, combined with (22), leads to

$$B = \frac{\mu_0 I}{2\pi r}\left(\frac{d}{\sqrt{d^2 + r^2}}\right) \tag{28}$$

The value of B given by (28) is the same as that obtained from (18) making $d_1 = d_2 = d$. Thus, the present example can be used to illustrate that, although the displacement current is used to determine B in (28), the same result is obtainable without resorting to the displacement current, using the Biot–Savart law.

Before closing this discussion, it is instructive to further compare the present example with the discharging capacitor circuit. Unlike the present example of the pair-of-charges (Fig. 10), the discharging capacitor circuit (Fig. 9) does not enable determination of $\boldsymbol{B}$ at every point in space due to lack of symmetry. However, in both examples, when r approaches zero, point P sees the approaching wire as infinitely long and thus symmetrical giving $B = \mu_0 I/(2\pi r)$, which is the value of (28) for $r \ll d$. For the pair-of-charges example the displacement current is dependent on r (through Ω) and is not spatially confined (e.g., see (23) and (25)) contrasting with the discharging capacitor example where such a current is constant, numerically equal to I, and confined within the plates of the capacitor. Also, by (22) with (27), while for the pair-of-charges example the effective current $I_{ef} = I + I_D$ in (20) gradually approaches zero as $r \to \infty$, for the discharging capacitor circuit the effective current has the constant value of $I_{ef} = I$, provided that r is not large enough for the dashed line in Fig. 9 to be intersected by S or S′, a situation beyond which we obviously have $I_{ef} = 0$.

6. Conclusion

The ampèrian and current loops' interchangeability, an intrinsic symmetry of ACL that is virtually ignored in textbooks, was discussed and illustrated. The limitations and generalization of ACL were addressed with instructive examples, paying particular attention to the concept of displacement current, which is a difficult topic regarding its intuitive interpretation. As far as is known to us, the discussion carried out here deals with issues that are often disregarded in textbooks. Given its educational nature, it is hoped that this paper will be useful for teachers and more advanced students.

References

1. P.A. Tipler and G. Mosca. Physics for scientists and engineers. 5th ed. Freeman, New York. 2004.
2. D.J. Griffiths. Introduction to electrodynamics. 3rd ed. Prentice Hall, New Jersey. 1999.
3. J.D. Jackson. Classical electrodynamics. 3rd ed. Wiley, New York. 1999.
4. R.P. Feynman, R.B. Leighton, and M. Sands. The Feynman lectures on physics. Vol. 2. 6th ed. Addison-Wesley, Amsterdam. 1977.
5. C. Nash. Topology and physics – a historical essay in history of topology. Elsevier, Amsterdam. 1999.
6. C.S. Wallace and S.V. Chasteen. Phys Rev. Spec. Top. Phys. Educ. Res. **6**, 020115 (2010). doi:10.1103/PhysRevSTPER.6.020115.
7. J. Guisasola, J.M. Almudí, J. Salinas, K. Zuza, and M. Ceberio. Eur. J. Phys. **29**, 1005 (2008). doi:10.1088/0143-0807/29/5/013.
8. C.A. Manogue, K. Browne, T. Dray, and B. Edwards. Am. J. Phys. **74**, 344 (2006). doi:10.1119/1.2181179.
9. H.A. Kalhor. IEEE Trans. Educ. **31**, 236 (1988). doi:10.1109/13.2322.
10. J.A. Heras. Am. J. Phys. **79**, 409 (2011). doi:10.1119/1.3533223.
11. J.W. Arthur. IEEE Antennas Propag. Mag. **51**, 58 (2009). doi:10.1109/MAP.2009.5433097.