Development of a dynamic model for twin hull ASVs

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***Abstract*—This paper presents an overview of a generalized 6 degrees of freedom model for surface vessels and explains how it can be extended for twin hull surface vehicles. The extended model takes into account the hull characteristics (dimensions and location), which are important to improve the accuracy of simulations and the performance of controllers.**

The method involves the calculation of the submerged volume of each hull, location of each hull's center of buoyancy and restoring forces/torques due to buoyancy contributions.

To evaluate the proposed model, some simulations were performed, using an example of allocation of propulsion system and realistic hydrodynamic coefficients (added mass and damping) and inertial tensors.

***Keywords*—Catamaran, Twin hull, Autonomous Surface Vessels**

I. Introduction

The main goal of this paper is to give an overview of the generalized model proposed by Fossen [1] and explain how it can be modified for twin hull ASVs (Autonomous Surface Vessels). Twin hull ASVs are preferred when the desired application requires high roll stability (for example, Bathymetric mapping). There are several scenarios in which more accurate models of the ASVs are critical for the systems performance, such as:

- High performance motion controllers;
- Complex maneuvers, like manipulation on floating base;

The approach developed estimates the value of the submerged volume of each hull, the location of each hull's center of buoyancy and the restoring forces/torques due to buoyancy contributions, according to some vehicle's state variables (z, roll and pitch) and hull's geometry.

Regarding the organization of the paper, the following section presents the generalized dynamic model proposed by Fossen [1]. Section II describes the extension of the dynamic model to twin hull ASV's, together with the allocation of the vessel's propulsion system into the dynamic model. Section IV discusses all simulations performed to evaluate the proposed dynamic model highlighting the benefits of the detailed approach. Last section contains the conclusions and some suggestions for future work.

II. Generalized model for surface vessels

The 6 DOF generalized model for surface vessels is fully described in [1] and widely adopted by scientific community. Nonetheless, we will briefly introduced it, since it facilitates the explanation of the performed modifications.

The vehicle's absolute position is given by the state vector $\eta = [x_I, y_I, z_I, \phi_I, \theta_I, \psi_I]^T$ represented in the inertial-frame (figure 1). Its velocities are described in the body-fixed frame by the vector ν=$[u, v, w, p, q, r]^T$. The kinematic relation between the two frames, is given by the Jacobian matrix ($J(\eta)$), as represented in equation 1.

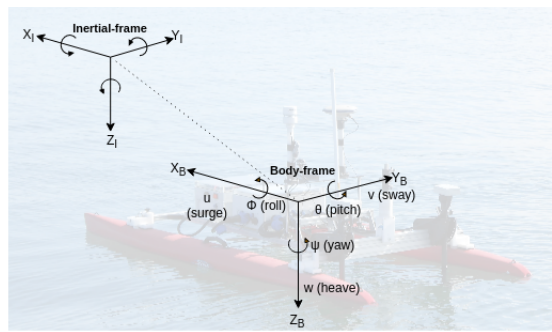

Fig. 1. Coordinate system associated with the kinematic model of an ASV

$$\dot{\eta} = J(\eta)\nu \tag{1}$$

The dynamic model reflects the application of Newton's Second Law for both linear and rotational motion in the DOF (degrees of freedom) considered by the model. This includes the M_{RB} and M_A matrices, which represents the mass and inertia associated with the rigid body (M_{RB}), as well as inertia and pressure effects (M_A) due to the movement of a body in a surrounding fluid [1]. Also, C_{RB} and C_A matrices incorporate the centripetal terms and the Coriolis vectors, respectively. The D matrix represents the hydrodynamic damping associated with the vessel's movement and can be divided into two main components, $D_l(\nu)$ and $D_q(\nu)$ (equation 2), according to their linear and quadratic relation with the vessel's speed, respectively [2]:

$$D(\nu) = D_l + D_q(\nu) \tag{2}$$

Lastly, the $g(\eta)$ matrix represents the contribution of gravity and buoyancy restoring forces and torques, as further explained in section III. Thus, combining all previous contributions, the vessel's dynamic equation is obtained:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \qquad (3)$$

where,

$$M = M_{RB} + M_A \qquad \text{and} \qquad C(\nu) = C_{RB}(\nu) + C_A(\nu)$$

III. Development of the dynamic model for twin hull ASVs

The metacenter procedure available in [1] is specially suitable for single hull surface vessels, since it only takes into account one center of buoyancy. In our approach, we increase the dynamic model accuracy by considering an independent center of buoyancy for each floater.

A. Assumptions

This new approach handles the full computation of $g(\eta)$ matrix, which consists in calculating the submerged volume of each hull, the location of each floater center of buoyancy and restoring forces/torques due to buoyancy contributions, according to the following assumptions:

- The vehicle can only experience small rotations in roll and pitch ($|\phi| \leq 10°$ and $|\theta| \leq 10°$), an inherent characteristic of a twin hull ASV;
- Only some vessel's states variables (z, ϕ and θ) have impact on $g(\eta)$ matrix;
- The hull's geometry was approximated by a rectangular shape extruded by the hull's length (frontal view - figure 2 and lateral view - figure 3);
- The vehicle exhibits symmetry in the $xy-plane$, relative to its center of mass;
- The vehicle's body-fixed frame was positioned on its center of mass, since it simplifies some matrices used on the calculation of vessel's dynamics;
- Parameter z represents the submerged height of the floaters (figures 2 and 3);
- Roll an pitch rotations are perceived by the vehicle as rotations of the waterline.

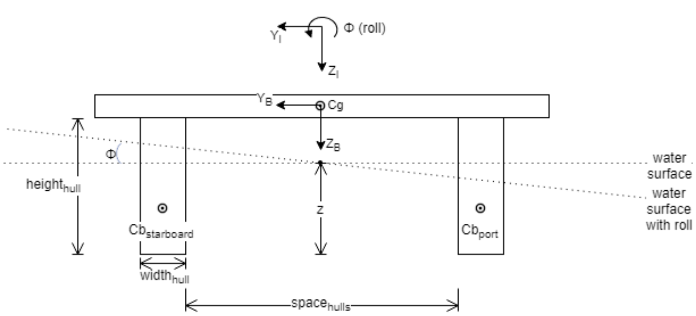

Fig. 2. Frontal approximation of the hull's geometry

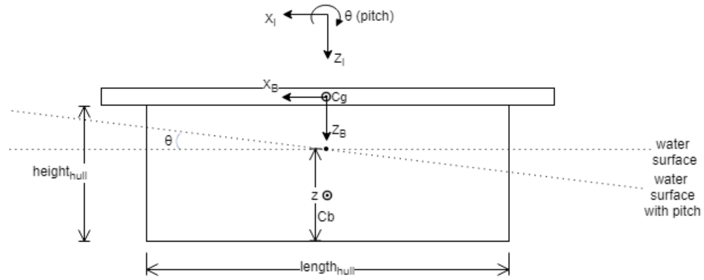

Fig. 3. Side approximation of the hull's geometry

These assumptions were chosen taking into account an additionally purpose, namely to achieve a procedure that allows accurate computation of $g(\eta)$ for twin hull ASVs and still possess enough simplicity to be implemented in numerical simulators or in any microcontroller commercially available.

B. Submerged volume

From the previous perspectives, it is possible to calculate an approximation of the submersed volume of each hull, for the case of small rotations in roll and pitch ($|\phi| \leq 10°$ and $|\theta| \leq 10°$). This approach (equation 4) relies on the multiplication of the floaters' submerged cross-section by their length. Thus, the contributions to the submerged volume are due to heave movement (z) and roll rotation (Δ_ϕ), since for any pitch rotation the amount that the bow rises is equal to the amount that the stern lowers and vice versa, according to the location assumed for the vessel's center of mass (section III-A).

$$\begin{array}{l} V_{port} \approx width_{hull} \cdot length_{hull} \cdot (z - \Delta_\phi) \\ V_{starboard} \approx width_{hull} \cdot length_{hull} \cdot (z + \Delta_\phi) \end{array} \qquad (4)$$

where,

$$\Delta_\phi = \frac{width_{hull} + space_{hulls}}{2} \cdot sin(\phi)$$

C. Center of buoyancy in $y - axis$

The center of buoyancy of each floater was calculated considering the motion of each floater's mid point with roll angle, as described in equation 5.

$$\begin{array}{l} Cb_{port} \approx -\frac{width_{hull} + space_{hulls}}{2} \cdot |cos(\phi)| \\ Cb_{starboard} \approx \frac{width_{hull} + space_{hulls}}{2} \cdot |cos(\phi)| \end{array} \qquad (5)$$

D. Center of buoyancy in $x - axis$

Since the total submerged volume remains the same for any pitch rotation, the center of buoyancy in $x - axis$ can be estimated by dividing each floater in two halves and then weight the difference (V_{diff}) with the total submerged volume (V_{total}). This procedure is given by equation 6 and remains suitable as long as $length_{hull} \gg height_{hull}$.

$$Cb_x \approx -\frac{V_{diff}}{V_{total}} \qquad (6)$$

where,

$$V_{diff} = 2 \cdot width_{hull} \cdot (0.5 \cdot length_{hull})^2 \cdot tan(\theta)$$
$$V_{total} = V_{port} + V_{starboard}$$

E. restoring forces/torques

The restoring forces/torques can be calculated by the difference between the corresponding gravitational and buoyancy components:

$$\begin{aligned} F_z &= m \cdot g - \rho \cdot g \cdot V_{total} \\ \tau_\phi &= -\rho \cdot g \cdot V_{starboard} \cdot Cb_{starboard} - \\ &\quad \rho \cdot g \cdot V_{port} \cdot Cb_{port} \\ \tau_\theta &= \rho \cdot g \cdot V_{total} \cdot Cb_x \end{aligned} \tag{7}$$

which results in:

$$g(\eta) = \begin{bmatrix} 0 \\ 0 \\ F_z \\ \tau_\phi \\ \tau_\theta \\ 0 \end{bmatrix}$$

F. Propulsion force allocation

After extending the dynamic model proposed by Fossen [1] to twin hull ASVs, the next step is the allocation of the vessel's propulsion system into the vehicle's dynamic model. As a case study, Zarco [3] was used. It has a 2-DOF propulsion system, which allows for surge and yaw motion. Thus, using equation 8 it is possible to map the forces produced by the actuation of each thruster (U matrix) into forces and moments acting on the vessel's center of mass (τ matrix) resorting to L matrix, as explained in [4].

$$\tau = L \cdot U \tag{8}$$

with,

$$L = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ dist_z & dist_z \\ dist_{port} & dist_{starboard} \end{bmatrix}$$

where $dist_{port}$ and $dist_{starboard}$ represent the distance between each thruster and the vehicle's center of mass in the $y-axis$ and $dist_z$ follows the same approach for the $z-axis$. The zeros in the L matrix represent the lack of actuation in sway, heave and roll.

IV. Simulation

To evaluate the proposed model, two tests were conducted. The first one consists in analyzing the behavior of the pitch component of the vehicle state, for different hull lengths ($length_{hull}$), when the thrusters are actuated in common mode. In the second one, the intention was to observe how the roll component varies with different space between hulls ($space_{hulls}$), when differential actuation is applied to the thrusters. Furthermore, all constant parameters assumed in the following simulations are:

- $height_{hull}$ equal to 0.5 m;
- $width_{hull}$ equal to 0.1 m;

All hydrodynamic coefficients (added mass and damping) and inertial tensors required for the dynamic model simulation are presented in table I. These were obtained using the approach referred by the authors of [5], which is based on semi-empirical formulas and experimental data, with the required adaptation to the geometry shape, number of hulls and for steady state conditions ($\phi = \theta = 0°$). Nonetheless, the values of the presented coefficients have already been estimated for other works in progress that focus on control approaches.

A. Pitch rotation vs hull length

Regarding the first test, this was conducted for hull lengths of 1.5 m, 2 m and 2.5 m, since these are typical lengths for the floaters used in Zarco ASV [3]. The results are presented in figure 4. To highlight the effects on the pitch angle we applied the maximum thruster force available in common mode.

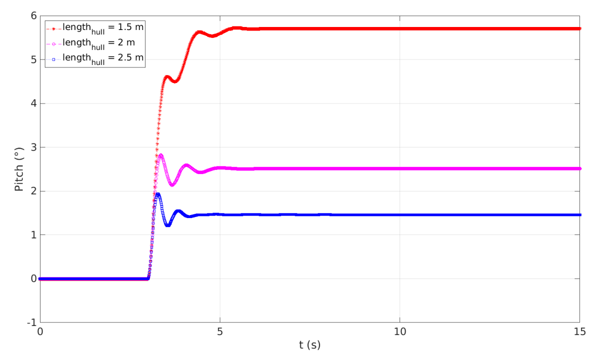

Fig. 4. Comparison of pitch angle, for different hull lengths, with common actuation of the thrusters at the 3 second mark (F_{port} = 125 N and $F_{starboard}$ = 125 N)

As expected, the pitch angle became smaller with the increase of the hull's length. This happens since the thrusters' torque arm ($dist_z$ in equation 8) remains constant for any hull length and a higher volume will be shifted, for the same pitch rotation. With the increase of hull lengths results an augmented pitch restoring moment and therefore a smaller rotation is needed to balance the thrusters pitch moment.

B. Roll rotation vs space between hulls

The results obtained for the second test are presented in figure 5. This has considered a space between hulls ($space_{hulls}$) of 0 m, 0.25 m and 0.5 m. The lower value will approximate the twin hull configuration to a single hull configuration [1]

[1]Although $space_{hulls}$ can be 0 m, there still exist some space between each hull's cb_y, due to the $width_{hull}$

and the higher one will highlight the main advantage of twin hull configurations - increased roll stability. We applied $F_{port} = 125$ N and $F_{starboard} = 0$ N to the thrusters, so that a coupling effect is noticeable in roll angle due to common and differential mode actuation.

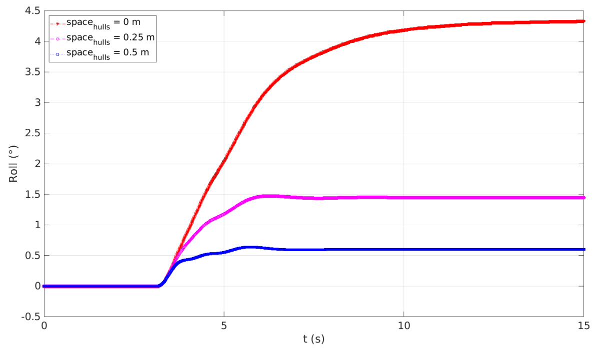

Fig. 5. Comparison of roll angle, for different space between hulls, with differential actuation of the thrusters at the 3 second mark ($F_{port} = 125$ N and $F_{starboard} = 0$ N)

From the above figure, it is possible to understand that the roll stability increases with the space between hulls. This happens because a higher space increases the torque arm of the roll restoring moment and therefore a lower rotation is needed to balance the resultant forces acting in this vehicle's state.

V. Conclusion and Future Work

In this paper, an extension of the generalized dynamic model proposed by Fossen was introduced for twin hull ASVs. This is in agreement with the kinematic model proposed by the same author and therefore with SNAME notation.

The behaviors occurred in roll and pitch are in agreement with the expected ones: the vessel's roll angle is dependant on the space between hulls, as well as the vessel's pitch angle is related with the hull's length. This means that, although the proposed model is simple to compute, which makes it suitable for real time calculations, it still has sufficient detail to capture these effects and provides a quantification of the magnitude of the occurrences.

Having a dynamic model with this kind of detail allows exploration of demanding operations with twin hull ASVs. The suggested extension defines some basis that can be used, for example, in floating base manipulation scenarios. For this type of operations, it is important to have a model capable of providing the vessel's attitude with some accuracy, in order to control the end-effector to reach the desired target. Another use for this framework is to provide a tool to select the vessel's appropriate parameters (e.g. space between hulls, hull's length, hull's width) in order to fulfill the desired purposes, for instance carrying some heavy payload or guarantee a proper stability in roll/pitch. Lastly, it also can be used as a more accurate model in the design of state-observers and fusion algorithms (e.g Kalman filter), for twin hull configurations.

Although the model already captures the relevant behaviors, it now requires field testing to calibrate the magnitude of the effects, which are intended to be addressed in the near future. Nonetheless, as the procedure presented was developed on the basis of decoupled submerged volume, center of buoyancy and restoring forces/torques, it makes it specially suitable for calibration only where required.

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Appendix A
List of Hydrodynamic Coefficients and Inertial Tensors Used in the Simulations

Inertial tensors		
$I_{xx} = 0.55$ Kg $\cdot m^2$	$I_{yy} = 8.284$ Kg $\cdot m^2$	$I_{zz} = 7.734$ Kg $\cdot m^2$
Added mass		
$X_{\dot{u}} = -5.5$ Kg	$X_{\dot{q}} = 3.028$ $Kg \cdot m/rad$	
$Y_{\dot{v}} = -67.39$ Kg	$Y_{\dot{p}} = -4.89$ $Kg \cdot m/rad$	$Y_{\dot{r}} = 3.365$ $Kg \cdot m/rad$
$Z_{\dot{w}} = -67.39$ Kg	$Z_{\dot{q}} = -5.00$ $Kg \cdot m/rad$	
$K_{\dot{v}} = -4.89$ $Kg \cdot m$	$K_{\dot{p}} = -10.82$ $Kg \cdot m^2/rad$	
$M_{\dot{u}} = 3.028$ $Kg \cdot m$	$M_{\dot{w}} = -5.00$ $Kg \cdot m$	$M_{\dot{q}} = -13.86$ $Kg \cdot m^2/rad$
$N_{\dot{v}} = 3.36$ $Kg \cdot m$	$N_{\dot{r}} = -13.86$ $Kg \cdot m^2/rad$	
Linear damping		
$X_u = -15.99$ Kg/s	$Y_v = -10$ Kg/s	$Z_w = -150$ Kg/s
$K_p = -80$ $Kg \cdot m^2/s$	$M_q = -150$ $Kg \cdot m^2/s$	$N_r = -5.369$ $Kg \cdot m^2/s$
Quadratic damping		
$X_{u\|u\|} = -30.66$ Kg/m	$X_{q\|q\|} = 0.0952$ $kg \cdot m/rad$	
$Y_{v\|v\|} = -129.0$ Kg/m	$Y_{p\|p\|} = 0.1916$ $Kg \cdot m/rad^2$	$Y_{r\|r\|} = 7.125$ $Kg \cdot m/rad$
$Z_{w\|w\|} = -331.9$ $Kg \cdot m$	$Z_{q\|q\|} = 7.205$ $Kg \cdot m/rad$	
$K_{v\|v\|} = -6.323$ Kg	$K_{p\|p\|} = -23.23$ $Kg \cdot m^2/rad^2$	
$M_{u\|u\|} = 0.7271$ Kg	$M_{w\|w\|} = 13.14$ Kg	$M_{q\|q\|} = -21.95$ $Kg \cdot m^2/rad^2$
$N_{v\|v\|} = 11.25$ Kg	$N_{r\|r\|} = -11.70$ $Kg \cdot m^2/rad^2$	

TABLE I. List of hydrodynamic coefficients and inertial tensors used in the simulations

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